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GRAPHING HAGAN CREEK

A Case of Relations in Sociomaterial Practice

INTRODUCTION

Over the past decade, it has almost become a truism to say that the mathematics in school and the mathematics in everyday settings are considerably different, and often incommensurable. However, it is much less evident what the competencies are that people bring to (implicitly or explicitly) mathematical tasks in everyday situations. In the following episode, Nadely, a beginning mathematics and science teacher visits a farm that is part of a movement in the valley to improve the health of a local creek and watershed. She meets with Karen, a water technician working on the farm and paid by – to a considerable extent – a grant from a governmental agency. Karen is also part of an environmentalist movement that has taken as its goal to improve the watershed and to design the creek, its riparian zones, and the practices of the people living in it.

One of the first things Karen shows Nadely is the water monitoring station, which essentially consists of a pen chart recorder that continuously inscribes water levels on a paper roll. In the following excerpt, Karen reads the graph and provides some explanations. In the process, she points and moves along the graph using deictic and iconic gestures (Figure 1).

K: So, this [b] is a twenty-four hour time period, so time's going this way on the graph, so a day ago [a] the flow was about fifty liters a second higher and each square going this way is about [b] thirteen liters a second. So, we were kind of, I guess we had a bit of rain yesterday [a] or the day before, and we've got a bit of a peak from that rainfall event [a]. In the summer, [c] the flow goes down to about [here and]
N: [Oh, whoa!]
K: that's equivalent to about. . . . This summer was pretty good, about twenty liters a second. But it can go down to about eleven, and that's, for fish to survive in this creek, we need about twenty.
N: Oh, okay.
K: So, when we get low, the fish will find a pool somewhere to hide and if it weren't for these deep areas, in little pockets, throughout watershed, they are sort of hiding and laying low until volumes have come up.

E. Teubal et al. (eds.), Notational Knowledge, 227–255.

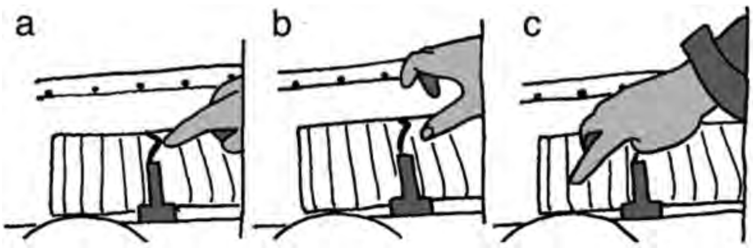

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Figure 1. Karen, the water technician, reads the graph to Nadely, the practicum teacher. The graph is "in-the-making", still on the pen chart plotter in the monitoring station at the farm where Karen works.

The first part of this reading (lines 01–10) is not unlike what one might expect a reading of a graph to look like. The reading appears to be able to exist by itself, no different from the decontextualized readings that are often required from students in research on graphing. But then it becomes quite clear that Karen's graph-reading practices exist with respect to much more.

Karen situates her talk about summer water levels (lines 08–10). Normally there are about twenty liters/second of water flowing past her monitoring site. In and of itself, this number does not tell us very much. Sense arises from the mediated relations that exist in activity settings. When summer levels can go as low as eleven liters/second, the value of twenty is already put into a new relation. Another time, she suggested that the volume, "last year was quite high, twenty-seven liters per second". For the innocent reader unfamiliar with the historical context of flow volumes, a graph that has volume values around twenty liters/second makes little sense. For Karen, on the other hand, the twenty liters/second do not exist independently of the other possible summer values, and the variations within and across seasons (see also Figure 6c). For her, the graph exists in relation to many other things she does and knows in this watershed and on her farm (Figure 2). In contrast to the eleven and twenty-seven liter/second volumes that are, in Karen's descriptive language, "low" and "quite high", we get a sense of the variability of summer flows. Furthermore, Karen's understanding does not stop with the numerical values of the volumes. She knows about fish and the conditions they need (lines 12–14). She is familiar with the temperatures that go with different water levels and, in turn, influence the conditions of the water as habitat. For Karen, all of this is part of what constitutes her competent reading of this graph at this time.

Karen also talks about fish, though these have little to do with her work on the farm. Yet the fish Karen talks about are not abstract objects somewhere out there. Rather, they are intimately related to her activities in her watershed-related world. She participates in capturing the trout in traps for measurement purposes,

Figure 2. Karen, the water technician, is intimately familiar with the valley (left), where her farm is located, and Hagan Creek (right), which she has studied closely as part of her involvement in the environmentalist group.

counts them making use of an electroshock device, and brings them to the surface by throwing small spitball-sized chewed paper into the pools. She knows that the trout prefer the areas below the riffles that she builds, where the water has a higher dissolved-oxygen (DO) content as indicated by her DO meter. She is familiar with the accounts given by elders from the nearby First Nations village. They still talk about the eighteen-inch cutthroat that they used to fish in the creek. Karen has read the notebook entries of the local priest who was able to capture a dozen trout in the course of one morning. That is, there is more to Karen's explanation than the graph as such. What she says here is only figure against a complex ground of embodied and distributed, practical understandings of this world. The object of her knowing is first and foremost the creek, knowledge *of* the creek, while the graph is *one* of the tools mediating her knowledge *about* the creek. Her talk is about the summer and about fish that need a certain amount of water. I suggest that Karen's reading of the graph – in fact, her competency – derives from a dense network of activities, practices, and facts. It is this dense network to other graphs, instruments, and practices that situates Karen's competency and allows these graph-related competencies to exist in the first place.

The excerpt shows two further aspects. First, without the gestures, it is virtually unknowable what the referent of Karen's talk is. So as a first step, we include these as part of our analysis. Furthermore, Karen's gestures and talk do not stand on their own but are over and about the graph. They are *about* the graph, that is, the graph is the topic of Karen's talk. But importantly, the talk is also *over* the graph in the sense that the latter serves as indexical ground. It therefore makes sense to use a cognitive unit of analysis that includes the entire performance, talk and gesture, and the graph. Second, graph reading exists in a social nexus: It matters whether a reading occurs as part of an interview for research purposes or whether it occurs as part of a person's ongoing work (Roth & Lee, 2004). In the present situation, the two participants contribute to establishing their mutual roles: Nadely constitutes herself, and is constituted, as a listener; Karen is the presenter, as she talks about her work and how the graph inscribes itself in the practices. But she does not just narrate irrespective of the listener. Nadely signals that she is still with the narrative at those points where breaks in the narration allow her to take a turn (lines 07, 11). This pattern is changed only once when Nadely expresses amazement about the extreme low levels of water during the summer months.

In the past, much research on mathematical knowing has focused on models of mind irrespective of the societal activity within which it occurs. This approach is being questioned, for it neglects the contexts that enable cognition to exist in the first place. Here, context is viewed as the historically constituted concrete relations within and between situations (e.g., Lave, 1988). I take the view that knowledge is not an entity that can be acquired but rather that knowing is equivalent to acting in the world; knowing is a process rather than a state. Knowing arises from historically constituted (concrete) relations within and between sociomaterial[1] situations and involves the individual body as much as the individual mind. The body-mind ensemble is an indissociable sociomaterial entity subject to be formed by the sociomaterial world to which it is connected and that embeds it. I am therefore interested in the position that "the relational dynamics [between sign and practice] are not created inter-subjectively in any simple sense, but are produced in relation to aspects of social practice which are culturally and historically specific" (Walkerdine, 1988, p. 12).

I begin by contextualizing my work in two ways. First, because graphs have sign functions, I provide a brief overview of a semiotic approach to graphing (see also Roth, 2003). Second, as the introductory analysis of Karen's work shows, graphs are embedded in numerous other relations requiring an expanded framework. I present one such framework derived from activity theory. After a brief exposition of the research context in which the data were collected, I provide detailed analyses of the relations that are constitutive elements in Karen's knowing. I end this chapter with a consideration of possible implications of this work for developmental issues in mathematical knowing.

