Societal Mediation of Mathematical Cognition and Learning[1]

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Abstract: Cultural-historical activity theory, originally developed by A. N. Leontjew in the 1960s and 70s, has been experiencing a revival in Western scholarship over the past two decades. Whereas the analytical category "activity" was offered up as a category that denotes systems that contribute to the production of things to meet generalized, societal needs, it tends to be used in mathematics education to refer to simple tasks and exercises − e.g., calculating the areas of geometrical figures. In this paper I argue, drawing on empirical examples from my own research, that the very strengths of cultural-historical activity theory are not realized, which lie in the affordance to integrate macro-sociological with micro-psychological dimensions of cognition and learning. Moreover, in the current way the theory is used, its potential as a critical theory is given short shrift in the service of emancipatory efforts that uproot the tendency for schooling to reproduce bourgeois society and its class structure.

Keywords: activity theory, mediation, reproduction, unit analysis

1 Introduction

The research on knowing and learning in mathematics education, consistent with the dominant constructivist epistemology, tends to focus on individuals and school classes while they do tasks and exercises − though there are indeed other efforts, such as ethnomathematics (D'Ambrosio, 2008) and critical mathematics education (e.g., Skovsmose, 2011), that are concerned with societal mediations of mathematical cognition and learning. The purpose of this article is to sketch out cultural-historical activity theory in support of an argument for using an inclusive theoretical framework that allows us to understand the totality of levels that characterizes the participation in mathematical practices or in practices that use mathematics as a means of production. Consider the following two vignettes, on which I draw as empirical examples in the course of articulating the theory. As part of a two-year ethnographic effort, my research team videotaped seventh-grade students in the course of their regular mathematics classes and while participating in an experimental science curriculum that made it possible for students to participate in an already existing environmental activist movement.

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Vignette 1: In one of the mathematics classes, the teacher had decided to support the teaching of science by introducing the students to graphs and graphing. The teacher had prepared a task sheet containing several columns with numbers. He explained that the task was to find relationships between any two columns of numbers of the students' choice. He also prompted the students by telling them that they "had learned about a variety of graphing techniques," and then he listed, among others, pie charts, bar graphs, and scatter plots. After handing out the sheets, he reminded his students to use pencils, which would allow them to make corrections to their work if needed. The camera follows Jamie, "one of the better students," and Davie. Jamie settles down to begin working on the task, whereas Davie queries the teacher as soon as the latter passes their desk.

Davie: What are we supposed to do? Like when is, um, um, like a bar graph? (*Points to an example of a bar graph in the book in front of him.*)
Teacher: A scatter plot graph and choose the speed and one of these other categories.
Davie: We are comparing this (*he points to one column on the task sheet*) and this (*points to a second column*) and this (*points to a third column*).
Teacher: So make a scatter plot that compares two things.
Davie: But how do I make a scatter plot? (*He restlessly gets out of his seat and appears to move away from it.*)
Teacher: You are not going to do it? (*He gently pushes Davie back into his seat.*)
Davie: Jamie is going to do it.
Teacher: And you are going to do the rest of it? Okay!

"What do we have to do?," Jamie asks Davie. "I thought you knew what to do," the latter responds. Jamie orients to the task by beginning to draw axes on his lined paper, whereas Davie gets up and throws some crumbled paper at another group of students. He then takes his bottle of lemonade and shares it with other students in the class. Every now and then he returns, watches Jamie for a while, and then continues walking about the classroom. At one instant while Davie observes Jamie, the teacher passes by their desk. Jamie uses it as a way of addressing the teacher: "We don't get this." But the teacher moves on. When he returns, Davie, who has been talking to a neighbor, again watches Jamie. The teacher asks Davie, "Are you contributing?" to which the latter responds: "Some." The teacher moves on again when a student from another group asks, "What kind of graph are you using?" Davie answers, "I don't know how to do it." He subsequently continues walking about the classroom, talking to others, sharing his drink, and at times watching them at work on the task. In the end, Davie will have spent contributing less than 2 of the 26 minutes that the teacher had allotted to the graphing task. The teacher and the teaching intern later tell me that Davie has "attention deficit hyperactive disorder" and "severe writing problems" and has been identified, for these reasons, to be "learning disabled."

In this instance, we observe a student (Davie) who, by all accounts, behaves in the mathematics class consistent with the label that the school and psychologists have pinned on him. Davie engages little with the task and, as a result, does not produce what the teacher wants him to produce. Repeatedly in the course of this lesson, Davie says that he does not know what to do and that his partner Jamie is going to do it. He spends much of the lesson "off-task," talking to other students, sharing his drink, and sometimes interrupting others by bugging them. He apparently knows little and, during this lesson, does not augment what he knows by learning what to do and how to do it when requested to analyze tables of numbers.

