Contradictions and uncertainty in scientists' mathematical modeling and interpretation of data

Wolff-Michael Roth
University of Victoria

MacLaurin Building
University of Victoria
Victoria, BC, V8W 3N4

Email: mroth@uvic.ca
Tel: +1 250 721 7808
Fax: +1 250 721 7598

Abstract

Contradictions have been recognized as important factors in learning (conceptual change), because they require students to engage in deep reflection that leads to accommodation and learning. However, in the face of uncertainty, confirmation bias and the theory-laden nature of observation may not allow the recognition of a situation *as* harboring a contradiction. In the present study, I analyze a meeting in which a scientific research team presents its results to an informed audience. I show that with hindsight, there are contradictions in the mathematical models that the scientists use and the graph interpretations that they produce. Because the contradictions went unnoticed, they could not become a determinant factor in the process. This has implications for thinking about the role of uncertainty and contradiction as factors in and of mathematical learning.

Key words: contradiction • uncertainty • mathematical modeling • natural phenomena • everyday cognition

Cite as: Roth, W.-M. (2013). Contradictions and uncertainty in scientists' mathematical modeling and interpretation of data. *Journal of Mathematical Behavior, 32*, 593–612.

1. Introduction

Mathematics educators have shown interest in processes whereby students acquire the data that they subsequently model and interpret (e.g., Cobb & Tzou, 2009). In part, this interest addresses the contention that students and scientists have difficulties interpreting data unless they are deeply familiar with the context from which the data have been sourced and the methods and tools that are involved in the data collection (e.g. Roth, 1996, 2003b). There is another reason that educators often mobilize in support of an argument for data generation or collection: to generate contradictions and uncertainty (e.g., Hadas, Hershkovitz, & Schwarz, 2000; Roth, 2012a). Contradictions and uncertainty as integral moments of learning processes have been of interest to educators under various guises: (a) as discrepant events that promote cognitive conflict and conceptual change (e.g., Kang, Scharmann, Noh, & Koh, 2005); (b) as a means to "push" students from one conjecture to another when there are tensions between results and findings in geometrical proof (Hadas et al., 2000); (c) as drivers of reasoning when examples and counterexamples differ (Giannakoulias, Mastorides, Potari, & Zachariades, 2010) or when argument and counter-argument come into conflict (Komatsu, 2010; Mueller, 2009; Yankelewitz, Mueller, & Maher, 2010); (d) as sources of new mathematical ideas arising from the contradictions between perception and beliefs (Singer & Voica, 2008); (e) as a resource of proofing by contradiction when they face dilemmas such as that regarding the parity of zero (Levenson, Tsamir, & Tirosh, 2007); or (f) as the driver for increasing students' meta-level awareness, especially features of their thinking (Ely, 2011).

Contradictions are said to promote learning because a "discrepant event *constrains the construction of the new model as well as creating dissonance with the old model*" (Clement & Steinberg, 2008, p. 110). The same may be said about examples and counter-examples, argument and counter-argument, that is, whenever two positions on the same issue appear to differ. Thus, mathematics educators have shown interest in "built-in contradictions" and in "personal images [that] may involve essentially contradictory features" (Tall, 2001, p. 199, 201). Others have noted the role of contradictory statements without also stating *for whom* there were contradictions and under what guises these appeared (e.g., Douek, 1999). Less attended is the fact that learners themselves have to recognize a situation as containing a contradiction for it to have any impact on learning (Balacheff, 1987). Thus, perceptions tend to be consistent with, rather than contradict expectations because observation is theory-laden (Hanson, 1958) and because of confirmation bias (Nickerson, 1998).

The purpose of this study is to contribute our understanding of the role of contradictions in the face of uncertainty in mathematical behavior generally and in the course of graph interpretation more specifically. To do so, I provide a detailed analysis of the role of contradiction and uncertainty in graph interpretation. The context is a scientific research project where a team measuring light absorption in photoreceptors of coho salmon tries to make sense of their data. The event took place in the middle of a paradigm shift, where the team was producing evidence that a 60-year, Nobel Prize-winning research—which had become the dogma—incorrectly described and theorized the variations in the absorption. In the scientists' work, there are contradictions in the way that the data are modeled graphically and algebraically that remained undetected. In the process, the scientists did not take into account the information that would have given them clues to contradictions and they maintained their new interpretation, which has become a new conceptual theoretical lens right through the publication of the work.

2. Contradictions, discrepancies, and conceptual change

2.1. The role of contradictions in learning

Mathematics educators noted that when high school students solved "equations resulting in an identity or a contradiction revealed the *superficiality* of their apparent flexibility" (Huntley, Marcus, Kahan, & Miller, 2007, p. 136, emphasis added). The lack of flexibility and adaptability or the "superficiality" thereof has been explained by stating that "[m]ore often than not, the pupil cannot cope with problems which do not yield to the standard algorithms" (Sfard & Linchevsky, 2010, p. 119). These authors further note that "although somewhat flat and one-sided, the student's do-it-yourself algebra is not devoid of a certain consistency" (p. 120). In both instances, students' engagement is characterized in negative terms. The question posed by the present study is whether such negative characterizations are justified given that learners do not yet know and therefore find themselves in a situation of uncertainty that not only precludes flexibility, depth, and consistency but also the recognition of what after the fact will have been contradictions. This was brought home to me in a study that attempted to answer the question about "What do scientists do when they do not know what they are doing?" That study revealed that experienced research scientists did not go about looking what was wrong with their equipment in the systematic ways that (science and mathematics) educators expect their students to go about in the face of unfamiliar tasks (problems) (Roth, 2004a). However, for participants in events who do not know the final (correct) result, contradictions may remain hidden because their comprehension "does not exclude, but to the contrary implies, that [they] could be most of the time in complete incomprehension, and even in the deepest incomprehension, with respect to what is 'going on'" (Romano, 1998, p. 84, my translation).

Contradictions may go unnoticed because observation is theory laden and because of confirmation bias. This was quite apparent in a design experiment conducted in an Australian 12th-grade physics class (Roth, McRobbie, Lucas, & Boutonné, 1997). This study showed—in a predict-observe-explain setting—that some students ($N = 18$) saw movement in a demonstration where others ($N = 5$) did not see movement Moreover, *every* student saw *precisely* what s/he predicted to see; they all provided reasons that to them explained their observations. As part of his instructional sequence, however, the physics teacher assumed that all students had observed no movement in a similar demonstration. That is, the 18 students who had seen movement were mistaken. Yet they had not noticed the contradiction. In fact, they understood the theory the teacher presented as explaining movement. There was therefore a double contradiction in the sense that 18 students saw movement where no movement was to be seen; and they understood a theory to explain movement when in fact the teacher used the theory to explain the absence of movement. On the other hand, the teacher and 5 students who already knew the correct theory expected and saw no movement consistent with the theory. This study therefore shows that the (new) observation has to be theoretically understood even though the current conception and confirmation bias subordinate perception. The issue of contradiction and discrepancy is more complicated than often portrayed in the literature. Even more interestingly, the new perception (imagination, presentation), leading to a new conception or theory, cannot but emerge from the current conception/theory. How or at what point are data seen so that a contradiction arises between "expected results and actual findings" (Hadas et al., 2000, p. 130) in the face of the well-known phenomenon of the theory-laden nature of observation and confirmation bias?

2.2. Contradictions in conceptions and conceptual change

Contradictions have emerged as a topic of study from two perspectives. On the one hand, contradictions, in the form of discrepant events or states (e.g., between an infinite 0.9999… or $0.\overline{9}$ and the finite 1), are offered to or created for students such as to encourage them to change their current ways of seeing or conceptions. On the other hand, contradictions have been observed between gestural and verbal representations while students provide explanations for mathematical (or scientific) phenomena. From an instructional point of view, educators can use this as an indicator of the readiness to develop a more advanced concept—even though the students themselves may not yet recognize the contradiction. I discuss both of these dimensions.

Mathematics educators have shown increasing interest in using conceptual change theory that already had been an important approach in science education (e.g., Merenluoto & Lehtinen, 2004; Tirosh & Tamir, 2004; Vosniadou & Verschaffel, 2004). They have also recognized that there is much to be done to understand conceptual change in mathematical learning (e.g., Greer, 2004). From a conceptual change perspective, the presence of discrepant data are supposed to constitute anomalies, which set in motion a process that leads from a state of crisis via the generation of new hypotheses, through testing and elimination, the acceptance of a new theory (Chi, 1992). The representations associated with conceptual change are structural shifts within a hierarchy of concepts, when the change is within ontological categories, and are changes in the overall structure, when the change are between ontological categories. The mechanisms associated with such structural changes, and the theoretical discourses that come with them, appear to suggest a sudden qualitative change such as associated with insight learning (Chi, 1992). In mathematics education, cognitive perspectives on contradictions focus on the latter as drivers of reasoning and changes therein that come about when examples and counterexamples differ (Giannakoulias et al., 2010)—which assumes, as discussed above, that the problem solver actually has some theoretical grasp of the contradiction rather than experiencing something as not working right.

There are studies about cognitive development and conceptual change that suggest the co-presence of multiple, contradictory ways of talking about pertinent issues. These contradictory ways are evident to the teachers as “muddle” (e.g., Duit, Roth, Komorek, & Wilbers, 2001) but not to the children themselves. For example, when psycholinguists study mathematical problem solving, they note that sometimes there are differences in cognitive levels between what children say, generally at a lower level, and what they gesture, generally at a higher level without nevertheless being noticed by the children as such (Perry, Church, & Goldin-Meadow, 1988). In other studies, science students were observed in natural school settings were gesturally presented conceptions preceded the corresponding verbal conceptions on the order of several weeks (e.g., Roth, 2003a). As a result, multiple, contradictory conceptions where expressed simultaneously. Although the result may sound like confused and muddled talk to an all-knowing observer, the contradictions between alternative conceptions expressed are not apparent to the speakers and become apparent only subsequently, after the new understandings that have emerged. Thus, even university professors may express alternative conceptions with respect to graph interpretations during their lectures in introductory courses (Roth & Bowen, 1999).

2.3. Contradictions and everyday mathematical knowing

The present study focuses on contradictions in the interpretation of data and graphs in everyday, non-school settings. Research of mathematical behavior in such settings tends to have a slightly different focus than research conducted in school mathematics settings. Thus, rather than understanding a situation in terms of a contradiction between different matters of affairs, adult problem solvers have been said to experience dilemmas that evolve and come to be resolved slowly by cautiously proceeding (e.g., Lave, Murtaugh, & de la Rocha, 1984; Levenson et al., 2007). Contradictions between different interpretations may exist in work situations when specific mathematical competencies are black-boxed such that operators in the laboratory can perform actions based on and with respect to graphs without being able "to explain the layers of crystallised mathematics to an outsider" (Williams & Wake, 2007, p. 334). In fact, graphs may not be helpful at all in locating where in a production process there might be trouble (Noss, Bakker, Hoyles, & Kent, 2008). In that study, one process engineer suggests that "you have to specifically go and look for [variations], it is not normally displayed. That's why none of the other shift leaders noticed" (p. 376). That is, merely looking at a graph does not reveal any contradictions or problems. One has to know what to look for, that is, know the relation of the graph to the work process and the relation of any *this* graph with what a graph looks like in general. However, in the discovery sciences, this knowledge about graphs does not exist at the outset because its construction is the very purpose of research: There therefore exists "radical uncertainty" about the nature of graphs and about their relationships with natural objects (Roth, 2009). The studies reviewed here suggest that the detection of contradictions and problems requires certain forms of knowledge. Even when this knowledge already exists, such as industrial work processes, one has to look for certain features to detect contradictions and problems. When this knowledge does not exist, such as when research scientists investigate new phenomena, contradictions and problems are undetectable.

3. Study background, data, and analysis

The purpose of this study is to contribute to the understanding of the role of contradictions and uncertainty to mathematical behavior generally and to graph interpretation more specifically. I do so with an ethnographic study of discovery science, where scientists engage in the interpretation of data and graphs in the face of uncertainty, that is, without knowing what is or should be the correct result. Such study focuses on the actual practices and behaviors of people in their everyday settings—in contrast to the controlled settings of structured interviews, think-aloud studies, and experimental laboratory settings.

