The relationality in/of teacher-student communication

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Abstract   In mathematics education, student–teacher communication is recognized to constitute an important dimension in/of mathematical learning. Significant effort has been made in recent decades to depart from a focus on the individual in which teachers and student simply use communication to express, to and for others, their private knowledge or thinking. In this paper, we continue this departure taking as a starting point the observation that (mathematical) communication is possible only when there is a relation with others: Communication is the relation with others. That is, we present a way of thinking about student–teacher communication in which geometrical being-in-the-know is conversationally produced. Using fragments of elementary classroom conversations involving three-dimensional geometry as a tool to flesh out this theoretical study, we illustrate (a) how being-in-the-know-with can be recognized in asking and responding to questions involving mathematical concepts and (b) how conversations are then the fine-tuning of being-in-the-know relations in which mathematical ideas can come forth even in those instances where not-being-in-the-know is asserted.

Keywords   Relationality · Communication · Knowing (being-in-the-know) · Teachers · Students

"Language" is not an instrument of communication, and communication is not an instrument of Being; communication is Being. (Nancy 2000, p. 92)

One could say even more simply that Being is communication. (p. 28)

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From individual subjects to relations

More than three decades ago, researchers in mathematics education exhibited a grow-ing interest in the relation between communication and mathematics teaching and learning (Austin and Howson 1979). Classroom conversations have since become of increasing interest to mathematics educators (e.g., Brown 2001; Pimm 1987; Zevenbergen 2000) with a growing interest in the fine-grain semiotic analysis of teachers' and students' utterances as well as gestures, body orientations and move-ments, intonation, rhythm, or sound, all of which constitute integral parts of mathe-matical thinking (e.g., Bautista and Roth 2012a, b; Radford et al. 2008, Bussi and Mariotti 2008). The importance of the collective aspect of communication also runs through the work of scholars who study language in classroom mathematics talk (e.g., Barwell 2012; Roth and Thom 2009; Sfard 2008; van Oers 2001).

In the past, semiotic and communicative analyses have focused on what individual speakers contribute to the conversation and how individuals "make meaning" on the basis of what has been said. But a conversation does not consist of independent utterances; rather, it is a collective phenomenon (Bakhtin 1924/1994; Vološinov 1930). The next step in the evolution of talk in mathematics classroom is, therefore, to try and consider how conversations in mathematics classrooms are irreducible social phenomena in their own right, meaning that they cannot be taken as an assemblage of individual inputs (e.g., Roth 2013c; Roth and Hsu 2010). Indeed, studies in the philosophy of language strongly articulate how words and utterances ought not be taken as "means" for the enunciation or the negotiation of ideas and concepts, but rather as the very fabric in which thinking takes place as a soci(et)al relation (Bakhtin 1975; Leont'ev 1969; Vološinov 1930; Vygotskij 2005 [1]). Not easily grasped, this idea actually implies that teacher–student conversations in mathematics classroom should not be taken as instances during which children and educators use mathematics (and everyday) language to share their (mathematical) thinking so that the former can conceptually develop. In this view, students do not learn mathematics language to clothe in words what they individually think and talk about as part of a collective endeavor, or learn to understand the mathematical meaning of a word so they can properly use it for thinking or speaking mathematically with others. Rather, this idea implies that conversations in the mathematics classroom are manifestations of relations that are the "higher-order psychological functions" (e.g., mathematical concepts or processes) subsequently attributed to individuals (Vygotskij 2005). These relations constitute being-in-the-know-with one another (Maheux and Roth 2011; Roth and Middleton 2006) with/in language, where the expression being-in-the-know (as the subsequently employed not-being-in-the-know) emphasizes the nature of (not-) know-ing as a form of being rather than as a substance or structure in the mind. These relations—a form of "being-with, as a being-many" (Nancy 2000, p. 12)—allow both students and teachers to develop (Roth and Radford 2010). We therefore propose considering mathematical thinking (doing, learning, communicating) in a new way: emerging from and existing in and as a social relation between speakers, between self and others, and this way recognize that at the foundation and heart

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[1] With citations, we spell Vygotsky's name as it appears on the book cover in romanized form; within the text, we use the normal English spelling.

of mathematical Being—as at the foundation of Being as such (Nancy 2000)—lies "the plurality of beings" (p. 12).

To articulate the change in perspectives that we propose here, consider the following classroom excerpt that the author had analyzed in terms of a teacher asking a mathematical question, followed by students providing answers which are successively evaluated by the teacher until he or she hears a satisfactory answer (Zevenbergen 2000, p. 212):

Excerpt 1

01 Teacher: Here is a diagram of a 3D shape—who can tell me what it is?

02 Student: A rectangular shape.

03 Teacher: Mmm, almost. John?

04 Student: A rectangular cube?

05 Teacher: Nearly, you are half right. Margaret?

06 Student: A rectangular prism.

07 Teacher: Good, that's it. A rectangular prism. Funny word, isn't it.

The author states quite cogently that language "provides a medium through which communication of ideas is made possible, and negotiation of ideas and concept is delivered" (p. 201). Such a perspective on classroom communication is generally recognized as "useful" because it allows researchers to characterize mathematical discourses, for variations in turn-taking patterns can easily be spotted from one classroom to another (e.g., Cobb and Tzou 2009; Sfard 2008). But from the relational perspective that we present here, the focus of such research—attending to what each party says without considering what others says and hear as integral part of each and every utterance—remains the individual. Not taking such conversations as something to which each individual adds in its turn means taking into account that teachers' or students' utterances (a) are always a/in response to something else and as offerings and (b) find their actual signification (as a question, an answer, an explanation, a justification, and so on) in how it is responded to (cf., Barwell 2012; Jaworski and Coupland 2006). In this case, the fundamental point would be that the teacher's first utterance is not a question in itself but turns out to become a question in the students' responses; and it does so only once everything is said and done (Roth and Gardner 2012). Those replies then appear as answers because they follow a teacher's utterance and because they are completed as such in what we recognize as "evaluations." Thus, the teacher alone does not terminate the conversation—e.g., when he or she is satisfied by the answer he or she gets. Rather, the termination of the exchange, because it is part of an irreducible social phenomenon (i.e., the conversation), is a joint achievement of teacher and students. Continuations other than the one actually observed in turn 07 are imaginable, for example, if a student had said, "this word isn't funny at all" or "isn't this just a prism that is a rectangle?".

We choose to present Excerpt 1 because it offers a strong contrast with one of the episodes that was recorded in the course of our research on primary students' mathematical investigations in three-dimensional geometry. In Excerpt 2, a student describes

a rectangular prism (actually a pizza box) as a "flat cube," a proposition which gives rise to a response that differs significantly from that in Zevenbergen's transcript:

Excerpt 2

08 Student: I think this one is more like a rectangular prism because this one's like longer than this one's. But it's like a flat cube.

09 Teacher: So a rectangle prism might be a flat cube?

10 Student: Um hm

11 Teacher: It could be like a cube that's been flattened?

12 Student: Uh hm but some edges are like long and some are shorter than the other ones

13 Teacher: Uh hm. So that makes it a rectangular prism as opposed to a cube, because if it was a cube what would it have to have, what would that box have to have to be a cube?

In this excerpt, although the student and the teacher discuss ideas similar to what we read in the preceding one, working on naming a rectangular prism while observing one of its instances, the descriptive, connective, and elaborative tone differs. From one turn to the next, there is an offering (a proposition to call a rectangular prism a "flat cube" based on features explicitly mentioned in the talk) taken on, reformulated, and glossed (a cube that's been flattened?) in a way that allows the conversation to keep moving and transforming: from one mathematical idea to another (what would that box have to have to be a cube?). This movement links and integrates these ideas in various ways (faces and edges as component describing prisms, qualitative distinctions made by length, necessary particularities of cubes). In both excerpts we can see joint achievement, but whereas in the first case a strong divide seems to emerge between apparent individual knowing, the second appears to assume (as in "suppose" and "take on" but also "takes toward," from Latin ad + sumere) a form of being-in-the-know-with rather than striving toward the construction of individual minds. But can we really make such observations based on a turn-to-turn analysis of actual pieces of teacher–student mathematical communication? What might enable us to do so? And what could be effective implications of doing so for the both the practice of teaching and researching in mathematics education?

