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The *emerging* and *emergent* present: a view on the indeterminate nature of mathematics lessons

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ABSTRACT: The notion of emergence has considerable currency in mathematics education. However, the notion tends to be used in a descriptive way rather than being theorized and developed as a phenomenon sui generis. The purpose of this article is to contribute to building a theory of *emergence*. After providing an exemplifying description and analysis of an episode from a second-grade mathematics classroom studying three-dimensional geometry, we discuss implications for theoretical and classroom praxis in mathematics education, especially for the curriculum planning and the preparation, training, and enhancement of teachers of mathematics.

Keywords: Emergence; indeterminacy; witness; intention; intuition; excess; sociality

> Without *emergence* there are no distinguishable events thanks to which time emerges. (Mead, 1932, p. 49)
>
> Th[e] present is the scene of that emergence which gives always new heavens and a new earth, and its sociality is the very structure of our minds. (Mead, 1932, p. 90)

Mathematical lesson plans and assessment that evaluates classroom events in terms of what the lesson plans have stated presuppose the idea that whatever is contained in the planned curriculum specifies, more or less accurately, what will actually happen in a lesson. Such presuppositions, however, contradict empirical findings showing that even the most highly trained professionals (e.g., engineers and scientists) cannot with any certainty anticipate their own always contingent, practical actions, which leads to the fact that there is a permanent gulf between plans and situated actions (Roth 2009; Suchman 2007). In mathematics education, this is viewed, for example, as a conceptual shift away from mathematics as content, where the notion of mathematics placed in the container of curriculum dominates, toward terms according to which mathematical ideas "emerge as the collective practices of the classroom community evolved" (Cobb 1999, p. 31). Complexity science has also been used to describe and perhaps “occasion” the emergence of collective possibilities in mathematics classroom that “arise and evolve without intentions, plans, or leaders” (Davis and Simmt 2003, p. 140). From an embodied, enactive perspective, accepting the emergent and emerging nature of classroom activity more specifically means to think lesson planning not in terms of selecting optimal traits to bring about pre-given possibilities, but “to uncover not what is ideal, but what is possible” (Davis et al. 1996, p. 166) and “use prospective logic [so that] the curriculum in classroom is large [and] each student participates in the curriculum in a way which co-verges [i.e. moves in a similar direction] with his colleagues” (Kieren 1995, p. 22). More generally, even those scholars who study

mathematics lesson plans in terms of "script" (e.g., Jacobs and Morita 2002) find the need to acknowledge the unpredictable nature of teaching and learning mathematics and, for instance, the role of the teacher "behind the general scripts for teaching" (Shimizu 2008, p. 947). This calls for flexibility in the face of what comes from the students, so that even the most experienced, Socratically operating mathematics teacher is confronted with the unforeseen (Wagenschein 1999).

That is, we can understand the existence of such a gulf between intentions and realization of mathematical curriculum activity in terms of *emergence*. The concept of emergence stands out in the work of philosopher George Herbert Mead, who suggests in the quotations opening this article that events can be distinguished (e.g. in time) only because emergence is *always* there to impose presence (the present) before us. He also describes the role of emergence as a happing where past and future are linked; and it is this linkage that constitutes the structure of human consciousness. In this nuanced view of what we take to be a category[1], "emergence" inherently surpasses any attempt to predict it: it is itself emerg*ing* giving rise to the emerg*ent*. The assumption that we can determine to any specific extent the enacted mathematics lessons through prior planning means "to assume a single determinate past to which every present must wholly conform," and this means "to deny emergence altogether" (A. E. Murphy, in Mead, 1932, p. xviii). At the same time, what is in the course of emerging—e.g., the living mathematics lesson—is not completely independent from the past—e.g., the lesson plan—for in the transition, neither the emergent thing nor the emergent order follow from the past. These grow out of the past, which is the ground upon which the new order (unpredictably) emerges and, in so doing, the ground that the new order transcends.

The category of emergence seems fundamental to understand the phenomena commonly referred to as "teaching" and "learning." A constructive phenomenology of learning in mathematics (e.g. Roth 2012) forces us to attend to the "phenomenon of emergence—how shifting patterns within a complex dynamic eventuate in new possibilities and how these new possibilities engender new relationships among all constituents of the whole" (Sheets-Johnstone 2011, p. xxv). It is only in looking back that we can use something like a lesson plan as a more-or-less fitting description of what has gone on (Amerine and Bilmes 1990). Our grasp (i.e., com*prehension*) *of* what has happened is always delayed with respect to the happening, a framing that contains precisely the emergence of time as Mead postulated it. That is, emergence *as* emergence needs to be approached and theorized *in statu nascendi*: in its birth to presence. In a constructivist view, the present (e.g., in the form of an object) is present to the subject, who has access to the world only through its "constructions" that are used in the interpretive process by means of representations[2] (e.g., von Glasersfeld 1983). In the present, the subject relies on the fit of these constructions with the objects as perceived, and, if the fit is low, the constructions are updated to achieve a better fit. Contrastively, the particularity of the birth to presence is that

[1] Categories are units of thought that embody an inner contradiction, which is to say that these reflect the flow typical of life (e.g. Il'enkov 1977).

[2] Representations literally allow us to make present again (re-) that which is away physically or temporally.

the "premium," that which has existed just prior to the emergent, what founds the fund/ament [*fond du fonds*], is actually dissolved "*in statu nascendi*: by this dissolution, by this birth, the identity of the fundament, all identity, simultaneously disintegrates and is precipitated (in the chemical sense of the word" (Nancy 1993, p. 232). Emergence is unanticipated and unintended birth; it denotes the "coming-to-be of something new or novel out of something already present" (Sheets-Johnstone, 2011, p. 229) whereas "what emerges dynamically in learning [. . . is] an expanding repertoire of 'I cans'" (p. 233). In terms of emergence, there is not a fixed self who "teaches" a lesson or "learns" some mathematical concepts (e.g., by re/organizing itself as in the Piagetian constructivist metaphor). What we call *learning* is rather conceptualized as the expansion of the virtuality where being can unfold, and *teaching* has to do with this linking of past and future at the heart of making present. Lessons and "interpretive process" then take a radically creative, even artistic twist in which scripts become lessons only in the interpretation teacher and students perform together[3], the outcomes of which remains ineffable until emergence is no longer consider in its flowing terms, but pined down killed in knowable forms (events, etc.).

As part of our research program around the theorization of the flowing nature of lived-life mathematics, and drawing on philosophical and mathematics education literature in which emergence is thematic, the purpose of this paper is to conceptualize mathematics lessons not just as flux, but as an *emerging* flux that can be understood as *this* (specific) flux only once it (flux) has disappeared. We begin by developing, following Mead, the idea of the emerging and emergent present; we then provide an analysis of a classroom episode from the mathematics lessons in a second-grade class studying three-dimensional geometry. Following a discussion of the emergent properties found in the lesson example, we develop implications of the category of emergence for thinking about teaching and learning, curriculum planning and teacher preparation and enhancement.

Setting the stage: emergence in philosophical and mathematics education literature

In the mathematics education literature, the concept of emergence rarely is examined in itself: How do we interpret emergence, its possibility and possibilities? Intuitively taking it as bringing about something "new," mathematics educators have nevertheless shown interest in the concept of emergence generally (e.g., Davis 2004) and specifically in the emergence of (a) problem solving strategies (e.g., Lautner 2012; Proulx 2013), (b) curriculum (Davis et al. 1996), (c) objects from mathematical practices (Cobb 1999; Font et al. 2013), (d) mathematical structures (Hegedus and Moreno-Armella 2011), (e) mathematical thinking (van Oers 2010), (f) identities (Black and Williams 2013), (g) mathematical meaning (Cobb and Bauersfeld 1995), or (h) perspectives and theories (Hershkowitz and Schwarz 1999). Our purpose here is not to condemn the current uses of a concept of

[3] Should it only be in the way dancers or musicians or actors perform with, and not only "to," their audience (e.g. Duranti 1986)

emergence, but rather to accept these as useful beginnings that give rise to something emergent that overcomes and sublates them. We do so by relating, contrasting or nuancing the concept as currently used with a scholarly literature that is generally external to mathematic education but that addresses the concept of emergence more directly.

In the introductory section of this article, we sketched a portrait of emergence in which *time* is given a central role. We might follow Sheets-Johnstone's (2011) analysis of movement in terms of emergence. Accordingly, movement, a typical emergent phenomenon, does *not* proceed from a subject's intention, but rather *precedes and conditions* an "I" that forms itself in moving: movement "moves before the I that moves forms movement" (p. 229). This implication of time presents an important conceptual step on a garden path laid down in walking. In a conversation with S. Kauffman, Varela (1995) confided that the question of emergence was the one question that he pursued all his life, a quest that can be divided into three periods marked by a particular inflection in the notion of emergence (Protevi 2009). The first period deals with emergence in terms of synchronicity, homeostatic part-whole relations "unable to foster the condition for diachronic emergence" (p. 95), a shortfall Varela began to address in the second period. Here, he considered the production of novel functional structures from a repertoire of potential or virtual behaviors, hence mostly focusing on individuals (virtual selves), only later on fully embracing "transversal emergence, the production of distributed and interwoven systems along brain-body-environment lines" (Protevi 2009, p. 96). In this move, consciousness became a central concept, together with a concern for the "living present" as a foundational dimension of moment-to-moment emergence and the "self-movement of the flow" (Varela and Depraz 2000). This allowed Varela to finally (explicitly) connect with Sheets-Johnstone (2011) in observing that: "emergence of the living present is rooted in and arises from *a germ or source* of motion-disposition, a *primordial fluctuation* [where] affect *precedes* temporality [and] can *only* deploy itself in time and thus *as* time" (p. 162).

In such conceptualizations, three intertwined key features are present that are required for a theory of emergence as category:

1. Emergence belongs to two very worlds, and in such a way that the newer world cannot be derived from the older one even though the later constitutes the condition of emergence; as category, it encompasses the whole transition between the two worlds.
2. Emergence cannot be predicted or grasped: reified in some "thing" said to emerge implies that emergence is no longer considered as emergence, is no longer living, thus assuming that everything has been said and done.
3. Emergence is not a homogeneous *thing* but heterogeneous, non-self-identical flow of transiting in which agential and pathic dimensions breed in a creative expression of mind/consciousness understood as social phenomena.