SIGNS AND SIGNING PROCESSES

In the past, philosophy of language assumed two separate domains – world and signs (symbols, language) – separated by a deep gulf. This gulf had to be transgressed through correspondences (Figure 2a). Scientists presuppose a structural isomorphism between structure of the world and mathematical structure (signs). This isomorphism, expressed in the form of Wilson's couplet "Fundamental Structure ↔ Mathematical Structure" (Lynch, 1991), is embodied in the mapping of Figure 3a. However, there is evidence that this isomorphism is an illusion and that the isomorphism may be the outcome of scientists' work rather than a pre-existing condition. Thus, cognitive scientists faced a problematic gulf in the guise of the question, "How do the symbols [which are the basis of information processing] ever come to relate to the things in the world?" That is, cognitive scientists came to identify the "grounding problem" as its major challenge. In the psychology of mathematics education, the problem surfaced as referential isolation, the fact that for many individuals, the mathematics of the classroom existed

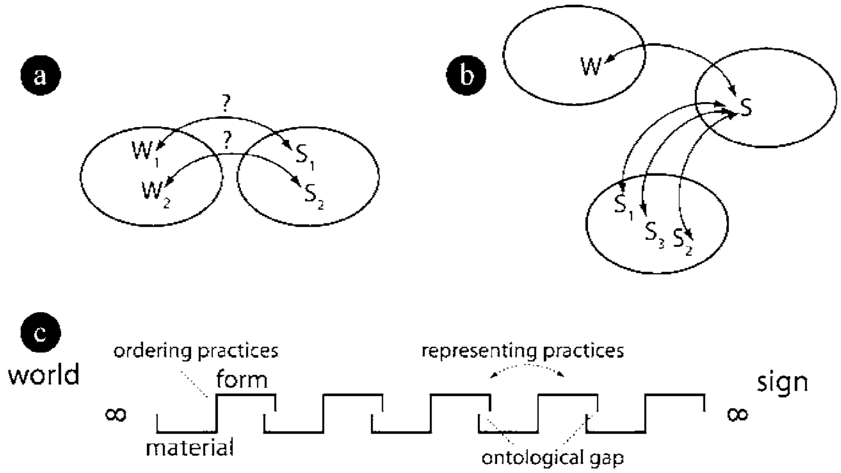

Figure 3. (a) Different views on the relationship between sign and the world. World and sign have long been considered to be separate domains, which were linked by correspondences, referential functions. (b) The relationship between a sign and its referent is elaborated by another relationship of the original sign with another sign, said to be its interpretant. Because there are potentially many interpretants, semioticians speak of an infinite process of interpretation, or infinite semiosis. (c) A potentially infinite chain of signification relates world to language (Latour, 1993). Each element is at the same time a sign for the previous element, and the referent for the subsequent element. Between elements, there is an ontological gap. Within each element, material can be given form. These gaps are navigated by means of shared social (representing) practices, which establish and control the relations.

separately from events in the world (e.g., Greeno, 1988). This separation was of particular importance in those situations where psychologists of mathematics education saw that mathematics could be applied (i.e., there was said to be a structural equivalency).

Researchers in the social studies of science have come to question the existence of such an isomorphism. They bracket Wilson's couplet and thereby make it to a phenomenon to be researched rather than to be accepted a priori. The resulting research focuses on the practices by means of which such things as soil samples, screaming rats, or defecating lizards come to be represented in mathematical (Roth & Bowen, 1999). At every step of the way, we encounter elementary forms of mathematical practices that always involve the physical body of the researcher. The gap seemingly disappears in the practices of the scientists who enact continuous series of transformations. This research aligns itself with semiotics, a line of research concerned with the relation between signs and things in the world relatively little consulted by mathematics educators.

For nearly one hundred years, semioticians (e.g., Peirce, Saussure, and Eco) have researched the relationship between world and language, signs and their referents (Nöth, 1990). They recognized that there existed an ontological gap between signs and their referents. Those following the path of Charles Sanders Peirce propose that while we cannot close the gap, we can always superpose another, sign-sign relation on top of the first relation. This second relation between a sign and another sign, its interpretant, elaborates the first relation, which remains inaccessible in principle (Figure 3b). For example, the sign /dog/ refers to some aspect of the world. The relation between the sign /dog/ and the class of entity, "dog", that it refers to can be elaborated by other signs such as a drawing, the equivalent in another language (e.g., chien [French], Hund [German]), a metaphoric use such as /fidelity/, and so forth. Here, each production of an interpretant sign constitutes a translation of the original sign. The process of translation (interpretation) is unlimited, because there are many, potentially an infinite number of interpretant signs (Figure 3b). Semioticians refer to this process as unlimited semiosis.

When we follow scientists (or any other individual involved in sign production), we begin to notice a series of (potentially infinite) translations. These translations turn, for example, living lizards caught somewhere in the mountains of the Pacific Northwest into a statement such as "there is a significant correlation between lizard sprint speed and leg length". This statement itself translates and is translated by a graph or statistical information (e.g., p, r, R^2). (For a detailed ethnographic study of such activities, see Roth, 2004.) For example, a two-column table of numbers and a Cartesian graph are equivalent because of established practices, not because there is an inherent logical connection between the two. Similarly, a mathematical function such as $f[x] = x^2 - 3$ and a parabolic curve on a Cartesian graph are equivalent because of established and shared mathematical practices, not because of some internal logical relation.

Following scientists around we come to see chains of elements – each of which plays the role of sign for the previous element and the role of thing/matter for the next, giving rise to a chain of signification (Figure 3c). That is, each element constitutes a map for the previous element, its territory; in turn, it becomes territory for the subsequent element. It is important to note, however, the consecutive elements are separated by an ontological gap. The links across each gap are established as a matter of sociomaterial practices common within and constitutive of particular communities. This view is commensurable with semiotic processes operating at multiple hierarchical scales in which elements to the right in Figure 2c are the objects of an element in the middle, which themselves are signs in a system of interpretants (Lemke, 2002). The semiotic processes relating to the lines of graphs themselves are inscribed in topological semiotics, whereas entities such as the variables (axes labels) inscribe themselves in typological semiotics (Lemke, 2000).

GRAPHING AND ACTIVITY SYSTEMS

Traditionally, sign-reference relationship was the primary object of research in mathematical cognition. More recently, it has been recognized that this relationship is not independent of the community of interpreters, which led me to a semiotic approach. But even considering the community of interpreters is insufficient to account for mathematical understanding; that is, a semiotic analysis is

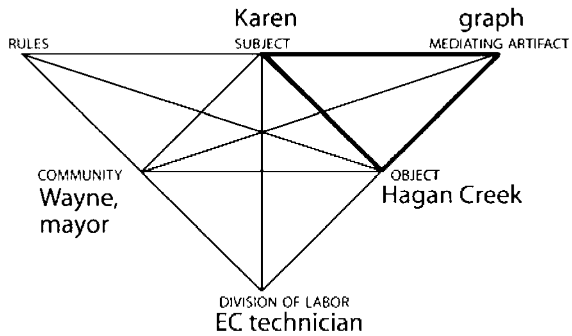

Figure 4. The activity constitutes the basic, irreducible unit of analysis in the present study: None of the parts can be understood on its own and each contributes to constituting all others. It shows the kind of mediations that exist between the elements isolated for heuristic purposes. Schools and research often only focus on the "primary" relation between subject, object (water level in Henderson Creek), and mediating artifact (graph).

therefore only part of the story. To capture the other parts, I draw on activity theory (e.g., Cole & Engeström, 1993). My introductory description and analysis of Karen in activity brought out a number of relations. Thus, Karen focuses on the creek that her farm needs as a water supply throughout the season, but this relation is mediated by the graph. Both the creek and the graph are historically and culturally situated. First Nations and other people have been drawing water from the creek for hundreds of years, and Western farmers have received licenses since the 1940s. The graph as the product of a pen chart recorder is embedded in scientific and technological culture as an important recording device. That is, the relations between graphs and some aspect of nature are not simply perceptual or functional. An important task therefore lies in carefully studying the way in which material, practical, and linguistic relations are produced in activity (systems).