This concern with offerings and responses begins to move us to thinking (about) a conversation as an irreducible social phenomenon, rather than thinking (about) it as a composite in which individual subjects are the basic elements. It moves us towards thinking mathematical conversation in terms of relations by taking into account that there are always pairs of turns that constitute the sense and significance of a word (Vološinov 1930). A conversation is something two or more do with (Lat. con[m]-, from cum, with, together) each other, thereby implying two or more interlocutors, who realize a social phenomenon in its own right that cannot be reduced to individual contributions (Durkheim 1919). Using methods such as conversation analysis (Atkinson and Heritage 1984) and giving greater attention to a variety of semiotic resources (e.g., gestures and intonations) found in teachers and students' utterances, researchers in mathematics education increasingly recognize the need for a shift from

the individual to the social. Sfard (2008), for example, makes a strong case for considering observable discourses rather than presupposed isolated individual thinking (carried out by means of words) as our object of study. She describes communication as collective activity, stressing that "interpersonal communication and individual thinking are two faces of the same phenomenon" (p. 262), whereas "[w]hatever is done by the learner constitutes a response to a discursive move of interlocutors and an invitation to yet another move on the interlocutors' part" (p. 270).

On the other hand, those who work within such "discursive approaches" to mathematics teaching and learning sometimes leave the reader with the impression that only words and other signs in themselves are of interest, as opposed to real, living people—teachers and students—who contribute to mathematical classroom talk. A case in point is the comment that a student's contribution is interesting only "insofar as it is interpreted by other members of the group as a sign, i.e., as having some meaning for them" (Sierpinska 2005, p. 207). Such an interpretation is possible because the relation in teacher–student communication is often presented as a methodological requirement, a way to "getting around" making (sound) assumptions about what teachers or students think or know (cf., Lerman 2001; Sfard 2008). It would be indeed quite hazardous, in the quoted transcription, to attribute the being-in-the-know of rectangular prisms to Margaret: Her answer emerged after a few other possibilities where discarded or hinted to (you are half right). Is she not, at the best, "the one who said it" and can we not say the same about virtually anything observed in a mathematics classroom? The collective metaphor may also evoke oppressive systems in which the possibility for individual and unintentional though is negated (Sierpinska 2005).

In this paper, we make a case for thinking about learning in mathematics classrooms in terms of relations that are the very forms of being-in-the-know that mathematics educators, at the end of a unit, might want to observe in and attribute to individual students. According to cultural–historical activity theorists, anything that is specifically human, the human life form generally, exists in and as societal relation (Leont'ev 1983; Vygotskij 2005). Such relations manifest themselves as acts of persuasive thinking that always "is half-mine and half someone-else's" (Bakhtin 1975, p. 158). Considering mathematical activity as an evolving relation between self and other, we develop in this paper a relational perspective on teacher–student mathematical communication. Although our proposal is valid for other curriculum areas as well, we pay particular attention to the mathematical aspects at play to assist our readers in appreciating the relevance of considering relations in mathematics education.

We divide the following sections of this theoretical article in two parts. In the first one, we present relationality as a way of thinking teacher–student mathematical communication through its relation with other mathematics education research on the topic. Although not exhaustive, the discussion aims at introducing our approach, giving heed to how relationality conceptually connects with, but also distinguishes itself from other perspectives on communication and cognition in mathematics education. In the second half of the article, we continue our theoretical study but this time turning to a concrete illustration of how relationality takes us to analyse teacher–student mathematical communication. We do so using short episodes (such as the one borrowed from Zevenbergen) from a second-grade lesson in geometry as occasions for thinking through the consequences of our proposal; and we explore what can be done with/in this approach. Our analyses thereby contribute to what has been termed "educating

awareness": an endeavor that alerts us "to something worth noticing because it then opens the way to choosing to respond rather than react with a more creative action than would otherwise be the case" (Mason 2009, p. 12). In this way, our study constitutes an important step toward the development of a new way of understanding communication that emphasizes soci(et)al relations as irreducible phenomena in their own right into mathematics education discourse (e.g., Maheux and Roth 2011; Radford and Roth 2011; Roth and Gardner 2012).

Thinking teacher–student mathematical communication relationally

> Communication would be precisely impossible if it should have to begin in the Ego, a free subject of which every other would only be a limitation. (Levinas 1978, p. 189)

> To paraphrase Marx: the psychological nature of man is the ensemble of societal relations shifted to the inner and having become functions of the personality and forms of its structure. (Vygotskij 2005, p. 57, original emphasis, underline added)[2]

Epistemologically speaking, the way one conceives of mathematical knowledge influences the possibilities and the manner of how one analyses and interprets mathematical communication (Steinbring 2005) and vice versa. For example, classroom talk was long considered in terms of information transmission, making mathematics something that exists outside of teachers' and students' experiences. Closer to constructivist epistemologies, communication as the interpretation of signs (a reception model) takes mathematics as essentially constituted within individuals, whose interactions are to support their own cognitive adaptations. In another way, processual models of classroom talk—familiar to those thinking in terms on negotiation of knowledge—present students and teachers as taking part in collectively organized mathematical conversations, where "learning mathematics or learning to think mathematically is learning to speak mathematically" (Lerman 2001, p. 107).

Recent work rooted in an analysis of students' (and teachers') experiences in mathematics classroom conceptualizes the mathematical activity in terms of relationality: being-in-the-know in mathematics classroom is always being-in-the-know-with (Maheux and Roth 2011; Roth 2013c). Thinking mathematics relationally means that students are not taken as meaning-makers in interaction with each others, acquiring or constructing knowledge. Rather, they are seen as approaching mathematical concepts at the moment of their sudden, unforeseen emergence in and through students' activity (Roth 2012b). Mathematical concepts or ways of doing then exist as such (i.e., for the students and the teacher themselves) not in the individual or between individuals (i.e., inter-individually), but as a relationship in which Self and Other co-exist and co-emerge (e.g., Roth and Jornet 2013). Self and Other are abstractions from and outcomes of relations, of being-with (Nancy 2000). Being-in-the-know of mathematics then does not amount to individual or collective constructs or the subjects' ability to make sense of a certain number of ideas: it reveals itself as a space of joint

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[2] All translations are ours.

action and attention (Radford and Roth 2011). Dynamic relationships define the individuals and their world based on the inseparability of the word and "consciousness-for-myself" and "consciousness-for-others" (Marx and Engels 1958, p. 30; Vygotskij 2005, p. 1017). Thus, "where there is a relation, it exists for me" so that "consciousness always-already is a societal product and remains as such as long as there are human beings" (Marx/Engels 1958, p. 30–31, emphasis added). Relationality is a way to recognize the vital role of the Other in mathematical being-in-the-know, situating the Self–Other relationship not only as a useful or necessary means to an end (being-in-the-know) but also as the very direction towards which being-in-the-know, including being-in-the-know-of-mathematics, is orienting us (e.g., Radford 2011; Roth and Gardner 2012). The relations that make mathematics lessons are the loci, forms, and developmental origins of all mathematical being-in-the-know-with that sometime later might be attributed to the individual.

How can this perspective on the mathematics of teachers and students reflect the way we conceive of classroom communication? In the first introductory quotation to this section, the philosopher Levinas states that communication is possible only as the result of a relation with others, rather than the other way around: that is, as something by means of which isolated individuals would come into relation. The second quotation completes Levinas' idea suggesting that "higher psychological functions"—such as inner dialogue, but also the visualization a rectangular prism or the mental search for a name for it (e.g., Excerpt 1)—first are societal relations subsequently attributed to the individual. This is so much so the case that any such individual behavior implies "to relate to oneself as to another" (Vygotskij 2005, p. 1022). These ideas, although rarely explicitly discussed, connect with a number of accounts on teacher–student communication that strive to depart from a concentration on individuals' struggles to achieve mathematical understanding in which students are said to "construct knowledge" inter-individually before constructing it for and within themselves (intra-individually). The semiotic turn we observe in the study of classroom communication—conceptualized as the active creation and use of signs from which mathematical significations[3] emerge (e.g., Radford 2011)—takes first steps in that direction. Challenging the theoretical separation of language and mathematics in use (Ongstad 2006), an awareness of how teachers and students live in a world of "signs" highlights the wide variety of semiotic resources used in the production of dynamical, embodied, enacted mathematical ideas or ways of being-in-the-know (Radford et al. 2009).