Vignette 2: The same class of student participated in an experimental curriculum that I had designed and taught, in this instance, together with the teaching intern. The curriculum allowed students to participate in and realize environmental activism. Encouraged by the call that an environmentalist activist group published in the local newspaper – concerning help required in finding out and doing something about the sorry state of the creek that drains the watershed in which the municipality is located – Davie and his peers decided to produce knowledge about the creek and to publish what they learn at an open-house event, which the environmentalists planned later that year. Davie and Stevie, another "learning disabled" student, emerged from this unit as widely recognized experts. That is, this science unit, which essentially existed in creating knowledge and services that were contributed to this farming community by working on a community-relevant problem, set up situations in which Davie and Stevie turn out to be functional and literate individuals. Nobody watching the videotapes without knowing their background actually would label them to be learning disabled. More pertinently, so-called learning-disabled students developed tremendous forms of knowledgeability with respect to scientific and mathematical representations that depicted Hagan Creek in its current state at the time. In this unit, they became experts on environmentalism in their school.

Davie and Stevie not only became competent participants in environmentalism but also were among the first to present their findings in another class of the same grade. They taught the teacher of that class on how to design and conduct research. They also signed up as mentors of their peers from another class when these, too, came to participate in environmentalism. Here, for example, although Davie and Stevie had not done well on the task of producing graphs during their mathematics lessons, they in fact tutored their peers on how to transform data that they had collected (Figure 1a) into a graph that showed the relationship between the width of the creek and the speed of the water at two different locations (Figure 1b).

My research team collected a lot of evidence for the mathematical competencies that were produced while Davie and Stevie assisted their peers (Figure 1b). That such would be the case is not self-evident, for even professors teaching undergraduate courses produce erroneous translations from one form of representation to a graph and vice versa (Roth & Bowen, 1999). However, the present results are consistent with other research: when students learn mathematical representations in the context of school-based tasks of their own design, which allow them certain levels of control over the object and the means to produce knowledge claims that they have to "sell" to their peers and teachers, then they become highly competent (e.g., Roth & Bowen, 1994).

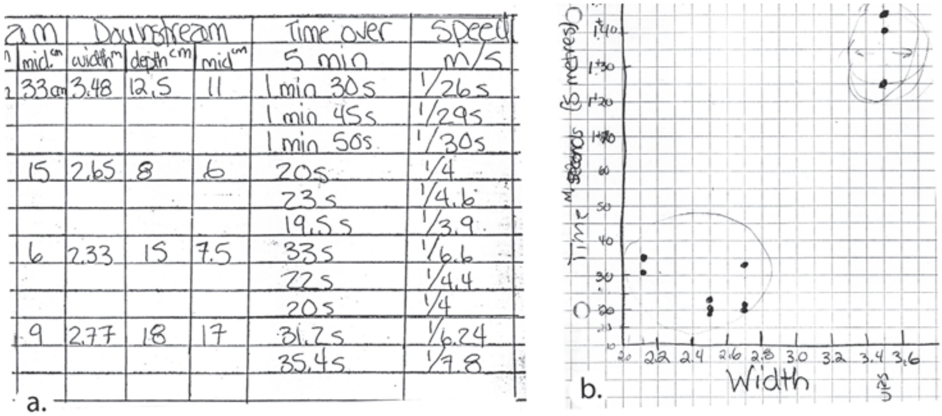

Figure 1 Davie has tutored a group of peers in another seventh-grade class in producing this graph from the data they collected at Hagan Creek.

Our study ended when Davie and Stevie took part in the open-house event presenting, alongside the environmental activists, their own mathematical representations that articulate features of the creek. Just as the water technician working at a local farm and member of the environmental activist group represents a graph exhibiting water levels and rainfall in the course of a year, the two students present their results in the form of graphs. Our video recordings show that adults actively engaged Davie while he talked about the findings of his and Stevie's research presented in the mathematical representations. It was precisely for this instant that they had worked: to contribute, in response to the environmentalists' call for community participation, to their municipality by informing its members of the not-well-known sorry state of the creek.

The differences between the two courses are striking. In one, Davie, Stevie, and students like them exhibit behaviors and performances of the type that warranted the school to treat them as "learning disabled." The result of their participation in the tasks led the school system to regard them as deficient. Whereas this also meant that the students received "special help," this help required them to leave their classroom context, and, therefore, interrupted any relation that they have had with others and which normally supported them in their participation. On the other hand, our videotapes testify to the tremendous forms of knowledgeability exhibited and developed wherever and whenever the two students appear. They were teaching and otherwise assisting others in doing research and representing research by means of mathematical representations. How are such striking differences possible given that the two vignettes derive from the same time period? Going theories have difficulties explaining these differences, because these pin knowledge and knowing to the individual mind. Cultural-historical activity theory, on the other hand, is capable of explaining the differences, which arise, in part, from the very structures of society. In the following, I sketch this theory by exemplifying pertinent aspects with materials from the same study. The theory does not only explain the differences but also provides the tools for critiquing and changing the (societal) conditions in which students learn.