3.1. Setting and data collection method

The data for this study was derived from two concurrent, 5-year ethnographic studies of scientific and mathematical knowing in the workplace. The two studies were conducted as part of a larger, interdisciplinary project that attempted to understand the social, cultural, and economic stress factors in communities on the east and west coasts of Canada (www.coastsunderstress.ca). Holding a professorial chair in applied cognitive science, my contribution to the overall study was to investigate knowledge exchanges between scientists and the coastal communities generally and pertinent organizations specifically (here, fish hatcheries). One of these was conducted in a scientific research laboratory investigating life history strategies of Pacific salmon species through the analysis of changes in the absorption spectra of photoreceptors in fish eyes. I had joined forces with the head of the laboratory to conduct both natural and social scientific aspects of the study. Holding a graduate degree in physics with expertise in optics, mathematical modeling, and statistical analysis, I had become a member of that laboratory. I participated in the design of the scientific experiments, contributed to the collection of data in the field and wet laboratory, data analysis, and publication of the results in scientific journals. My second ethnography investigated mathematical knowing and learning in fish hatcheries. I spent about 3 days/month at Rodney Creek Hatchery (pseudonyms are used throughout) where the laboratory sourced most of the salmon that we studied. The members of the scientific laboratory and the fish culturists repeatedly met, for example, when the researchers reported findings back to the hatchery personnel. I also spent time at Kittila Hatchery, run by a First Nation and supported by a federal funding scheme. Two non-indigenous biologists served as scientific advisors to this hatchery. Both hatcheries agreed to participate in the natural and social scientific parts of the study.

The members of the scientific team included (a) the head of the laboratory, a full professor with more than 30 years of experience doing that kind of research; (b) Shelby, a doctoral student whose PhD is based in part on this study; (c) a research associate with a physics background responsible for developing data analysis and data modeling software and conducting data analysis; (d) a graduate student doing his Master's degree responsible for assisting Shelby in conducting most of the measurements; (e) a postdoctoral fellow responsible for field relation, research management, and design of experiments; and (f) myself, collaborating with the biologists—in the design of experiments, conducting measurements, interpretation of data, and mathematical modeling of light rays traversing equipment and samples—while conducting a study of knowing and learning. In the fish hatchery, there were two main informants, an older fish culturist with over 20 years of experience at the job and his mentee, Erin. In the meeting analyzed below, the assistant manager of the institution, Ray, an individual who was regularly interviewed, also participated.

In both settings I used ethnography as apprenticeship approach (e.g., Coy, 1989; Roth, 2005). In this form of ethnography, the researcher learns the practices under study by working alongside everyone else in the site doing their daily tasks. That is, the researcher comes to know a culture while actually contributing, as apprentice, to the actual work. In the fish hatchery, this meant participating in and the learning of such activities as (a) culling female fish to collect eggs, (b) "cracking" males to retrieve the milt, (c) fertilizing eggs, (d) feeding fish, (e) collecting dead specimen, (f) gathering samples and measuring weight and size, and (g) entering data into a computer data base.

In the course of the two ethnographic studies, I not only participated in the work at both sites, but also videotaped the work (e.g., data collection, interpretation, processing) and interviewed fish culturists (or had one of my research assistants do so). (The laboratory members also were part of the social science research team, and we presented together papers at international conferences on knowing and learning in the sciences.) I collected evidence in the form of photographs of mathematical representations used in both sites in an ongoing manner, or produced in the course of explaining a particular aspect of the work. I also included the PowerPoint slides used during presentations to a variety of audiences (research team, seminars, hatcheries) or posters presented at scientific conferences and the marked up drafts of scientific papers.

In the present study, I analyze one meeting in particular. In this meeting, the scientific research team presented its results to the assembled fish culturists and managers of the hatchery. I made this selection because here (a) most contradictions existed in the same situation and (b) the talk was oriented towards non-biologists in contrast to our laboratory meetings, and, therefore, presupposed less scientific background. In this hatchery, only the manager had a university degree and only one of the remaining 8 fish culturists had attended tertiary education courses without obtaining a degree (Erin). Nevertheless, most of the fish culturists had designed, assisted by their peers, and conducted scientific experiments to improve various aspects in the production of salmon smolts. They also had developed sophisticated levels of understanding of biology, the life cycles of different salmon species, diseases, and ecological knowledge. The presentation of the research team presupposed this understanding. It was precisely in this meeting that contradictions in the interpretation of data became salient, which was a further reason for the selection for the episodes reported below. An additional consideration in restricting the selection of episodes was that it made unnecessary description of further settings and participants.

3.2. Scientific context

The scientific research team was in the process of conducting tests whether the shifts that had been observed in the levels of one particular chemical involved in the absorption of light in the eye—i.e., porphyropsin, the purple vitamin-A_2 based pigment typical of the freshwater stages of certain fishes and amphibians—could be used to make decisions about the release time of the young, hatchery-raised salmon to maximize their return rates several years later when they would ascend the rivers, spawn, and die. The freshwater stages of salmon include the *alevin* (the fish have hatched, but the yolk sack is still attached), *fry* (developing stage without yolk sack), and *parr* (when they feed until they reach the size required for migration). They then undergo physiological changes, which includes the silvering of the skin, at which stage the fish are referred to as *smolt* and they begin their seaward migration. The “dogma” (the lead scientist’s term) was established in the 1930s and 1940s and subsequently received a Nobel Prize. It said that the freshwater pigment porphyropsin would be, in the process of outward migration, completely replaced by another purple pigment, the vitamin-A_1-based rhodopsin, typical for saltwater (e.g., Wald, 1957). This would lead to a step-like graph (Fig. 1), which the team members repeatedly sketched when they described their work to others or gestured with hand/arm movements. If the dogma held, then the readiness of coho salmon for migration could be determined from the changes in porphyropsin levels. This information might have been helpful to fish hatcheries—currently confronted with great variations in the return rates of adult salmon—in producing more consistently high return rates and to the local economy that relies on salmon fishing (commercial fisheries, tourism industry, and aboriginal sustenance fisheries).

«««« Insert Fig. 1 about here »»»»»

During the back migration, a reverse change was to be observed. The ratio of vitamin-A_2 based to vitamin-A_1-based rhodopsin is derived from the position of the maximum of the light absorption spectrum when plotted against wavelength (i.e., λ_{max} or “lambda-max”); the width of the absorption spectrum at half its height, “half-max-bandwidth”) is also used in the determination of the porphyropsin to rhodopsin ratio. Five years prior to our own study, a scientific paper reporting the results of a study conducted in the same geographical region supported the dogma. Our research hypothesis stated: If the dogma held up, it should be possible to correlate porphyropsin levels at release time with return rates. As it happened, our study, which had collected almost two orders of magnitude more data than previous studies, would overturn the dogma suggesting that there are annual variations independent of developmental stage and type of water.

3.3. Data analytic orientation

This study draws on conversation analysis and ethnomethodology to analyze episodes from the research process concerned with the interpretation of the data that the scientists had collected and the graphical and mathematical representations that they generated from these. Conversation analysis takes as its minimal unit of analysis the turn pair (Have, 1999). Doing so prevents analysts from imposing their interpretations of what someone has said and to focus on how the recipient of a locution in the episode takes up and responds to a preceding locution. This requires analysts to be able to hear a conversation in the way that the members to the setting do, especially to hear the double entendres, allusions, in-jokes, and other aspects of a conversation hidden to novices to a culture (Schegloff, 1996). I was a contributing member to both laboratory and fish hatchery for five years. I therefore had developed the competence of participating in the meeting analyzed without having to ask others (as I had to do during the first few months of the research) what they meant. Ethnomethodological precepts focus us on what people really do rather than what they might be saying to have done afterwards. What is apparent to participants and the nature of the appearance comes to the fore in situation when there is trouble or a dilemma, at which point participants make available for each other their respective ways of seeing and understanding (e.g., Suchman, 1987). That is, the ethnomethodological approach exhibits the forms of orderliness that *all* participants to a setting both produce and exhibit to each other (Garfinkel, 2007).

While viewing the videotapes recorded over the two-year period, I came to realize that in the final publication from the biology laboratory, some data had not been included. I then noted that there had been numerous meetings where the team discussed the data but had trouble understanding some aspects. In the early phases of the research, the team thought that its data supported the canon—a step-like function of the porphyropsin levels around the time that the young coho smolts were released into the river. However, in later phases of the research, the data apparently supported annual variations following a cosine function. For the purpose of this paper, I analyzed all episodes where the team engaged in the interpretations of their data or presented their interpretations for the purpose of identifying the sources of difficulties that the team had.

4. Contradictions in data modeling

In this section, I present a scientific team's interpretation of the data that it was in the course of collecting. It became apparent to me, a member of the scientific team, only several years after the research that there were (undetected) contradictions. These did not manifest themselves as such and, therefore, *could not* influence our interpretation of data. Rather, the team experienced dilemmas, uncertainties, (minor) troubles, situations and things that did not totally rhyme or that participants could not "put their finger on," difficulties of "wrapping the head around," unexplained uneasiness, confusion, riddles, things not working, failure, or more general breakdowns of the ordinary ways of going about things. Today, from an after-the-fact perspective that operates with 20/20 hindsight, the problematic interpretations of the data are especially salient during a meeting where the scientists presented preliminary results of the research to the fish culturists in one of the participating hatcheries. In the following, I present and analyze seven episodes from that meeting and the interpretation that the scientists provided in the publication that resulted from this work. After the fact we can identify contradictions that were not salient as such to the research team or audiences of their presentations at any moment during the research process. In the following, Shelby is sometimes cited without the acknowledgment of what a subsequent speaker has said. However, prior to the meeting in the fish hatchery, there already had been a laboratory meeting (also on record) where the entire presentation was given and discussed; the team as a whole has already accepted what Shelby is explicitly saying or implying.

4.1. "I'm getting wild fish, hatchery fish, from different populations, and they're doing the same thing"

The scientific canon stated that the level of vitamin-A_2-based porphyropsin levels decreases substantially at the time that (anadromous) salmon prepare for their seaward migration (see Fig. 1). This implies that salmon at different stages in their life cycles (juvenile fish [parr] vs. migrating fish [smolt]), from different locations along the migration route, and from different ecological systems (where the onset of migration differs), ought to have different levels of porphyropsin. In this episode, Shelby reports that all fish "are doing the same thing." This, therefore, constitutes a contradiction between the data of this team and the canon, which would have predicted different porphyropsin levels for coho salmon en route to the ocean. But, as the episode shows, the contradiction is not evident in such a way that it would have forced scientists to change their understanding or conception. But because the scientists do not know what their data will support, that is, in the face of (radical) uncertainty (Roth, 2009), they cannot intentionally look for the problem in the way that industrial engineers can who know what they should be looking for to locate trouble (Noss et al., 2008).

During the presentation of results to the staff at Rodney Creek, the important aspect of the graph concerning parr and smolt to fall onto the same curve is explicitly presented and subsequently raised and talked about. In his presentation at the hatchery, Shelby, speaking on behalf of the research team, points out that all the data points follow the same line, including one "ocean capture point." He adds the data sets from different origins one after another: hatchery smolt at release, hatchery smolt in estuary, wild smolts in estuary, hatchery smolts in the tanks at the university (freshwater), hatchery smolts in the tanks at the university (salt water), hatchery juveniles in ocean, wild coho juveniles in the ocean, hatchery parr in hatchery ponds, wild parr in Rodney Creek, wild coho juveniles in ocean in winter, and hatchery alevins in hatchery ponds (Fig. 2). According to the canon (Fig. 1), there should have been considerable differences ranging from the parr in the hatchery with about 90% porphyropsin (10% rhodopsin) to the smolt in the ocean with about 5% porphyropsin (95% rhodopsin) of the pigment make-up of the rod. Shelby notes during the meeting that out of all these data, there is only one point that he characterizes as not following the line (the only filled triangle in Fig. 2, recorded during the month of November), which is exactly the one that he had talked about earlier in the laboratory, the point caught from a research vessel of the Department of Fishery and Oceans during the month of February. He suggests to the hatchery personnel: “I can’t tell you exactly why they’re not on the same line.” Despite claims to the contrary (Gainsburg, 2007), the uncertainty about the claims that the scientific study would ultimately support left scientists with an intractable problem despite the empirical data they had at hand. Shelby expands by saying that he would like to collect some of the “missed bunch of points” to fill in the picture concerning the coho adapted to ocean life.

«««« Insert Fig. 2 about here »»»»»

Episode 1 (020619, 21:23)

```
→ 01 so what i can say from that is that salinity ITself doesnt
  02 change the visual pigments. which is interesting because
  03 thats not what other people had suspected. because thats
  04 why theyre called marine pigment and freshwater pigment.
→ 05 bcause people thought that well if you go to marine water
→ 06 then that has some effect on the fish and stuff like that.
  07 so thats kind of interesting. now it dOESnt mean that going
  08 to the SEA doesnt have an effect cause the sea is a
  09 different light environment. and i didnt change that
  10 environ all i changed was the salinity. so all i can say
→ 11 from this for sure is that salINIty itSELF doesnt have an
→ 12 effect. so going back and so going back and forth between
  13 brackish and freshwater itself dOESnt have an effect.
```

In this first episode, Shelby notes that there is a difference between the results presented in this meeting and “what other people had suspected” (line 03), that is, that going to seawater has an effect on fish (lines 05–06). He concludes: “salinity itself does not change the visual pigment” (line 01). He describes what others anticipate or have observed. He says, however, that this does not exclude going to sea (lines 07–08) and salinity as a factor driving the change between the two opsin forms (lines 11–12) because he changed salinity but not the light environment that comes with travel to the ocean habitat (lines 09–10).