In the following sections, we begin by describing a research project in a second grade mathematics classroom and by providing an exemplifying analysis of an actual classroom mathematics lesson in terms of the three features articulated above. This sets the stage for elaborating a view on emergence by contextualizing and elaborating on these three features making links to mathematics education

literature where pertinent ideas are evoked. We conclude with an outline of possible implications for curriculum planning and teacher preparation. In this way, we contribute to developing a new approach to teaching and learning in mathematics education research.

Research background

Classroom context

The exemplifying episode provided below was defined by the designation of Thomas as the next one to have his turn at talking about edges, and it ended when one of the two teachers, Mrs. Wade, asked him to sit down again. Thomas was part of a 22-student, mixed ability, and culturally diverse second-grade class during a special three-dimensional geometry curriculum unit that was based on the ideas of a mathematics educator working in the area of embodiment (Roth and Gardner 2012). The unit lasted 15 days that extended over a three-week period, with 70 minutes per day. The children engaged in a series of tasks, such as collectively evolving a classification of three-dimensional objects or modeling hidden but touchable objects from Plasticine.

The unit emphasized children's engagement with actual materials and in their verbal and gestural communication. The two teachers often provided the children with written prompts on the chalk board that were intended to assist them in producing descriptions ("It doesn't have . . .," "It can . . .," or "It can't . . .") and explanations, reasons, or conjectures (e.g., "It could be . . . because . . ." or "It can't be . . . because . . ."). On the tray of the chalkboard, there were models of the different geometrical forms—including triangular prism, cone, cube, rectangular prism, and pyramid—that teachers and students drew on as part of their communicative efforts.

Prior to the episode described below, Mrs. Tee (a university mathematics education professor and former elementary teacher), standing next to the regular teacher Mrs. Wall the second teacher in the room, had pointed to the poster that she had prepared with the characteristics of three-dimensional objects (Fig. 1). These characteristics had emerged during and from the previous week's activities, which included student talk about some of the different things that they were learning. In preparation of the discussion, Mrs. Tee had held up three different rectangular prisms and asked students to think about those characteristics that distinguished between these apparently similar objects. She had asked students to raise their hands if they thought the edges were the same. She then asked those students to raise their hands who thought that the edges of the three rectangular prisms were different (turn 01).

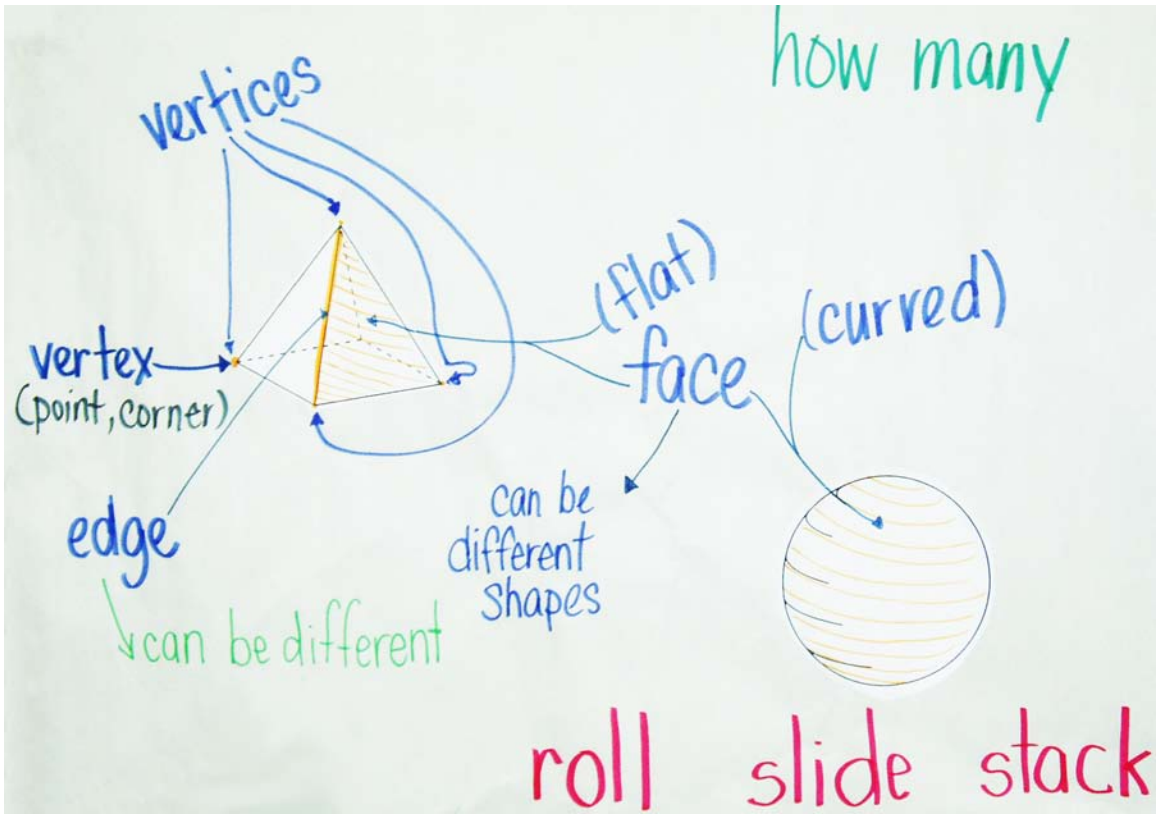

Fig. 1. The poster Mrs. Tee had prepared to list the characteristics of three-dimensional objects. The children were to use these to describe objects in the classroom.

Theory and praxis of analysis

The analysis that follows realizes our pursuit of achieving an ethnographically adequate description (McDermott et al. 1978), with a particular attention to the perspective from the "shop floor" where irreducible *social* phenomena are produced in the joint (*collective*) actions of participants (Garfinkel 2002). To get at an irreducible social phenomenon sui generis, we follow the recommendation to take a minimum analytic unit that is social rather than individual in nature (Durkheim 1919). This means that our categories have to include all aspects of the situation at once, human beings and their material and social environment, that is, the physical-practice, intellectual, and affective moments of the human life form (Dewey 2008; Vygotskij 2001) in the ways these have shown to be relevant to STEM education (e.g. Roth and Jornet 2013b).

Mathematics lessons as a whole and its moments (parts) are irremediably social phenomena that are in excess of any individual's intention. This leads to the fact that individuals are not just contributing to the making of the event but also are subject to it. Joint (social) actions are in excess of individual actions, and *what* is happening has emergent properties to the extent that it is not contained in, and in excess of,

what can be anticipated before (Durkheim 1919).[4] Thus, in the following, every turn is considered only in its context, that is, as the response part completing a preceding turn and as the solicitation part setting up the succeeding turn. But with the preceding turn and its determinations, each turn is connected historically to the unfolding conversation; and with each succeeding turn, its signification is continuously held open. Text and context mutually specify and produce each other (Roth 2010).

In classroom talk, every turn only is a moment of a larger unit, so that its signification is a function of this unit: the conversation as a whole. But because every turn is part of two turn pairs simultaneously, each of which is again part of turn pairs that both connect the Saying to earlier and later instances, it is non-self-identical. It is one of those things that have "the capacity of being several things at once" (Mead 1932, p. 49). Each turn, therefore, is a form of sociality, a term used to refer to "the situation in which the novel event is in both the old order and the new which its advent heralds" (p. 49). This is precisely the same conceptualization that we find in societal historical theories, where new, for the learner emergent psychological functions exist *in*, *for*, and *as* societal relations (Vygotskij 2005). We follow Mead in the sense that "what we seek in the environment is a statement of the world out of which the *emergent* has arisen, and consequently the conditions under which the *emergent* must exist" (Mead 1932, p. 42). We do so with the understanding that *what* has emerged has led, through its appearance, to a newer world that itself is a passage of still newer worlds. Therefore, to understand the emergent as emergent, we must not take the perspective of the Monday morning quarterback, that is, an after-the-fact point of view, but position ourselves, together with the actors, at the point in time where the future course of what will have been an episode specifically and the lesson generally was unknown. Thus, nobody could anticipate what would happen when Thomas was called upon, and how whatever ensued would be ending. At the instant of talking, any speaker and all those who listen *cannot know* what *will have been said* when Thomas returns to his seat after all the saying has come to an end (Roth 2013a, 2013b). With Mead (1932), we assume that "if emergence is a feature of reality this phase of adjustment," then that "which comes between the ordered universe before the emergent has arisen and that after it . . . must be a feature also of reality (p. 47). The emergent as feature of the reality in the living mathematics lesson is the focus of the following analysis.

A fragment from a mathematics lesson: attention to its emerging and emergent properties

Some readers may wonder why a simple mathematics lesson should bear similarities with the emergence of new languages during periods subsequently

[4] In mathematics education, elements of a theory of joint action (which also appeals to Mead) have recently been presented (Sensevy 2011). However, in the use of the theory, individual contributions are emphasized over the irreducible nature of the *joint* action. Characteristically, key concepts such as *emergence* or *evolution* are not present or emphasized.

denoted as "scientific revolutions." Without (almost imperceptible) changes at the moment-to-moment level, however, changes of a historical order would neither come about nor be understandable (Bakhtin 1981; Vološinov 1930). We therefore have to conduct microgenetic analyses to understand developments at the historical level. In the following account, we observe many exchanges before something like the reply completing a question | response turn begun in turn 07 *will have come* to a completion. The entire conversation has emergent character, as per the three interlinked features stated above. This is so because if the teachers had known what their actions would transpire, they could have formulated the opening turn in a way that Thomas's next turn could have completed it. The teachers would not have had to try, as they did, ask and re-ask over and over again before the apparently desired response had come forth. On the other hand, if the student (Thomas) learned what he did not know and could not have intended in and as a consequence of the episode, then this, too, was entirely the result of emergence and has to be theorized as such.

When one just follows the fragment from the beginning to the end, it is not quite certain where this happening is moving and what its underlying intent would be—if there were any. At the end, when the Saying has momentarily concluded and Thomas walks back to this seat, he will have contributed to the identification of a *round* and a *straight* edge. There will have been other answers that he will have provided by the very fact that the teacher questioning | student answering continued for a while. In fact, the poster later will be including information about the type of edges shown on the poster, that is, that edges not only "can be different" but also that this difference exists because there are round (curved) and straight (Fig. 1). That is, one of the traces of this event was an addition to the poster: the specification of the nature of the edges. The entire episode from the specification of Thomas to have a turn to the accepted request of sitting down, which realizes the end of the turn, is presented in the following six fragments.