2 Cultural-Historical Activity Theory

Cultural-historical activity theory was created to understand the specific nature of human consciousness, the psychic reflection of material reality that emerges in the course of labor and produces ordinary and everyday society in the way we are familiar with it (Leontyev, 1981). It theorizes the psyche as having developed in the course of evolution, where existing animal forms of cooperative behavior led to a generalized division of labor during the time of anthropogenesis. Generalized division of labor means to participate in labor activity in a way that a split is achieved between the goals of immediate actions and the motives of the overall activity. Thus, in the early forms of division of labor, individuals contributed to the provision of food in various ways: in the collective hunt, some participated as beaters and hunters made the kills; others remained at the campsite to maintain the fire or produce the tools required in the hunt. There is therefore a network of activities that together meet the dietary needs of the group (Figure 2). By contributing to the generalized control over collective conditions, hominids expanded their individual control over life conditions and provision of personal needs. This, then, created an important double relation: humans create the conditions under which they live. This also means that they can change these conditions. In the course of cultural history of human collectivities, the increasing division of labor led to societies that exist in the form of highly diversified networks of activities.

2.1 Definition of activity and activity levels

To begin, a clarification is in order. In English, as in other languages, two different German (Russian) concepts important to activity theory, and which must not be conflated (Leontjew, 1982), are rendered by the same term *activity*. Thus, *Tätigkeit* (*deyatel'nost'*) refers to a collective formation that produces something important to the generalized provision of human needs at the level of society. Typical activities include farming, producing tools, and environmental activism. An *Aktivität* (*aktivnost'*), on the other hand, refers to a vital process – e.g., in narrower form the functions of the brain or the vocal cords. In English, both of these terms are referred to as "activity." From here on, the term activity is reserved for use according to the former sense. Activities have conscious societal object/motives. Applied to the educational context, activity in the sense of *Tätigkeit* (*deyatel'nost'*) refers to schooling whereas activity in the sense of *Aktivität* (*aktivnost'*) refers to enacting tasks that students do not necessarily understand (Roth & Lee, 2007). In schooling, the practical object/motive are the diplomas that permit/hinder students to enter

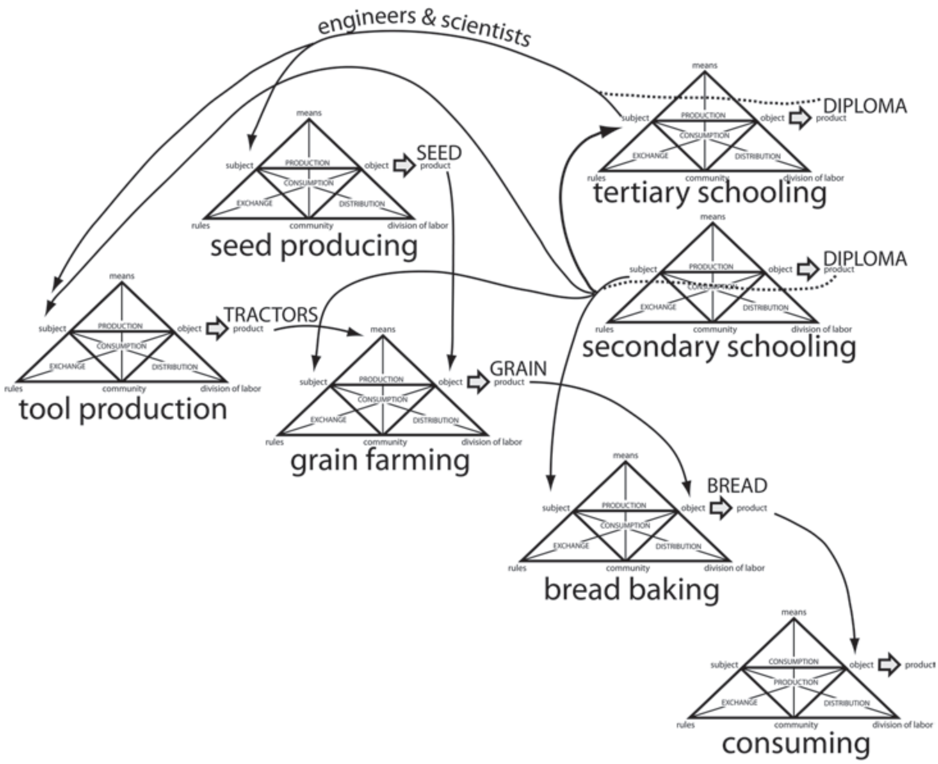

Figure 2 Society is network of activity systems, each of which contributes to meeting the satisfaction generalized human needs (here food). Division of labor can lead to the generation of new forms of activity from existing activities; and new forms of need may be created, also leading to new forms of activity. This synchronic perspective emphasizes the structural dimensions.

other forms of activity (see below). In the theory, *activity* is an analytic category that circumscribes the smallest unit that makes sense. Although we may identify *moments* of an activity (i.e., apparent sub-structures) these *moments* do not make sense independently of the total activity, which also manifests itself in other moments. That is, we must not think the system in terms of the sum of its manifestations because the relation arising from a summation would produce only *external*, arbitrary relations between the different manifestations. Cultural-historical activity theory was designed to think the phenomenon in terms of the *inner* relations of activity.