At this point in the presentation, a clear picture appears to emerge: all data points but one from the Rodney Creek hatchery or its natural environment—which includes samples from very different contexts (see legend of Fig. 2)—follow the same circannual pattern. Most importantly, also contributing to the plot are those fish sourced at the same hatchery and kept for nearly a year in the university laboratories (triangles pointing left or right). Half of the fish were kept in saltwater tanks, the other half in freshwater tanks. All tanks were at a constant temperature of 15±1 °C. Although Shelby explains some of the differences between the different environments, he, speaking on behalf of and consistent with the remainder of the team, does not talk about the temperature difference. Yet it is clear today that the team claimed an effect of temperature and simultaneously plotted the data as if there were no temperature effect—e.g., in the comparison between hatchery and wild coho from the river near Rodney Creek Hatchery, which live in water kept at different temperatures at least part of the year.

4.2. "You can see, there's a very tight correlation"

At the time of the meeting, the research team hypothesizes: temperature drives the process of smoltification, the beginning of migration, and the adaptation of the photoreceptors for the extended stay in a marine environment. However, during the presentation at Rodney Creek, contradictory evidence emerges: the wild coho in the river near the hatchery and the hatchery-raised coho do not live at the same temperature. Yet Shelby suggests to the audience of fish culturists: "You can see, there's a very tight correlation." The hatchery draws its normal water supply for the coho ponds from the creek that gives the hatchery its name and in which at least some wild coho live. However, in the summer time, the temperature is high so that the hatchery mixes the warmer creek water with cooler water from the bottom of a large nearby lake before allowing it to flow into the coho ponds. As a result, the hatchery-raised coho live in much cooler water during the summer time than do the wild coho that serve as a direct comparison case.

During the meeting, Shelby first presents the tight correlation between the water temperature and the "freshwater fish" (line 02). He describes the correlation and then—suggesting that someone pointed this out to him at a recent conference (line 06)—adds that there was a jump in porphyropsin (line 12) that correlated with drop in temperature (line 09). While talking, he circles the two data points from the month of July with the cursor (Fig. 3). He articulates hoping for something like a natural experiment during the next month, where, if the temperature were not to drop (lines 17–18) with corresponding lower porphyropsin level, he would expect the data to "follow the curve a little bit more closely" (lines 18–19).

««««« Insert Fig. 3 about here »»»»»

Episode 2 (020619, 00:25:11)

```
  01 and this gets really interesting because you can see there
→ 02 theres a very tight correlation ((Fig. 3)). its a
  03 negative correlation; its the reverse. but its a vERy tight
  04 correlation that what appears to be hAPpening is that the
  05 visual pigments are following the temperature and whats
  06 neat about this is; i took this data down to florida and i
  07 hadnt really had a chance to look at it closely and someone
  08 said to me wow this is following really closely because in
→ 09 twothousandandone you just happen to have a cool spell in
  10 late july ((circles the temperature dip in July)) and sure
  11 enough look at that the pigments ((circles 2 data points in
→ 12 July)) jumped up a little bit just like youd expect if the
  13 temperature dropped a little bit. which is really neat. now
  14 i dont have a lot of data points around that to follow up
  15 and down. but theyre sitting on the same line. so its its
  16 pretty nice. and what i=ll see this year hopefully is
→ 17 twothousandandtwo we probably wont have that little cool
→ 18 spell and i=ll see whether they follow the curve a little
→ 19 bit more closely.
```

A closer inspection of the graph shows, however, that there are at least four contradictions in this description. First, most of the data points displayed (Fig. 3) derive from coho smolts kept in freshwater tanks at the university, where the water is kept steady at 15±1 °C. The temperature, as shown in the next section, is that of the ponds in the fish hatchery. That is, the porphyropsin levels are said to follow temperature even though the temperatures are constant for a large segment of the data displayed. Second, the water temperatures in the estuary are not the same as those of Rodney Creek, the hatchery, or the river that connects both into the estuary, where the briny water already corresponds to a mixture of river and ocean waters. Whereas the river water fluctuates considerably with season, the ocean water temperature remains fairly steady around 6–7 °C. Third, if there is the kind of relation between porphyropsin levels and temperature, then the rise in the former *does indeed* follow the expected pattern—which requires a deviation from the curve that does not take into account the natural variations in temperature. Fourth, in the preceding presentation, the team had shown the same curve modeling all fish, which means, the curve describes a pattern that is independent of temperature as it provides a trend for fish living in very different ambient temperature regimes. (The climates at the hatchery and the university laboratory are considerably different with several degrees differences in summer and winter average temperatures.) How is it possible that a team of scientists does not notice what today appear to be contradictions? It may well be that what is valid for industrial engineers (Noss et al., 2008) also is valid for scientists: to see contradictions, one has to know what to look for. But in the discovery sciences, this knowledge is the goal of the research. Just as in some cases of structural engineering (Gainsburg, 2007), the phenomenon does not (yet) exist leading to intractable problems.

At this point, the team has modeled the data by abstracting it from the context and, therefore, from the particular temperature regimes that characterizes the different geographical locations and settings. Shelby will say a little later about the added photoperiod information—i.e., day length (Fig. 3, fat solid sine curve)—that it, too, is correlated with the porphyropsin levels. But this correlation is "not as tight so that when day length starts to go up so does temperature, but temperature lags for longer after [length of day] reaches its peak" (00:26:54). On the other hand, his statistical analysis shows that "the correlation for temperature and visual pigments is highly significant" (00:27:58).

4.3. "We were actually cooling our water during that period"—not hearing contradictory evidence

About two minutes after the beginning of the preceding episode, a discussion unfolds in which temperature differences are articulated between the hatchery ponds, normally directly fed by water from the creek but cooled during the summer period, and the creek ("We were actually cooling our water during that period"). Because the research team believes that temperature drives porphyropsin levels, temperature differences inside and outside the hatchery would imply differences in porphyropsin levels between the wild fish and those in the hatchery. There are also temperature differences between the fish kept in tanks at the university versus those that are kept in outdoor earthen ponds in the hatchery. The differences come to be marked in the interaction with hatchery personnel, but do not enter the considerations of the scientists, who, having shifted to this new model for understanding the variations of porphyropsin levels in the life cycle of the coho salmon, do not seem to hear evidence to the contrary. That is, evidence for a possible contradiction is articulated in the discussion. But the discussion unfolded as if the team of scientists did not hear what the fish culturists were saying—possibly because of confirmation bias that is prevalent even in the sciences (Nickerson, 1998) or because their observations were shaped by the dominant theory (Hanson, 1958).

The episode begins with Ray asking about the nature of the information plotted in the graph featuring temperature and porphyropsin levels and the graphical model thereof (Fig. 3). Shelby does not appear to know exactly what he means, as he asks Erin, from whom he has received this information (turn 04). Ray then points out, the emphases stressing the contrast of his statement with what appears to be the case assumed, that they are cooling the water (turn 07). As a result, the temperatures in the earthen ponds where the coho are raised do not follow the photoperiod. In the interaction with Erin, who suggests the pond temperature to be between 13 and 14 °C, Ray says that otherwise the temperature would be somewhere between 20 and 25 °C in the ponds, which draw their water directly from the creek (turn 12). My ethnographic study shows that the water is channeled from the creek to the top of the ponds and released from the bottom back into the creek at a lower point. Shelby appears to hear Ray saying that "this [graph] looks close to what you guys are doing then" (turn 13), to which Ray responds by reiterating that it "naturally would be up around 25 [°C]" (turn 14). Erin confirms this with a constative utterance repeating the temperature value (turn 15).

Episode 3a (020619, 00:27:14)

```
  01 R:  whats the temperature of
  02 S:  whats the temperature of?
  03 R:  yea
  04 S:  ah thats a good question; erin its the creek?
  05 E:  rodney creek
  06 S:  rodney creek?
→ 07 R:  so we were ACtually cOOLing our water during that period so it
```

doesnt really correlate with [photo]period
08 S: [well] so youre cooling when
this is at its peak? ((*circles the temperature in the April*
through October period))
09 R: yea
10 S: whAT do you cool to.
11 E: between thirteen and fourteen degrees generally right?
→ 12 R: yea we we=re from from twenty to twentyfive degrees
13 S: okay so this is not hitting that high though because this is
sitting at ten fifteen ((*moves cursor from curve to ordinate on*
right)) so this looks like its pretty close to what you guys are
doing then. thats ten and thats fifteen so you guys would be
sitting right around there anyways; right?
→ 14 R: yea but in naturally it would be up around twentytwentyfive
15 E: twentyfive.
16 S: right. huh thats interesting so is that because last year it didnt
get as hot? or is this?

Shelby appears baffled, perhaps confused, and suggests that during the preceding year, the one displayed on the graph, “It didn’t get as hot?” (turn 16). The intonation toward the end of this utterance is rising, as it typically would in a question. The intonation also is rising in the fragment of a sentence that Shelby then produces, a fragment that also has the grammatical structure of a beginning question (“or is it this?,” turn 16). Erin then explains that they—she and her mentor Mike—attempt to keep the pond temperature between 12 and 13 °C. My ethnographic work in the hatchery revealed that at higher temperatures, certain diseases, such as bacterial kidney disease (BKD) increase exponentially and, with them, the morality rates. Moreover, coho salmon grow best between 9 and 15 °C. The interaction between Shelby, Erin, and Ray then ascertains the source of the temperature readings as coming or not from the ponds (turns 20–26), which both Ray and Erin ascertain to be from the ponds. Both assert that the water is “artificially cooled” (turns 29, 30).

Episode 3b (020619, 00:28:07)

16 S: right. huh thats interesting so is that because last year it didnt
get as hot? or is this?
→ 17 E: we we always try tmaintain between twelve and thirteen degrees
mike likes it thIRteen I like it twelve we have a big fight no hh
18 S: right.
→ 19 E: jokin. but like ray had said like it could get up as high as
twentyfive and we sort of turn ON and shut dOWn pumps in the creek
to maintain that [temper]ature.
20 S: [right]
[yeah. so this data you sent me then]

```
  21 E:  [throughout the summer months       ]
  22 S:  is not from your ponds its from
  23 R:  its frOM the ponds
  24 E:  its frOM [the ponds]
  26 S:           [oh it is ] from the ponds
  27 E:  yea but
  28 R:  but the what i=m saying is the
→ 29 E:  artificially cooled
→ 30 R:  its artifi[cially cooled]
  31 S:             [oh I see okay] yea yea alright
  32 R:  so it doesnt really correlate with phOTOperiod then like like
  33 S:  not necessarily
→ 34 R:  normally that temperatures [r much higher]
→ 35 S:                             [except that  ] it would go even
         higher; like your temperatures are higher in july and august.
```

In this exchange, the difference in temperature between the hatchery and wild fish for the summer months comes to be established. The temperature in the creek could get "up to twenty-five," which is the place that the wild coho are sampled. That is, there are the same kind or even more radical differences in temperature between hatchery and wild coho in and around Rodney Creek than between all other fish samples used in the research. There is therefore a contradiction because the team continues to collate all the data and modeled them by means of the same rather than different curves. On the other hand, as the next subsection shows, the team clearly separated out all of those data for another hatchery that are "off the chart" and therefore would introduce very large variations into the picture.

In the present instance, the fish culturists have raised a question about the source of the temperature plotted in the graph. They are very familiar with the hatchery setting, its conditions compared to the conditions outside of the hatchery; they know about the temperatures of the creek, the river, and the surface and bottom temperature of the nearby lake, where the cool water is pumped from to the channel that feeds the fishponds. It is out of this familiarity that they could question the representation of the temperature. Although not articulated explicitly, the normal temperatures of Rodney Creek are co-articulated, and thereby, the different temperature regimes between the "natural" (turn 14) or "normal" (turn 34) and the artificial temperature environment produced by means of "artificial cooling" (turns 29, 30). Shelby initially marks to be understanding (turn 31), then explicitly states that the temperatures "would go even higher" (turn

35), and finally adds that the "temperatures are higher in July and August" (turn 35), where it remains uncertain whether the temperatures are those in the ponds or creek.