Language and communicative acts are no longer considered as peripheral to our grasp of mathematical concepts (or, more generally, of the world we live in): They are collective products of relations in and through which human beings constitute and articulate the lived life of mathematics, society, and the worlds they inhabit. The development of "higher mental functions"—such as mathematical concepts or processes—is then essentially rooted "not within the individual, but in the communication [rečevoe obščenie] between individuals, in their relationships between each other and in their relationships with the objects created by people" (Davydov 1991, p. 14–15, our emphasis). Through mathematical language (use), students constitute the special

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[3] We use the term signification—which some translators incorrectly and inconsistently render as "meaning"—which better reflects K. Marx's German Bedeutung and F. de Saussure's French signification, which Vološinov (Bakhtin) and Vygotsky have taken up in Russian as značenie.

signification of words such as "edge," "face," or "rectangle" within a mathematical context (e.g., Lerman 2001; Zevenbergen 2002). Participants in mathematics classroom talk are linked together in and through communicative acts (Van Oers 2001), jointly producing the "discursive demands" that determine the actual mathematical requirement of a given task (Barwell 2012).

Running through these conceptualizations, the relational nature of teacher–student communication comes forth. Research in the embodied nature of mathematical cognition (e.g., Davis et al. 1996; Lakoff and Nunez 2000; Roth 2011) also informed our reflections on being-in-the-know-with and its relationship with languaging (Maheux and Roth 2011; Roth and Thom 2009). Embodied cognition first helps us to recognize how mathematical activity emerges from early bodily experiences shaped by social interaction in such a manner that they become an integral part of our living bodies rather than product of a conscious intellect. For example, children learn very early on how to manipulate objects such as a prism or cylinder, looking and touching them but not tasting them, and taking advantage of the flatness of their faces to put them in place even when the given face is not directly visible and without the need to "think" about it. During instruction, the embodied multi-modal aspect of mathematical communication is consider to play an important role in the "grasping" of mathematical meaning in the course of socially motivated activity (e.g., Williams 2009). In the enactivist tradition, knowing actions are observed to be occasioned by the actions of others, arising in transactions[4]: A person brings forth a world of significance with others within a sphere of behavioral possibilities from which patterns can be distinguished and interpreted as "doing mathematics," whereas "an enactivist turn on the question of [mathematics] knowledge would be to ask how we are knitted together in this particular body" (Davis 1995, p.2). Teacher–student mathematical communication is then an instance of third-order coupling, that is, of coordinated languaging within which meaning arise as a relationship of linguistic distinctions.

Taking societal relations as the origin of all speech activity invites us to go further. Seeing mathematics as emerging in and as teacher–student relation evokes Levinas' perspective on language, in which "speaking makes the world common [and] language lays the foundation for possession in common" (Levinas 1978, p. 74). With such an approach, one can foresee mathematical being-in-the-know-with as a relation that materializes in the words and gestures of teachers and students. A perspective that also sits well with a cultural–historical perspective on language and child development, working from the idea that any form of communication—a word, a gesture, an act—is "absolutely impossible for one person, but is possible for two" (Vygotskij 2005, p. 1018, emphasis added). Mathematics classroom conversation as a possibility for two allows us to turn our attention to how and when opening opportunities may arise to keep concrete mathematical activity at work. This is so because "from the very earliest stages of the child's development, the factor moving his activities from one level to another is neither repetition nor discovery." The source of development in/of these activities lies in the "social environment of the child and is concretely expressed in

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[4] Transaction is a category that does not reduce a relation to the collaboration or interaction of individuals but rather constitutes a unit that encompasses all actors and their world (Dewey and Bentley 1999), which makes it a better category for understanding situated actions and situated cognition (Roth and Jornet 2013).

specific relations" (Vygotskij 1984, p. 29), which in Vygotsky's case are those with the experimenter but which we extend to others in general, teachers and peers.

Thinking (about) communication in terms of irreducible societal relations also means that no utterance can be attributed to a speaker or recipient: The former always is speaking for the benefit of the latter, the latter actively listening to the former (Vološinov 1930). All mathematical talk in the classroom therefore belongs to both speaker (who is saying the word) and recipient (who is hearing the word) simultaneously. This constitutes a clear departure from approaches to language and mathematics education that conceptualizes the pertinent issues "from the individual learner's perspective as he builds an understanding of mathematics" (Brown 2001, p. 132). Communication presupposes common understandings, which continue to develop in (ongoing) and as societal relations (Vološinov 1930; Vygotskij 2005). This suggests a need to investigate how any specialized, specifically mathematical vocabulary may become opportune as a matter of being in relation for the purpose of continuing a rapport or disposition to one another carried on by teachers and students in mathematical classroom talk (Roth 2012a). But this investigation is not to be in the sense of identifying the specificity of mathematics language to help teachers "plan what they are going to discourse about in their classrooms, and prepare them to better understand and capitalize on students' often awkwardly worded contributions to this discourse" (Sierpinska 2005 p. 22). The view proposed here is radically different by not taking "mathematical thinking [as] both a collective and an individual endeavour" (p. 23) but by sublating[5] both in the relation from which they can be derived, with an emphasis on how teachers and students offer and respond to one another while making their relation a specifically mathematical one.

The nuance is, conceptually, an important one. We do not theoretically separate teacher and students, giving one the role of providing product or process help through their interaction while insuring that classroom communication allow students to "express their ideas in order to effectively develop mathematical concepts" (Pijls and Dekker 2011, p. 380). Rather, the concept of relationality emphasizes teaching and learning as two sides of the same coin. This is entirely in Vygotsky's spirit, who employs the Russian word obučenie (обучение), which simultaneously refers to education and instruction (e.g., Roth and Radford 2011). In this view, "help" is a mutual and cogenerated action, conversations (from the Latin conversare we can render as "turning recurrently toward the other") in which teachers learn how to teach just/only as much as students learn mathematics (e.g., Roth and Radford 2010). Hence, talking (about) mathematics "reveals how the learning of mathematics and classroom mathematics can be jointly constructed by a teacher and his/her class in quite different ways" (Ingram et al. 2011, p. 37, our emphasis) according to what emerges as the task at hand. We take into account such tensions as inherent to teacher–student talk in mathematics classroom—understood as the result of the coexistence of distinct and conflicted voices within language present in each and every utterance because of the intrinsic social nature (Bakhtin 1975) and not to be overcome and eliminated as awkward mathematical contribution (Barwell 2012)—and embrace these as the driving forces for mathematical classroom talk to develop. We welcome tensions not in the form of expressed

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[5] The verb "to sublate" translates Hegel's German term aufheben, which has the contradictory senses of "to do away with" and "to keep."

ideas but as outcomes of societal relations in which mathematical ideas and ways of doing materialize, hence positing the Other as the very condition of a relation where thinking/communicating mathematically exist in an immanent manner.

A relational view on teacher–student communication can provide us with important nuances in regard to what researchers before us develop on the subject. For example, the idea that a teacher's role has to do with offering the student conventional mathematical vocabulary while helping to elicit their thinking and align their knowledge with the conventions of the discipline (Lampert 1990) would need a reformulation. We might rather talk about teachers and students as finding themselves in their joint mathematical activity employing a scientific mathematical lexicon. Such nuances aim at keeping relationality at the centre of our conceptualization of teacher–student communication, while remaining as consistent as possible. Hence, whereas we agree that learning should be considered within the frame of relationships while emphasizing the centrality of processes of communication and language in students' learning (Renshaw and Brown 1997), we disagree with speaking in terms of "providing space in the classroom for the voice of the students [and] supporting them to communication their ideas in their own words" (p. 201). With our concept of relationality, teachers and students are not taken as individuals coming in and out of partnerships (each being entitled to and responsible for his/her part). Rather, they find themselves as co-producers of the classroom discourse and as co-proprietors of the words and ideas that create the communicative space in which they evolve.