In the course of the first fragment, what will have been a question and a response are offered (on the part of the teacher) and invited and accepted (on the part of Thomas). Two themes begin to emerge, where *theme* is understood as a momentary, never constant and never repeated signification (Vološinov 1930). We can distinguish two voices, one insisting on "edges" and the other slowly articulating itself "against"/in response to the first one around the word "vertex." There are repeated invitations to *describe* the edge of a cube (turns 07, 12). Then there is a request to run the finger along the edge, and following it, lengthened by the walk from desk to the chalkboard, we hear the verbal articulation of the "vertex" and the deictic selection of the part of the poster where there is a plural version of the word with arrows towards the vertices of a pyramids (turns 14–16). There are also long pauses, which mark the giving | taking of time available to both speakers who apparently take the stage at the instant.

Fragment 1

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01  T: okay (0.63) wHO had their hands up for some edges to be different
       from other edges.
02     [(1.81)                                                        ]
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       [((Mrs. T looks around the classroom, points towards Thomas.))]
03  T: tHOmas.
04     (2.62)
05  t: everyone.
06     (2.36)
07  T: thomas; hOW would you [describe this edge.                ]
                             [((Runs finger along edge of cube.))]
08     (1.31)
09  t: <<len>well;>
10  T: [i ask you to run your finger along
       it;                          ]
       [((Mrs. T reaches the cube toward
       Thomas, who leans forward))]
11     [        (0.79)                  ]
       [((Thomas gets cube from Mrs. T))]
12  T: [how would you descrIbe=it.
13     [          (0.58)          ]
       [((Thomas gazes at cube →))]
14  t: well=no (0.85) because u:mmm; (2.30)
       [<<confidently>because its like a
       sort=a (0.60) vERtex> (0.39) like
15     (2.30)                       ]
       [((gets up, walks to board))]
16  t: [like this one.  ]
       [((points to the word vertices in
       the diagram on chalkboard with a
       pyramid on which the different parts
       of an object →))]
17     (0.52)
18  T: yES. ((Thomas still points))
19    (0.79)
```

There is perhaps a first *surprise* apparent in the unexpected "vertex" when the offered topic of talk apparently concerns edges. We say surprise because, as the following analysis shows, there will have been a question | reply sequence followed by a statement that begins with the oppositives "but" and "instead" in the context of the "vertex" (turn 22a), itself articulated with rising intonation (turn 20). There is an emphatically articulated "yes" (turn 18), followed by a pause. At this point, Mrs. Wall, surprisingly perhaps, takes over from her colleague. We do not know of any individual intentions, which might include the realization on Mrs. Wall's part that she is in a better position than the guest teacher to help Thomas, who has been in her class for nearly 6 months.

In the second fragment, the event is characterized by invitations to identify edges as distinct from vertices, which are the endpoints of the edge. There is a movement in which Thomas's finger follows the edge from one to another vertex of

the cube in Mrs. Wall's hand, followed by an utterance grammatically structured as a question about what it feels like (turn 22c). Another long pause is developing, which *is giving time* to Thomas, who, simultaneously, *is taking his time*. It also allows anyone else to take a turn, including Mrs. Wall. From a social perspective, we just witnessed an offered but unanswered question, followed by an apparent attempt to try something else ("all right, *this* time," turn 24). Trying *again* is the marked attempt at "repeating" "something" (e.g., an "'instructional' 'sequence'") that cannot be the same, as what happened initially has shown not to lead to an acceptable result (as per the repetition, which also marks that a repetition is needed). That is, what we subsequently understand as having emerged is a situation, in which an instructional sequence has not been not been successful—as per the apparent, because announced attempt to do it over again ("this time").

Fragment 2

20 W: <<p>vERtices?> ((*She points to the top vertex in the diagram.*))

21 t: yea. (0.36) like this one ((→) [mine]

22a W: [but your] instead of your if you start on one verticee; put that finger on that verticee. ((→ *a*)) (0.82) your finger ((*grabs finger* → *b*)) (0.26) point (0.62) ((*holds finger to vertex* → c)) one verticee (0.48) and rUN:it along that edge to the other verticee (0.77) <<p>okay?>

b (1.24)

c [whats that feel like; what kind of edge is that.] [((*Mrs. W runs his finger along the edge*))]

23 [(4.22)] [((*scratches his head* →)) ((*Mrs. W gets the cylinder*))]

24 W: kay, () where (0.57) all right thIS time; (0.68) put=jer=finger OUT

25 [(1.12)] [((*Thomas puts finger out, Mrs. W takes it and places it on the edge of the cylinder.*))]

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26  W: [what=does thA::T one feel:like.
       [((Moves his finger along the edge. →))
27     (0.88)

28  t: it feels like=um;
29     (4.26)                    ]
       ((Mrs. W still moves his finger repeatedly
       around the circumference.))]
```

In this new attempt, Thomas's finger—held/ guided by Mrs. Wall's hand—follows the full edge of the cylinder repeatedly completing the entire circumference (turns 26). This leads to an incompletion, because the reply merely reiterates the second half of the querying statement without further words coming forth that would allow some sense of completion to emerge and set in. Again, there is an instructionally very long pause.[5] There are many possibilities for how different social actors experience the pause—being at a loss on the part of Mrs. Wall in finding the next instructional strategy, being at a loss on the part of Thomas to provide an answer that warrants going on to the next student, the attempt to allow Thomas all the time he needs to produce an acceptable answer, or the "invitation" (on the part of Thomas) to the teacher to provide another hint. Despite repeated attempts, therefore, an answer that would have allowed Thomas to sit down and other students to take their turn or the teacher to segue into another part of the lesson has not yet occurred. It is also not evident what will have to have happened that would justify an invitation | acceptance to sit down. This is evident especially in the fact that the questioning continues, with the apparent selection of Thomas as the currently designated respondent. Giving it another try provides new future possibilities (in addition being hearable as a sign of not having achieved some possibly desired result). With each new try, new paths are laid in walking. From all of those (as in Fig. 1b), one path *will have* emerged—i.e., it has been in excess over anything anyone might have intended ("Two roads diverged in a wood and I–/ I took the one less traveled by/ And that has made all the difference" (Frost 1966, p. 112).

The long pause ends as Mrs. Tee takes a turn, which turns out to be the first part of a question | reply joint action pair that offers and asserts a difference in the "feel" (turn pair 30 | 32). The reply turn offers different possibilities. Here the one that is realized both accepts the statement and seeks clarification of the content (i.e., the *what*) of the different feel (turn pair 32 | 34). Spaced by a pause, the answer that completes the question | reply turn distinguishes one of the designated entities as round and the other one as square. Is this turn pair such that it provides a sense of momentary completion and therefore offers the opportunity to move on? A longish pause develops, while Thomas gazes at Mrs. Wall, with a facial expression as if asking for confirmation. The pause might also be an indication that Mrs. Tee now is at a loss, or, in fact, that both teachers are—if the preceding question | reply

[5] Long because teachers have been reported to leave only 0.8 seconds on the average for a student response, with exhortations to extend this amount (Tobin 1987).

sequence had been perceived as not corresponding to what was to have occurred.

Fragment 3

30 T: ((*Thomas features questioning look* →))
((*Asking from behind Thomas*)) does it feel
the sAME or does it feel dIFFrent;
31 (0.32)
32 t: feel dIFFrent.
33 (0.93)
34 T: what is dIFFrent about those two EDges.
35 (0.77)
36 t: because um thIS one is rOUnd and thIS one
is ap (0.48) isa square.
37 [(1.63)]
((*Thomas looks up to Mrs. W, as if looking
for confirmation*))]

This time the speaking pause comes to an end with Mrs. Wall's voice, apparently articulating an instruction to "do just *one* of them" (turn 38), followed by an articulation of presumedly "the same" question already offered up in turns 22c, 26, 30, and 34. In the meantime, Mrs. Wall and Mrs. Tee have taken turns, as if (a) both were coming to the help of the other and as if neither one could achieve some intended goal—not yet apparent to Thomas—on her own or (b) there is still "more" that can be said and in this saying more, the topic itself of the conversation could become apparent. Whatever they have done or might have intended, the next turns exhibited the unexpected and surprising: something other than the anticipated reply. This repeats itself, in a different way, in Fragment 4 as well. Here, the query about how the edge feels come to be paired with the reply "triangle" (turn 42). That is, even if Mrs. Wall had intended Thomas to focus on the straightness of the edge, she did not achieve it. What the reply highlights is the (tri-) angular nature of the edge (turns42, 46a, 46c)—in itself a possibility that arises from the offered query (turn 40). In fact, the reply is repeated (turns 46a, 46c) even though Mrs. Wall gets a pyramid, points to one of its faces, and says, "*that* would be the triangle" (turn 44). There are pauses (turns 46b, 47), the second one coming to an end with what will have been an evaluative statement about what an appropriate reply *cannot* be, or rather, what cannot be used to constitute the reply: a shape word, including triangle, circle, or square (turn 48). There is another pause, which provides a new opportunity to Thomas for completing what would have been a query | reply sequence with a statement about how "it [edge]" feels; and then the apparent invitation to others to "help out" in describing the edge without using a shape word (turn 50). The turn is not just asking for someone else to respond but, at least momentarily as it turns out, to deselect Thomas as the next speaker. There apparently is an implied answer, which heretofore has not been stated.

Fragment 4

38 W: [if=you=jUSt=did ONE=of=them]
[((*moves finger up and down edge of cube*))]

39 [(0.79)]
[((*Thomas nods*))]

40 W: whAT would it (0.64) f::eel like.

41 (1.19)

42 t: [feels like um this one (0.65)
[((*moves finger up and down the edge →*))]
is um () triangle,

43 (1.83) ((*Mrs. Wall fetches pyramid from chalk tray*))

44 W: <<p>okay> thAT would be the triangle.]

45 (0.82)

46a t: [triangle and (0.57) and thIS one feels like um;]
[((*moves around the triangular face of the pyramid*))]

b (1.14)

c is=a triangle.

47 (0.95)

48 W: right; not U:sing a sh:ape word. (0.22) so we=re not gonna use a
[tRIangle CIRcle or squARE.]
[((*Rhythmically counting, beat gesture*))]

49 [(1.80)]
[((*Moves finger along edge of cube.*))]

50 can someone help us out?