In activity-theoretic terms, I focus on relations that arise from triplets of heuristically isolated entities including subject, object, mediating artifact, community, rules, and division of labor (Figure 4). These entities are not "elements", for this notion implies reducibility of the whole to smaller parts (Vygotsky, 1986); these entities function as "go-betweens" between two other entities, and therefore as mediators – in one sense of the term mediation.[2] For example, the graph (mediating artifact) mediates the relation between Karen (subject) and Hagan Creek (object), which means that Karen does not just know the creek through her embodied dealing with it as a material thing but that she *knows* it *in terms of* the graph. This relation is equivalent to the semiotic relations featured in Figure 3b. Here the subject, object, and mediating artifact find their correspondences in interpretant, referent, and sign, respectively. However, the point of an activity theoretic perspective is that this primary relation is stabilized and made possible by other mediating relations – in fact all of the relations that exist on the inside of the irreducible activity unit, which in Karen's case are constituted either in her work on the farm or through her participation in the environmentalist group. The division of labor with an Environment Canada technician mediates the relation between Karen and the creek. The different mediated relations displayed in Figure 4, as an ensemble, including material and social dimensions, constitute the basic unit of analysis of human behavior, an activity system.[3] The fusion of the material and the social (discursive) produces relations of signification and the individuals that are positioned, qua subjects, within practices.

Activities are oriented toward something and driven by something. This something, the object/motive, exists on two planes: as part of the sociomaterial world (*being-in-itself*) and in consciousness (Hegel, 1977). The object/motive is constantly in transition and under construction, and it manifests itself in different forms for different participants in the activity. We have also seen that objects appear in two fundamentally different roles, as objects and as mediating artifacts or tools. In both activity-theoretic and semiotic terms, there is nothing in the material constitution of an object that would determine which of the two roles it has in an abstract and decontextualized sense. It is the activity that determines the place and sense of the object (Marx, 1976).

In the present study, I show that Karen's reading of the graph in terms of the water levels of Hagan Creek is inscribed in many other relations, some of which are made thematic in Figure 4. For example, another configuration mediates the primary Karen – Hagan Creek – graph relation: the community, graph, and Karen. Yet another relation made thematic in this chapter is that between the Mayor Walter, the Hagan Creek watershed, and Karen. Still another mediated relation is that between Karen, the water levels, and the Environment Canada technician who comes to establish the calibration curve that allows Karen to read her graph in terms of liters/second although it really displays water levels. In another relation, the graph replaces Hagan Creek as the object and is, in turn, mediated by other graphs.

Graphs provide the basis whereby particular physical relations are inscribed as relations within the organization of practice. In such cases, we cannot simply speak of "representation", because signs represent more than physical relations (Walkerdine, 1988). As beings, we always come to a world where graphs, as signs in general, are always and already social (Heidegger, 1977). In Karen's work, the graph taps many other practices (signifying and material) within the valley, irrigation, damming the creek, fishing, habitat maintenance, building impervious surfaces, building riffles, planting trees in riparian zones, oxygenating the creek. It is in relation to these other practices that Karen's graphing exists. Looking at her graph reading in the absence of everything else, we could come to the conclusion that she competently reads it. But this is of little help for understanding the relations that make this graphing competence possible in the first place.

BACKGROUND OF THE STUDY

The data presented in this chapter derive from a large three-year study on the representation practices among scientists (almost exclusively ecologists) and environmental activists.[4] This study included both formal interview situations in which participants were asked to read and elaborate on graphs that we had culled from introductory university ecology courses and textbooks. My research team also conducted multiyear ethnographies in field research settings and in an environmental activist group that focused its activities on the Hagan Creek watershed and the community of Central Saanich. One of these activists was Karen, a water technician employed by a local farmer with funds from a government grant. The farmer had environmental concerns related to the water resources in the Hagan Creek watershed where his farm is located, and especially in regards to the creek from which he obtains much of the water for irrigating his fields. Among others, Karen operated a device that continuously monitors the water level in the creek by means of a pen chart recorder.

In addition to the research among the environmental activists, we also designed and enacted a science curriculum in which elementary children have opportunities to engage in (mostly mathematical) representation practices. The children constructed representations about the creek and its environs with the ultimate goal of feeding their understandings back into the community during an open house organized annually by the activists. As part of this work, Karen and the activists worked with teachers, such as the new teacher Nadely with whom I worked, to familiarize them with her work on the farm and among the activists.

The materials used in this chapter derive from four videotaped situations where Karen explained the water level graphs and talked about the graphs with teachers and visitors at the open house. In addition, I spent considerable time with Karen walking in and alongside the creek, studying the habitats from the mouth of the creek to its beginnings. We spent time talking about the watershed while standing far above the creek, or walking the fields of the farm. Furthermore, we spent considerable time together working with seventh-grade students in the creek, teaching them how to collect data, make observations, and how to understand the watershed as an ecosystem. Our conversations were recorded in the form of videotapes, audiotapes, and fieldnotes. Some materials on which I draw derive from grant proposals written by the activists to garner funds for their activities. I further draw on understandings deriving from my ethnographic work among the activists.

KNOWING GRAPHING: RELATIONS IN PRAXIS

In this section, I describe some of the relations within my unit of analysis. My primary focus is Karen, the subject at the base of the mediating triangle (Figure 3). My research on graphing suggests that relations such as those described here are fundamental to the constitution of competence. My detailed analysis shows that Karen was highly competent and each feature of the water level graphs provided her with a window into her world, the Hagan Creek watershed (Roth & Bowen, 2001). Here, I do not want to return to that analysis but rather provide evidence for the different kinds of relations that go with the knowledge that we had documented earlier.

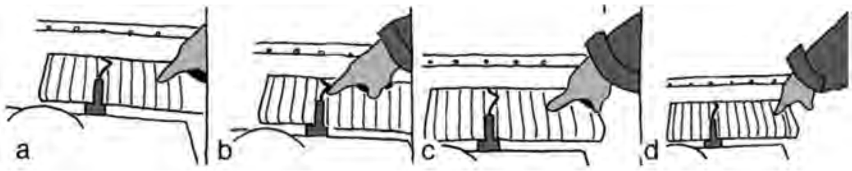

Figure 5. Karen explains where the graph would be given a major rainfall both today (d) and sometime in the past (a, c). As her finger moves back and forth on the page, it embodies the movement of the pen in response to the changing water levels.

Historical Context of Hagan Creek and Central Saanich

Karen and the graph she reads to the visitors on the farm and the yearly open-house event organized by the environmentalists do not exist in a vacuum. Rather, Hagan Creek and the community located within its watershed boundaries have their own political, social, and economic histories. Commuting into the community and working with the people that inhabit the watershed provides Karen with many opportunities to find out about past events and the historical evolution of water-related contexts. Thus, although she had been around for only four years, and although the water level monitoring station has existed for about the same time, she can make then-and-now comparisons of the amount of water coming through the creek after a specific rainfall event. (The positions of Karen's hands at four points are given in Figure 5.)

K: But in the winter, the rainfall times [a] what historically might have been, a [b] say [c] a [b] two-inch [c] rainfall [c] event might have gone up to about three thousand liters a second. What we're getting [d] is, we're getting up here almost, almost off the graph paper, or up to five thousand liters a second. That is, in a major rainfall, when all the watershed is saturated, nothing else is soaking in, either off the grass cover or off the pavement.

For Karen, the graph does not exist in and of itself. Rather, it is mediated by the historical evolution of the watershed (lines 15–18). It is further situated with respect to other, larger watersheds and with respect to the season (winter) when such rainfall events occur contrasting (extremely) low water levels in summer when the farms need it most for irrigating their fields. Furthermore, elsewhere in the transcript Karen situates the increase from 3,000 to the projected flow rate of 5,000 liters/second, on the one hand, relative to small and large watersheds, on the other hand. Smaller watersheds suffer from flash floods, especially with the large number of impervious surfaces and straightening and channelizing of the stream (e.g., Figure 2, right) that Hagan Creek and the community as a whole have experienced in the past. Finally, the graph does not just show 3,000 or 5,000 liters/second, but these values exist in relation to the physical characteristics of the watershed, the grass cover and pavements (lines 20–21). Karen also knows that if more than twelve percent of a watershed is covered by impervious surfaces (e.g., pavement), its health will be seriously affected. In this watershed, the impervious surfaces have increased tremendously over the past two decades. The mediating relation of the community and its history is further highlighted in the following excerpt:

> K: There are about twelve licenses on the whole creek. And that was all made, all these decisions were done like in the late forties based on zero knowledge of this creek, this watershed. So, we finally decided well, they don't have the funds, so we're just gonna pay for one. The municipality has gotten their own station just down stream at Stella's Road and Wally's Drive. So, they're monitoring up there for changes in the fluctuations of the flow, we're monitoring down here, so we've pretty much covered the whole watershed.