In the sense just developed, our conceptualization of relationality is much closer to an embodied cognition perspective in which, as we work out our lives in languaging, acts of knowing contribute to continuous becoming that we bring forth with others (Maturana and Varela 1987). But as we argued elsewhere, enactivism focuses on "interaction" and sees relations as that which binds together entities, rather than thinking the relation first (Maheux and Roth 2011). Although full length discussions of how our perspective concurs or differs with other theories—and, in this, is affiliated with or opposed to them—is beyond the scope of this study, a brief clarification can be useful. In a nutshell, we find conceptualizations supportive to our theoretical development in various non-representational approaches of mathematical cognition and communication, but insist on making nuances whose importance precisely comes about when a relational perspective is assumed. Inasmuch, we refer to findings of scholars such as Sfard (2008), despite her insufficient attention to the bodily aspect of thinking and communicating; other research we reference take embodiment as an existential to any attempt at conceptualizing mathematical talk (as we do). These conceptualizations that fully include the material body often tend, however, to separate thinking from speech, a perspective that we, with Sfard, find to be untenable.[6] Here, however, our purpose is to discuss and illustrate the basics of conceptualizing relationality in and of teacher student communication (while other aspects are discussed, for example, in Maheux and Roth 2011). In the following section, we analyze episodes from a second-grade classroom lesson in geometry to highlight what our nuanced perspective contributes to approach teacher–student communication.

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[6] Precise mechanisms for the emergence of language from physical movements have been provided in the study of Bonobos (Hutchins and Johnson 2009) and for mathematical activity (Roth 2012c, in press).

The relationality of teacher–student communication in elementary mathematics lessons

In the preceding section, we propose rethinking teacher–students communication from the perspective of the irreducible relation with/in which they together bring forth mathematics in the classroom. In this second part of our study, we illustrate how this relational perspective plays itself out in a concrete, moment-to-moment analysis of classroom fragments. Our analyses are not intended to constitute the evidence from which the theory was derived by means of an inductive scientific process. Instead, we offer the analyses as a methodological apparatus to flesh out this relational perspective in concrete practice. Fragments from elementary classroom geometry lessons in which teachers and students discuss three-dimensional objects (cubes, cylinders, rectangular prism, and so on) are analyzed in the way philosophers and mathematicians conduct "thought experiments" (e.g., Gendler 2000): as a tool for thinking through the consequences of our propositions. In our analyses, we tend to look for the emergence of classroom language that articulates geometry and, in so doing, makes explicit teacher–student mathematical communication as an irreducible societal relation consistent with the concept of being-in-the-know-with (Maheux and Roth 2011). Because we are interested in relations, our unit of analysis reflects this as an irreducible societal phenomenon, which requires a minimum unit that is societal rather than individual in character. We focus on the semiotic resources (words, gestures) in a scene that are available to everyone present as thoughts and ideas on the outside and in the exchange (Vološinov 1930). To understand the flow of a conversation, every word is approached as something said and heard simultaneously (Roth 2013a): It is only because recipients have actually heard a word that they can reply.

From such an angle, we can already examine and compare Excerpts 1 and 2 in a new light. In Excerpt 1, we see a teacher offering to her students a "diagram of a 3D shape," the enunciation ("who can tell me what it is?") being realized as a question about the object ("A rectangular shape," turn 02) as if the teacher (only) asked what it was. And the student's response, itself an offering, is visibly taken up by the teacher, yet returned to the class ("Mmm, almost. John?", turn 03) where another student's utterance orients the collective thinking-aloud toward naming the object ("A rectangular cube?", turn 04) and so on. Words in this "language game" are truly ideas on the outside and thoughts in the exchange. Is not saying "a rectangular shape" or "a rectangular cube" the actual taking into account of observations that we usually render as "thinking"? Is it not affirmative of directing oneself and others toward something (words, a name), the active formation and connection of ideas? And is not this back and forth of offerings involving mathematical concepts (diagram, shape, cube, prism) a particular orientation of teacher–student communication in which we recognize mathematical talk, but also the emergence of "individual (not) knowers" when the teacher's use of words such as "almost" and "nearly" is, apparently, collectively taken an evaluation that requires other attempts at finding the right word?

As a tool for thinking, Excerpt 2 contrastively encourages such a reading of the transcript—where distinctions with Zevenbergen's analysis exist in nuances. There, talk coming from a student comments on a rectangular prism that "it's like a flat cube"

(turn 01), an idea which becomes a possibility ("might be," followed by a "Um hm," turn 02 and 03) when heard and responded to in the teacher's voice. And as ideas keep flowing, the teacher–student relation, which enables and constitutes this mathematical conversation, does not appear to advance knowing and not-knowing positions in the way it can be seen to occur in the first excerpt. Contributions are conceptual rather than informational in that they offer to unpack the way in which elements are taken together in the conceptualization (from Latin conceptus "collecting, gathering") of cubes, prisms, edges, and surfaces. But they also seem to bear witness of the movement of being-in-the-know-with one another as the teacher's and the students' voices are similarly salient in the polyphonic development of this mathematical talk around a moving topic (e.g., the offer to engage on "what would that box have to have to be a cube" in the last utterance). The proposition "it's like a flat cube" observably was heard as requiring a reformulation such as "it could be like a cube that's been flattened" (turn 04), to which the final state distinction in edge size had to be repeated. The repetition is apparently important enough to be articulated in the opposition between rectangular prisms and cubes, and so on. It is not so clear anymore what should count as a question and an answer, as knowing and not knowing, as teaching and learning. All these categories, so often taken for granted, require deep investigation when teacher–student mathematical communication is approached from our relational perspective.

With the teacher–student relation as the object of attention, we now specifically discuss two questions central to the topic: (a) What are questions and instructions in teacher–student communication from a relational perspective? And (b) what can this perspective tell us about the role of conversation in mathematical learning? We give attention to how teacher and student utterances are, and solicit, responses to preceding and subsequent locutions, others' orientations to what is said, objects made present (again), and so on. Conversation analytic methods (e.g., Atkinson and Heritage 1984), considering the production and effect of language act, are used to scrutinize the conversations, which enables tracking the appearance of "growth points" and "catchments" of mathematical ideas (see McNeill 2000; Pozzer-Ardenghi and Roth 2008). There is no rigid, explicit analytical procedure to follow in such investigation, as our analytic method has to be identical to the ethno-methods of participants in the conversation (Roth and Gardner 2012). But we did in fact follow the classical loops of video analysis: (a) repetitive individual and group viewings, (b) identification of interesting/significant events, (c) precise transcription including gestures, (d) individual and group analysis, and (f) writing a storyline/narrative. The following subsections present two of those narratives, each beginning with an assertion theoretically contributing to the development of a relational perspective on teacher–student communication, an assertion that we then develop, explain, and illustrate with empirical materials.

When are questions and instructions?

Assertion 1: Moving conversations forward, being-in-the-know-with can be recognized in teacher–student communication asking and responding to questions involving mathematical concepts. Teachers and students could not produce

mathematics as moment-to-moment lived dialogue if question and reply were not two manifestations of the same societal phenomenon: being-in-the-know-with.