Scientists tend to be held to be rational and model problem solvers not only with respect to their field but also with respect to the interpretation of graphs (e.g., Tabachneck-Schijf, Leonardo, & Simon, 1997). However, in the face of uncertainty, the mathematical behaviors may not as rational as that displayed in think-aloud sessions with well-behaved problems (e.g., Roth, 2004a). In this instance, a possibility existed to revisit the earlier presented data, or to discuss the earlier articulated drops in the water temperatures and the corresponding increases in porphyropsin levels. It might have been an opportunity to revisit the question of the role of temperature and to ascertain those that constitute the natural environment of the wild coho salmon that led to the data on the graphs. But such opportunities may exist only from the a posteriori perspective, because at the time, none of the scientists or hatchery personnel present raised the question. The possibility exists perhaps only from the after-the-fact perspective, because if it had been noted as a possibility *in this situation*, one or the other participant (there were 10 individuals attending the meeting) might have brought it out.

4.4. "This confuses me a little bit"

The data and graph interpretation did not completely rhyme. However, rather than experiencing contradictions that might have driven a conceptual change, Shelby felt confused ("This confuses me a little bit"). Following the presentation of the data derived from coho salmon sourced at Rodney Creek Hatchery where the presentation took place, the research team's PowerPoint presentation moves on to share the results from the fish sourced at the second hatchery participating in this study (Fig. 4). This hatchery (Kittila), as an associated slide had shown earlier during the meeting, geographically is located much further north in the same province with a very different climate, geography, and ecological context. Some of the data from this hatchery clearly contradict the claims made about the data from Rodney Creek.

««««« Insert Fig. 4 about here »»»»»

Shelby describes the pertinent features of the hatchery and the graph, especially that it exhibits a cycle despite the fish being kept at constant temperature. He says that he is “confused” by some aspect of the graph (line 02). The confusion arose from the fact that one could expect, if temperature were linked to porphyropsin, colder temperatures at the Kittila hatchery should lead to higher porphyropsin levels. Whereas during the “colder times of the year,” Rodney Creek water temperatures are indeed lower than those at Kittila and, as anticipated, the porphyropsin levels are higher, the same relation is not the case in the summer, when the Kittila temperatures “are overall colder” (line 09). In this situation, the Kittila best fit and the Rodney Creek best fit both have a value between 20–21 °C. On the other hand, the “wild [coho] are off the chart” (line 12), which is interesting and could be anticipated, “because those wild fish came from very cold, glacial creeks” (lines 13–14).

Episode 4 ((020619, 00:29:40))

```
  ((presenting the slide in which the only left graph
→ in Fig. 4 was presented)) thIS confuses me a little
→ bit because if their temperatures are generally cOLDer then
  i wouldve expected if its linked to porphyr, linked to
  temperature that the percent porphyropsin would be hIGHer.
  i would expect that if theyre cOLder then ((pause)) in the colder
  times of the year you guys show a higher percent porphyropsin.
  so i wouldve thought these guys would be higher when because
→ theyre overall colder. but thats not the case. and so there
  may be something else at play here.
  . . .
→ the wild are off the chart. and thats really
  interesting too because those wild fish came from ah very
→ cold ah glacial creeks with low nutrient levels, where they
  believe the coho overwinter for two years rather than one.
```

Shelby then elaborates on the statement of confusion stating that it is “really hard for me to interpret these data ((circles the data from the wild coho)) because their life history strategies are so different” (00:31:04). That is, there are big differences between the wild and hatchery coho related to the Kittila hatchery. He adds, “I can tell you for sure that the temperatures ((points to the data from the wild)) are cooler” (00:31:08). It can be “correlated that *these* ((pointing to the wild data)) are higher up”; but Shelby insists that the team cannot make predictions about the relationship and differences between wild and hatchery fish from that setting because the conditions at the hatchery are so "removed from the wild conditions" (00:31:22).

In this episode, we observe a contradiction between the two graphs even though these are displayed together. On the left-hand side, only the data from the hatchery, both parr and smolt stages, are aggregated and modeled by the sinusoidal curve—later, in the scientific publication, modeled as a sine curve and correlated with the photoperiod, with respect to which it was shifted by about 3 months. In the Rodney Creek-related part of the slide, however, the data from fish at very different temperatures regimes are compiled and modeled by means of the same curve. That is, the scientific practices underlying the two parts of this PowerPoint slide are different, because the equivalent temperature differences in the Kittila situation are not modeled simultaneously with the same graph, because "the wild are off the chart" (line 12). There is therefore a contradiction, which, however, neither the scientific research team nor the hatchery personnel in attendance discovers. That is, there is a blatant contradiction that despite all of its salience once noticed (e.g., in the way a teacher might notice or set up a contradiction) is not noticed by those discussing the data in this setting and during their laboratory meetings.

The final point discussed is the fourth outlier, which comes from the main river into which the glacial creeks feed. This point may reflect data from coho that already are in their second year. (In particular cold conditions, salmon have been reported to remain in the freshwater environment until their eighth year following emergence from the egg.) Shelby then talks about a study conducted by DFO scientists, who, as he says, believes that the fish in this river system stop migrating in the bigger river to overwinter there and only leave the river for the ocean during a second year of migration. This would mean that there is "a two-stage system" (00:31:01). He summarizes: "so it's really hard for me to interpret this data because their life history strategy is considerably different" (00:31:03). He ascertains knowing for sure that (a) the temperatures are cooler at Kittila Hatchery than at Rodney Creek Hatchery and (b) outside the former, the fish are "higher up" in porphyropsin. But then he concludes, "outside of that I can't really make a lot of predictions to what their wild fish are doing relative to the hatchery 'cause it seems their hatchery is so far, much farther removed from the wild conditions" (00:31:14). That is, the team simply leaves the problem rather than actively dealing with it the contradiction—an approach reminiscent of the adult shoppers who sometimes abandon problems when these become intractable (Lave, 1988).

4.5. “It would be even more dramatic if it was our actual if it was in our wilds in the river”

During the meeting, participants appeared to be pursuing contradictory ideas without that these contradictions became apparent. Shelby compiled all Rodney Creek data into one graph, which assumes that temperature does not make a difference. But Erin talks about dramatic variations in porphyropsin levels if the scientists had plotted them for wild, river-based coho—which the team actually had included. Thus, Shelby presented these data together with hatchery fish, modeled by one and the same graph whereas Erin suggested that the wild coho should be dramatically different. This contradiction is evident in presentation of a comparison between the coho salmon from Rodney Creek and Kittila. The amplitudes of the two graphs are rather different so that the variations in temperature do appear to make a difference in the sense that when the temperature is held constant, there are still annual variations but with lesser amplitude, as the comparison between the two hatcheries shows (Fig. 5). There are also temperature-related differences possible between the wild fish at Kittila—i.e., the 3 first outliers in Fig. 5 not modeled by the sinusoidal curve—living in glacial rivers and temperatures near 0 °C for 5 months of the year, and Kittila Hatchery, where the temperature is held at a constant 7 ± 1 °C. When Shelby talks about the two graphs, he suggests that “temperature” “seems to be what’s mostly manipulating this” (turn 01).

«««« Insert Fig. 5 about here »»»»

Episode 5 (02-06-19, 34:40)

01 S: so HERe i=ve just compared the two for you ((*shows the two graphs in Fig. 4*)) thats the constant temperature ((*points to left graph from Kittila*)); its a very subtle curve and theres ((*points to the right graph pertaining to the Rodney fish*)) the varied temperature and you can see its higher here. so i=m pRETty sure that we=re talking about a temperature effect here. that seems to be what whats MOStly manipulating this.

```
02     (0.34)
03 E:  and it would be even more drAmATic if it was our actual if it was
       in our wilds in the river
04     (0.46)
05 E:  <<p>that would be [dramatic.]>
06 S:                    [yE:A:    ] yEA if temperature is [chang]ing
07 E:                                                      [yea  ]
08 S:  more thats right. now the thing though tOO is that (0.57) they
       seem to pEAK bottom out at twenty percent (0.34) and i hAVEnt seen
       a fish go much below about fifteen percent. (0.33) so if the
       temperature gets up to twentyfive (0.32) it would be cOOl to see
       maybe they do drop down to zero. i doubt they do.
```

The stark differences between the temperatures within Rodney Creek Hatchery-raised and Rodney Creek wild coho became the topic again in the context of this PowerPoint slide. Immediately following the brief presentation of the slide exhibiting the difference between the "very subtle curve" for the hatchery with "constant temperature" and the "higher [curve]" when the temperature is "varied" (turn 01), one of the fish culturists suggests: "it would be even more dramatic if it was . . . in our wilds in the river" (turn 03). That is, the variation would be more dramatic if the curve modeling the wild coho were shown. Erin repeats that this would be more dramatic (turn 05), and Shelby agrees, restating that this would be the case "if the temperature is changing more" (turns 06, 08). That is, there is a collective agreement articulated here whereby greater variation in porphyropsin levels should be observed when the temperature variations are larger. There is then a contradiction: in a preceding slide, *all* fish from *all* contexts, including river, estuary, and the open ocean were lumped together and modeled by the same curve (Fig. 2).

Shelby elaborates the issue by stating that he has not seen porphyropsin levels below 20% in this study and that "it would be cool to see" whether they would "drop down to zero" "if the temperature gets up to 25 [°C]" (turn 08). A little while later during the presentation, Shelby also instructs the fish culturists at the hatchery to keep the fish at the temperature that these were at when caught; that is, for example, to keep the coho salmon in a fish trap floating in the creek or in the river that the creek flows into. Thus, he articulated wanting the fish culturists to hang the fish that had come from the river in a net in the river until these would be shipped in Styrofoam containers to the university laboratory where the fish normally were processed within 48 hours when, as the following subsection shows, the longer-term changes in porphyropsin levels that would come with a change in temperature have not yet markedly shown up.

In this episode, therefore, the expectation that different temperature regimes should lead to different porphyropsin levels is articulated again, both by a member of the research team and, as an inference, by one of the fish culturists. An all-knowing theorist might suggest that this should have rendered problematic the aggregation of data from different temperature regimes in the Rodney Creek case: but it has not, and the contradiction has not been or become salient. There is therefore a logical contradiction in the data modeling—seen now after the fact—that has gone unnoticed during this meeting and even later in this scientific research project right into the final paper that the team publishes.

4.6. "Because our water was warmer than yours, they started to drop"

Although Shelby had compiled all salmon from Rodney Creek hatchery (wild and hatchery) into one data set modeled by the same curve, which assumed that temperature does not make a difference, he also suggested that temperature does indeed make a difference. The context for this contradiction was a comparison between the results from the coho salmon raised in the hatchery and those from the coho kept in the laboratory. Shelby said, "because our water was warmer than yours, they started to drop."

After having presented the results of the study on the annual changes in the porphyropsin levels of hatchery-related coho salmon, the team of scientists talks about extensions to the research. The team reports some relatively sudden changes that had occurred while conducting an experiment on the effect of hormones on the visual system together with another member of the wider team. These included the test that exogenous hormones (hormones applied on the outside of the fish) would shift the photoreceptor optimal absorption (λ_{max}) to longer wavelengths, for rods (associated with increased porphyropsin levels) and for red and green members of the double cones in fish eyes. Contrary to what had been expected, the porphyropsin λ_{max} levels increased rather than decreased, as the "dogma" at the time suggested. As part of this presentation, the team showed results that clearly are interpreted as evidence for the effect of temperature on the fish when these were kept for a while in the university-based fish tanks that were at a higher temperature than the ponds in the fish hatchery where these fish were captured and transported to the university. They had sourced the fish from the Rodney Creek Hatchery, and had kept these in the university tanks for two weeks (line 01). As a consequence, “because our water was warmer than yours they started to drop in percent porphyropsin” (lines 02–04). On an accompanying slide that showed the different trends for hormone-treated and untreated fish, the drop in porphyropsin of the untreated fish, which Shelby attributes to the differences in the ambient water temperature (line 08), is clearly visible especially because the treated fish significantly increased in porphyropsin levels (Fig. 6).

««««« Insert Fig. 6 about here »»»»»

Episode 6 (02-06-19, 00:46:39)

01 we kept them for two weeks ((*cursor moves from first point*
→ 02 *[red] down to second point [green]*)) and because our water
→ 03 was warmer than yours they started to drop in percent
→ 04 porphyropsin . . . so the visual pigments are a gradual
05 shift. they dont change overnight. so i dont have to worry
06 about keeping fish in a different temperature of water for
07 one night. but after two weeks i know that it can have an
→ 08 effect, cause it happened here ((*cursor moves from first*
09 *point [red] down to second point [green]*)) in two weeks.

Shelby then emphasizes that the fish “do not change overnight,” which means, he does not “have to worry about keeping fish in different temperatures” (lines 05–06). But there will be an effect—as exhibited in the plots and highlighted by the laser pointer moving sharply down from the first to the second data point plotted—“after two weeks” (line 07). That is, in this episode, a temperature dependence of the porphyropsin levels is explicitly acknowledged and substantiated by the data on display. This is so not only for the type of photoreceptors used in the study pursuing the life history changes in porphyropsin levels (rods) but also for other types of photoreceptors (red, green) studied as part of other research conducted in this laboratory.