Although Mrs. Wall has called for others to step in, following a pause just a bit shorter than the 0.8-s average pause teachers on average tend to leave for a response, she takes another turn, with a grammatically incomplete utterance that may also be heard as a query about what Thomas has said, which is only completed when, following an extended pause, Mrs. Tee will have offered one of the adjectives that Thomas has uttered previously: round (turn 54). What follows is not only an acknowledgment that Thomas has used this adjective but also that the speaker (Mrs. Wall) had forgotten that Thomas has actually used the word. Another pause emerges without the possible completion of the preceding query (about what the edge of a cube feels) and without a re/statement of some request. Then there is a restatement of the query and the selection of a respondent. In turn 58a, the preceding turn is de facto accepted and directly leads into what can be heard, in its repetition of earlier parts of the conversation, that "this one" (whatever is understood and perceived when the finger moves along the edge of the cube) is straight and, following a pause, with an analogical hand gesture around the edge of the cylinder, "this edge is round" (turn 58). Subsequent to a pause, there is a query | reply turn—initiated now by the other teacher, Mrs. Tee—that sought and received agreement with "that," which, given how the indexical term is heard, may refer to the statements about the edge of the cube being straight and that of the cylinder being round.

Fragment 5

51 (0.61)

52 you said this ones=s
53 [(1.79)]
[((*moves finger around circumference of cylinder* face))]
54 T: rou[::nd]
55a W: [i for]got what he said rOU:nd.
b (1.09)
c [what would this one bE:] (0.30) alycia
[((*Moves finger along cube edge*))]
56 A: straight
57 (1.06)
58a W: can we agree? (0.35) this one [is straight.]
[((*Follows edge of cube with finger → a*))]
b (1.00)
c [this edge is round.
59 (0.77)]
((*Follows edge of cylinder with finger → b*))]
60 T: would you agrEE with that thomas.
61 t: o::h: yes.
62 T: okay
63 (0.31)

At this point, then, the conversation has established what the two descriptions of edges would be. In turn 54, Mrs. Tee repeats what Thomas already has said in the context of the cylinder (turn 36): that there are round edges. There is then a question | reply sequence (turn pair 55c | 56) in which a second property of edges is articulated: there can be straight edges. There is also a query and affirmation sequence involving Mrs. Tee and Thomas (turn pair 60 | 61). The episode could have easily ended here, and, in other situations, might have ended precisely there. If Thomas had accepted an invitation to sit down, this might have realized this possibility and, thereby, given rise to new possibilities to continue the lesson. But, as shown by what follows, this is not the only possibility. Another question | reply turn makes the event continue—perhaps unexpectedly from the perspective of everyone other than Mrs. Wall. The reply to the query is not appropriate, as shown in the unfolding sequence that constitutes the first and second gesture as having moved through the center of the face rather than along the straight edge. That is, despite the apparent query about the particular place where the gesture has occurred—as if saying, "Do you really mean down here?"—it occurs again. What will be an evaluative statement follows, which not only will have described the gesture as "going straight through the face" but also will have constituted it as inappropriate or incorrect—in any case, as to be revised. In the hand of Mrs. Wall, Thomas's finger moves along the edge of the cube, which is associated with the descriptive statement "here's our straight edge" (turn 70a). Another query | reply turn follows, in which the identification of a round edge is invited and successfully delivered, as shown in the evaluative turn (turn 72).

Fragment 6

64 W: so which one is the straight (0.84) edge?
65 (0.43)
66 t: [this one.]
[((*Runs finger down face of cube →*))]
67 (0.46)
68 W: is it [down here?
[((*Mrs. W runs finger down center of face of cube.*))
show put your finger down a straight edge
69 t: [(1.57)]
[((*again runs finger through the center of cube face*))]

70a W: now when your [doing this were going right through the f:A::ce (0.50)]
[((*holds Thomas's finger and moves it through center of face*))]
b [heres our strA:I::ght edge]
[((*moves Thomas's finger along edge of cube →*))]
c now go to the rOU::nd edge ((*Mrs. W holds out cylinder to Thomas*))
71 [(0.93)]
[((*Thomas runs finger along cylinder edge*))]
72 W: <<p>good for you.>
((*Asks him to sit down.*))

With the final evaluative turn 72, the episode ended, as determined by the participants themselves. In the course of calling another student to order, we observe an invitation | acceptance pair, where Mrs. Wall asks Thomas to take a seat, and the student responds in walking back to his seat and sits down.

The lesson is emerging, always *in statu nascendi*: a first reading

The lesson constitutes an emergent presence. If at any point in time readers had been asking themselves, "Where is this conversation going?" then they will have borne witness to our phenomenon of interest. As a whole, what kind of lesson this will have been once the talking has come to a close—unsuccessful, successful, eventful, and so on—is unknown to any or all participants. For example, neither Mrs. Wall nor Mrs. Tee, or, for that purpose, any of the research personnel present, com*prehends* the episode (as a successful or unsuccessful one, who learned what, if anything, at which point in time); and once "the episode" is something that can be comprehended, it is no more happening but already is some denotable thing that has passed. The precise nature of Thomas's turn can be comprehended only when it is completed, lies in the past, and therefore no longer *is*.

When readers look back at what has been playing out before their eyes while looking at the episode, they might not have anticipated the precise trajectory of the event or even the general outcome, though it is likely that someone having taught a

lot might have some sense of what was being asked of Thomas. On the other hand, we know from our own experience of observing lessons without having talked to teachers beforehand that their stated intentions and the nature of the desired responses often is not recoverable from the lesson itself, especially if it is subsequently denoted as an unsuccessful one. As observers of mathematics and science classrooms, we have been repeatedly in the situation of not knowing some answers while the teacher observed was asking them. When we do look at the end of *this* episode, we can understand it as realizing the intention to have Thomas identify curved (round) and straight edges. This end could have occurred any time before. If this end could have occurred before, this also means that the happening was emerging, and the episode, grasped when all had been said and done, can be characterized as occasioned and emergent. From the perspective of the two teachers, they had no prior knowledge of when this end would come about. In fact, Mrs. Tee could not know what would unfold when she called on Thomas. Thus, the lesson could have unfolded in many other ways as well; and, while Mrs. Tee and Mrs. Wall planned for this lesson, this end would have been unpredictable to and unanticipated by them.

Although we can look back at the episode, now a whole, namable thing, doing so will not get us at the living experience of the participants. If we want to understand the happening from the shop floor itself, especially, if we want to understand its *emerging* and *emergent* properties, then it would be unwise to theorize and analyze what is happening in terms of any individual or collective intention that Mrs. Wall or Mrs. Tee might have had. We must not see it as a teleological accomplishment. Rather, following Mead (1932), we need to understand it in terms of an emergent form, each turn or action *de facto* realizing a possibility (i.e., $p_x = 1$) (even if unknown or unintended), thereby also not realizing other possibilities (i.e., $p_{y \neq x} = 0$). At the same time, there are new possibilities generated at every instant, with every word, gesture, movement, or action of another type. Thus, Mrs. Wall could not know that Mrs. Tee would step in and address Thomas in turn 34 and neither did Thomas; in fact, Mrs. Tee might not have known even seconds before that she would be stepping in. But in and through this stepping in, the unfolding lesson had gone through a bifurcation point, making disappear some possible futures ($p_x = 0$) and enabling other future developments ($p_x > 0$). What we call after the fact "the episode" now was on a new trajectory. But it does not take talk to make a lesson an emerging and emergent phenomenon. Even not talking is a form of action, and, consequently, changes the probabilities of the different opportunities that arise. Thomas apparently did not know what he is being asked, for at no point does he say what it takes to bring the episode to an earlier close. He comes to be confronted with the unknown with each new utterance, asking something, which the subsequent turns show he responds to in ways that do not allow the episode to conclude.

Even if its outcomes are unpredictable, what will be know as the episode ("Thomas's turn") does not unfold entirely willy-nilly. It does not just take any completely unknown course, but it is path dependent. That is, once Thomas was selected to have the next turn in producing a response to the general inquiry about the features of three-dimensional objects, this particular form of societally specific

event—i.e., school mathematics lesson—had to be brought to a close before another form of societal event could occur. There would be a (transactional) event that had to run its course before another type of event could begin. Thus, throughout this unit on geometry, a child would get its turn until it came to an end so that another child could get a turn.

In this situation, Mrs. Tee, Mrs. Wall, Thomas, and Alycia *together* achieve—i.e., as a result of irreducible joint action—the episode as one that we might denote as "Thomas's turn." Our account emphasizes its dramatic form, where the heroes come to meet the unknown future. Thomas's turn did not have to be that way, for, to take one example, we can easily envision a situation where nobody would have uttered "straight," as Alycia had done (turn 56). It could also have been the case that Thomas eventually gets to sit down without the sought-for response to have come forth. In fact, there is a little of that when Mrs. Wall moves her finger through the center line of the cube face saying "doing this we are going right through the face" (turn 70a) and then moving along the edge of the cube while saying "here's our straight edge" (turn 70b). Although the unfolding of the episode shows that "straight edge" constitutes something like the ideal-type response to the query "how would you describe this edge" (turn 07) to complete an appropriate question | reply turn pair, it takes some 70 turns to complete it. Everything in between, and including the end, unforeseeably is emerging for all participants while they are in the midst of things.

Attempting to understand and theorize this episode specifically and, for this purpose, any lesson generally, we have to choose a different approach—or thus Mead (1932) recommends. We need to understand it as emerging and understand the emergent as sitting astride two worlds, the one that is ending in and with the emergent and the one that is beginning in and with the emergent. This is precisely why we need a *category* of emergence, which emphasizes the co-belonging to two very different, logically contradictory orders (Il'enkov 1977). Our analytic approach is realizing this suggestion in that we focus on the dual role of each turn—and, because of each turns connection to a preceding turn, to the conversation as a whole—as completing one turn pair and opening up another. We also focus on the multiple perspectives that arise from taking the turn pair specifically and the whole historical situation generally as the minimum unit of analysis. Each word, sound, phrase, gesture, or movement is produced by someone *and* simultaneously perceived by others. Each of these minimum units, therefore, inherently is plural. It also is stretched out in time (technically: dehiscent) or, rather, in its unfolding, is producing time.