Here, Karen's work with graph inscribes itself in a situation that has historical roots to the 1940s. At that time, her farm as well as eleven others received their water licenses, although, as she emphatically points out, little is known about the ecological complexity of watersheds, their watercourses, and the underground aquifers that feed them. Furthermore, what happens at her monitoring station is also linked to, and interacts with, what happens at other water monitoring stations. Thus, Karen's reading of the graph becomes meaningful in the ensemble of mediated relation to other currently existing practices and to the watering and communal water distribution practices from which they have evolved.[5] Karen's activity inscribes itself in a historical context – consistent with cultural-historical nature of activity theory, we cannot understand what she does and knows in her actions unless we consider these historical relations as well. The years she has spent as a water technician in the watershed have given her many opportunities to talk with farmers (other than her employer), First Nations people, and other local residents who have been living next to the creek for more than half a century. As a member of the environmental activist group, Karen also has access to the historical records that speak of plenty in terms of water resources (e.g., people used to canoe in the creek) and trout sizes and quantities no longer heard of.

There are farms with water licenses that take water from the creek between a monitoring station placed by the community and her own. The differences between Karen's readings and those coming from the station of the municipality are also important to her work. The water monitoring station and the graph it churns out also exist in the context of the entire water budget of the watershed. In fact, Karen and the other technicians and engineers she collaborates with have done calculations of the total rainfall on the watershed and compared it to the amount of water that flows out of it at the station, which is only 300 meters from the mouth of the creek.

Karen further is familiar with the relationship between the amount of rainfall and the response by the watershed in terms of the amount of water that will be shown on her graph. Here we have, embodied in her practices, another translation and semiotic connection which contribute to Karen's competent reading.

One of the questions asked by visitors related to the minimum water flow required by the fish, "Does the water level affect temperature or oxygenation levels?" Karen first did not address either temperature or oxygenation but responded that fish needed a minimum depth for navigating the creek. Then she picked up the question of temperature and oxygenation.

29 K: Yeah certainly, as the flow gets lower the temperature would get higher.
30 Because they only got this much water to heat up, it's gonna all get
31 warm throughout it. It but if it's this deep and there's a pocket that is
32 covered into the bank you can. . . . You gonna have a nice little hiding
33 spot. I think they need anywhere from eight to eleven parts per million
34 of oxygen. Most fish do. We have little sticklebacks that, actually they
35 eat them. They can survive at two parts per million oxygen completely
36 exposed sunlight areas. So where there is a food supply, there's still no
37 cutthroat trout because they are not gonna follow them into those areas.
38 In terms of temperature, anything around thirteen is a really nice tem-
39 perature, they will, the bigger granddaddy kind a cutthroat that hang out
40 here. In the summer, it's like twenty-seven. In this fully exposed area.
41 But down at the bottom, they've got a seven-foot depth to hang out
42 when the dam is in. And they got cool temperatures down at the bottom.

Karen is very familiar with the creek. We have walked along its bed numerous times, including other individuals interested in the restoration of Hagan Creek or consultants. Karen has constructed riffles from local rocks to improve the oxygenation rates, and she planted trees that eventually would provide habitat. She has used a dissolved-oxygen meter to determine the oxygenation of the creek water both above and below the riffles she single-handedly built on the farm property, or in collaboration with the other activists in other parts of the creek. When she reads the graph, it is against all of these experiences that have left indelible traces in her body and mind.

In this excerpt, we can recognize Karen's familiarity with the creek as an ecological and a physical system. First, the water flow will affect water temperature (lines 29–31), the temperature lowers the dissolved oxygen (lines 33, 35–36), and there are temperature gradients in deep water. All of these factors affect where cutthroat trout and sticklebacks (trout food) can live during different parts of the year and especially during the summer months (lines 37–41). Thus, Karen's reading of the graph is not independent of her familiarity and practical understanding of the creek, the physical characteristics of heat capacity, and ecological relationships between species that have different requirements on their physical environment. Karen's reading of the graph during the summer months exists in and as of the mediating relations in respect to the creek.

In the extensive network constituted by these relations, Karen navigates between the different representations of the creek (i.e., signifiers that characterize knowledge about the creek) and integrates the tools and history. Yet we must not forget that this integration is achieved in and through Karen's activity. We need to remind ourselves that the different representations (signs) Karen uses in her

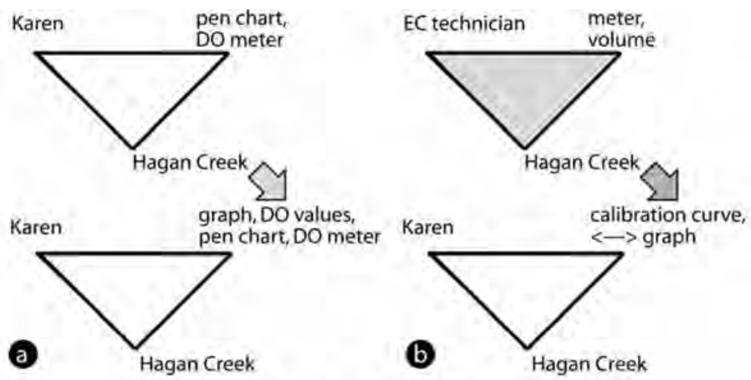

Figure 6. Changes in mediational relations. (a) Karen uses the tools at her hand to produce graphs, outcomes of her activity at some point in time. These graphs increase the number and range of mediational tools thereby constituting subject and object in and through new forms of relations. (b) As an aspect of the division of labor, the Environment Canada technician uses her tools to produce a calibration curve for water levels water volume conversions. This curve becomes one of the tools appropriated by and available to Karen, mediating her own relation to creek and the water level.

activity are nevertheless distinct and arbitrary. As she works in the creek, her familiarity with the setting increases. Furthermore, the outcomes of her activities provide additional mediating tools for understanding the creek. For example, Karen measures the water level in the creek using the pen chart recorder connected to the measurement device. As an outcome of this activity, she gets graphs that monitor the water level throughout the year. These graphs then become new tools that mediate her relationship with the object and in fact constitute Karen as a "more knowledgeable" subject and the creek as a "better-known" object. This development over time is represented in Figure 6a.

Mediating Artifacts, Tools, and Division of Labor

In all instances where Karen read the graph in public, she talked about water volumes. Yet the graph is directly proportional to water level but not to volume. Karen's reading therefore involves a translation, which is not linear in terms of the water quantity. The graph stands in a linear relationship with the height of the water in the central pipe, which is transferred from the floating device to the pen by mechanical means. However, because of the shape of the creek, the height-volume relation is not linear but some complex function.

Her work exists in the context of all activities of the watershed, the other farms, the efforts of the community to monitor the water usage in the community, etc. In part, of course, the work is divided up among people. On the farm, Karen is responsible for the work of monitoring water levels, replanting riparian zones, or building riffles while others are responsible for operating the pumps, drawing the water, and irrigating. Similarly, in the activities of the environmentalists, Karen may produce graphs but others write the proposal in which the graphs are used to get further funding for Karen's position. Furthermore, although Karen shows how to get the volume data from the water level data, she did not establish the calibration curve that allows water level data to be translated into water volume curves. Here, there is a clear division of labor involving a different organization.

43 K: What the Environment Canada technician does is, he or she, comes
44 down three or four times a year, gets into the creek, and measures the
45 area across the creek, and based on the corresponding water levels . . .
46 And eventually get a calibration curve which means that someone like
47 me can come down and say that means *X* volume. And for example, at
48 one point to the line was here we got seventy-one liters a second.

The Environment Canada technician also gets into the creek, establishing cross section data, maps these against water levels, and constructs a calibration curve (lines 43–46). These curves are themselves an outcome of an activity and become mediating tools in Karen's work. Karen is familiar with major markers established by the calibration curve. In fact, when she reads the graph, she talks about water volumes rather than the water levels displayed. But she does not actually produce the water volume graphs (e.g., Figures 7a–c) herself. This, too, is done by the technician from Environment Canada, and as she repeatedly emphasizes, takes about a year to get done.

49 K: It kind of takes a year after the information collected to process it and for
50 six thousand dollar cost. So, an Environment Canada technician comes
51 down, scoops the whole roll up, takes it back to his office and calculates
52 what that line means.