The porphyropsin changes as a function of temperature changes observed here would be consistent with the one data point where a dip in the ambient temperature of the ponds (July) appeared to be a corresponding increase in the levels of porphyropsin (see above). It would also be consistent with the data from the Kittila hatchery, where the wild coho from glacial waters (for five months about 0 °C) exhibit much higher porphyropsin levels than the hatchery-raised salmon in parr and smolt stages kept at 7±1 °C year-round. But the data contradicts the indiscriminate aggregation of the data in the two main graphs displayed that summarize the findings of this research (Fig. 2, 3, 5 (right)). In the conceptual change approach, teachers (are asked to) use “discrepant events” to promote learning as if the events themselves could give rise to a contradiction. However, the conceptual change approach in mathematics also assumes that there “must be dissatisfaction with existing conception” (Vosniadou & Verschaffel, 2004, p. 446) and a new conception must be available, intelligible, plausible, and embody “the possibility of a fruitful program” (p. 446). In the context of the present research, although there were inconsistencies and issues that the team could not interpret, the problems were apparently insufficient to create dissatisfaction with the present theory. It may be that the team lacked sensitivity—i.e., the awareness and interest in the novel cognitive aspects of the phenomenon—that is required to lead to conceptual change (Merenluoto & Lehtinen, 2004). In fact, it is because of reliance on current theory that the problems may not have led to a level of dissatisfaction sufficient to seek for alternative theoretical approaches.

4.7. Contradictions in the final publication

In the final publication, the team presents the data reflecting seasonal changes of coho salmon from different sources side-by-side without explicitly addressing the absence of variations in the temperature-controlled hatchery other than suggesting that photoperiod may drive the system in this case. Omitted in this journal article are the obvious changes that had been observed between wild and hatchery-raised coho from the Kittila setting or the changes observed during the thyroxin-treatment experiment. In its first graph presenting the results, the scientists collate all data from all locations *except* the wild Kittila into the same graph, that is, independent of the hypothesis that temperature makes a difference. They then present a graph in which the data for Rodney Creek Hatchery and Kittila Hatchery are displayed separately. The paper then displays three graph panels in one plate, the first panel showing temperature and porphyropsin levels for Rodney Creek Hatchery versus time of year, the second panel showing day length and porphyropsin levels versus time of year, and the third panel displaying day length and porphyropsin levels versus time of year for the Kittila Hatchery. The associated main text reads:

> Monthly mean percent A_2 values were more closely correlated in time to temperature than day length at [Rodney] Creek hatchery (Fig. 5a, b). For fish collected from Rodney Creek hatchery, temperature and percent A_2 were negatively correlated (Fig. 5a). Percent A_2 peaked in late February; 1 month after temperature reached its annual low. Cross correlation between percent A_2 and inverse temperature was highest ($r^2 = 0.821$) when the cosine function was phase lagged by 1 month relative to temperature (Fig. 5a). Cross correlation between percent A_2 and day length was highest ($r^2 = 0.855$) when there was a 2-month phase lag between percent A_2 and inverse day length (Fig. 5b). For fish collected from Kittila Hatchery, a cross correlation with temperature was not possible since this hatchery maintained a constant rearing temperature year round. However, a cross correlation between percent A_2 and inverse day length peaked ($r^2 = 0.977$) when the curve fit to percent A_2 data was advanced by 3 months relative to day length (Fig. 5c). The variation in percent A_2 found in ocean-going coho also showed an inverse relationship to ocean-water temperatures (see Fig. 6 in following section). (Temple et al., 2006, p. 308)

The shift of the cosine curves seems to be temperature dependent, so it may surprise that the RCH wild and hatchery data are collated and modeled together, much like all the data from the experiment were added in one of the figures of the ultimate journal publication. The report suggests a correlation with temperature—even though, as shown in this article, the temperatures are not the same for hatchery and wild salmon in the summer—but indicates that a correlation could not be calculated because the temperature was held constant in the second hatchery.

4.8. Contradictions resolved

I was a member of the research team and an author of the final study. At the time, the contradictions articulated in the preceding subsections were not apparent to me, which may have been so because of multiple reasons that I could only speculate on today from a perspective with hindsight. Today I can say that the porphyropsin level P that the team published in the end might have been modeled by a function of the type

$$P(T, t) = P_1(T)\cdot\cos(\omega t + l(T)) + P_0(T)$$

where T = temperature, t = time of year in months, $A_1(T)$ = amplitude of cycle, $\omega = 2\pi/12$, $l(T)$ =

lag time, and $A_0(T)$ = vertical offset. Such an equation not only accounts for much of the present data but also for those that remained unpublished and apparently supported the canon. The onset of the changes may be temperature related, as the minimum is earliest at a nearby hatchery—where the previous study supporting the “dogma” had been conducted—where the temperature hits 9 °C in March, whereas the same temperature was reached at Rodney Creek Hatchery only in the latter part of April, whereas it was never attained in Kittila Hatchery, where the temperature is at a constant 7 °C. But because of the different ecological systems, different life history strategies may be at work, too. A Japanese study, as Shelby pointed out one day in the laboratory, suggested small amounts of porphyropsin in another salmon species at precisely the time that Shelby had found in the coho compared to the 0% levels during the remainder of the year. That is, the levels turn out to be lower on an absolute scale in the open ocean where the temperatures are higher on average and more constant than at Rodney Creek Hatchery.

One might even conjecture that different curves modeling the different sources would have become suggestive if the standard errors of the means rather than the standard deviations had been plotted. Each data point was endeavored to consist of 20 photoreceptors from 10 coho. In this case, for example, the error bars would have decreased by a factor equal to the square root of number of data points, that is, by a factor of about 14 if $N = 200$. The parr and smolt from the Kittila would no longer follow the same curve but, as can be seen from Fig. 4, the parr might be seen to dip in October rather than in the July–September period, when the smolt appear to be at their lowest. However, given that the within-fish variability is very high, the team chose to plot error bars that indicate ± 1 standard deviation.

5. Discussion: contradictions in scientific research

The purpose of this study is to present a detailed description and analysis of the contradictions and uncertainties that may exist in the modeling and interpretation of data in scientific discovery work. Because contradictions play an important role in conceptual change theory, this study of scientists’ data and graph interpretation therefore has the potential to be of value for understanding their role in the presence of the uncertainties that learners face. With hindsight, as this study shows, we can say that contradictions have been inherent in the scientists' interpretation work. But although the leading professor specifically and his various teams more generally have been highly successful in obtaining grants and publishing their work, the contradictions never became salient as such and therefore never promoted a conceptual change that would have allowed creating an understanding of all the data that the team had collected. These contradictions therefore never became a driving force in and to the research. In the face of the uncertainty about what will be the new knowledge accepted in the community, most we can observe the presence of unease, dilemmas, uncertainties, (minor) troubles, situations and things that did not totally rhyme; there were things that participants could not "put their finger on," difficulties of "wrapping the head around," unexplained uneasiness, confusion, riddles, things not working, or failure. In the team's final publication, the problematic data from Kittila Hatchery were not included so that the contradictions could not become apparent. For the actors to address the contradictions in the relationship between graphs and the natural phenomena they denote, these contradictions must be apparent in the conscious awareness. To note an existing contradiction appears to require that scientists are familiar with graph, the details of scientific equipment and data collection process, and the natural phenomenon in its habitual context (e.g., Roth, 2004c; Roth & Bowen, 1999). But even when scientists note something odd, they, much like people in other everyday contexts (e.g., Lave, 1988), may simply sidestep intractable problems.

This study underscores the possibility that multiple concurrent descriptions contradicting each other exist within a research team. It is interesting that a team of highly trained scientists or their representation- and context-savvy audience (i.e., the fish culturists) recognized the contradicting descriptions as such. Perhaps even more surprisingly, some of these contradictions have made their way through the scientific peer review process without raising the ire of experts in the field. In the present study, depending on the context, the scientists claim the existence of a correlation with temperature and the non-existence of such a correlation. They claim that temperature makes a difference—as when they point to the differences in porphyropsin between fish from glacial creeks or rivers and a nearby hatchery—and act as if temperature does not make a difference—when they aggregate the coho smolt from within Rodney Creek Hatchery, where the smolt live in artificially cooled water as compared to the creek from which the water with the normal temperature is taken.

The main part of their findings—the annual sinusoidal variations of porphyropsin levels independent of the salinity of the environment—overturned the cannon that predicted sharp decreases and increases in porphyropsin levels when the salmon migrated from freshwater to saltwater (Fig. 1) and saltwater to freshwater, respectively. The scientists were quite aware of the fact that the results that they actually reported overturned the dogma that had driven their research because it had promised to be a tool in determining the readiness of salmon for migration and, therefore, to fish hatcheries the determination of release dates that optimized return rates. Natural scientists who make major discoveries that overthrow a canon often aim at publishing their work in *Science* or *Nature,* the two journals with the highest impact factors. It would be pure speculation, however, to suggest that the team chose a journal with a lower impact factor (5-year index = 2.1, Rank 46 in the field) because of the contradictions. Shelby later suggested that the "impact of that work to the broader community was just too low" (June 10, 2011, email to the author). In fact, the final publication did not emphasize the overturning of the canon but rather stated that it reported "the first description of seasonal variations in A_1/A_2 ratios in ocean-going coho." It suggested that the small amplitude in the variation was similar to one other study of coho salmon; and seasonal variations rather than dramatic shifts during the migration-related changes between saltwater and freshwater had been reported for other species.

Analysts with after-the-fact information might exhibit indignation about the fact that what I have shown to be contradictions in the scientists' models and interpretation of data had not been recognized at the time. In fact, historical analyses of scientific research suggests that contradictions may remain present and be accepted for some time within a community until that point when a certain threshold is reached, at which point some "scientific revolution" will introduce a theory that allows viewing data in a new way (e.g., Kuhn, 1970). Other pragmatic descriptions of the conceptual revolutions in a number of fields—poetry, science, politics—suggest gradual changes rather than changes based on decision making: “Europe did not *decide* to accept the idiom of Romantic poetry, or of socialist politics, or of Galilean mechanics. . . . Rather, Europe gradually lost the habit of using certain words and gradually acquired the habit of using others” (Rorty, 1989, p. 6, original emphasis, underline added). That is, prior to and during the change process to a new conception, the subjects of activity cannot have the criteria for making decisions between the relative usefulness of two conceptions because the second one is only emerging. Transitional discourse inherently contains contradictions (Rorty, 1989). The period of change therefore may be characterized as “muddle” as a positive concept to denote the fact that multiple different discourses co-exist simultaneously and the contradictions that come with this co-existence. There would be a period of *uncertainty* when it is not clear which aspects of the old discourses need to be abandoned and what would be the new tools, because the difference between the two forms of discourse is undecidable *during* the period in which the change actually occurs. There are multiple interlocked theses, each of which may develop independently, and only after some time “Europeans found themselves speaking in a way which took these interlocked theses for granted” (p. 6). From this perspective, then qualitatively new forms (e.g., of talking) emerge from quantitative changes in the old form. Both dialectical materialist logic and catastrophe theory have evolved discourses that handle qualitative changes as the result of quantitative change of a number of different dimensions simultaneously.

One might ask why the contradictions, however these manifested themselves, did not lead to a conceptual change in data and graph interpretation. A lack of sensitivity, i.e., awareness and interest in alternative conceptions (Merenluoto & Lehtinen, 2004), is one explanatory possibility. As far as the team of scientists was concerned, there were some confusing aspects in the data that constituted a dilemma but not a major problem that should have prevented publication, and a little uneasiness about the way in which the existing data were modeled. But there was no salient contradiction as such. At the time, the scientists did not act and talk as if they were dealing with a contradiction. There was therefore also no need for changing what the team developed as an alternative to the current "dogma," which had predicted, as shown in the work of another recent study, approximately logistic decrease of porphyropsin levels around the time that the coho salmon would migrate to sea and an approximately logistic increase of porphyropsin levels at the time that the coho salmon migrate up-river to their spawning grounds. The scientists felt that the porphyropsin levels varied in sinusoidal form with season highly correlated to temperature.