Relevant to understanding the movement of a lesson is the take up of an utterance as a question or instruction on the part of a recipient. But questions and instructions do not just exist in themselves: a minimum unit of "turn pair" is necessary to determines question and answer, instructing and following instruction, simultaneously (Roth and Gardner 2012). The following fragment derives from an episode featuring a teacher and 23 second-grade students in the process of reviewing some of the geometrical ideas that they had explored during the preceding week. After talking about how solids can be described according to their faces, the concept of "edge" appears in the conversation (a growth point we identified). According to Piaget and Inhelder (1956), the concept is an important mathematical one for second-grade students' constitution of a "topological" space and its articulation with a "projective" approach. First essentially playing out as boundaries with the articulation of topological properties related to closure or continuity for instance, edges slowly emerge—as unforeseen and unintended, emergent properties (Roth 2012b)—from the visual or tactile exploration of objects and construction (e.g., drawing, molding), in terms of properties relating to its direction (e.g., straight, round). But as Piaget himself often notes, the verbal manipulation of all those properties is a challenge of its own, when compared with how children use them during sorting tasks. A teacher–student conversation about edges in a second-grade classroom offers a demanding context for both the recognition of edges as distinctive feature of geometrical solids (involving comparative perceptual activity) and as an increasingly analytical approach (giving attention to properties such as lengths and directions in description or comparison) (e.g., Gutiérrez 1992). From a relational perspective, it would be interesting to see how these mathematical ingredients actually come about and move (in) teacher–students communication, that is, when considering that questions and instructions are in essence "reality for two." Fragment 1 illustrates what happened on the day when the (material) sound /ɛd /[7] (heard in English as "edge") first appears in the classroom:

Fragment 1

01 Teacher: What about edges, what about when we were talking about edges? Can we describe edges as being different? Are there some edges that are different from other edges? Hands up if you think some edges are different from other edges.

02   ((Moves her hand up and down while pointing to a poster showing parts of a pyramid, then picks up a cube and slides her finger back and forth along an edge, Fig. 1. Finally, raises her right hand.))

03 Students: ((Four or five students raise their hands.))

04 Teacher: Okay. How many people think edges are all the same?

05 Students: ((Seven or eight students raise their hand, including Tobin.))

[7] The transcription of the sound makes use of the language-independent conventions of the International Phonetics Association.
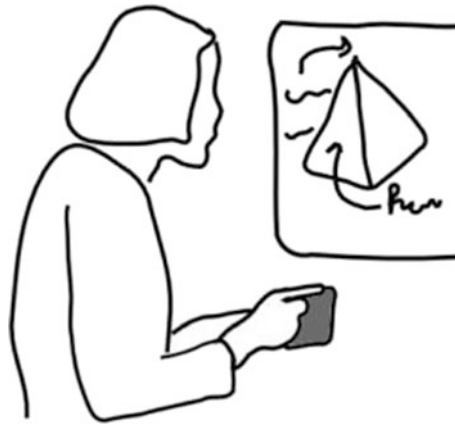
Fig. 1 The teacher slides her finger back and forth along an edge of the block (grey). Next to her, there is a poster on the wall featuring a pyramid on which essential parts—e.g., edge, vertex, surface—are named

No matter how tempting it is to ascribe "questions" to the teacher (turns 01, 03) and, separately, "answers" to the students (turn 02, 04), understanding the movement of the conversation demands the uptake of the utterance on the part of an interlocutor. From the perspective of the conversation as social phenomenon, question/answer, invitation/ acceptance, invitation/rejection, instruction/following, etc., always come in pairs, where each part is a part only in relation to the other part: a question is a question because there is an answer, and an answer is an answer because there is a question (Roth and Gardner 2012). Thus, although the teacher produces the sounds that we hear as words forming questions, these have to be designed for the students, in whose ears they resound to exist as such. The sounds, therefore, are the students' as much as they are the teacher's. There would not be student responses if the words were not theirs.[8] We can see this effect when a first turn pair is completed, e.g., by students' responses (raising a hand) to the utterance "Hands up if you think some edges are different from other edges" (turn 01 and 02). On the surface, we hear this as an instruction that has a certain effect, but this effect, even though it occurs while the teacher is speaking, is also available to her only in and through the raised hands of the students. Students for whom the teacher's utterance denoting edges, constitutive part of a geometry lesson in which they are asked to contribute with their observations, can also only appear as such in/ through the (collective) reaction and the teacher's response to that reaction.

To make our point more explicit, we consider what one might have observed if there had been "Why are you insulting us?" in place of the raised hands in turn 02. From the perspective of the conversation, therefore, there would have been something like an insult/complaint pair, which likely would be followed by a conversation in which this turn pair (the insult) itself becomes the subject (Roth 2013c). The turn pair, therefore, would not have been a growth point for the mathematical conversation that was actually recorded, where the growth point relates to the conversation as a seed relates to the tree that comes from it (Pozzer-Ardenghi and Roth 2008). We find being-in-the-know-with exemplified here: It is only in and through the status of the hands that the

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[8] It has been shown that even where a sound-word appears for the very first time, it is inherently shared (Roth 2013b). A word, even when it has never occurred before, is a reality for two (Vygotskij 2005).

utterance and its subject (content) appear to have been intelligible for the children in whose language the utterance was produced and to whom the language therefore returned. Moreover, we realize that because no "secured meaning" can be attributed to what we might take as a question about the mathematical concept of edges, teacher and students have to co-produce, generate signification. From a relational perspective, signification is not something "shared" or "negotiated," something each party previously held, contribute to, or recognized. On the contrary, the first movement in/of this piece of conversation is the apparition of the concept as something between the teacher and her students: a "common understanding" (in the etymological sense of standing on a common ground that is below, from the Proto-Indo-European n̥dhos , "under, below" + standan, "to stand").

This understanding in common is and constitutes a relation. For students to be able to raise a hand in reply to something, they need to have attended to the teacher's words, heard and be affected by them before having taken up the words in their answers. The response therefore comprises active listening—in the form of inner dialogue as counter-speech (Vološinov 1930)—and replying. But the word the recipients hear appears in the turn attributed to the first speaker (here, teacher). Thus, the response extends across two turns (Roth 2013a). What we theorize as being-in-the-know-with on the part of the teacher or the students is the result of this dialogical interplay of turns in conversation. Thinking in terms of relationality, and thus not reducing dialogue (literally: "two-voices") to the sum total of two independent, individual locutions, demands that in each turn, both parties are integrally involved: acting (speaking, raising hands) and attending (active listening, watching) crossing over into one another. Edges as pertaining to the comparison/analysis of geometrical solids (the announced theme of the lesson) are, in talk, recurrently made publicly available so that the interlocutors—teacher and students—may actively attend to them (in talk). That is, speech here has an apophantic function in that it "lets something be seen (φαίνεσθαι [phainesthai]), namely what is being talked about, and indeed for the speaker (medium) or for the interlocutors. Speech "'lets something be seen' from itself, ἀπό [apo]… what is being talked about" (Heidegger 1977, p. 32). Because the second half of a turn in conversation is also the first part of the next one, there is an already existing relation that permits the concept to move into the conversation despite the challenges second-grade students may be confronted with when hearing the sound /ɛd /—which competent speakers of English here to be the word "edge"—for a first time. Whereas one might be tempted to suggest that the teacher was doing much of the talking here, our framework orients us the relation that appears across the turns.

In this analysis, we insist on the processual nature of communication because it is precisely that which makes the questions asked not belonging to the teacher (and the answer to the students) but existing in/as a relation between the interlocutors. Attending to the variety of semiotic resources at play in the beginning of this lesson also helps noticing signification as "the effect of the interaction between speaker and receiver that imposes itself on the material of a given sound complex" (Vološinov 1930, p. 104). During the first utterance, the teacher points to a poster on the wall showing vertices, faces and edges on a pyramid, and slides her index finger back and forth along what culturally competent individuals recognize to be the edge of a cubic block (Fig. 1). In addition to the words, intonation, sound levels, and overall orientations toward the students, these communicative resources are all irreducible parts of a mathematical

conversation about edges. The possibility for those resources to contribute in such a way reveals that in teacher–student communication, mathematical questions exist because children are already competent to hear utterances in both senses of the verb (i.e., to perceive and understand). The gestures, the objects, the cultural dispositions (including prosody and posture) all are part of the material in which the irreducible teacher–student relation realizes itself while involving mathematical ideas to be responded to. This call and possibility to act upon the idea enables engaging in the process of coordinating perceptual and analytical approaches to geometrical figures despite the immediate challenge to so (some) students might experience in those circumstances (as we will see in the next fragments).