Any comprehension of what has just been said—once the emerging saying has come to an end—is itself emergent, a function of the speaker and the recipient. Thus, for example, it is only in turn 70 (Mrs. Wall) that the comprehension of turn 66 (Thomas) becomes apparent. At the microlevel of the locution, the recipient has to await the end of the saying before being able to com*prehend* (i.e., grasp) precisely what the saying will have said once it is over. Comprehension, therefore, cannot ever be simultaneous with the saying, even for the speaker himself—unless, perhaps, s/he were to read aloud a completely prepared text. But, already known to ancient wisdom, "He who grasps loses" (Lao Tsu 1972, §64): in grasping the Said, the saying has been lost. Even then, his/her own comprehension may change; and

even then the affective apprehension and appreciation will differ (e.g., Vološinov 1930). This is why even university lecturers become better not only at lecturing but also in their subject matter (Roth 2010). That is, the comprehension of an emerging event—before what is happening is *this* or that namable event—is delayed with respect the unfolding thereof; and this delay is the origin of time and temporality itself (Heidegger 1977; Husserl 1980; Mead 1932; Romano 1998).

Emergence in theoretical praxis

In the two preceding sections, we provide the description of an episode from a second-grade mathematics lesson and a first reading thereof in terms of those characteristics that exhibit its emergent characteristics. Our account specifically points to those features that clearly are evidence of the unforeseen and unforeseeable, that is, the emergent. Such accounts, therefore, provide us with empirical materials to theorize and think emergence as a category of experience.

Emergence belongs to two different worlds

All too frequently, scholars interested in teaching or learning processes stop short of attending to their emergent aspect, and let alone to this emergence is accomplished, and what shape it takes. As part of our analysis, we state that emergence belongs to two worlds, two different orders. But saying that emergence belongs to two very different worlds simultaneously, first situates emergence as the center of our investigation, and secondly conceptualizes this center in terms of an encounter. In fact, *encounter* is Vygotsky's understanding of the term category, which he applied to higher psychological functions and societal relations that are in continuous development (e.g. Roth and Jornet 2013b; Veresov 2010); and the term category embodies the encounter of, and belongs to, two different worlds (Il'enkov 1977). Emergence is characteristic of life generally and consciousness specifically, because living organisms and consciousness actively and recursively relate to the surrounding world. In a strong sense, therefore emergence is not essence[6]. That is, organisms are not merely determined by the environment but act in, transform, and are transformed by the environment. Thus, "emergent life changes the character of the world" (Mead 1932, p. 65). Mead, as does von Weizsäcker (1973), uses the term *sociality* to refer to these mutually relations; the educational philosopher Dewey denotes the mutually determining relations as *transactions* (Dewey and Bentley 1999). Sociality is reflected in our episode, where there is a relation, an encounter, a Vygotskian *drama* where presumably new psychological functions emerge asymmetrically symmetrical for mathematics teachers and their students (Roth and Radford 2010). In an appropriation of Maturana and Varela's enaction theory to the understanding of mental mathematics problem solving processes, this to-ing and fro-ing of living organisms and their environment has been articulated: "as coming from, emerging in, the interaction of the solver and the environment in which the

[6] In the recent philosophical literature, one might have found the alternate expression, where the "is" has been stricken through as in "emergence ~~is~~ essence."

solver is immersed" (Proulx 2013, p. 6). This leads to the situation that we cannot locate mathematical activities "inside either the solver or the task: they emerge from the solver's interaction with the task" (p. 6). That is, a transaction naturally belongs to *both* organism and its environment "just as mind-and-world are made inseparable by the body, knowers-and-known world are brought together in a body of knowledge . . . co-emerging with the world we inhabit and which en-habits us" (Davis et al. 1996, p. 165).

For Kant (1956), time is "a necessary representation at the foundation of all intuitions" (p. 78); for Piaget (1937), time is the result of a construction. In a theory of emergence, the role of time is different: it becomes present at the same time as the consciousness awareness of the world (Romano 1998). Time is not external to experience, a condition thereof, but internal, being produced in the to-and-fro where participants *give, make*, and *take* time. The world is changed in and through emergence, and this different world arises from the old during emergence (Mead 1932). In our episode, this is seen in the new and very different possibilities that arise when Mrs. Tee steps in and actively participates in the exchange. The older order, its possibilities, makes possible the change and is both overturned and survives in the newer order, the new possibilities, just as the text on the origins of mathematics (Husserl 1976) is re-written, erased, and sedimented in the introduction that Derrida (1962) provides to the French edition of Husserl's text. As a result, "[t]he old system is found in each member and in a revolution becomes the structure upon which the new order is established" (Mead 1932, p. 52). This temporal dimension is intimated at in enactivist reading of mathematical activity, when the "structures" (students and their environment) are considered as results of a historical evolution. But this literature does not sufficiently appreciate that we grasp the novel event only when we can see it both in the old and the new order (world), which was brought about by the emergence of the former (i.e., novel event). For this reason, "the *passage* from the past into the future the present object is both the old and the new" (Mead 1932, p. 51). Passage, drama, and encounter all are of the type of phenomena that we lose in grasping.

We present the event from the second-grade mathematics classroom *in statu nascendi*, from the perspective of the witness who cannot grasp what is happening until the happening has come to a close. The temporality of emergence *in statu nascendi*, as an articulation of old and new words (rather than entities interacting in "the" world), requires us to take further steps. Emergence is subject to the conditioning character of the immediate past present, and it also both overcomes and keeps this across the emergent in the new. It is for this reason that the emergence of a new conceptual order, such as new scientific (mathematical) understanding does not abolish the old but continues to exist, sedimented in the new practices: if Thomas comes to understand in and through the episode, then the new understanding has both emerged from and overturned what he understood before. Despite being antithetical to the new understanding, it exists as one of its conditions (Husserl 1976). That is, emergence does not merely belong to two systems, two worlds: "its presence in the later system changes its character in the earlier system or systems to which it belongs" (Mead 1932, p. 69). Starting points for employing such an approach can be found in mathematics education, where the

evolutionary nature of mathematics classroom practices is considered from which mathematical ideas emerge (Cobb 1999). They also exist in characterizations "the [problem] posing/solving is emergent, emerging through this continual flow of interaction between solver and task" (Proulx 2013, p. 15). This "continual flow" takes us at the heart of emergence as belonging to two different worlds, old and new. Thus, there is a circularity whereby "what is new evolves from what is old and, at the same time, what is old is modified (re-structured) by what is new" (Davis et al. 1996, p. 167).

This transformation is important to keep in mind to avoid narrowing emergence to the "adhocness," mostly stressing the "tailored" and "good-enough" nature of problem solving processes (Proulx 2013). Emergence allows us to focus on momentariness, and the tailo*ring* and the valu*ing* of what comes to hand *while* the relational drama in which Mrs. Tee, Mrs. Wall, and Thomas participate is unfolding. Without this transfigurative and transcendent dimension of emergence, it would be impossible to explain why the newer world cannot be derived from the older one even though the later constitutes the condition of emergence. The future is unpredictable not only because we do not know enough about the past or the present, but because past and present are *always* open-ended, always re-emerging.

Emergence cannot be predicted or grasped

Our description of the episode makes salient that not even the teachers, Mrs. Wall and Mrs. Tee, were in a position to know what will unfold. We suggest that if they had known that Thomas could not provide an answer to the questions initially raised, they would have made a different start. Thomas would have been exposed for a much shorter period to a situation that mathematics students often find embarrassing (e.g. Cobb et al. 2010). In the hope to observe change and effect, research in mathematics education long mostly focused on "before" and "after" in terms of states to be compared, assuming that "something new"—expected learning outcomes—would show in relation with what took place in between. Typically, descriptions prior to learning (e.g., pretest, interview) and after (post-test, interview) are taken and constituted as features of reality, thus giving little heed to the change itself, and, for example, considerations as to the difficulty to describe the change if it is itself unavailable to the participants—lest it be after the fact.[7] Thinking about emergence from the after-the-fact point of view does not give us the "reintegration of the affair as it went on, for it is undertaken from the standpoint of the present emergence" (Mead 1932, p. 48). In this article, we push the current thinking on emergence in mathematics education, addressing it not from after the fact but from within and thus in its own terms. In doing this, we elicit an important distinction borrows from our studies on movement, to contrast emergence with the illusion of emergence: it's recreation. This requires a change from the observer with

[7] Sometimes referred to as "the learning paradox": one cannot intend to learn some mathematical concept unless she already knows what that concept is because it cannot otherwise become an intentional object (Roth 2012).

a theoretical gaze, who grasps, to the mathematics teacher and learner as witness, who neither does nor can thematize (Roth 2013a).

In our episode, Thomas cannot be said to construct whatever he might be said to know in excess to what he has known: He is a learner precisely because he does not yet know what he will know after the event. He has not yet the language for talking about what he is doing, which will have led him to know something unforeseen, because such a language comes into being during the learning (Rorty 1989). Paradigmatic examples of emergence are changes subsequently denoted to have been conceptual revolutions. Thus, for example, during the approximately 100-year emergence of the Copernican worldview, European scientists spoke a language neither that was neither Ptolemaic nor Copernican but had all the characteristics of "inconclusive muddle" (p. 6). After the fact, the language sounds like inconclusive muddle because it was "in-between" two more stable forms of talking. Yet this inconclusive muddle led and gave rise to the Copernican worldview. This is so even though scientists, such as Galileo, were "typically unable to make clear exactly what it is that [they] want to do before developing the [Copernican] language in which to do it" (p. 13). At the classroom level, the emergence of new conceptual language that occurs at the collective and individual levels in physics classrooms has been described in the same terms (Roth 2005). Here, too, the lesson particulars constitute constraints on the activities globally so that—although many "garden-paths" are possible—the actually emerging forms of language lie within a certain range (e.g., as measured by the frequency of certain concept terms).

Grasping *what* is emerging (consciousness of emergence), however, requires an after-the-fact point of view, because it is only then that the emerging happening can be com*prehended* as something against the past that it has been. Initially, Thomas does not appear to understand what the teachers are asking him; this *what* will be available to him only at the end of the episode, after the teachers have tried repeatedly. This is so because one can *grasp* a happening (an event) as some*thing* only when it is complete, so that it can be made present again (requiring a representation) (Romano 1998). This implies a delay between undergoing and witnessing a happening (as witness) and understanding *what* has happened in the same way that there is a delay between saying and understanding what has been *said*. The latter is available only when the saying has ended. This means that the *emergent present* never is present to us through *re*presentations. The new that will be known to Thomas always is in excess of construction, for it is "in the *stride*—of thinking that it itself is changed" (Heidegger 2006, p. 109). But from the perspective of the past, in the situation itself, the emergent present cannot be grasped (e.g. by means of representations), though we witness and undergo what is going on in a participating way (von Weizsäcker 1973). Even the two experienced teachers, Mrs. Tee and Mrs. Wall, cannot know whether they are witnessing a successful instant of teaching. There is foundational indeterminateness in and to the consciousness, easily appreciable considering the "what" that occurs in what we subsequently call *a* (definite) "event."