6. Implications for school mathematics

This study has implications for mathematics education in the sense that it calls for much more caution in denoting a situation as containing a contradiction or of identifying contradictions between theory and data. The study shows that in a situation of uncertainty, when the new knowledge in the process of being created is unknown, the learners (researchers) may not be sufficiently sensitive to detect contradictions. Because the ultimate knowledge does not yet exist, the learners have no criteria for judging the quality of their data and graph interpretations. This is consistent with the results of a phenomenological analysis of mathematical learning, which suggests that because the knowledge in the making is not yet known—unseen and therefore unforeseen—there is an inherent uncertainty in the nature and quality of the mathematical learning process (Roth, 2012b). In the present study, even a group of successful research scientists does not detect, during a period characterized by uncertainty, what a posteriori can be seen as contradictions. That is, the contradictions are not "built in," as some mathematics educators suggest (e.g., Tall, 2001), ready to create cognitive conflict. The study thereby throws into relief the role of contradictions in some learning theories—e.g., to create cognitive conflict in Piagetian constructivism and conceptual change theory. As other mathematics educators have suggested, a naïve reliance on creating cognitive conflict is insufficient (Greer, 2004; Merenluoto & Lehtinen, 2004). This is especially the case because the present study shows that in a situation of uncertainty, contradictions may be insufficiently salient to lead to cognitive conflict. Something recognized at some later point as a contradiction by the learner—student or scientist creating new knowledge—cannot be an aspect of scientific decision making before that point precisely because it is not noted as such. It cannot therefore drive any conceptual change, revisions in the argument, or contribute to the development of a new theory.

One important reason for not noticing a contradiction as a contradiction until after the fact is the different way in which an event, such as the process of making and reporting a scientific discovery, can be grasped only after the fact (e.g., Romano, 1998). That is, what evidences itself with an unfolding event as unease or dilemma may subsequently, once scientists grasp the event as a whole, be recognized as a true contradiction. Without contradiction explicitly stated, however, a change in thinking may not be required. Thus, for example, students interpret velocity–time line graph intersections as instances where two cars collide on the racetrack (“iconic interpretation”) (Clement, 1989). What mathematics educators recognize as misconception or contradiction, however, *cannot* appear as such and therefore, cannot direct student learning and the regulation thereof. It is only when the person knows what is to be learned that the contradiction can become salient as such. This has implications for the role of self-directed learning.

Mathematics educators have shown interest in self-regulated learning (e.g., Berger & Karabenick, 2011; Throndsen, 2011). In this approach, metacognition is of interest because it bears on learners’ active selection of strategies for solving problems. Self-regulated students are said to be good learners because they solve the tasks, conceptualize their opinions, and adapt their strategies to task demands (Kramarski & Revach, 2009). In contrast, previous research on other aspects of the research process in the discovery sciences showed that even highly trained and successful research scientists grope about in the dark rather than make decisions about best strategies (Roth, 2004a); in practice, even researchers of metacognition may not exhibit high levels of metacognitive behavior (Roth, 2004b). In the present study, too, the process of these research scientists more resembled a process of groping about rather than engaging in the kind of metacognitive behavior that are expected from mathematics students. Thus, rather than attempting to come to grips with the totality of their data, this research team dropped those data from consideration that resisted integration into their model. The present results therefore are more consistent with other research on mathematical behavior outside schools, where abandoning rather than persisting with problems and difficult issues is one of the legitimate choices individuals make (Lave, 1988). Mathematics educators also noted that partially correct constructs are useful to understand students' inconsistent answers on mathematical tasks (Ron, Dreyfus, & Hershkowitz, 2010). However, the present study suggests that mathematics learners, precisely because they do not yet know the end result of the learning process, cannot use partial correctness or inconsistency to select learning strategies.

Students are frequently asked to interpret data and graphs—as part of research (e.g., Preece & Janvier, 1992) or on international tests such as PISA or TIMMS—in which relationships between natural variables are shown. For example, Tairab and Khalaf Al-Naqbi (2004) asked high school students, among others, to interpret a graph that featured the concentration of dissolved oxygen, numbers of fish, and numbers of bacteria in a river over a distance of 50 km. As Preece and Janvier before, these researchers not only noted a high rate of incorrect responses (> 30 %) but also commented on the "*inability* to interpret interactions among variables" (p. 129, emphasis added). Two other studies among scientists showed that to make any sense of the graphs they generated, the scientists had to rebuild the biological and ecological context that decontextualization of the data sources had accomplished (Roth, 2012a, 2013). Lack of familiarity with the contextual particulars would prevent scientists even more from detecting discrepancies and contradictions. However, other than speculating about students' lack of practice with interpretation, previous research with school students did not address familiarity with the scientific phenomenon or the uncertainty associated with a lack of familiarity or background knowledge that also affect scientists' interpretations (e.g., Roth & Bowen, 2003). Thus, middle school students familiar with an ecological and empirical setting may outperform university graduates with B.Sc. and M.Sc. degrees unfamiliar with the setting on data interpretation tasks (Roth, McGinn, & Bowen, 1998).

The present study shows that a lot of scientific and empirical background knowledge is required for interpreting the data and resulting graphs. A great deal of familiarity with the data—the geographical and ecological origin thereof and the processes of data and graph generation—is a condition for reaching an appropriate, contradiction-free interpretation. This became even more salient to me in the comments of one reviewer, who complained that "are the extensive scientific discussions regarding salmon, chemistry, and hatcheries that the reader must go through in order to make sense of what is going on. I got lost in the science and found it difficult to focus on the main issue." This study therefore suggests that mathematics educators should not expect high level, contradiction-free interpretations of data and graphs unless students are very familiar with the empirical setting—which is consistent with the call for beginning mathematical learning with "sound concretizations" that are familiar to learners (Greer, 2004). In the context of data and graph interpretation, some mathematics educators have recently begun to address this issue by involving students in the generation of the data that they are subsequently to interpret (Cobb & Tzou, 2009; Roth, 1996). It may take experimental studies in mathematics classrooms to find out the relationship between levels of familiarity with the empirical context of data and the quality of the interpretations that learners can provide. Learning process studies may be able to better understand how familiarity mediates not only the interpretative process but how it mediates the process of the changing nature of data and graph interpretation.

Acknowledgments

The research reported in this study was funded by grants from the Natural Sciences and Engineering Research Council of Canada and the Social Sciences and Humanities Research Council of Canada. I am grateful to my team members and the hatchery staff for their collaboration in this study and for teaching me all I know about hatching fish and conducting research on fish vision.

References

Balacheff, N. (1987). Processus de prévue et situations de validation. *Educational Studies in*

Mathematics, 18, 147–176.

Berger, J.-L., & Karabenick, S. A. (2011). Motivation and students' use of learning strategies: Evidence of unidirectional effects in mathematics classrooms. *Learning and Instruction, 21,* 416–428.

Chi, M. T. H. (1992). Conceptual change within and across ontological categories: Examples from learning and discovery in science. In R. Giere (Ed.), *Cognitive models of science* (pp. 129–186). Minneapolis, MN: University of Minnesota Press.

Clement, J. (1989). Learning via model construction and criticism. In G. Glover, R. Ronning, & C. Reynolds (Eds.), *Handbook of creativity: Assessment, theory, and research* (pp. 341–381). New York, NY: Plenum.

Clement, J., & Steinberg, M. S. (2008). Case study of model evolution in electricity: Learning from both observations and analogies. In J. J. Clement & M. A. Rea-Ramirez (Eds.), *Model based learning and instruction in science* (pp. 103–116). Dordrecht, The Netherlands: Springer.

Cobb, P., & Tzou, C. (2009). Supporting students' learning about data creation. In W.-M. Roth (Ed.), *Mathematical representation at the interface of body and culture* (pp. 135–171). Charlotte, NC: Information Age Publishing.

Coy, M. W. (1989). Being what we pretend to be: The usefulness of apprenticeship as a field method. In M. W. Coy (Ed.), *Apprenticeship: From theory to method and back again* (pp. 115–135). Albany, NY: State University of New York Press.

Douek, N. (1999). Argumentation and conceptualization in context: A case study on sun shadows in primary school. *Educational Studies in Mathematics, 39,* 89–110.

Duit, R., Roth, W.-M., Komorek, M., & Wilbers, J. (2001). Fostering conceptual change by analogies: Between Scylla and Charybdis. *Learning and Instruction, 11*, 283–303.

Ely, R. (2011). Envisioning the infinite by projecting finite properties. *Journal of Mathematical Behavior, 30*, 1–18.

Gainsburg, J. (2007). Mathematical disposition of engineers. *Journal of Research in Mathematics Education, 38*, 477–506.

Garfinkel, H. (2007). Lebenswelt origin of the sciences: Working out Durkheim's aphorism. *Human*

Studies, 30, 9–56.

Giannakoulias, E., Mastorides, E., Potari, D., & Zachariades, T. (2010). Studying teachers' mathematical argumentation in the context of refuting students' invalid claims. *Journal of Mathematical Behavior, 29,* 160–168.

Greer, B. (2004). The growth of mathematics through conceptual restructuring. *Learning and Instruction, 14,* 541–548

Hadas, N., Hershkowitz, R., & Schwarz, B. B. (2000). The role of contradiction and uncertainty in promoting the need to prove in dynamic geometry environments. *Educational Studies in Mathematics, 44*, 127–150.

Hanson, N. R. (1958). *Patterns of discovery: An inquiry into the conceptual foundations of science*. Cambridge, England: Cambridge University Press.

Have, P. ten (1999). *Doing conversation analysis: A practical guide*. London, UK: Sage.

Huntley, M. A., Marcus, R., Kahan, J., & Miller, J. L. (2007). Investigating high-school students' reasoning strategies when they solve linear equations. *Journal of Mathematical Behavior, 26,* 115–139.

Kang, S., Scharmann, L., Noh, T., & Koh, H. (2005). The influence of students' cognitive and motivational variables in respect of cognitive conflict and conceptual change. *International Journal of Science Education, 9,* 1037–1058.

Komatsu, K. (2010). Counter-examples for refinement of conjectures and proofs in primary school mathematics. *Journal of Mathematical Behavior, 29,* 1–10.

Kramarski, B., & Revach, T. (2009). The challenge of self-regulated learning in mathematics teachers' professional training. *Educational Studies in Mathematics, 72,* 379–399.

Kuhn, T. S. (1970). *The structure of scientific revolutions* (2nd ed.). Chicago, IL: University of Chicago Press.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge, UK: Cambridge University Press.

Lave, J., Murtaugh, M., & de la Rocha, O. (1984). The dialectic of arithmetic in grocery shopping. In B.

Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 67–94). Cambridge, MA: Harvard University Press.

Levenson, E., Tsamir, P., & Tirosh, D. (2007). Neither even nor odd: Sixth grade students' dilemmas regarding the parity of zero. *Journal of Mathematical Behavior, 26,* 83–95.

Merenluoto, K., & Lehtinen, E. (2004). Number concept and conceptual change: Towards a systemic model of the processes of change. *Learning and Instruction, 14,* 519–534.

Mueller, M. F. (2009). The co-construction of arguments by middle-school students. *Journal of Mathematical Behavior, 28,* 138–149.

Nickerson, R. S. (1998). Confirmation bias: A ubiquitous phenomenon in many guises. *Review of General Psychology, 2*, 175–220.

Noss, R., Bakker, A., Hoyles, C., & Kent, P. (2007). Situating graphs as workplace knowledge. *Educational Studies in Mathematics, 65,* 367–384.

Rorty, R. (1989). *Contingency, irony, and solidarity*. Cambridge, England: Cambridge University Press.

Perry, M., Church, R. B., & Goldin-Meadow, S. (1988). Transitional knowledge in the acquisition of concepts. *Cognitive Development, 3,* 359–400.

Preece, J., & Janvier, C. (1992). A study of the interpretation of trends in multiple curve graphs of ecological situations. *School Science and Mathematics, 92*, 299–306.

Romano, C. (1998). *L'événement et le monde* [The event and the world]. Paris, France: Presses Universitaires de France.

Ron, G., Dreyfus, T., & Hershkowitz, R. (2010). Partially correct constructs illuminate students' inconsistent answers. *Educational Studies in Mathematics, 75,* 65–87.

Rorty, R. (1989). *Contingency, irony, and solidarity*. Cambridge, UK: Cambridge University Press.

Roth, W.-M. (1996). Where is the context in contextual word problems?: Mathematical practices and products in Grade 8 students' answers to story problems. *Cognition and Instruction, 14,* 487–527.

Roth, W.-M. (2003a). From epistemic (ergotic) actions to scientific discourse: Do gestures obtain a bridging function? *Pragmatics & Cognition, 11*, 139–168.

Roth, W.-M. (2003b). *Toward an anthropology of graphing: Semiotic and activity theoretic perspectives*.

Dordrecht, The Netherlands: Kluwer Academic Publishers.

Roth, W.-M. (2004a). “Tappen im Dunkeln”. Der Umgang mit Unsicherheiten und Unwägbarkeiten während des Forschungsprozesses [“Groping in the dark: Handling the uncertainties and vagaries during the research process]. *Zeitschrift für Qualitative Bildungs-, Beratungs-, und Sozialforschung, 5*, 155–178.