At the end of the first turn at talk, before a pause and the appearance of several raised hands, the teacher articulates what we customarily hear—because of the grammatical construction—as an instruction: "Hands up if you think some edges are different from other edges." Waiting for what can be seen as the effect of the utterance, we know what the teacher says has been heard as an instruction if we see at least one hand up. It is precisely then that the pair instructing|instruction-following is a good description of the relational nature of this event. Far from simply "carrying across" the "concept" of edges or "putting it out there" for co-construction, the utterance and the rising hands already make the material sound-word a "reality for two." To hear the sound/hænds-ʌp/(hands up) as instruction implies acting upon the mathematical content of what has been heard: to raise the hand if "some edges" are different from "other edges." This requires hearing that a difference is made within the totality of edges, for example, the different edges pointed to or gestured by the teacher. A difference is made in the pairing of (at least) two contributions (e.g., a question and an answer). Through this movement (that on making a distinction), we recognize in this talk about edges which is as much for the teacher as it is for the students the "practical consciousness-for-others and, consequently, consciousness-for-myself" (Vygotskij 2005, p. 1017). Co-articulating mutual intelligibility, teacher and students are involved in a being-in-the-know together (being-in-the-know-with) that includes one of the fundamental concepts of early geometry (edge), now addressed as a generalized entity (edges as opposed to a specific boundary of a specific object) about which claims can be made.

If one might want to evoke them as a "propeller" for mathematical thinking/conversing/being-in-the-know-with to emerge, question|reply pairs need to be taken as irreducible phenomena in their own right in the light of the movement they generate. In these first turns of a conversation, the teacher articulates with the students what we may hear as "ideas concerning edges." But what can we say about the mathematical concept therein? The teacher articulates the sound-word "edges" seven times together with gestures, using a poster, and a block (Fig. 1). If there is any sense to saying that the teacher teaches the concept of edges, then this concept exists materially in the public arena of the relation between the teacher and the students. When the conversation unfolds smoothly—such as in the turn pair "Hands up if…" and raised hands—the concept is not "taken-as-shared" or "negotiated" on the basis of private ascriptions to an object of observation, but is truly shared as the material embodiment of an idea co-emerging between and for teacher and students simultaneously. The speech itself "summons" an idea in ways that has to be intelligible for all, even if students do not yet "know" the concept. Signification is exhibited, for the benefit of all, in turn pairs (such as question|answer, invitation|acceptance, etc.). It requires and takes the form of

being-in-the-know-with. Moreover, when we focus on teacher–student communication as a relation, we can drop "understanding" and "meaning" from our consideration because "to teach the meaning [Bedeutung] of a word is called to teach its use" (Wittgenstein 2000, Ts-213, p. 31v). Use is completely out in the open, where it constitutes a reality for two and it is the ambiguity and shifting nature of signification that makes it possible to learn by appropriating words that are already there and come from others. In this way, question|answer pairs create and are created by mathematical attention that cannot be simply attributed to the use of a specific term (edge). It is also (made) present in/through a "formal gesticulation" that appears to have nothing mathematical and would normally be qualified as a rather poor indicator of (mathematical) being-in-the-know taking place.

The preceding analysis shows that because each conversational turn belongs to producers and recipients simultaneously. To communicate is to share with, which inherently implies a relation within which utterances concretely produce mathematical ideas. Communication presupposes being-with because "being-with is 'explicitly' shared in speech, that is, it already is, only unshared as something not grasped and appropriated" (Heidegger 1977, p. 162). The comprehension of the question of edges is a geometrical endeavor in itself (especially with young children having their first experiences in geometry), a concretization of an instant in the continuous development of a mathematical way of thinking that emerges on the ground of mundane activity (Husserl 1939). Speech here is an overarching and encompassing phenomenon in which being-in-the-know-together spreads across all those present to a situation. To make sense of this teacher–student communication as an unfolding, moving lesson in mathematics, whatever sound comes from the teacher's mouth has to be viewed as the-teacher-being-in-the-know-for-and-being-in-the-know-with-the-children. Moreover, because the teacher, too, once has been a second-grade student, this being-in-the-know also re-lived in historical time, it is a societal relation in which the history of (geometry and geometrical) being-in-the-know-with resides and is made. Hence, we can also think of mathematics classroom conversations as relations with past/future distant others as well.

When is lack of being-in-the-know?

> Assertion 2: Because teacher–student communication is simultaneously a condition for and a result of mathematical activity, (not) being-in-the-know is but the result of a relation. Conversations are then the fine-tuning of being-in-the-know relations in which mathematical ideas can come forth even if a student says, "I don't understand."

At least since Plato and his endorsement of Socratic dialogue as exemplified in the well-known Meno episode (in which a slave "realizes" that a square can be duplicated in area using its diagonal), the potential of conversations for teaching/learning mathematics is greatly appreciated. For Socrates, such conversations permit the "remembering" of mathematical ideas. Nowadays, it is the argumentative possibility of dialogue that comes to mind (e.g., Ernest 1994). Evocative of perspective in which conversation serves the "negotiation of knowledge," such a view requires conversationalists to individually announce being-in-the-know (e.g., something about edges being all the same or different). But from a relational perspective, it is not possible to talk about individual forms of being-in-the-know that come to be confronted and negotiated in

conversation. From the relational perspective, communication always implies being-in-the-know-with (and relative to) others. What then can be said about the role of teacher–students communication and its contribution to teaching/learning? The following fragment, which continues the preceding episode, will help us flesh out the idea of how relationality enables us to consider moments in which a student attests to not being-in-the-know.

Fragment 2

06 Teacher: Okay. Who had their hand up for some edges are different from other edges?

07 Students: ((A few students raise their hand, including Tobin.))

08 Teacher: Tobin.

09 Tobin: ((3 s)) I don't know.

In this fragment, Tobin utters "I don't know" after the teacher nominated him in relation to the utterance "Who had their hands up for some edges are different from other edges?" (turn 05). What became a question here permitted a surprising answer, since Tobin raised his hand, the visible part of a response that would normally be followed by another response indicating that he understood a question and has something to say. From a relational perspective, the important phenomenon here is how the teacher and the student bring about, together, being-in-the-know and not-being-in-the-know in a geometry lesson. When Tobin says, "I don't know" in response to the teacher's invitation, the response, as a relation to the teacher and her utterance, problematizes the general criterion of being-in-the-know invoked by the preceding utterances. The expression of not-being-in-the-know, whether it comes from the student or from the teacher, sets up a change in the direction of the unfolding relation that not only makes explicit how much engaging in a mathematical conversation (e.g., about edges) is itself an educational endeavor, but also inherently a movement of being-in-the-know mathematically. In this conversation (rather than a pathological one where the two participants do not speak each other's language), Tobin's saying "I don't know" suggests that he heard the invitation, realized that it asks of him something that he in fact does not know, and made the result available publicly in an intelligible manner. That is, the articulation of not-being-in-the-know presupposes a considerable amount of shared being-in-the-know just as the articulation of what some call a "misconception" presupposes shared being-in-the-know and intelligibility (Roth 2013a).