It takes a number of goal-directed *actions* to realize an activity. That is, an activity exists only because there are actions that realize it, but the action is mobilized only because there is an activity toward which it is directed. Schooling requires students to engage in tasks (e.g., constructing a graph given a table of numbers), but these tasks are chosen precisely because they realize schooling (i.e., lead to grades, grade reports, and diplomas). The activity and the actions that realize it, therefore, are mutually constitutive: they stand in a whole–part relation. An action (e.g., producing a graph) may contribute to different activities (e.g., a task in mathematics class vs. an environmentalist exhibit); it therefore takes its specific sense from the activity. This allows us to understand that Davie knows and understands what to do to produce line graphs and bar charts as part of the environmentalist activity, whereas he "does not know how to do it" in mathematics class. The action of making a coordinate system would have been different if Davie, Jamie, and Stevie had had access to one of the computers available in the mathematics class rather than to paper-and-pencil only.

Actions are realized, in turn, by contextually determined operations. Jamie may have had the goal to draw the axes of the graph but he did not have to think about how to hold the pen, which constitutes an operation. But this operation is mobilized only because there is an action that calls for it. An action and the operations that realize it are mutually constitutive. In the course of individual development, an action – e.g., scaling and labeling the axes of a graph – may become an operation, no longer requiring conscious attention. In fact, tools may embody actions in a crystallized way, such as when a computer makes axes automatically once students choose "line graph" from the appropriate menu.

The advantage of cultural-historical activity theory over other theories is this: it recognizes that the analysis of any instant of object-oriented human praxis requires the conjoint attention to all three levels simultaneously because of the mutually constitutive relations that bind them together. We may not analyze, therefore, what students do when tasked to produce a graph from data presented in tabular form independently of the activity of schooling, on the one hand, and independently of the range of the embodied operations that students have developed as part of their personal histories, on the other hand. Moreover, two instances of praxis separated in time cannot be compared without taking into account the historical nature of societal activity, so that doing graphs or mathematical modeling phenomena in the 1980s, in the absence of rapid and powerful personal computers, was very different from what it is or can be today: a simply click on a pull-down menu and selection of the option "line graph" is all that is required to plot the data presented in two columns of a spreadsheet.

2.2 The structure of activity

As noted, modern societies structurally are the result of a continual division of the labor oriented toward the satisfaction of basic and complex human needs. This labor process involves *production*, the overarching process, which inherently includes *consumption*, *exchange*, and *distribution* as correlative processes, each of which can be understood as a productive activity (Engeström, 1987). This underpinning of cultural-historical activity theory implies that we cannot understand productive activity without also understanding its orientation toward the consumers of its products, the processes by means of which the products come to be exchanged and accumulated within society as a whole. As part of the environmental activist movement, Davie and his peers do not just produce any representation but they orient what they produce towards the anticipated visitors to the open-house event. The completed worksheet from their mathematics class, however, ends up in the garbage can. What most students correctly realize is that the only thing that really counts are evaluated pieces of work that contribute to their overall grades and grade reports. Though not generally included in the current (mathematics) education literature, this four-fold nature of productive activity allows us to understand why students and teachers are oriented towards grading, grades, and grade reports rather than towards mathematical knowledge and understanding. This instrumental focus on grades is the direct result of a capitalist exchange economy, in part held up by schooling, one of the constituent activity systems of Western societies.

The triangular representation of an activity system (Figure 2) expresses the mediated nature of activity. One can also say that the triangle exemplifies the distributed and societal nature of the knowledge that is required by and goes into the production of goods. That is, the representation suggests that we cannot attribute some product independent of the activity generally and its constitutive moments particularly. In a very strong sense, therefore, we may not attribute the product to the subject, who is only one of the *irreducible* moments of activity. We have to attribute it to the totality of the concretely realized activity, itself understood as an unfolding event. This totality involves the tools, division of labor, rules, and community. Thus, for example, if the mathematics teacher had encouraged his students to use the readily available computers and a graphing program, what Davie, Jamie, and Stevie produced would have looked very different. Rather than asking the teacher how to make a scatter plot, Davie simply could have chosen the scatter plot option from the menu. In this way, the use of computers would have realized schooling in a different way, leading to a more advanced activity system; and the difference between the pencil-and-paper- and computer-using systems would be theorized as a *tertiary* contradiction. In fact, the very problems observed in Vignette 1 may have been caused by the technology that the *rules* allowed, i.e. paper-and-pencil, which may have disabled Davie; this disabling relation between Davie and the tool is theorized as a *secondary* contradiction.