Roth, W.-M. (2004b). Theory and praxis of metacognition. *Pragmatics & Cognition, 12*, 157–172.

Roth, W.-M. (2004c). What is the meaning of meaning? A case study from graphing. *Journal of Mathematical Behavior*, *23*, 75-92.

Roth, W.-M. (2005). *Doing qualitative research: Praxis of methods*. Rotterdam: SensePublishers.

Roth, W.-M. (2009). Radical uncertainty in scientific discovery work. *Science, Technology, & Human Values, 34*, 313–336.

Roth, W.-M. (2012a). Data generation in the discovery sciences – learning from the practices in an advanced research laboratory. *Research in Science Education*. doi: 10.1007/s11165-012-9324-z

Roth, W.-M. (2012b). Mathematical learning, the unseen and unforeseen. *For the Learning of Mathematics, 32* (3), 15–21.

Roth, W.-M. (2013). Undoing decontextualization or how scientists come to understand their own data/graphs. *Science Education, 97*, 80–112.

Roth, W.-M., & Bowen, G. M. (1999). Complexities of graphical representations during lectures: A phenomenological approach. *Learning and Instruction, 9*, 235–255.

Roth, W.-M., & Bowen, G. M. (2003). When are graphs ten thousand words worth? An expert/expert study. *Cognition and Instruction, 21*, 429–473.

Roth, W.-M., McGinn, M. K., & Bowen, G. M. (1998). How prepared are preservice teachers to teach scientific inquiry? Levels of performance in scientific representation practices. *Journal of Science Teacher Education, 9*, 25–48.

Roth, W.-M., McRobbie, C., Lucas, K. B., & Boutonné, S. (1997). Why do students fail to learn from demonstrations? A social practice perspective on learning in physics. *Journal of Research in Science Teaching*, *34*, 509–533.

Schegloff, E. A. (1996). Confirming allusions: Toward an empirical account of action. *American Journal of Sociology, 102,* 161–216.

Sfard, A., & Linchevski, L. (2010). The gains and the pitfalls of reification—the case of algebra. In P. Cobb (Ed.), *Learning mathematics: Constructivist and interactionist theories of mathematical development* (pp. 87–124). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Singer, F. M., & Voica, C. (2008). Between perception and intuition: Learning about infinity. *Journal of Mathematical Behavior, 27,* 188–205.

Suchman, L. A. (1987). *Plans and situated actions: The problem of human-machine communication*. Cambridge, UK: Cambridge University Press.

Tabachneck-Schijf, H. J. M., Leonardo, A. M., & Simon, H. A. (1997). CaMeRa: A computational model for multiple representations. *Cognitive Science, 21,* 305–350.

Tairab, H. H., & Khalaf Al-Naqbi, A. K. (2004). How do secondary school science students interpret and construct scientific graphs? *Journal of Biological Education, 38,* 127–132.

Tall, D. (2001). Natural and formal infinities. *Educational Studies in Mathematics, 48*, 199–238.

Temple, S. E., Plate, E. M., Ramsden, S., Haimberger, T.J., Roth, W.-M., & Hawryshyn, C. W. (2006). Seasonal cycle in vitamin A_1/A_2-based visual pigment composition during the life history of coho salmon (*Oncorhynchus kisutch*). *Journal of Comparative Physiology A: Sensory, Neural, and Behavioral Physiology, 192*, 301–313.

Throndsen, I. (2011). Self-regulated learning of basic arithmetic skills: A longitudinal study. *British Journal of Educational Psychology, 81,* 558≠578.

Tirosh, D., Tamir, P. (2004). What can mathematics education gain from the conceptual change approach? And what can the conceptual change approach gain from its application in mathematics education. *Learning and Instruction, 14,* 535–540.

Vosniadou, S., & Verschaffel, L. (2004). Extending the conceptual change approach to mathematics learning and teaching. *Learning and Instruction, 14,* 445–451.

Wald, G. (1957). The metamorphosis of visual systems in the sea lamprey. *The Journal of General Physiology, 40,* 901–914.

Williams, J., & Wake, G. (2007). Black boxes in workplace mathematics. *Educational Studies in Mathematics, 64,* 317–343.

Yankelewitz, D., Mueller, M., & Maher, C. (2010). A task that elicits reasoning: A dual analysis. *Journal of Mathematical Behavior, 29,* 76–85.

Appendix: transcription conventions

For the transcriptions, I use the following conventions. As a basic principle, everything is written using small letters and the transcription is phonetic. Sound-words that run into each other are transcribed that way unless the run-in sign "=" is used—e.g., when it would be difficult to distinguish pronunciation (e.g., "a=one").

Notation	Description	Example
(0.14)	Time without talk, in seconds	more ideas. (1.03) just
((*turns*))	Verbs and descriptions in double parentheses and italics are transcriber's comments	((*circles 2 data points*))
[ ]	Square brackets in consecutive lines indicate overlap	07 R: with [photo]period 08 S: [well]
<<p> >	Piano, lower than normal speech volume	042 T: <<p>that would be dramatic>
prETty	Capital letters indicate louder than normal talk indicated in small letters.	Now it dOESnt mean
–,?;.	Punctuation is used to mark movement of pitch (intonation) toward end of utterance, flat, slightly and strongly upward, and slightly and strongly downward, respectively	C: okay; save that. (0.27) do you want me to blEACH it?.
=	Equal sign indicates that the phonemes of different words are not clearly separated	i=ll

Captions

Fig. 1. The scientific dogma predicted porphyropsin levels during a short period of time (6 weeks) prior to and at the beginning of migration from about 90% to about 5% vitamin-A_2.

Fig. 2. All data points from all sources related to the Rodney Creek Hatchery are plotted on the same graph and modeled by a single sinusoidal curve.

Fig. 3. PowerPoint slide of porphyropsin levels in coho salmon at different stages in their life cycle (smolt, parr) and from different settings (hatchery, estuary, university tank) are plotted together with temperature and day length.

Fig. 4. In this PowerPoint slide, the team presents the data from the Kittila data, with only the hatchery-raised data modeled by the curve and the other data treated as “off the chart.”

Fig. 5. In this PowerPoint slide, the team presents side by side the data from the two hatcheries for both wild and hatchery-raised coho salmon.

Fig. 6. Two graphs, one for rods (left) the other one for red cones (right), from the PowerPoint presentations showing the results of porphyropsin and λ_{max} measurements in the thyroxin experiment.