Using something they have in common (e.g., words), Tobin articulates not-being-in-the-know and the conversation now has to address this. As a relation, this not-being-in-the-know expressed in speech cannot but emerge from being-in-the-know itself and in a particular way, for "speech is existentially equiprimordial with attunement and understanding" (Heidegger 1977, p. 161). Not-being-in-the-know, too, entirely depends on the other. This is so because not-being-in-the-know is but one aspect of intelligibility that becomes figure against an always already known since "being-in-the-know emerges in establishing just what the boundaries are of what-it-is-not-to-be-in-the-know" (Roth and Middleton 2006, p. 76). The being-in-the-know relation is always of a special kind, and mathematics (or geometry) is one specific way of being-in-the-know. That is, if the utterance does not stand for Tobin's own personal knowledge and

struggle to understand the concept of edges, it can be analyzed as the continuation of a relation in which geometrically being-in-the-know and not-being-in-the-know about edges opens a world of possibilities. Both being-in-the-know and not-being-in-the-know are constitutive parts of the relation (together-being-in-the-know/doing-geometry), and they are constituted by it. Still resounding in their ears, the "I don't know," a moment of geometrically being-in-the-know-with, now provides an opening for the teacher. Precisely because it is impossible to ascertain what Tobin "wants" or "intends" to say, or to predict what will happen next, relationality takes us to examine how the interlacement of speakers and listeners (exhibiting the effect of the preceding utterances and setting up the succeeding ones) manages to produce a discussion in which geometrical being-in-the-know is realized.

The obligation to complete the utterance (even by not replying to it) that makes teacher–student communication relational is an injunction for instant coping, to work out the "I don't know," and preferably (in the context of a lesson in geometry) towards geometrical being-in-the-know. The matter at this point is not one of "negotiating" the meaning of the word "edge" or the geometrical concept it calls upon, which demands both parties to affirm knowledge and try reaching an agreement or compromise by mean of mathematical argumentation. Although such situations certainly can and may occur, they represent but a fraction of most teacher–student communication. The lesson fragment we discuss here is particularly interesting for bringing about this range of phenomena otherwise overlooked, and provides us with a resolution this apparent impasse through the articulation of teacher–student communication as simultaneously a condition for and a result of being-in-the-know geometrically:

Fragment 3

10 Teacher: ((3 sec.)) Tobin how do you describe this edge? ((Takes the block she was previously holding and slides her finger, as in Fig. 1))

11 Tobin: Well.

12 Teacher: I ask you to run your finger along it. ((Moves toward Tobin, stretching her arm with the cube in his direction, Fig. 2.)) How would you describe it?
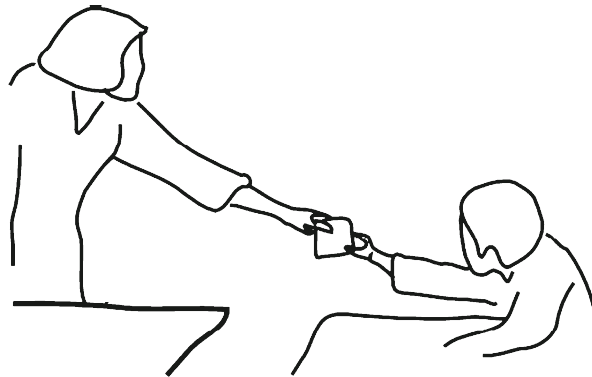

Fig. 2 Tobin and the teacher coordinating one another as she hands him the cubic block

13 Tobin: Well… because um because it's like a sort of vertex. ((Grabs the block, looks and touches it, but does not slide his finger along the edge.)) Like this one ((points to the word 'vertices' in the diagram)).

14 Teacher: Yes.

In this fragment, we clearly see how (not)-being-in-the-know is constituted from turn to turn, when the effect of an utterance stating "I don't know" is a call for being-in-the-know (as a description) taken up in the geometrical realm (mentioning "vertex") which is then in the process of being confirmed (turn 13) in its appropriateness to the lesson. For one, edges are made thematic from a mathematical perspective, although the signification of the term itself remains highly implicit and vague. After Tobin indicates that he does not know, the teacher reaches for the block she previously used as part of talking about edges. In this instance, there is a shift of level in the talk, from more general referents (sets of edges and differences between them) to one specific edge (this edge)—reminiscent of Piaget and Inhelder (1956) observations about children's usual abilities to exhibit understanding about specific objects in gesture. Inviting a gestural, experience-based appreciation of an edge to accompany the talk "about edges" in general, the relation keeps conversationally moving in the geometrical realm. The expression responding to what seemed the affirmation of not-being-in-the-know turned instead into an opportunity to offer a new entry point to the question ("can we describe edges as being different" versus "How do you describe this edge?"). In return, Tobin articulates himself by attending to the question (well), sustaining the relation and therefore the means for the possibility of (new) mathematical being-in-the-know to appear. Indeed, the utterance "it's like a sort of vertex" (turn 12) points to the presence of vertices on a cube, which is part of what mathematically distinguishes straight from circular edges (as on a cone or a cylinder, where no vertex is associated to the edge)—something also often observed when young children begin to make distinctions between curved and rectilinear boundaries (Laurendeau and Pinard 1970).

From a relational view, the episode also leads us to consider teacher–students communication beyond words and gestures (and other communicative resources) to include various forms of "interchanges" as integral part of "communicatively" (from Latin commūnicāre, to share with, take a share in, i.e., to make generally accessible) being-in-the-know-with. When Tobin and the teacher coordinate one another as she hands him the cubic block (Fig. 2), they both literally reach out and toward the other, embodying this mutual reaching in which mathematical being-in-the-know exist as joint (geometrical) attention (Radford and Roth 2011). In this case, the experience of the cube serves as a common ground for the conversation to become mathematical being-in-the-know-with, an experience that also emerges conversationally (turn 11, "I ask you to run your finger along it," in its pairing with turn 09, "how do you describe this edge?"). With and within the conversation, the common understanding—which is both the prerequisite and result of the relation to the other (Heidegger 1927/1977; Nancy 2000; Ricœur 1986)—also moved the relation forward on the question of being-in-the-know solids through edges. Moreover, because not-being-in-the-know became part of the conversation, it is not only the concept of edges that the dialogue enlightens (the presence of vertices as a way to describe straight edges as oppose to round edges), but also what it means to know about mathematically with (and about) edges! When signs of being-in-the-know are produced as an effect of a statement of not-being-in-the-know, a conversation materializes as the

locus of an epistemological shift. The object thus appears not merely as a "tool" (as an instrument serving specific ends), but as relations in which teacher–students communication can realize mathematically being-in-the-know-with. That is, the object or tool is an irreducible part of the being-with of the relation. Relations, because they imply joint action, also have to be "realities for two." We observe such a relation arriving at joint attention to the geometrical aspects of a plastic cube when the teacher and the student respond to and set up one another addressing structural aspects of the object rather than other perceptual properties (e.g., color or texture).

From this observation that mathematical being-in-the-know exists in and as a relation, a togetherness of being-in-the-know emerges where there is never a complete "lack of being-in-the-know." We then can see conversations in mathematics classroom as the fine-tuning of being-in-the-know relations. The dynamical process of significa-tion illustrating the relational aspect of teacher–student communication as a way of being-geometrical-with-one-another shows that being-in-the-know-with stems from the affordance to think that is due to our common existence with others: consciousness always already is a societal product (Marx/Engels 1958). Being-in-the-know in and as a relation becomes mathematical, making mathematics an "emergent property" of the teacher–student communicative relation rather than simply being its subject matter. Being-in-the-know or not-being-in-the-know ([about] edges) is among the (possible) outcomes of actual, material relations between teacher and students. Whether there is mathematical being-in-the-know-with in a rich, or conceptual, or aesthetic, or critical, or argumentative way is then a question of the moment-to-moment, turn-to-turn tuning. This is also showing us that a conversation can never taken as mathematical in itself, or that from one point it will certainly continues along the same way. Growth points for teacher–student mathematical communication are after the fact analytical constructs indicative of something "around" which a conversation appears to navigate, but never reaches and grasps in a definitive way. Through its inner movement, relationality points to this continuous adjustment (from Latin ad- "to" + juxta "near") allowing but never securing the mathematical becoming of a classroom conversation.