By contrast, during the environmental activism, the division of labor, available tools, and surrounding community were enabling, which allowed Davie to participate competently in mathematical practices while transforming the creek into a variety of representations. In the context of schooling, provided with a task the goal of which is not at all evident to him, Davie behaves such that the label "learning disabled" easily can be attributed to him. In the context of environmentalism, Davie produces something for the local society, as he engages in the knowledge exchange opportunities during the open-house event. Activity theory forces us to consider these products as the results of systems rather than those of the individual subjects that are only moment of the system as a whole. Moreover, as he recognizes at the end of the unit, the adults in the community learned *from him*, which means, he recognized the differential distribution of knowledge pertaining to the creek, as a result of which he becomes a knowledge provider.

The foregoing description makes immediately clear the ways in which current mainstream epistemologies differ from cultural-historical activity theory when it comes to locating cognition and learning. In constructivism, *individuals* are the seats of mathematical knowledge, which they construct for themselves and test for viability in the world (von Glasersfeld, 1987). Social constructivists do not differ significantly, as they assume that knowledge, once extra-psychologically constructed with others subsequently is constructed intra-psychologically by individuals (e.g., Cobb, Yackel, & Wood, 1992). In cultural-historical activity theory, on the other hand, any product is the result of the activity system as a whole and, therefore, is marked by all its constitutive dimensions. The fact that the practice-generating dispositions of any subject are formed in and through participation only emphasizes the importance of *activity* as the relevant unit and minimal category of analysis.

In any activity, the actions performed by the (individual, collective) subject are oriented towards some material object, which, through a series of actions, is transformed into the product. It is evident that not any action will do. Actions are a function of the ultimate product, already *envisioned* at the beginning of the work process. That is, whereas the object exists in material form, and is *ideally* reflected in consciousness, the future product initially exists only in ideal form. It constitutes the (collective) motive. Thus, cultural-historical activity theorists understand activity to be oriented to (material, ideal) object and (ideal) motive simultaneously. We may therefore speak of the *object/motive* of activity, an expression that makes salient this dual orientation of activity toward the sensual-material and ideal world. In fact, activity has to be understood in terms of its simultaneous existence at the sensuous-material and ideal levels (Leontjew, 1982). This category allows us to understand that there are considerable differences between the practical work of mathematicians and those of individuals who use mathematics as *means* for transforming their material objects into products. Thus, for example, the water technician in the Vignette 2 has as her object the creek, which she envisions (motive) to transform into a healthy feature of this part of the world. Similarly, our ethnographic study in a fish hatchery showed that the fish culturists did not consider themselves doing mathematics, despite the fact that there were many mathematical representations used and even mathematical modeling of fish populations was occurring, but rather considered themselves as raising fish; they used mathematics as a *means* in their productive labor to ascertain a healthy brood. Both technician and fish culturists were adamant about the differences between what they were doing and mathematics. The forms of consciousness associated with these activities are very different. The forms of consciousness with respect to the graphs that Davie, Jamie, and Stevie (not) produce are very different when they participate in normal schooling, where it is the object, versus when they are part of environmental activism, where these graphs are but means in/of their communicative efforts.

2.3 Society as network of activities

As a result of the progressive division of labor, modern societies have evolved into highly variegated networks of activities that are linked (a) by the exchange-related movement of products and (b) by the movement of persons. Figure 2 sketches some of the activity systems involved in the production of bread, which, as the final stage of production, steps outside of it to become the object of individual need satisfaction. In the overall process, the production process is distributed across the contributing processes of tool manufacture, seed production, harvest, and bakery. Second, individuals participate, in the course of their everyday lives, in multiple activities, for example, working as research scientists or laboratory assistants during the day, being shoppers in the bakery in the evening, and engaging in family life at night. The figure also allows us to understand that what Davie produces in the mathematics class is not part of these networks: it ends up in the garbage can. At other times, the results of such a task may lead to the production of a grade, which is recorded and included with other forms of evaluation, in a summary grade at the end of the school year. These grades and the associated diplomas are the actual material productions of schools. The activity in which Jamie and Davie participate while being in mathematics class is schooling rather than doing mathematics.

Davie does participate in a different activity system as part of his science curriculum: doing environmentalism. This activity *transforms* not only the world but also the social world, not in the least because of events such as the open-house activity. As a subject in this activity system, he contributes to producing mathematical representations that are exchanged and enter other activity systems that constitute society as a whole. The visitors to the open-house event are the “consumers” of the representations. Davie does not do mathematics in the strict sense, however. Instead, he uses mathematical representations to do what he intends to do: representing aspects of Hagan Creek to the general public that attends the open-house event, and those who read associated reports online and in the local newspaper where the results of the children’s work are presented.

There is further indication as to the appropriate unit for the analysis of any material praxis that involves mathematics. Thus, to access their jobs, scientists and technicians require diplomas that they obtain in formal schooling (Figure 2). The production of these diplomas is an outcome of, and therefore characterizes, formal schooling. These diplomas are the products that travel into other systems, opening the doors to potential employees independent of what they may actually know. There is evidence to show that not what someone has learned and knows in mathematics and science comes to count in accessing further education and career but the grades received in the courses that they had taken. That grades rather than knowledge or competent practice matter can also be seen from the fact that some students “cheat” on mathematics examination, for it is only the grade that counts rather than the knowledge that the examination is supposed to measure.