A typical situation showing that there has been something in excess in emerging life exists when someone, like Mrs. Tee or Mrs. Wall, despite having made a most rational decision while relating to Thomas possibly will say afterwards, "If I had

known, I would have done [something different]." Here we see that at the crucial instant, the best decision option was chosen. Subsequently, however, after the fact, "more appropriate" alternative ways of acting are envisioned that would they think would not have gotten them where they are at the point of making the reassessment. The concept of *excess* helps us see that there is more than the algorithmic recovery you get when recreating emergence after the fact. This is why there always is a gap between plans (e.g., recipes, instructions) and situated action (Suchman 2007), so that even highly experienced scientists are confronted with the radical uncertainty concerning their own actions (Roth 2009).

In the episode, we see how Mrs. Wall and Mrs. Tee are confronted with replies that they evidently did not foresee. But our conceptualization of emergence has to go further. Not only does emergence lead to the new and unforeseen. Emergence *itself* cannot be predicted or grasped other than that it is ever present. But once reified in some "thing" said to emerge implies that emergence is no longer considered *as* emergence, is no longer living, thus assuming that everything has been said and done. It is in this latter way that we observe much of the mathematics education literature use the term. In contrast, in proposing emergence as a category, we highlight that:

1. when everything has been said and done momentarily, whatever has been emerging no longer is, and, therefore, is no longer living (e.g., *living* curriculum but one that has been *lived*);
2. emergence can be grasped only once everything has been said and done, when the associated saying and doing is momentarily over and when the emergent appears within the newer order to which it has given rise; and
3. emergence disappears as soon as we attempt to essentialize it ("It is . . .") as an "it."

The constructivist adaptation/reorganization metaphor, overemphasizing the agential dimension of human experience over its passive counter part, contradicts the fact that "phenomenality *is not* comprehended; it is *received*" (Marion 1997, p. 364, emphasis added). The category of emergence, on the other hand, points us to something novel that is in excess of what we are able to grasp in that instant, and which we can only experience through exposure and being affected (von Weizsäcker 1973). This requires us to theorize emergence *in statu nascendi*. We postulate that in this way, we arrive at the sense of birth, which, in fact, is indistinguishable from the birth of sense. That is, paraphrasing Marion (1996) for the present context, sense, coming among its own had to render itself visible and capable to be intended. The impossibility to grasp manifests itself in the phenomenon of *surprise*—from Lat. *sur-*, over, beyond, + *prendre,* to take, seize, grasp—which refers to experiencing (something in) the present as having been *beyond* our *grasp*. The surprise of the newly known is in excess of what constructivist explanations can provide, for what we construct is, because of the transitivity of the verb, already in our purview and, therefore does not surprise.

Emergence is not a homogeneous thing but heterogeneous like flow

As recent work in theoretical psychology shows, the true appreciation of the role of emergence to psychology is only at its beginning (Marsico and Valsiner in press). Emergence requires us to think in terms of a category that denotes difference in itself, not unlike other, related terms such as hybridity, heterogeneity, and non-self-identity (Roth 2008). In chaos and catastrophe theories, bifurcations and folds are typical ways in which emergence manifests itself, and therefore appears through the illusion of emergence. It is widely known that the state *after* emergence is not predictable from any of the parameters of the before emergence.[8] But the point of emergence in the present is spread across the old and the new orders; it covers the excess of the new over the old. In chaos theory, emergence is observed in a simple recursive system such as $x_{i+1} := \lambda x_i(1-x_i)$ is a function of λ (Fig. 2a). For values $\lambda < 3$, there is 1 limit value for x. But when λ passes 3, the line breaks into two. A bifurcation is occurring. This is a nodal point from which newer structural relations arise, which is equivalent to new forms (i.e., morphogenesis) and a new world having emerged. The bifurcation point belongs to both orders, the one where there is only one solution for x and the one where there are two solutions. It is precisely at this point where all forms of classical logic break down—especially those of constructivist thought—and that only a dialectical logic can handle (Il'enkov 1977). Thus, any "theory constructed according to the rules of [classical] logic, must give a picture of the object withdrawn . . . from the power of time" (p. 130) and, for this reason, precisely misses the phenomenon of emergence *as* emergence. As we show in our analysis, every turn at talk is a bifurcation point, which therefore manifests itself differently, depending on whether it is seen within one—i.e., effect of a preceding turn pair—or the other order—i.e., cause of the next turn pair (Roth in press). But any such difference that is associated with perspective is an expression of an inner difference of the phenomenon: it is not identical to itself (Il'enkov 1977). Mead (1932) had already noted this feature of the emergent: "In the present within which emergent change takes place the emergent object belongs to different systems in its passage from the old to the new" (p. 65). This is so because the emergent belongs to the orders before and after its arrival.

The two orders, however, are different so that the instant of emergence—which occurs at any instant, and with every each act, of perceiving and moving—also constitutes a crisis (von Weizsäcker 1973), an elementary catastrophe in qualitative mathematics (Thom 1981), or, to refer to a poetic expression of this situation, to a crossroads, which "both that morning equally lay/ In leaves no step had trodden black" (Frost 1966, p. 112). Our description and analysis highlights this continuous traversal of crossroads, where the conversationalists could have made a different statement. The appropriateness of this figure of thought also has been shown in the description of artifact design described as the emergence of virtualities (Roth 2001). Thus, at a problematic juncture in their curriculum task (Fig. 2b, A), three students articulate three possibilities for the continuation of their design of an earthquake-proof building (Fig. 2b, v_1-v_3). After considerable debate, which itself constitutes a passage, they take a first step in the realization of the third alternative (v_3). Then, as

[8] An example from the physical world is the Raleigh-Bénard convection leading to Bénard cells, a pattern of regular convections in a shallow liquid heated from below.

soon as they had taken a step (Fig. 2b, B), a new virtuality v_4 had emerged termed "Empire State Building." Yet one step toward the future branching point separating virtualities v_3 and v_4 (Fig. 2b, C), yet another virtuality (v_5) termed "Eiffel Tower" emerged. Yet just before A (Fig. 2b), just before running into trouble, none of the five virtual futures even existed.

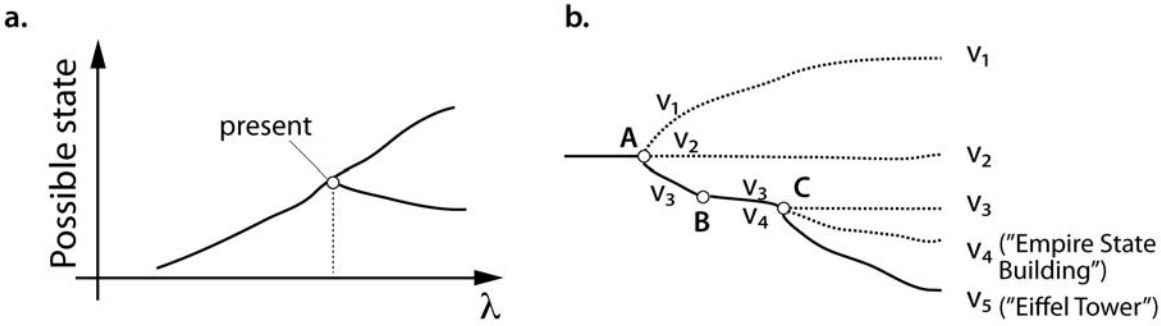

Fig. 2. a. Emergence in a chaos system perspective. For the system going from left to right, there is bifurcation, and the states after the bifurcation cannot be derived from the states prior to bifurcation. b. As the design process unfolded, 3 possibilities (virtualities) emerged at point A (v_1, v_2, and v_3), but as soon as the next development step was taken, an alternative (v_4, "Empire State Building") arose, and another one (v_5, "Eiffel Tower") after a further step (C) in the design.

In the example of chaotic systems, the bifurcation arises as a feature of recursion. Recursion also is at heart of Mead's articulation, even though he has not named it as such. In his view, the recursion arises from the relation between organism and environment. More importantly and thereby going beyond other enactivist and constructivist framings of emergence, the separation of organism and environment is itself a product of such recursion and impossible without it (e.g., Mikhailov 2002). The characteristics of the environment constitute the values and significations, and, isolatable as such, they found the basis of communication (Mead 1932). Emergence, therefore, relates to the form of thought and behavior and the environment. The human life form, characterized by communication, is a "realm of continual emergence" (Mead 1932, p. 85). The "sociality [of the human life form] . . . is the principle and form of emergence" (p. 85).

Following emergence, we do not just perceive the older reifications in a newer way. In contrast to the way in which Piaget has described it, perception itself has changed—which is the fundamental point of critique that has been launched against (Piagetian) constructivist ways of conceiving learning and development). Thus, for example, "the unsophisticated thinking of our earliest years remains as an indispensable acquisition underlying that of maturity, if there is to be for the adult one single intersubjective world" (Merleau-Ponty 1945, p. 408). Pregnance, the emergence of the new is "completely outside of the alternatives of Piaget" (Merleau-Ponty 1964a, p. 259). Whereas Piaget's work on the child's construction of the real makes it appear as if "the passage to a higher type of perception and conduct can be explained simply as a more complete and more exact registering of experience," what we see is rather that any "new" object or dimension "presupposes a

reorganization of the perceptual field and the advent of clearly articulated forms" (Merleau-Ponty 1964b, p. 104). Emergent events "become nodal points from which a new structure of relations arises" (Mead 1932, p. 97); they lead to a different world rather than to the previously existing world with new objects added and a more precise perception of this world. The actual history of any (cognitive) system, individual or collective (e.g., a classroom populated with teacher/s and students), traces out a path. Its future development is path dependent. And yet, the phenomenon of emergence does not make the past a determinate of the future.