Nevertheless, the resultant graphs come back to the community to be used by Karen, the farmer, and the environmentalists for a variety of purposes. For example, graphs may become part of a proposal that seeks further funding so that Karen can continue her work. Figures 7a and 7b show graphs taken at different places in the watershed. In respect to this figure, the main body of the proposal reads:

> Discharge measurements are generally 6 to 10 times greater at downstream site than at a flume site. The downstream site, a water survey station on Central Saanich Farm is roughly 2 km below the flume site on the Gooding's Farm. In between these sites, 7 small tributaries feed the main creek, yet there is negligible flow in them during the summer period. The inflow is believed to be due to the influence of the nearby bedrock aquifer just to the north of the valley. Bedrock is observed to form sections of the main streambed. (From proposal to fund Karen's position.)

Here, there is more discharge down river than at the upper site. As the activists' proposal states, the contributions by the seven tributaries are negligible: the differences are due to the aquifer. These differences constitute up new relations in that another graph relates to the amount of water in the aquifer (Figure 7c). Here, the graphs qua social objects embody the material properties of inscriptions. Inscriptions can be layered, transformed, juxtaposed to other graphs, and inserted into

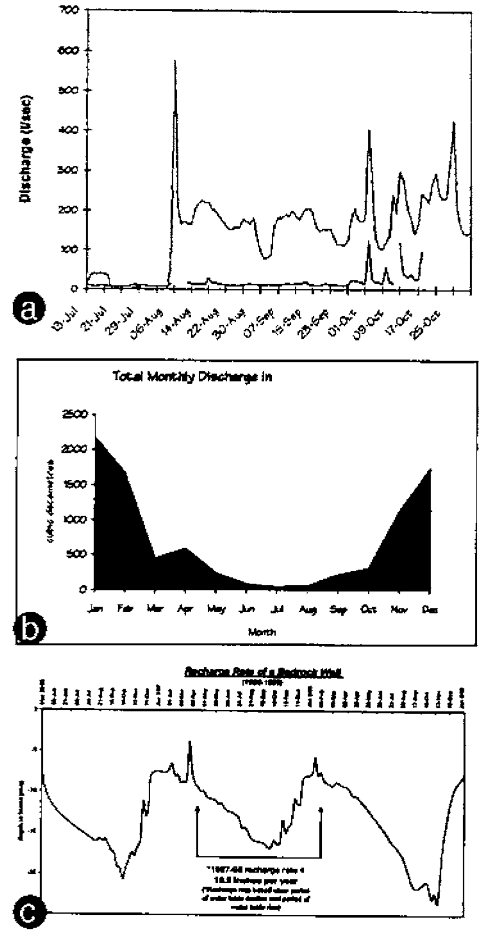

Figure 7. Artifacts mediate the understanding Karen has of the original graph. (a) The water volume graph constitutes a translation and was produced by another technician using a calibration curve. (b) Further translation produces a monthly discharge graph. (c) Changes in ground water levels parallel the seasonal changes in discharge levels [b]; ground water accounts for the differences in two other graphs [a].

documents. For example, there are relations to the third graph in that the discharge rate curve displayed in Figure 7b has maxima and minima that correspond to those in Figure 7c, both being related to the amount of rain fall onto the watershed. Figure 7c plots the depth of water in local wells, and therefore inscribes itself in the practices of drawing water for irrigation purposes. Figure 7b arises from integrating the transformed graphs over time, that is, $D = \int v(t)\,\mathrm{d}t$ for one-month periods.

These graphs derive, in part, from operations that characterize inscriptions, including translations (non-linear), layering, scaling, and integrating. For example, graphs such as those in Figures 7a and 7b can be constructed from Karen's original graphs by transforming them using the calibration curve and translating the water levels into volume. (Mathematically, this volume v as a function of time is given as $v(t) = C_{v \to h}(h(t))$ where $h(t)$ is the water level and C the calibration function that maps height onto volume.)

In concluding this section, three points are to be noted. First, there exists a division of labor concerning the focal object of the activity (Hagan Creek). The actions of different individuals are interdependent all contributing to the overall project of coming to know the creek. Second, as part of this activity, Karen, the Environment Canada technician, and others produce representations that subsequently become part of the set of artifacts mediating the relation between subject and object. These activities have outcomes that change the relations in the mediational triangle with Karen as the subject (Figure 6b). Third, the graphs are themselves objects mediated by other graphs (mediating artifacts). That is, the graph Karen explains to Nadely (the teacher) or Walter (the mayor-visitor to Open House) exists in relation to other graphs that Karen, her activist colleagues, and others in the community work with. The three graphs displayed here exist in and through their relation with the graph at Karen's hand, but also with respect to each other. Evidence such as that provided in this section supports the contention that Karen's graphing competence is embedded in many other mediated relations (Figure 4). These mediated relations constitute the very context that establishes the sense of the graphs.[6] However, Karen's competence also involves a very physical, embodied component, which is described in the following section.

EMBODIMENT

Knowing the source of the data and the instruments by means of which data are collected was an important aspect of scientists' determining the level of competency (Roth & Bowen, 2001). When scientists were unfamiliar, their readings often involved mis-readings that shared similarities with those of high school students. This is also the case for Karen. In this subsection, I show that reading graphs is an embodied activity, against the graph as a ground. Gestures are used together with language so that the three constitute a communicative ensemble that is much more complex than talk would be by itself. It is not only the text, but Nadely can see embodied in Karen's gesture an iconic representation of the pen (shaded inverse "T" next to curve in Figures 1, 5) moving across the paper, inscribing the line that is the focus of the present interaction.

Karen knows the instrument that records the water level graphs and in particular the mechanism by means of which the curve is being taken. She knows the instrument so well that her body participates in communicating the functioning of the device. Consider her presentation featured in Figure 8a. Here, Karen's hand moves from right to left along the horizontal direction of the paper while uttering "hours go this way". Her hand follows the direction of the paper in the pen chart recorder, taking the same trajectory. The gesture therefore stands in an iconic relationship with the tracking paper. Time is not just a label on the axis, but is

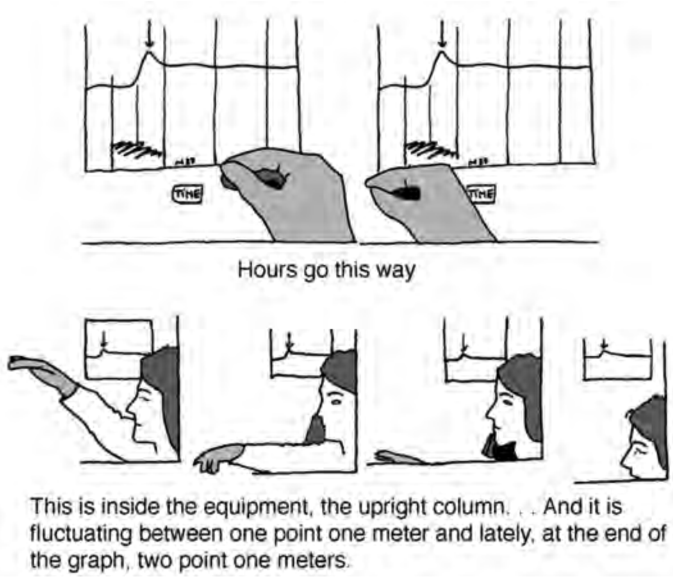

Figure 8. The gestures of Karen embody the movement of pen and paper, and the dynamics of the recording device. (a) The hand shows 'time" as elapsing in the direction in which the paper moves in the recording device. (b) The hand embodies the pen tracking across the paper while the body moves up and down in the way the floating body in the stilling well that ultimately drives the pen.

something that continuously unfolds and is indexed by the turning wheels and moving paper. Karen has a very embodied understanding of time as it pertains to her graphs. Interestingly enough, indicating the time through a bodily movement may depend on the circumstance, for in another situation, Karen's gesture described a trajectory in the opposite direction. Here, her gesture described the trajectory of the graph as it unfolds on the paper under the pen. Here, Karen's hands track the apparent direction of the pen across the paper. In both situations, time was something embodied in the (apparent) motion of the paper or pen, and embodied in her gesture.