Society continually evolves, which, because of the constitutive nature of its moments, implies that the relations between the activities and all the activities continually change. In a very strong sense, therefore, we have to take into account the history of the activity system and that of society as a whole. As a result of a five-year ethnographic effort in a fish hatchery, we showed that the current observable practices, including those that involved mathematical representations, were strongly related to the 30-year institutional history generally and to the 120-year history of fish hatching in the area more specifically. Thus, we found specific mathematical representations that did not exist in other hatcheries, which was a function of the work processes within the key hatchery's history; and we found younger employees using mathematical representations to model fish populations, whereas the older staff members unfamiliar with computers did not use these. That is, the mathematical performances we observed were mediated culturally and historically, whereby the latter had to be considered at different institutional levels, that is, the history of the field and society, the history of the particular institution, and the personal history of the people.

2.4 Subjectification and personality

Davie is not just a subject in schooling activity but also is part of a family, a shopper, and a participant in environmentalism. The cultural-historical activity theoretic framework presented here allows us to understand Davie as a person simultaneously characterized by very general and highly particular features. Thus, as participant in schooling, doing environmentalism, and shopping, he is a constitutive subject in the associated activity systems. But because the nature of the subject is a function of the activity system, the different subject positions that the person occupies are not equivalent. The longer Davie participates, the more familiar he becomes with the attendant activity as a whole, the more competent his practices become, and the more knowledgeability is associated with his participation. This process of developing as a subject in a particular activity system is denoted by the term *subjectification*. Drawing on ideas from the philosophy of politics, I understand subjectification as "the production . . . of a body and a capacity for enunciation not previously identifiable within a given field of experience" (Rancière, 1995, p. 59). The identification of this body, which occurs "through a series of actions" in and by this body – here those that Davie performs as part of the praxis that leads to the product of activity – "is thus part of the reconfiguration of the field of experience" (p. 59). As the subject of any activity system, Davie also is subject to and subjected to the same field of experience. This leads to the fact that he develops the same practice-generating dispositions modified by minor variations that are characteristic of the phenomenon. That is, as the subject of the activity, Davie takes on highly shared, societal features. He is recognizable, in his forms of participation, *as* student, environmentalist, or shopper. That is, as a participant in each of these activities, Davie realizes the corresponding object/motives, which are common to all corresponding subjects; and, as a participant, the subject Davie also changes together with all the other moments of activity. This is so, not in the least, because the activity, as a societal field, is constitutive of the habitus that generates the practices characteristic of activity.

As any person, who Davie is exceeds the shared, collective features that allow us to recognize him as a participant in a particular activity. But there is something highly singular about him that allows us to distinguish Davie from all other people who engage in the same and similar activities, including his classmates. In cultural-historical activity theory, *personality* is used to understand the integration of the societal features into a unique hierarchical "knot-work" of collective features (Leontjew, 1982). These "'knots' that unite the individual activities are not gathered up as the effects of the subject's biological and mental capacities that lie within him but are created in the system of relations that the subject enters" (p. 178). The knot-work that we denote by name "Davie," therefore, is a partial image of society as a whole, including, most specifically, all those activities that Davie is a member of. Although the individual moments of the resulting hierarchical knot-work are societal through and through, the knot-work as a whole is highly individual (singular), differing from other knot-works by the relative position of and bonding strengths between the contributing object/motives. Thus, for example, for Davie the object/motive of society-transforming environmentalism is dominant at the time of the science unit. The dominance of this object/motive leads to his participation in preparing another teacher to teach this unit, in the supervision of his same-age peers in another class, and to his participation in the open-house event, where he teaches adults and young children alike.

Davie is a working-class student, for he shares with his working-class family and peers a large number of object/motives. This is so because as part of his life in family and peer group, he participates in those activities that are also characteristic of the others; and he tends to value highly those object/motives that also are valued within his inner circle of family, friends, and acquaintances. Thus, schooling generally is a less-favored activity, and he does not subscribe to its object/motive (grades, diplomas) to the same extent that his classmates from bourgeois families do. Among the school subjects, mathematics in particular played a lesser role so that it mattered fairly little to him how well he did. Among the farmers and workers that inhabit the semi-rural municipality, doing mathematics often receives a much lesser esteem than most other activities. In fact, as shown by those families who live on my street, even working-class people do quite well for themselves without knowing any mathematics at all. More specifically, our research among local craftspeople, technicians, and sailors shows that the mathematics they learn in school and college is all but irrelevant in and to their working places for which the formal institutions are supposed to prepare them (e.g., Roth, 2012). For example, the environmentalist who worked as a water technician at a farm alongside the creek learned to handle very different mathematical representations and even produced some of them on her own. Yet she had found the college mathematics courses quite annoying and useless. There is therefore little incentive within the culture that would emphasize the need for participation in mathematical activity or the use of mathematics as a means of production. The forms of participation described in the opening vignettes are therefore not merely expressions of some biological disorder but are phenomena mediated through and through by the structure of the schooling activity and the manner in which it is knot-worked to the other activities.