Figure 1
Click here to download high resolution image

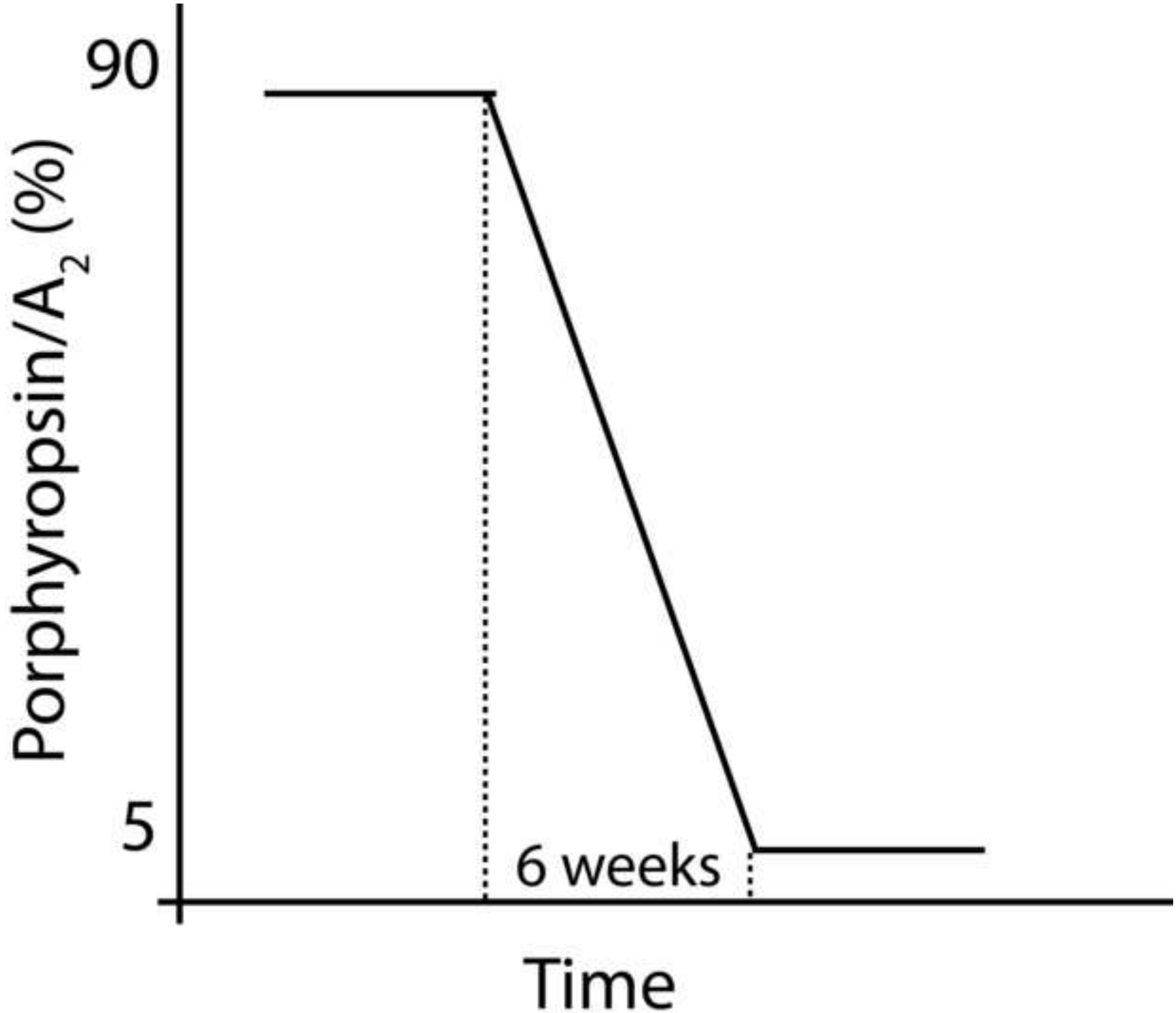

Figure 2

Click here to download high resolution image

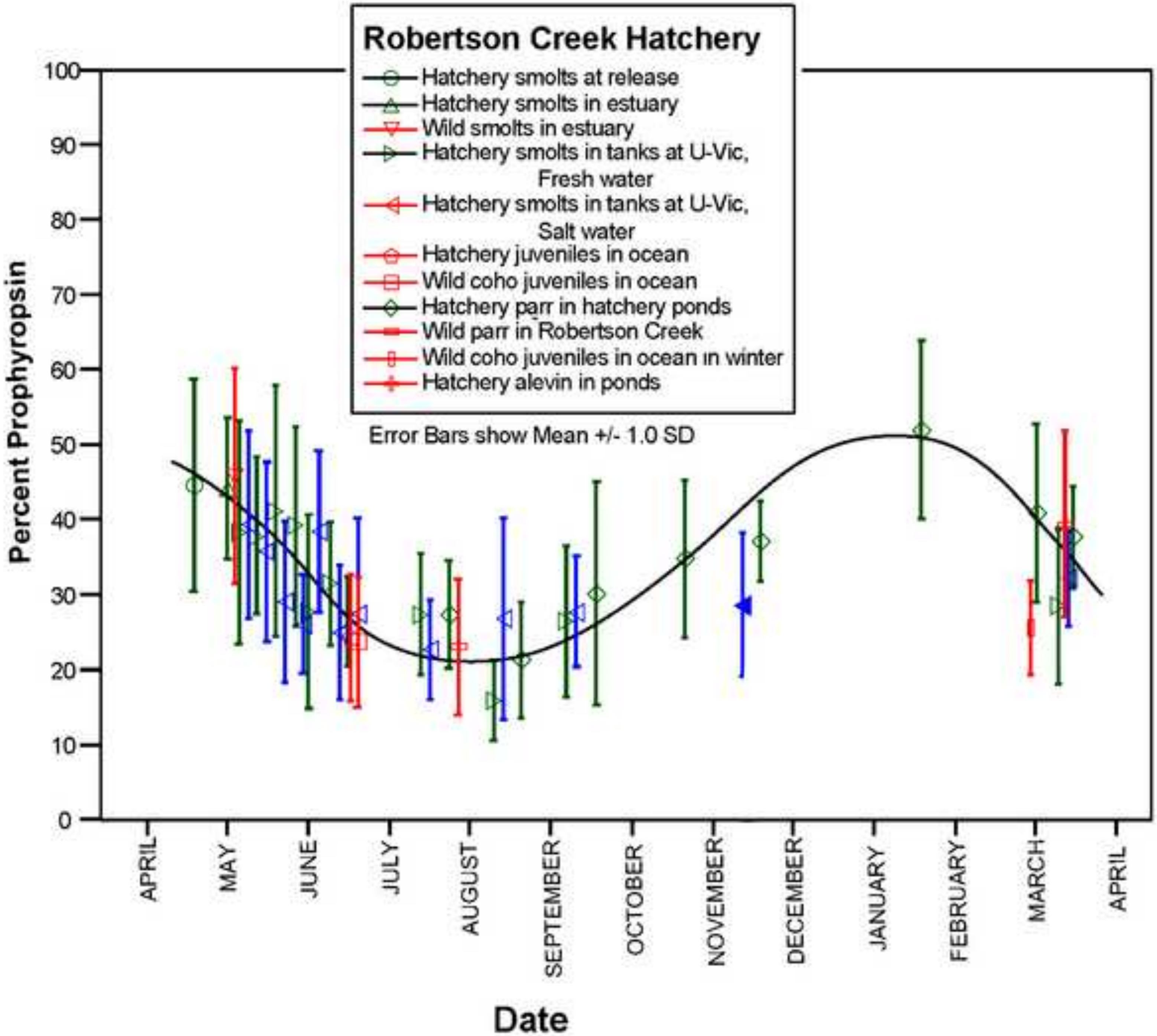

Figure 3
Click here to download high resolution image

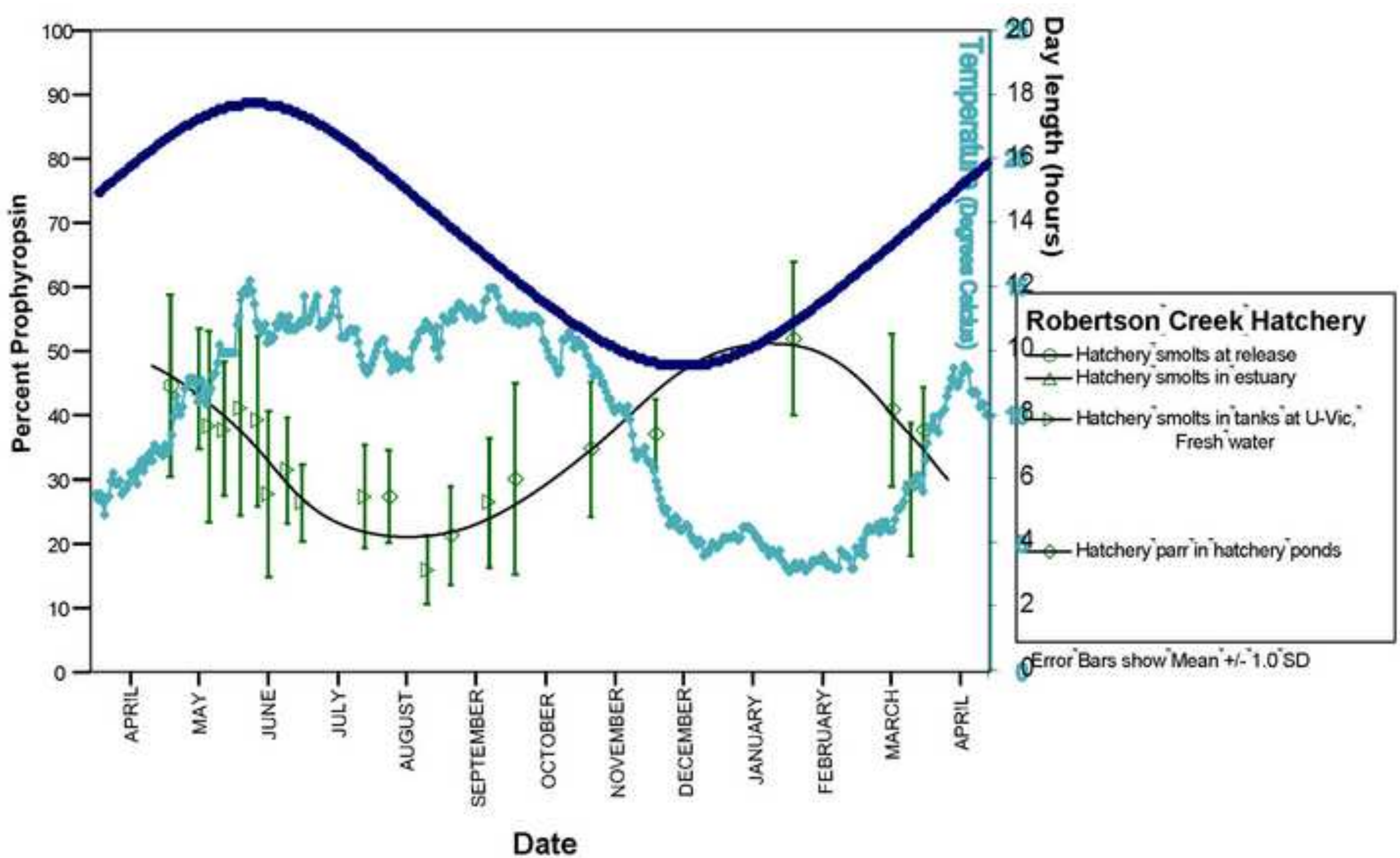

Figure 4
Click here to download high resolution image

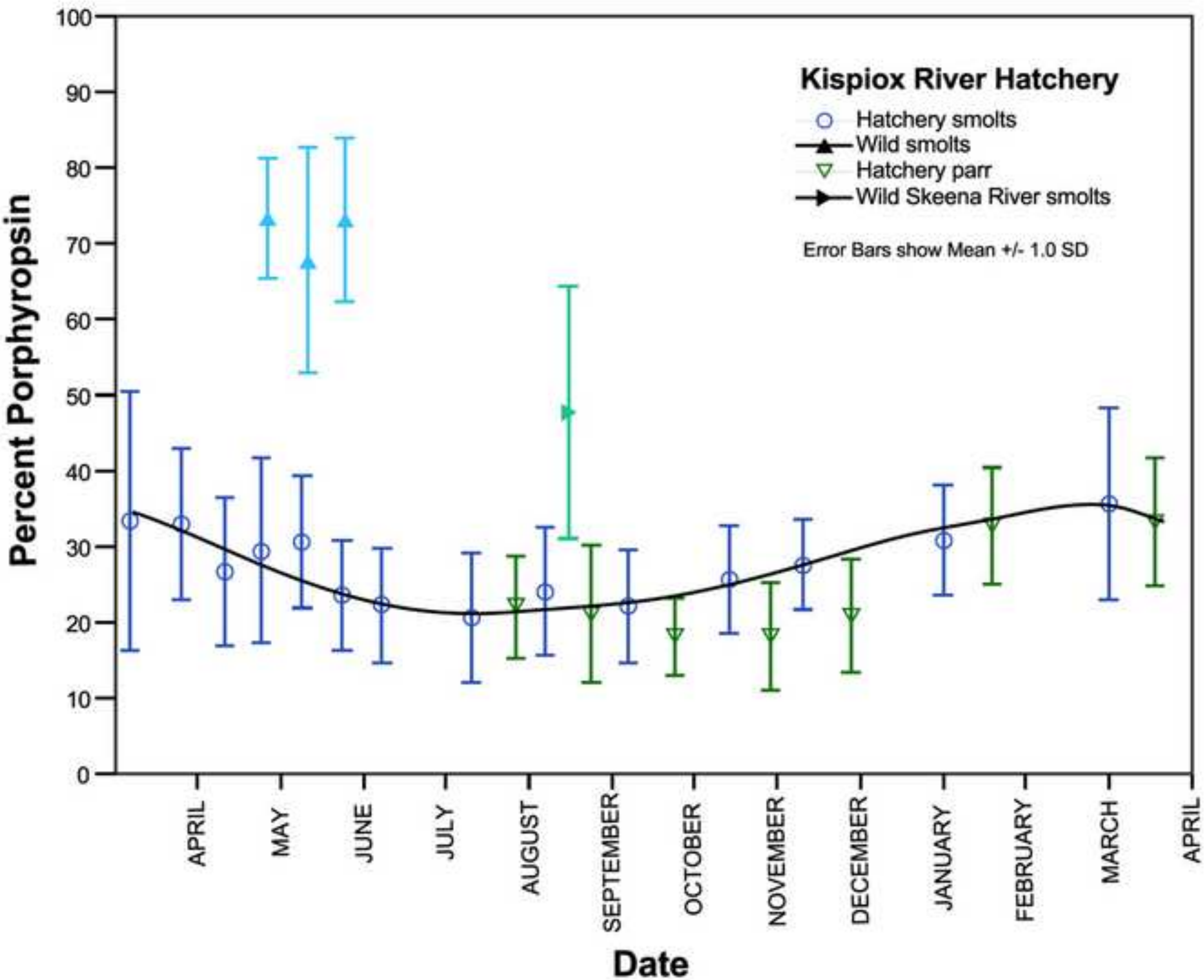

Figure 5
Click here to download high resolution image

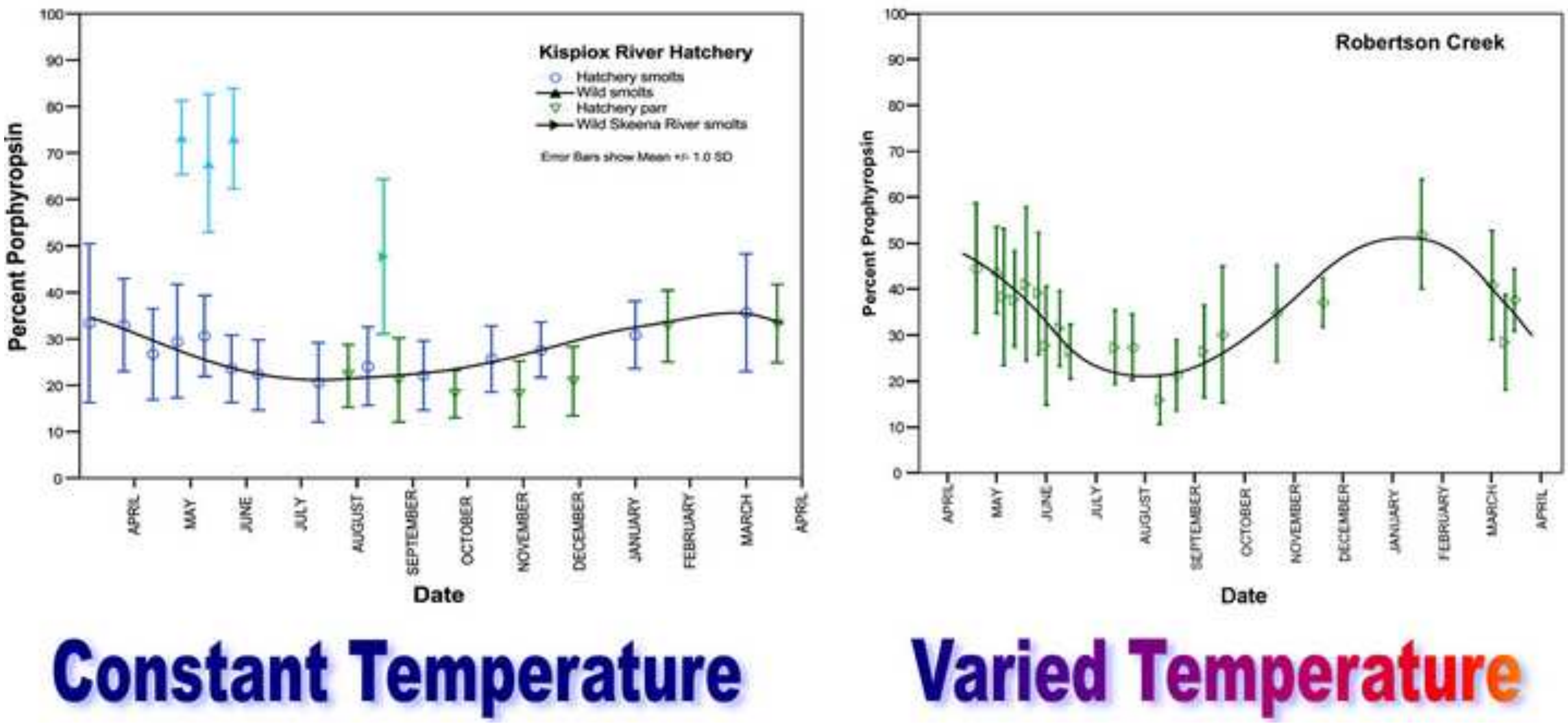

Figure 6

Click here to download high resolution image