Karen does not just know the relationship between some sign (point on graph) and the amount of water. Her gestures embody the working of the pen. As Karen talks, her hand-finger movement (sequence Figures 5a–5d) moves along a trajectory similar to that of the pen. The trajectory of her finger, therefore, stands in an iconic relation to that of the pen similarly to her movements portrayed in Figure 1. Interrupted by the movement and gesture that indicated the distance on the paper that amounts to one day, her finger moves, similar to the pen, across the paper. In the same way, the to-and-fro movement of the pen is embodied in her hand moving up and down along the now vertical side of the chart.

The involvement of her entire body in representing the recording in terms of sensori-motor experience is even more pronounced in the following episode. Here, Karen explains the source of the graphs to a visitor to the Open House (Walter, the mayor of Central Saanich).

53 K: So, the way this works is there's pipes going across the creek and the
54 water comes into the still(ing?) well area and this is, the fluctuations in
55 that water level drive this wheel here and then this pen works.

As Karen talks about the well in which the water fluctuations drive a floater, her arms form a circle; she bends her knees, so that, as a whole, she describes a drumlike object. Immediately thereafter, Karen enacts the graph recording with her entire body, hand and body standing for (i.e., representing) the pen and floating device, respectively (Figure 8b). She begins to bend her knees so that her body moves downward like a piston "inside the equipment" in "the upright column", while her right hand follows in and amplifies the downward and upward movement of her body. But in this, Karen's sensori-motor actions are not just indexical to the floater, but to the fluctuations of the water level (the OBJECT in Figure 4) throughout the year (see utterances in Figure 8b). Karen performs the recording, her right hand embodying the pen movement; her entire body then enacts the up and down of the floater in the stilling well, her right hand constituting an iconic relation to the recording device.

In these examples, we see Karen not just talk about but move (parts of) her body through trajectories that stand in iconic relations to the graph, paper, or recording device. The movements of these entities, which she observed frequently over the four years preceding the episode, exist as sensori-motor representations in Karen's experience, and are available in public to her listeners. The movements of pen and paper, which exist in the material trace of the graph, are literally embodied. The meaning of "time goes this way" and Karen's up and down movement with her body, followed by her hand is simultaneously built on two types of meanings. First, it built on the sensori-motor action involving Karen's finger (hand) over (in front of) the plotting paper and the graphical space it defines. Second, Karen draws on the symbolic meanings associated with the marks and lines on the paper of the conventional graphical signs. The sensori-motor processes therefore constitute an important aspect of collective processes of meaning making, and the witnessing of the Other's subjective understandings. The graphs thereby constitute subjects and objects in referential ways as simultaneous, co-existing participants in the described events.

Sociologists and philosophers, and more recently artificial intelligence and cognitive scientists, view learning as the structuring of mind, which is fashioned during bodily interactions with the social and material world (e.g., Merleau-Ponty, 1945). Linguistic studies suggest that our language is deeply grounded in and arises from the mid-level (not too tall, not too small) entities that we encounter in the world (e.g., Roth, 2000). Here, my recordings of Karen constitute an exemplary case. Here understanding of the graphs is deeply linked to her understanding of the water level recording device.

To summarize, Karen's graphing competencies have a strong physical component, which I exemplified here in terms of the relation between her gestures, the recording device, and the graph. However, other physical relations also exist, for Karen's understanding of the creek is tied to her in-creek activities, building riffles, catching trout, planting trees, etc.

GRAPHS IN FACE-TO-FACE INTERACTIONS

Before discussing the relevance of this work to mathematical cognition and development, I articulate an important aspect of graphing as it has arisen from my work. Until now, my description focused largely on different relations involving Karen and other aspects of the sociomaterial setting of Hagan Creek and its watershed. However, I have not yet addressed the role and importance of graphs in face-to-face interactions. Here, graphs can become sites where social interactions occur over issues that are relevant to the lives of the people living in the area. Both with the teachers on the farm and with the visitors to the Open House, Karen explained the graphs and her work. She was in what we might term a *knowledge display mode* (e.g., Roth & Middleton, 2006), and her audience provided but continuers, that is, turns at talk that allow the current speaker to continue speaking, such as “Yes”, “Okay”, or “Is that right”. However, there are many instances in my transcripts with interactions beyond continuers. She has prepared her exhibit at the Open House of the environmental activist group. She intended to show and explain to people of what her work consists, how it inscribes itself in the life of the community, and what plans the activists have for improving water quality and quantity in Hagan Creek.

When a graph is a public object, there is always the potential that other persons will contribute their readings. In the following episode, a visitor to the open house (Walter, the Mayor of Central Saanich and principal of local middle school) does not just let her continue, but contributes in an active way. He begins to talk while Karen is still going, thereby making a bit to take a turn at talk. Walter injects his hypothesis based on his own reading of the graph. As a consequence, the episode does not just constitute mere knowledge display, and is not only about the graph. Rather, the episode is an exchange that takes the graph as its starting point and elaborates many related issues.

56 K: ((Gesture as in Fig. 9)) And then there is superimposed rainfall up there
57 and [one of these]
58 W: [Does it say?]
59 K: Pardon?
60 W: Does it say, “Now I know when that will peak?”
61 K: Aha, good. So, one of these little squares is two millimeters.

Karen did not expect Walter’s interjection and she may not have heard what he said: her “Pardon?” requests a restatement. Walter thereby obtains a turn in which he elaborates his passing theory about the topic at hand. Karen then acknowledges his theory, but continues with an explanation of the conventions (scale) rather than addressing the relation of the rainfall with the peak of the water level.

Presenting her work in public, Karen navigates the tension between interacting with the audience and presenting the other with what she might consider to be the foundational knowledge required for being able to read the graph. Yet, there is always a tension involved in such relations. Here it is one of who owns the interpretation of the graph, and therefore the speaking platform. But there is an

Figure 9. . Karen points to a second inscription layered onto her graphs. These inscriptions signify the amount of rainfall. As she begins her explanation, Walter interjects his own hypothesis as to an inference that can be drawn about the relation between the two graphs.

additional tension arising from the fact that the conversational topic can be shifted to be about something else, here the entire valley and its water resources. Karen began in display mode, and continued in her attempt to retain the knowledge display mode rather than engaging in an open interaction. This changes when the topic moves to consider the watershed itself rather than the sign that mediates the knowledge about it – at least in as far as Hagan Creek is concerned. A few minutes after the above exchange, Walter indicates that he also lives in the valley and that he has one of the water licenses. Subsequently, the interactions between the Karen and Walter change in kind. They begin to talk not only about the graph, but also about the issues for which the graph stands in a reflexive relation to the water, valley, history of settlement, and changes in farming. At this point, both own the issues, and thereby construct each other as equal contributors.

The following episode begins when Karen talks about irrigation and vertical jumps in the graph that stand in a reflexive relationship with irrigation practices.

K: These very, you know, ninety degree angles in the lines, that's definitely straight, straight drops. That's definitely irrigation that decreases, people are all stopping at the same time, starting at the same time. And the conditions, it's dry for a while here. ((Gesture as in Fig. 9))
W: Yeah, a lot of hay, people are into the hay.
K: Yeah, a lot of people cut it at the same time.
W: Further, you go towards the Fellow's farm. Down Hagan Creek. Because once you get past Fellow's, it stops. There is corn. But of course, nowadays, there is late corn, too.
K: Yeah, they grow different varieties.
W: I think they grow mostly early corn on the fields that are around Hagan Creek.
K: Corn has a lot, requires lots of water, doesn't it? Compared to hay.

In this episode, Karen introduces the topic of irrigation, which goes with particular vertical discontinuities (jumps) in the graph. But it is not just that these jumps are signs that stand in a signifying relation to the water level changes caused by irrigation. Rather, irrigation also stands in relation to the second, layered graphical information on the top border of the graph (Figure 9). Presently, Karen and Walter stand in front of that part of the roll that was recorded during the summer. There are no rainfall events marked on the top part of the paper roll. Thus, the jumps attributed in this episode to irrigation exist in relation to the time of recording (summer), the number and size of the (here lacking) rainfall events. Most importantly, the topic in the episode is not some feature of the graph, but the farming practices and irrigation that obtain in the valley at the present time. Understanding now concerns the speakers' organic relationship in and with the valley (Figure 2), its people, their livelihoods, and the climate. That is, faced with the graph, their understanding has been reflected back onto itself: "consciousness pf am "other", of an object [here graph] in general, is itself necessarily *self-consciousness*, a reflectedness-into-self, consciousness of itself in its otherness" (Hegel, 1977, p. 102). Yet all this is part of the thick layer of knowledge and experience that brings forth the extraordinary competence in the first place. Here, and as a seventeen-year inhabitant of the valley, Walter is a knowledgeable conversation participant. He is as familiar as Karen with the hay farming that goes on in the summer, which requires the dry conditions of (late) summer in this part of the world. Karen then suggests that many farmers begin and stop irrigation at about the same time, a fact again related to the weather patterns in the valley and haying practices that require a dry period for each harvest.