Davie's way of participating in schooling leads to the production of a final grade report and diploma that only offer him possibilities to become a worker. In and through his participation in schooling, he therefore has contributed to reproducing class society. Although activity inherently offers the possibility of transforming the world, schooling actually stabilizes society rather than changing it into a more equitable one. That is, society changes because it is living. But, in the same way as the human body, society changes slowly leaving intact much of its structural features. Cultural-historical activity theory actually allows us to understand what is happening in terms of change: both the tendencies of transforming reproduction of the world and the reproductive transformation of the world. Mathematics, as means of production and object, may contribute to either process: making this a better society or reproducing it with all its inequities.

3 Opportunities Arriving from the Cultural-Historical Activity Theoretic Framework

Cultural-historical activity theory, as sketched out here, provides us with a means of (a) conducting research in mathematics education and (b) understanding mathematical cognition and learning in ways that other current theories do not. In my earlier work, I had focused, as others do, on individual cognition as if it could be reduced to itself and abstracted from everything else. In the late 1990s, I began an extensive ethnographic study in inner-city schools of Philadelphia, USA (e.g., Tobin & Roth, 2006). Most of the students attending these schools, predominantly African American and from the poorest parts of the city, would end up during their later life in the same conditions that they grew up in.

As a result of my research, I came to understand that in these schools the iniquitous nature of society is reproduced; I also realized that this reproduction occurs in and through the forms of participation in schooling practiced in these institutions. The fault could be placed neither on students or schools nor on schools and students; rather, the reproduction is the result of a dialectical negation that students and schools produce and that hinders students and schools in achieving their transformative potential. It would have been easy to relegate the reproduction to the school level, as sociologists tend to do it. But this is not the whole answer, because forms of participation also reproduce social class even when there are mixtures of working- and middle-class students. This means that the societal reproduction, a macro-level phenomenon, occurs in and through the everyday face-to-face encounters and relations that are reproduced and transformed at the classroom level.

Whereas much sociological research tends to focus on the macro-issues independent of everyday face-to-face work and psychological research tends to focus on interactions and individual engagement independent of more encompassing levels, cultural-historical activity theory *explicitly* theorizes the constitutive relations that exist between the very micro-level of practice and the very macro-levels, as well as every level that we may find in between. It therefore is a framework for researching and understanding how in everyday relations in the mathematics classroom, macro-level phenomena such as societal structure are reproduced. Most notably, this occurs because mathematics tends to be taken as an indicator subject, so that having succeeded well in specific mathematics courses becomes a criterion of entry to particular university programs. Whereas this is known, little studied are the everyday ways of doing mathematics lessons that allow more working-class students to fail more frequently than middle-class students. A cultural-historical activity theoretic perspective, however, might encourage mathematics education researchers to study questions such as how the emergent personalities, with differential emphases given to mathematics as object/motive, mediate the participation in specific mathematics lessons. Such studies require detailed documentation of individuals, the institutions they work in, and the history of schooling in a particular jurisdiction.

Another area of interest to mathematics educators might be the role and importance of "basic skills." In back-to-basics movement, it was assumed that there are basic skills that an individual needed to exhibit to do mathematics. Evidence shows, however, that an improvement of "basic skills," especially among the poorly served students of the US, did not lead to higher levels of knowledgeability that was required for university entrance. In fact, there is no evidence to support claims that the skills for requiring doing a long-hand division are the same that are required for doing a division using a slide rule, hand-held calculator, or computer (Roth, 2008). This contention makes sense within the cultural-historical approach, which emphasizes the mediational role that tools and their history in a changing society have on shaping the products of activity. Instead of thinking about the subject as the seat of knowledge, mathematics educators are encouraged to view the system as the bearer of knowing and the individual subject as constitutive moment. The form of consciousness characteristic of the activity is a function of the tools, and the operations called upon are a function of the actions, which differ when previous skills and actions come to be crystallized in the new means of production. This requires new forms of actions and skills, just as the spreading of cars required new actions and skills and decreased/abolished the need for proficiencies in walking and horse riding. Using the cultural-historical approach, therefore, allows asking questions about the role of particular curriculum elements that are currently assumed to contribute to mathematical knowledge without actual proofs existing to show that these presuppositions bear out. Thus, in very practical terms, we might ask about the need to include curricular topics such as longhand division, factoring of polynomials, or apply exponent laws?