Emergence in mathematics education praxis

> **Prescribed Learning Outcomes**
> *It is expected that students will:* (B1) use expressions to represent general terms for arithmetic growth, and apply these to expressions to solve problems.
> **Suggested Achievement Indicators**
> *Students who have fully met the prescribed learning outcome are able to*: generate the next three terms, given four consecutive terms of an arithmetic sequence. (BC Ministry of Education 2006, p. 37)

Mathematics education praxis continues to be governed by the deterministic conceptions of the curriculum—at least in as far as the political arena is concerned. Thus, as shown in the introductory quotation to this section, even the most recent official curriculum guidelines state *prescribe learning outcomes* and suggest indicators suitable for measuring the a priori described achievement. This is a non-emergentist approach to mathematics education even though already two decades ago, a path was opened for thinking mathematics education in terms of emergence when the "co-emergent curriculum" was recognized to be "not something which needs to be invented, created or predetermined" (Davis et al. 1996, p. 166). Such a curriculum is not the result of method or a particular set of methods, including texts and activities. Rather, "the co-emergent curriculum emerges from an understanding of human existence and cognition . . . as processes of natural drift in which human subjects co-emerge with the environments which contain them" (p. 167). These authors also point out that the view of the mathematics curriculum as emergent focuses our attention on the pedagogical relationship that binds students and teachers in what also has been called joint action [Fr. *action conjointe*] in (mathematical) didactics (Sensevy 2011). In our reading, however, there remains a reduction of emergence to certain a "*kind* of learning" (Davis et al. 1996, p. 167, our emphasis). Conceptualizing emergence in its own terms, emergence *as* emergence, and for the analysis of a somehow "mundane" classroom lesson render explicit an important nuance: *all* learning takes place while being immerged and submerged. Just as the proverbial fish immerged in water are the last to recognize *what* they are swimming in, so humans do not know what precisely is emergent from the current happening until they can look back onto the situation in which they had been immerged and from which they have emerged.

This study was designed to contribute to the development of a theory of

emergence. The theoretical sections of this paper emphasize the emerging and emergent nature of mathematics lesson, which, once accomplished, have emergent character. Emergent means that new courses of actions (garden-paths) unpredictably arise in and from a continuous unfolding, just as new system states arise in a bifurcation point of chaos and catastrophe theoretic models (Fig. 2). Our case study exemplifies how the course of the lesson unpredictably arises from the transactional relation of participants-acting-in-setting none of whom has or can have a theoretical understanding (god's eye view). The lesson thus has all the characteristics of a garden-path laid in walking. Specifically, the teachers could not have known that they would find themselves in *this* episode or that, once Thomas was designated as the next speaker, the episode would unfold in any *this* manner. They did not call upon Thomas to step forward. It was Thomas himself who had come to the poster on the chalkboard—without the moving being disallowed, and, inherently therefore, accepted on the part of the teachers—to point to the inscription and drawing of vertices. He did this rather than point to the edges, the content of Mrs. Tee's question or, for all that matters to any other inscription and drawing available, such as faces or shapes. In his pointing, one possibility came to be selected and others left aside so that, if these would have been addressed at all, the would have been so *also* (repetition) rather than initially.

If, however, mathematics lessons are *emerging*, then the teachers cannot be technicians who implement teaching strategies given specific and specifiable conditions according to a plan. Acting in and co-constituting an ever-*emerging* mathematics lesson, teachers, as students, are more like bricoleurs, who artistically create the unfolding happening as they go along. After the fact, they can look back and say that the teaching or learning was well done or unsuccessful on this day, whether the official curriculum guidelines have been met or not, or anything else between. Mathematics teaching no longer can be considered to be a repeatable routine—as textbooks on teaching *methods* and *practices* portray it—but must be viewed as a creative endeavor. To be understood after the fact as having reached its goals, successful teaching means inventing forms of conduct, "'on the spot,' 'in the twinkling of an eye,' 'in the heat of the moment'" (Bourdieu 1980, p. 137) "that will best satisfy the exception required by solicitude, by betraying the rule to the smallest extent possible" (Ricœur 1990, p. 312). *Urgency* is one of the key characteristics of practice; and it is a feature of participating in a game of which one inherently does not know the outcome and that cannot be won through the intellectualism implied in a constructivist approach (Bourdieu 1980). That is, emergence implies urgency because the mathematics teacher has to act without ever being able to know where the garden-path is going to lead. Such a perspective "offers a specific way of talking about strategies, avoiding ideas of possession . . . in favor of issues about emergence, flux, movement, interactions, relations, actions, and so forth" (Proulx 2013, p. 8).

In terms of teacher preparation, however, thinking a mathematics lesson as continuously emerging and, when everything is said and done, as having been an emergent phenomenon does not mean that nothing could or should be done in mathematics teacher education. One approach that explicitly oriented towards emergence was developed by the German mathematics and science educator

Wagenschein (1999), who recognized that a planned curriculum "can branch into several previously planed ways, but it cannot ever anticipate the unforeseeable and flowing, continuous plenitude of possibilities that a strongly Socratic conversation brings to light in a wake, self-coordinating group" (p. 98). Moreover, "the teacher cannot know at which turns of the road he will say something. For children, when their thinking has been awakened, think in surprising way and, often, also surprisingly well" (p. 98).[9] However, we know from every kind of conceivable game—athletics, team sports, chess, or checkers—preparation makes all the difference. But preparation does not mean determination so that every game is inherently open with respect to its unfolding and outcomes.[10] But despite all anterior preparation, deliberation, discussion, or modeling, "when it comes really down to the details of responding . . . you effectively abandon the plan and fall back on whatever embodied skills are available to you" (Suchman 2007, p. 72). The purpose of the preparation and the plan that issues from it is "to orient you in such a way that you can obtain the best possible position from which to use those embodied skills on which, in the final analysis, your success depends" (p. 72), without any preparation being able to make the "best possible option" even come close to what will be happening. As shown in the context of science teaching, preparation creates a *Spielraum* [room to maneuver, as in a game], which tends to be more extended for more experienced and for more prepared teachers (Roth 2002). The greater the Spielraum, the more possibilities for acting the teacher has for responding to the unforeseen that every instant of teaching brings with itself. In fact, what will have been successful teaching involves actions that increase the Spielraum, the network of virtual actions [Fr. *réseau d'actions virtuelles*] of the teacher-acting-in-situation (Masciotra et al. 2008).

It is quite evident that students, such as Thomas, find themselves on a path that they cannot comprehend because—not knowing yet what they will have come to know as a result of going through what they are going through—the lesson is continuously emerging and because they can understand the *why* of the mathematical activity only once they have learned what the lesson plan had laid out (Roth and Radford 2011). Students know they are participating in a lesson, but they do not know *what kind* of lesson it will be: boring, exhilarating, one in which they "didn't get the point," one in which they had an important mathematical insight, and so on. If Thomas could have known what Mrs. Tee and Mrs. Wall asked and wanted him to respond—if they had indeed made presented such an intent prior to asking him—then the entire episode would have been unnecessary and it is unlikely that it would have occurred. This is why Wagenschein (1999) recommends a genetic approach, grounded in students' current understanding and genetically following the leads that arise from the emergent nature of the curriculum.

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[9] Wagenschein provides some recommendations for orienting the curriculum towards emergence, including project-based curriculum, lots of student-student interaction, taking as much time as necessary to follow up leads that come from students, and so forth.

[10] In games where the outcome would be (nearly) predetermined by antecedent conditions, there are handicapping rules, such as special point systems in golf or additional weights to be carried in horse races, or competition classes (male/female, light/feather/heavy weight).

Students are involved in lessons precisely because governments and schools write curricula for students to "learn" something that they "do not yet know" and, therefore, to which they cannot intentionally orient (e.g., Roth 2012). Changes brought about by emerging and emergent processes, such as mathematics lessons, do not come as the "result from applying criteria (or from 'arbitrary decision') . . . as a result of either applying criteria or of *actes gratuits*" (Rorty 1989, p. 6, original emphasis). This also makes inappropriate a certain craftsperson-related metaphor of the "construction" to describe learning. This is so because the "craftsman typically knows what job he needs to do before picking or inventing tools with which to do it" (p. 12). The auto/poietic agent, a poet, "is typically unable to make clear exactly what it is that he wants to do before developing the language in which he succeeds in doing it" (p. 13).

Emergence requires descriptions and theories of practical action—teaching and learning—that take into account the fact that it unfolds in time, that comprehension of practice produces time and temporality. That is, such theories have to account for the fact that "one has no chance of providing a scientific accounting of practice . . . unless one knows the effects that scientific practice produces by the mere fact of *totalization*" (Bourdieu 1980, p. 138). The category of emergence works against totalization by the very fact that it denotes the unfinished and incomplete. During an emerging happening, we never know *what* emerges, the emergent, and, therefore, precisely what kind of emergence one participates in. During the mathematics lesson, we cannot ever be sure whether it will be successful or unsuccessful or in which ways success and failure will have been expressed. We do not know to any exact extent who (students and teachers) learns what and when. That is, we cannot anticipate in advance what precisely Mrs. Tee or Mrs. Wall themselves will have learned, how their practice of teaching has evolved as a consequence of the event presented here in an exemplifying fashion.

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References

Amerine, R., & Bilmes, J. (1990). Following instructions. In M. Lynch & S. Woolgar (Eds.), *Representation in scientific practice* (pp. 323–335). Cambridge, MA: MIT Press.

Bakhtin, M. M. (1981). Dialogical imagination. Austin, TX: University of Texas Press.

Black, L., & Williams, J. (2013). Contradiction and conflict between "leading identities": becoming an engineer versus becoming a "good Muslim" woman. *Educational Studies in Mathematics*. doi: 10.1007/s10649-013-9481-7

British Columbia Ministry of Education (2006). *Principles of mathematics 10 to 12: Integrated resource package.* Victoria, BC: Author.