The perspective is particularly productive because it permits us to include, in a common framework, those instances of teacher–student communication in which the typical features of mathematical discourse are not (yet) present and those where these are indeed (at last) present. Whereas traditional approaches to the analysis of mathe-matics classroom talk tacitly presuppose the ability of talking mathematics, our frame-work allows us to theorize how something entirely non-mathematical gives rise to mathematical conversation. This integration is of particular importance when it comes to discuss elementary mathematics education where such features are rarely present, although it is as a continuation of such early experiences that they can be developed. Most importantly, it derives from being-in-the-know-with that this development is not merely a matter of adopting norms of conversation in the struggle toward an ideal speech community (e.g., Taylor 1996), but as the refinement of being-in-the-know relations through which being mathematical with another is material and embodied.[9]

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[9] We subscribe here to a socio-phenomenological understanding of embodiment, which draws an important distinction between the material body and the flesh (e.g., Roth 2011), stressing that there would be no embodiment without the experiences of other material bodies, whereas the flesh is what is within those experiences.

Conclusion

In this theoretically motivated study, we move forward the reversal of the tendency to treat being-in-the-know as something individuals have (own) and share with others in communication by articulating the relationality of teacher–student communication. From this perspective, we found how students (like Tobin in our fragments) are an integral part in societal relations in which geometry is reproduced and transformed. These relations "turn out" to be concerned with geometrical concept, as this concept "turns out" to be what the discussion is all about, a view that accentuates mathematical being-in-the-know in and as teacher–student communication. Thinking mathematical activity from the relational point of view, we thus describe it as an irreducible collective phenomenon in its own right. In the relational approach, the explanation of mathematical activity cannot be grounded in the interests, goals, beliefs, or knowledge of individuals; all of these, if relevant at all, come to be understood as the outcomes of the being-with relation.

From an educational perspective, this study is significant because it informs educators about a dimension with new ways to attend to mathematical being-in-the-know. For example, re-analyzing Episode 1 from a relational perspective might take us to consider how the conversation moved to the layer of naming a 3D shape (rather than describing it, or comparing it with others) and perhaps attending to this "funny word" as a propriety of a class objects attainable from a diagram. A reading of the conversation is qualitatively different from that author's interpretation in terms of a teacher providing "a medium through which communication of ideas is made possible, and negotiation of ideas and concept is delivered" (p. 201). That is, we offer an alternative to the study of mathematics classroom communication thinking in terms of established meta-rules (e.g., van Oers 2001), socio-mathematical norms (e.g., Cobb and Tzou 2009), or power differential (e.g., Zevenbergen 2000) by means of which mathematics is (pre)defined (inside or outside the classroom). We suggest to work from the idea that mathematical activity unfolds in everyday classroom conversations and that it cannot, and does not need to be, reduced to its explicit dimension. In Episode 1 or in our fragments, as in the vast majority of mathematics teacher–student communication reported on in the literature, neither the teacher nor the students demonstrated having noticed the occurrence of "mathematics" as this form of being-in-the-know they together bring forth. This is important because the research that takes discourse as a paradigm is often concerned with the question of how classroom communication can be turned into a mathematical one. At the opposite end of the spectrum, being-in-the-know-with suggest that mathematics takes place as an emerging property of teacher–student relations that might be constrained by implicit or explicit regulation, but most importantly consist of a way of being with others: not a science but an art (from the Proto-Indo-European root, ar-ti, a manner, a mode).

Readers may find it useful to consider yet another piece of data to keep on reconsidering teacher–student classroom conversations from a relational perspective. In the following Episode 3, students were invited to search the classroom for objects that where "the same" as a geometrical solid they randomly picked from a bag. Ethan's group was required to gather "cube-like" objects, and they then presented on a mat a collection containing a cardboard box, wooden blocks of various sizes, a yellow plastic brick, all cubical, plus three bundles of sticky notes that had a square surface but were

not cubic. In the conversation that followed, an inquiry on the question of "what makes a cube at cube" rendered those sticky notes bundles problematic:

Episode 3

01 Teacher: Okay, so if we take the one you started with and talk about it as being a cube because it has all squares faces and they're all the same, that's what you said they were all square faces, and we looked at each one of the objects that they had… can we see that every single one of the objects that they found is a, is like this? Is the same?

02 Many students: No.

03 Ethan: No, please, these ones are paper ((shakes a sticky notes bundle)) and they don't really look like this.

04 Ben: And because they're, um, they're flatter.

05 Ethan: And these ones are like, they still just like don't look like those.

06 Teacher: Okay, so what do you want to do?

07 Ethan: This one.

08 Teacher: These ones ((gets up, moves the sticky notes off the mat)), what about this one? ((holding the last one in hand))

09 Ethan: Well this one's not really like a square because it doesn't match the square one like the thing like.

Some mathematics educators might want to use episodes like this as evidence that these children are unable to properly articulate themselves in the language of geometry (although they did observe some significant geometrical distinction brought up by the teacher). It could be added that while showing confusion in the use of mathematics lexicon to express their thinking, it can be hoped that the teacher would provide students with the words formally used to communicate such ideas, and require them to do so in a appropriate manner. From a relational perspective, however, such reading of the episode focuses on particular individuals instead of on the relationships from which they emerge in and as being-in-the-know-with one another. Considering such mathematical talk as an irreducible social phenomenon is not about commenting teachers and students as knowers, communicators, bodily experiencers, or performers, and their relationships with the (teaching and learning) environment. We rather steer our readers into the direction of the "turning out," the becoming of such teacher–students communication with/in which a mathematical lesson can be produced. The being of being-in-the-know-with evokes the living, changing, flowing nature of existence as opposed to the static thingness of things (beings) from which subjects and predicated (the teacher ask, the student answer, etc.) also proceed (Heidegger 1977). Letting go of taking entities (beings, things) as the starting point of the inquiry in favor of relations takes us to a geometrical way of attending to artifacts, which are themselves relations (i.e., "work of art" offered to us by others) creative of a (teaching/ learning) situation. We see emerging in the flow of words the sticky note bundles as some things about which geometrical thinking, in the form of conceptual descriptions can be made, a work of language where mathematical ideas are indeed articulated thanks to the

open-ended nature of the word (a reality for two) rather than resting on the enclosed "meaning" of a specialized vocabulary. As a result of this being-in-the-know-with one another—a with which, we need to insist, includes the teacher as well as all of those who listen to the conversation without, for example, objecting to such or such affirmation—the collection of objects attended to is transformed into a collection of cubes as defined through the cultural–historical development of western geometry.

As an intended practical implication of this study, we emphasize and reflect upon the "art of saying" that includes attending to the offering of the utterance over and beyond its content, yet without ascribing knowing, thinking, or intentions outside of the talk. In a very practical manner, taking conversations as elementary unit means, for teachers, worrying less about the exact words and more about the art of responding in ways that keep mathematical thinking—the conversation!—going. This allows the emergence of new and unforeseen ideas (a flat cube, strait edges as edges-with-vertices). From a relational perspective, our preoccupation is not on how to "increase and improve" student–teacher mathematical communication if this means better enclosing it in predetermined expectations to cover some topics in specific ways with language as a tool. On the contrary, in the view presented here, mathematics' language is not an instrument of communication but a way of being-in-the-known-with (Nancy 2000). In responding, responsibility acts not merely forward, for the consequences of "individual" actions, but also backward: for the action of another, since our responses are determinant in what the other has said or done. Theorizing in this way leads us directly into a fundamental, yet rarely addressed, question for mathematics education: that of ethics in the moment-to-moment of teaching and learning (e.g., Roth 2013c).

Responding to the Saying of the other (Levinas 1978) highlights a practical ethical responsibility where the entire background of mathematics as a human, culturally and historically developed activity, as a way of being-with-in the societal-material world needs to be taken into consideration (Maheux and Roth 2010) as opposed to any narrowed set of concepts or processes such as does generally laid out in a lesson plan. The challenges of living up to this view are substantial, just as the question of ethics generally is, and vividly and directly bear on research in teacher–student communication once we accept that "communication is Being" (Nancy 2000, p. 92, our emphasis). Addressing this dimension, toward an ethics of relationality in mathematics education, is one of the next important moves to further develop the concept of being-in-the-know-with. Another task will have to be the reflection on the implications of this perspective not only for research but also, for example, pre-service and on in-service teacher education, which needs to orient itself toward a more sympratical (i.e., collective and sensually lived) nature of the teacher–student relation.

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