Cultural-historical activity theory also constitutes an appropriate framework for asking questions about what we teach and how we teach it in mathematics classes. Thus, for example, those mathematics educators with ethnomathematical interests tend to report findings about how mathematical representations are used or show up in a variety of often-mundane practices (e.g., Pinxton & François, 2011). Those advocating classical forms of mathematics study how students engage with mathematical objects. The cultural-historical approach, as discussed above, makes evident that the forms of consciousness and knowledgeability in the two approaches are very different. Those who use mathematics as means in the pursuit of non-mathematical object/motives relate in very different ways to the field than those whose object/motive are mathematical objects transformed into other mathematical objects for the purpose of producing mathematical knowledge. The question, then, is not one of different epistemologies – our practical epistemology allows us to understand the differences that will result when the object/motives differ.

Most importantly, in the context of this special issue, cultural-historical activity theory provides a framework for studying the change of mathematical teaching and learning in the context of a critique of bourgeois society and the societal changes that arise from such critique. This is so because the theory has all the characteristics that are required. First, because schooling is an integral and constitutive part of a network of activities that we denote by the term society, changes in society mean changes in the entire network of relations and within the individual nodes (activities). This is so simply because of a whole-part relation between society and individual activities – easily modeled, for example, in artificial neural networks or constraint satisfaction networks. Here, changes in one relation or one node ripple through the entire network, and changes at the network level inherently mean changes in the individual nodes and links.

Second, cultural-historical activity theory explicitly links the structure of society and its moments to history, that is, it understands society as a living phenomenon. When properly using the theory, researchers inherently attend to the structural (synchronic) and temporal (diachronic) nature of this human life form. It is inconsistent with the theory to study students like Davie by giving them a psychological test away from the societal and structural resources that normally characterize his life and label him "learning disabled," with grave consequences for what happens to him and his developmental possibilities. The theory forces us to study the synchronic and diachronic relations that end up being grossly attributed to individuals. Societal changes are not the result of outside forces but emerge from the society itself much as a (a) meeting that begins as an expert think-aloud session over mathematical representations may turn into a tutoring session as a result of the practical relations and activity and (b) frustrated and failure-laden engagement in a mathematical task may turn into enjoyment and success. Consequently, cultural-historical activity theory is an ideal framework for understanding how the structural properties of society not only tend to be reproduced but also offer opportunities for overcoming existing societal barriers instituted by achievement on mathematics tests. It offers a way of understanding the person as a unique "knot-work" of societal object/motives. And, perhaps of the greatest importance of all reasons, the cultural-historical approach allows us to understand, theorize, and promote the role of mathematics education in the self-transformation of society into one that is consistent with its democratic ideals rather than one characterized by the capitalist market exploitation at the expense of the most vulnerable members of society.

References

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. *Educational Studies in Mathematics, 23*, 99–122.

D'Ambrosio, U. (2008). Reflections on mathematics as practiced in schools and in academia. *For the Learning of Mathematics, 28*(3), 16.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki, Finland: Orienta-Konsultit.

Leontjew, A. N. (1982). *Tätigkeit, Bewusstsein, Persönlichkeit* [Activity, consciousness, personality]. Cologne, Germany: Pahl-Rugenstein.

Leontyev, A. N. (1981). *Problems of the development of the mind*. Moscow, Russia: Progress Publishers.

Pinxton, R., & François, K. (2011). Politics in an Indian canyon? Some thoughts on the implications of ethnomathematics. *Educational Studies in Mathematics, 78*, 261–273.

Rancière, J. (1995). *La mésentende: Politique et philosophie* [Misunderstanding: Politics and philosophy] Paris, France: Galilée.

Roth, W.-M. (2008). Where are the cultural-historical critiques of "back to the basics"? *Mind, Culture, and Activity, 15*, 269–278.

Roth, W.-M. (2012). Rules of bending, bending the rules: The geometry of conduit bending in college and workplace. *Educational Studies in Mathematics*. doi: 10.1007/s10649-011-9376-4

Roth, W.-M., & Bowen, G. M. (1994). Mathematization of experience in a grade 8 open-inquiry environment: An introduction to the representational practices of science. *Journal of Research in Science Teaching, 31*, 293–318.

Roth, W.-M., & Bowen, G. M. (1999). Complexities of graphical representations during lectures: A phenomenological approach. *Learning and Instruction, 9*, 235–255.

Roth, W.-M., & Lee, Y. J. (2007). "Vygotsky's neglected legacy": Cultural-historical activity theory. *Review of Educational Research, 77*, 186–232.

Roth, W.-M., & Radford, L. (2011). *A cultural-historical perspective on mathematics teaching and learning*. Rotterdam, The Netherlands: Sense Publishers.

Skovsmose, O. (2011). Critique, generativity and imagination. *For the Learning of Mathematics, 31*(3), 19–23.

Tobin, K., & Roth, W.-M. (2006). *Teaching to learn: A view from the field*. Rotterdam, The Netherlands: Sense Publishers.

von Glasersfeld, E. (1987). Preliminaries to any theory of representation. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 215–225). Hillsdale, NJ: Lawrence Erlbaum Associates.

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