Walter subsequently adds that not all of the farms grow hay, but that they also grow (different types of) corn. He even provides a description of the specific farm where the corn crops begin to dominate the fields (lines 68–70). Karen then makes a statement – which can be heard as a question to Walter who had previously already talked considerably about strawberry farming practices – that corn takes more water to grow than hay. This, in turn, would have significant impact on the irrigation practices (especially if there are different types of corn) with a resultant effect on the water levels and Karen's charts.

This episode shows that graphs are not just signs standing in a unique and unambiguous relationship to objects. In this situation, the graphs are both topic of and ground for their interaction, an important aspect in aligning the interaction participants and in achieving intersubjectivity. These graphs are not just objects of knowledge in the way that past developmental research in mathematics treated them. Rather, as the analyses show, they are the primary means of interaction between Walter and Karen is the creek and the surrounding valley in which it is located. The graph is but an object that anchors the social space in which the two collectively engage in the construction of the watershed for the purposes at hand.

TO KNOW GRAPHS

In the foregoing sections, I articulated different aspects of knowing graphs. These different episodes show that mathematics in everyday practice is constituted in a noisy field of practical action and discursive relations. In these practices, graphs do not exist as ideal Platonic objects with definite structure and elements. Rather, in the context of particular practices, graphs make available what is necessary in the situation at hand. Other structural aspects that a theorist may identify remain undisclosed. For example, Karen needs neither to read individual data values nor to identify the slopes of the graphs. What matters in the context of her work are those differences that make a difference; that is, differences that contribute to the relations that constitute the heuristic entities in the mediational triangle (e.g., subject, object, tools, and division of labor). Thus, it matters in the Karen-creek-graph relation that a particular graphical feature arises from a clogged pipe in the instrument or the lifting of a dam rather than some other event in the watershed.

The graphs do not just serve to express something about nature (in this instance the watershed). Rather there are very different, economic and personal matters in which these graphs are inscribed. Here, the proposal from which the three graphs in Figure 7 were culled was seemingly written to seek funding for monitoring the water budget in the watershed. But at another level, the technician to be employed with the funding received is Karen herself. The graph that Karen reads to different audiences is not just a representation: it is lived as a relation within a range of practices and her life of being a water technician; it is part of her everyday working life, just as the pen chart recorder, the tractors on the farm, the trout she can make jump, and the trees that shade the creek. This graph exists in relation to the ending contract that provides for her subsistence and a new contract with the prospects of continuing her work at a place that she has come to be very familiar with.

"Context" has been one of the focal points in the discussion of how to make mathematics more relevant. We may ask, "Where is the context in contextual word problems?" The present chapter contributes new answers. It is evident that situated cognition does not mean that people think differently in different contexts. Although signifiers (graphs) may be the same in different contexts, they in fact contribute to constituting the context in different ways. In Karen's everyday practice, the many different relations contribute to an over-determination of any individual relation and the objects and tools involved. At the same time, unlike former assumptions concerned only with the relation between sign and world (Figure 3a) or sign, world, and interpretant community (Figure 3b), the change in objects and tools also changes the way in which subjects are constituted. Thus, Karen's subjectivity changes over time (Figure 6a), through interactions related to division of labor (Figure 6b), and in individual interactions. Thus, Karen is constructed differently as a subject in her interaction with Nadely (expert) than with Walter who, along a number of dimensions, is more familiar with and knowledgeable about the valley, community, creek, farming practices, or watershed. Without considering the relations, we would draw inappropriate inferences about the nature and extend of someone's knowledge based on analyses of the data we have at hand.

From a phenomenological point of view, what imports is the lifeworld, the world perceived and acted in by the person-in-activity. This familiar world of practical understanding constitutes the very ground that allows a person to work with a graph, only to be reflected into itself but now, augmented by the explanatory effort, in a more articulated way. The chapter showed that the concrete embeddedness and meaning of activity could not be accounted for by analysis of the immediate situation because the concrete social institutions and relations were characterized by historically emerging contradictions. But they emphasize, at the same time, that objectively existing social structures do not have a determinate effect. Any meaning is socially constituted in relations between activity systems and persons acting in the world. Meaning always has this relational character.

It therefore does not bring us much further if we view context as a container that can be grafted intact onto cognition or cognitive development. The social is more than a container of the psychological, but each of the two arises from complex dynamics by means of which they are constituted in actual practice. Signs are produced and used within the dynamic intersection of actions, objects, and speech within a practice and therefore function as relations within the practice. Signification, therefore, cannot be reduced to representation. Participants themselves become in and through the relations in which they are embedded. Karen is who she is in relation to her employer, the water conservation and creek restoration efforts of her activist group, and the new teacher Nadely, who has sought her as a consultant for a school-related project. "Karen" also emerges from the relations with the First Nations people, the creek that she so intimately knows, and the Mayor, Walter, who participate in the reading of graphs in relation to the community.

In the episodes featured earlier, Karen navigates the ontological gap that exists between (features of) the graph and those features of the Hagan Creek watershed with which it stands in an indexical and reflexive relationship. When she talked with Walter about the irrigation, neither individual had a problem talking about the vertical jumps in the graph in terms of the irrigation. In fact, the conversation shifted and was concerned with the object, Hagan Creek (watershed), the knowledge about which is mediated by the graph. That is, the ontological gaps do not exist in praxis: they disappear in the movement consciousness to the object (graph) and back into itself, both constituting but contradictory moments in the movement of understanding (Hegel, 1977). In Karen's work and interactions with others, the graph seems to be transparent, providing her with a window on the world of the watershed. At this point, a troublesome question demands attention. How did Karen get there? This question has to be central to the activities of mathematics educators, for if, as pointed out earlier, there are ontological gaps between representations and the things they are said to stand for, then how do individuals ever come to the point of using graphs as if these gaps did not exist?

IMPLICATIONS FOR LEARNING AND DEVELOPMENT

Heeding the Ontological Gap

Past research on graphing made (sometimes implicitly, sometimes explicitly) use theoretical frames in which graphs (as sign, symbol) have an implicit relationship to the world or some other sign. Thus, in the research on graphing, investigators often ask students to interpret a graph or select among graphs the one that referred to some situation. These students were frequently untutored (not instructed) in graph use, and had few opportunities to engage in representation practices. A typical task asks students to walk across the room and return to their starting point upon which they are asked questions about graphs or asked to select that graph which best represented their trip across the room in terms of distance or position and velocity. Not surprisingly, large numbers of students answered inappropriately or selected graphs that were inconsistent with scientific practices. In other studies, a winch was used as a pedagogical device to allow students first-hand experience with a phenomenon that can be modeled mathematically by using linear functions – i.e., height $= f(\text{turns})$ (Greeno, 1988; Kaput, 1988). Many mathematics teachers and educators assume that exposure to such (hands-on) activities is sufficient for learning mathematics. This assumption is justified if we assume that there exists an isomorphism between the world (here the winch) and mathematics. Counting turns and measuring the length of string wound onto the winch share deep structure; measuring length can be reduced to counting equidistant intervals. However, turning and getting a bucket of water from a deep well are not inherently mathematical. If this assumption does not hold, we have little reason to expect that students infer mathematical knowledge from interacting with the device. What educators forget is that our network of (discursive, mathematical, and material) practices is so extended and so habitual that we no longer remark the ontological gaps that separate them.