Bourdieu, P. (1980). *Le sens pratique*. Paris: Les Éditions de Minuit.
Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematics Thinking and Learning, 1*, 5–43.
Cobb, P., & Bauersfeld, H. (Eds.). (1995). *Emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale, NJ: Lawrence Erlbaum Associates.
Cobb, P., Yackel, E., & Wood, T. (2010). Young children's emotional acts while engaged in mathematical problem solving. In E. Yackel, A. Sfard, & K. Gravemeijer (Eds.), *A journey in mathematics education research: Insights from the work of Paul Cobb* (pp. 41–74). Dordrecht, The Netherlands: Springer.
Davis, B. (2004). *Inventions of teaching: A genealogy*. Mahwah, NJ: Lawrence Erlbaum Associates.
Davis, B. & Simmt, E. (2003). Understanding Learning Systems: Mathematics Education and Complexity Science. *Journal for Research in Mathematics Education, 34,* 137–167.
Davis, B., Sumara, D., & Kieren, T. (1996). Co-emergence, cognition, curriculum. *Journal of Curriculum Studies, 28*, 151–169.
Derrida, J. (Ed., Transl.). (1962). *Edmund Husserl: L'origine de la géométrie*. Paris: Presses Universitaires de France.
Dewey, J. (2008). *Later works vol. 10: Art as experience* (J.-A. Boydston, Ed.). Carbondale, IL: Southern Illinois University Press. (First published in 1934)
Dewey, J., & Bentley, A. F. (1999). Knowing and the known. In R. Handy & E. E. Hardwood, *Useful procedures of inquiry* (pp. 97–209). Great Barrington, MA: Behavioral Research Council.
Duranti, A. (1986). The audience as co-author: An introduction. *Text, 6,* 239–247.
Durkheim, É. (1919). *Les règles de la méthode sociologique*. Paris: Felix Alcan.
Font, V., Godino, J. D., & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educational Studies in Mathematics*, *82*, 97–124.
Frost, R. (1966). The road not taken [1915; 1916]. In L. Herrig, H. Meller, & R. Sühnel (Eds.), *British and American classical poems* (p. 112). Braunschweig: Georg Westermann.
Garfinkel, H. (2002). *Ethnomethodology's program: Working out Durkheim's aphorism*. Lanham, NY: Rowman & Littlefield.
Hegedus, S. J., & Moreno-Armella, L. (2011). The emergence of mathematical structures. *Educational Studies in Mathematics, 77,* 369–388.
Heidegger, M. (1977). *Sein und Zeit*. Tübingen: Max Niemeyer.
Heidegger, M. (2006). *Gesamtausgabe I Abteilung: Veröffentlichte Schriften 1910–1976 Band 11: Identität und Differenz*. Frankfurt/M: Vittorio Klostermann.
Hershkowitz, R., & Schwarz, B. (1999). The emergent perspective in rich learning environments: Some roles of tools and activities in the construction of sociomathematical norms. *Educational Studies in Mathematics, 39,* 149–166.
Husserl, E. (1976). *Husserliana Band VI: Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*. The Hague, The Netherlands: Martinus Nijhoff.
Husserl, E. (1980). *Vorlesungen zur Phänomenologie des inneren Zeitbewusstseins*. Tübingen: Niemeyer.

Il'enkov, E. V. (1977). *Dialectical logic: Essays on its history and theory*. Moscow: Progress Publishers.

Jacobs, J., & Morita, E. (2002). Japanese and American teachers' evaluations of videotaped mathematics lessons. *Journal for Research in Mathematics Education, 33*, 154–175.

Kant, I. (1956). *Werke Bd. II: Kritik der reinen Vernunft*. Wiesbaden: Insel.

Kieren, T. (1995). Teaching in the middle: Enactivist view of teaching and learning mathematics. Plenary lecture at the Queen's/Gage Canadian National Mathematics Leadership Conference for Teachers, Queens University, Kingston, Ontario.

Lao Tsu (1972). *Tao te ching* (G.-F. Feng & J. English, Trans.). New York, NY: Vintage Books.

Lautner, A. (2012). Der Einsatz des Mathematikmaterials von Maria Montessori und dessen Auswirkung auf die Entwicklung des Zahlbegriffs und die Rechenleistung lernschwacher Schülerinnen und Schüler im ersten Schuljahr. Dissertation at the Ludwig-Maximilians University, Munich. Accessed May 24, 2013 at http://d-nb.info/1031631089/34

Marsico, G., & Valsiner, J. (in press). Mind the border! Experiencing the present and reconstructing the past through the boundaries. In R. Säljö, Å. Mäkitalo, & P. Linell (Eds.), *Memory practices and learning: Experiential, institutional, and sociocultural perspectives*.

Masciotra, D., Roth, W.-M., & Morel, D. (2008). *Énaction. Apprendre et enseigner en situation*. Brussels: de Boeck.

McDermott, R. P., Gospodinoff, K., & Aron, J. (1978). Criteria for an ethnographically adequate description of concerted activities and their contexts. *Semiotica, 24,* 245–275.

Mead, G. H. (1932). *The philosophy of the present*. London: Open Court.

Marion, J.-L. (1996). *La croisée du visible*. Paris: Presses Universitaires de France.

Marion, J.-L. (1997). *Étant donné: Essai d'une phénoménologie de la donation*. Paris: Presses Universitaires de France.

Merleau-Ponty, M. (1945). *Phénoménologie de la perception*. Paris: Gallimard.

Merleau-Ponty, M. (1964a). *Le visible et l'invisible*. Paris: Gallimard.

Merleau-Ponty, M. (1964b). *Sens et non-sens*. Paris: Gallimard.

Mikhailov, F. T. (2002). The "other within" for the psychologist. *Journal of Russian and East European Psychology, 39*(1), 6–31.

Nancy, J.-L. (1993). *The birth to present*: Stanford, CA: Stanford University Press.

Piaget, J. (1937). *La construction du réel chez l'enfant*. Paris: Delachaux et Niestlé.

Protevi, J. (2009). Beyond autopoiesis: Inflections of emergence and politics in the work of Francisco Varela. In B. Clark & M. B. Hansen (Eds.), *Emergence and embodiment: New essays on second-order systems theory* (pp. 94–112). Durham, NC: Duke University Press.

Proulx, J. (2013). Mental mathematics, emergence of strategies, and the enactivist theory of cognition. *Educational Studies in Mathematics*. doi: 10.1007/s10649-013-9480-8

Ricœur, P. (1990). *Soi-même comme un autre*. Paris: Éditions du Seuil.

Romano, C. (1998). *L'événement et le monde*. Paris: Presses Universitaires de France.

Rorty, R. (1989). *Contingency, irony, and solidarity*. Cambridge: Cambridge University Press.

Roth, W.-M. (2001). Designing as distributed process. *Learning and Instruction, 11*, 211–239.

Roth, W.-M. (2002). *Being and becoming in the classroom*. Westport, CT: Ablex/Greenwood.

Roth, W.-M. (2005). *Talking science: Language and learning in science*. Lanham, MD: Rowman & Littlefield.

Roth, W.-M. (2008). Bricolage, métissage, hybridity, heterogeneity, diaspora: Concepts for thinking science education in the 21st century. *Cultural Studies in Science Education, 3*, 891–916.

Roth, W.-M. (2009). Radical uncertainty in scientific discovery work. *Science, Technology & Human Values, 34*, 313–336.

Roth, W.-M. (2010). *Language, learning, context: Talking the talk*. London: Routledge.

Roth, W.-M. (2012). Mathematical learning: the unseen and unforeseen. *For the Learning of Mathematics, 32*(3), 15–21.

Roth, W.-M. (2013a). To event: Towards a post-constructivist approach to theorizing and researching curriculum as event*-in-the-making. *Curriculum Inquiry, 43,* 388–417*.*

Roth, W.-M. (2013b). Toward a post-constructivist ethics in/of teaching and learning. *Pedagogies: An International Journal, 8*, 103–125.

Roth, W.-M. (in press). Working out the interstitial and syncopic nature of the human psyche: On the analysis of verbal data. *Integrative Psychological and Behavioral Science.*

Roth, W.-M., & Gardner, R. (2012). "They're gonna explain to us what makes a cube a cube?" Geometrical properties as contingent achievement of sequentially ordered child-centered mathematics lessons. *Mathematics Education Research Journal, 24*, 323–346.

Roth, W.-M., & Jornet, A. (2013a). Situated cognition. *WIREs Cognitive Science, 4,* 463–478.

Roth, W.-M., & Jornet, A. (2013b). Toward a theory of *experience*. *Science Education*. DOI: 10.1002/sce.21085

Roth, W.-M., & Radford, L. (2010). Re/thinking the zone of proximal development (symmetrically). *Mind, Culture and Activity, 17,* 299–307.

Roth, W.-M., & Radford, L. (2011). *A cultural-historical perspective on mathematics teaching and learning*. Rotterdam: Sense Publishers.

Sensevy, G. (2011). *Le sens du savoir: Éléments pour une théorie de l'action conointe en didactique*. Brussels: de Boek.

Sheets-Johnstone, M. (2011). *The primacy of movement* (expanded 2nd ed.). Amsterdam: John Benjamins.

Shimizu, Y. (2008). Exploring Japanese teachers' conception of mathematics lesson structure. *Zeitschrift für Didaktik der Mathematik, 40*, 941–950.

Suchman L A. (2007). *Human–machine reconfigurations: Plans and situated actions* (2nd ed.). Cambridge: Cambridge University Press.

Thom, R. (1981). Worüber soll man sich wundern. In K. Maurin, K. Michalksi, & E. Rudolph (Eds.), *Offene Systeme II: Logik und Zeit* (pp. 41–107). Stuttgart: Klett-Cotta.

Tobin, K. (1987). The role of wait time in higher cognitive level learning. *Review of Educational Research, 57*, 69–85.

Vygotskij, L. S. (2001). *Lekcii po pedologii*. Izhevsk: Udmurdskij University.

van Oers, B. (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics, 74,* 23–37.

Varela, F. J. (1995). The emergent self. In J. Brockman (Ed.), *The third culture: beyond the scientific revolution* (pp. 209–222). New York: Simon & Schuster.

Varela, F. J., & Depraz, N. (2000). At the source of time: Valence and the constitutional dynamics of affect. In S. Gallagher, S. Watson, & P. Brun (Eds), *Ipseity and alterity: Interdisciplinary approaches to intersubjectivity* (pp. 153–174). Rouen: Université de Rouen.

Veresov, N. (2010). Forgotten methodology: Vygotsky's case. In A. Toomela & J. Valsiner (Eds.), *Methodological thinking in psychology: 60 years gone astray?* (pp. 267–295). Charlotte, NC: Information Age Publishing.

Vološinov, V. N. (1930). *Marksizm i filosofija jasyka: osnovnye problemy sociologičeskogo metoda v nauke o jazyke*. Leningrad: Priboj.

von Glasersfeld, E. (1983). On the concept of interpretation. *Poetics, 12,* 207–218.

von Weizsäcker, V. (1973). *Der Gestaltkreis: Theorie der Einheit von Wahrnehmen und Bewegen*. Frankfurt: Suhrkamp.

Wagenschein, M. (1999). *Verstehen Lehren: Genetisch – Sokratisch – Exemplarisch (5. Auflage)* [Teaching understanding: genetically, Socratically, exemplarily, 5th ed.]. Weinheim: Beltz.