A central assumption in cognitive research on mathematics is that structures are inherent across contexts. Thus, any linear function would be constant if it involved turning the crank of a stilling well to bring up the water bucket a certain way, increasing the velocity of a ball as it rolls down an incline, or increasing the height of a stack by adding books all. However, this chapter shows that we need to examine these relations as relations of signification. It then becomes evident that each practice is different though relations between them can be specified. In this case, situated cognition is not something people do when they think in different contexts; situated cognition means that subjects produce different outcomes in different settings. Certain transformations are therefore necessary to turn non-school practices into school mathematics practices.

When we take a traditional perspective and assume logical relations to exist between different sign systems, or even within sign systems, we might come to the conclusion that these students are not able to derive the relationship between walk and graph. That is, these students lack the skill or capacity for making logical inferences. They then are said to be stricken with cognitive deficiencies, mental deficits, misconceptions, and so on. It is easy to understand such conclusions, because they derive from sign-referent relationships that are taken to exist a priori. We come to entirely different conclusions, however, if we take a sociomaterial-practice perspective on ordering and representing activities. There are, therefore, no inherent logical grounds that link a sign (graph) and the world, but merely negotiated and shared ways of engaging in particular activities where the signs (graphs) are an integral part. From such a perspective, we do not expect individuals to derive the relation between two representations or between worldly events (walking across the room, turning a winch) and some mathematical representation. Rather, we might ask questions about the extent to which these individuals have participated in the practices. If there had been little prior participation, we might expect to see little resemblance between established practices and the activities that the individuals engaged in. It is not surprising, then, especially within the psychological frame applied, that this research focused on the deficiencies people bring to the task of relating signs and aspects of the world.

From Inside to Outside

Graphs are social objects in at least two senses. First, graphs only exist and have a sense in relation to the place they take in some sociomaterial practice. They are used in such places as poster displays, scientific articles, newspapers, or books. Here, readers who have previously participated in reading and graphing practices disclose through the process of their reading what the author intended the graphs to communicate: but in this reading, they really disclose their own practical understanding of the familiar world that they take as shared with the graph's author. As such, graphs only exist in and as of their relation to sign-related practices. Second, graphs can become the site of face-to-face interactions between people who negotiate, in real time, what graphs are meant to express, how they inscribe themselves in the issues at hand, and so forth. Thus, important understandings of graphing practices arose in the context of a micro-analytic study of mathematical representation practices in an eighth-grade classroom where students transformed nature into different sign forms, and interpreted the sign forms created by their peers (Roth, 1996). It turned out that these students developed considerable competence in transforming the material-form elements from Figure 3c in both directions: on tests, students easily moved from mathematical representations to their understanding of the natural world, whereas in their research, students easily moved from the natural world through cascades of representation to some final inscription. That is, on the one hand, they developed increasing competence in using graphs and statistics as a way to construct and express understandings about 35-m^2 plots of nature. On the other hand, they equally developed a tremendous competence in interpreting existing representations, that is, to create verbal descriptions of natural situations that could have been the origin of the graph.

In the context of this work, I conducted a quantitative study to compare their competencies to those of college science graduates enrolled in a fifth-year teacher education program. The task was based on pairs of numbers (light intensity, plant density) that were entered in each section of a subdivided plot of land; participants were asked whether there was a relationship between the two measures and how they could support their answers. There was a statistically higher use of graphs and statistics in the eighth graders' responses than in those college science graduates provided. I do not use mental deficiency to explain why college students do less well than eighth graders on data interpretation tasks. Rather, I suggest that the eighth-grade students were much more familiar and had more opportunities to enact the representation practices.

My research on graphing began in science classrooms assuming that scientists were experts that we could use to constitute a normative frame for expertise. The interviews with thirty-seven scientists (ecologists, physicists) taught me to rethink my assumptions, for there are many instances in my database where scientists do not provide expert interpretations. Rather, although the graphs have been culled from undergraduate textbooks in their own domain, scientists often read graphs in ways that mathematics research has come to denote with "iconic errors", "slope-height confusions", and more generally, with "misconceptions" (e.g., Roth & Bowen, 2003). Furthermore, rather than engaging in an inductive process where the referent of a graph is disclosed in an unfolding manner, we observed a dialectical process in which ecologists articulated knowledge as the outcome of a continuous movement between familiar ecological systems and tentative articulations of graphical signs: repeated return trips from perceived signs to memory traces of past experience and back to the perception of signs. At the same time, we found profound differences in the graph reading activities when scientists talked about graphs that arose from their own work. Here, they began by providing minute details of the local situations they had investigated, instruments they had used, and transformation that their data were subjected to. In these accounts, the graphs were transparent means, placeholders for an extended experience in the field and laboratory. That is, even scientists do not transfer skills from one domain of graphing to another; transfer of cognitive and linguistic operations across contexts is not as frequent as some educators and researchers would lead us to believe.

From my (sociomaterial) practice perspective, "individual" cognitive development is deeply bound up with changes of participation in sociomaterial practices of a culture: in their actions, individuals realize cultural possibilities in concrete ways, but, because an action "is indivisibly the action of one as well as of the other" (Hegel, 1977, p. 112), they do so because the cultural possibilities are available at a general, collective level. Schools are primary institutions for bringing about and fostering the enculturation into practices. Received conceptions of graphing competencies have led to educational misconceptions of what students should know and be able to do at particular points during their schooling. These misconceptions have led to ill-conceived pedagogical practices. If there is an ontological gap, relations between two domains cannot be derived on logical grounds. These relations are grounded in and given by the practices enacted by competent individuals; as shown, these relations are developed over time by participating in practice. We need to keep in mind here that out-of-school mathematical practices are inherently different from in-school practices because of the differences in the products of the practices, the relations of signification, the regulation of practices, the positioning of subjects, and the emotional investment. Calculations within everyday out-of-school practices exist in a different way than in school; but the calculations are often not the point of the activity. This leads to a new fusion of signifier and signified.

Everyday Mathematics, Inside

In relation to schooling, it is often assumed that students can learn practices independent of the settings in which they are used. The notion of "authentic mathematics" has been in circulation for some time. However, this notion makes little sense when we regard what Karen does as authentic mathematics (at least in relation to graphs). Let us assume that some teacher introduces the graphs Karen works with in order to make her classes more "authentic" or more "contextual". We can already say that she would not be able to bring children anywhere near to what Karen represents, for most of the relations present in Karen's activity will not be present in that of the children. What school children do seldom has a relation to other practices that we observe in Karen's case. One alternative to traditional teaching is to expand the range of activities and related practices that provide longer chains and denser networks of signifying practices. Graphing would then still be a school-related practice: it would no longer stand on its own but exist in relation to many other practices (material, signifying). The implication of my work is that out-of-school mathematics cannot become in-school mathematics, because the relations instantiated outside are so dissimilar from those instantiated inside schools. The best we can do is provide rich contexts in which activity structures are set into motion that allow new sets of (desirable) relations not-yet existing in the schools. The real trick is not one of finding everyday problems that bear some correspondence to school problems, but in finding problems that are truly problematic and therefore engaging to children. This involves considerable levels of control that children have to have over framing problems and solutions, even if this sometimes means that they abandon framing and solving altogether.

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NOTES

[1] As is custom in the social studies of science, I choose to link the social and material rather than the social and historical, or the social and the cultural because I view all social practices as historically contingent and embedded in some culture. However, the material aspects of cognition are seldom enough emphasized or, as in traditional cognitive science completely left out of the modeling of knowing and learning.

[2] In dialectical logic, the activity system constitutes the middle term, the irreducible whole, which expresses itself one-sidedly in any element identified (Hegel, 1977). These elements therefore have metonymic relations to the whole (activity system).

[3] Traditional graphing research, which focuses on the relation between sign and some referent only therefore misses most aspects that constitute the competence in graphing among the scientists, technicians, and students in our studies.

[4] My former graduate students Michelle McGinn and Michael Bowen participated in this large-scale study of scientific representation practices.

[5] Two other relations are notable in the transcript. First, Karen's reading of the graph is mediated by another graph not present in the setting. As a form of division of labor, Environment Canada technicians have determined this function and also convert the water level chart into volume charts each year. Second, Karen's hand-finger movements across the paper stand in an iconic relation to the movement of the pen. That is, Karen's reading of the graph also exists in the context of her embodied understanding of the instrument that records the graphs. Both of these relations are discussed below.

[6] Such dependencies force us to reconsider the notion of context in school mathematics.

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Wolff-Michael Roth
■■*Affiliaton missing*■■