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The stakes of movement: a dynamic approach to mathematical thinking

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Abstract

Standard approaches to thinking in the mathematics curriculum depict it as the result of some stable constructions in the mind of the person, constructions that are the results of individual efforts in the mind of subjects or of collective efforts that are then appropriated by and into the mind of individuals. Such work does not appreciate what Vygotsky actually said about thought: that it is one part of a self-moving flow that relates to another part, speaking, without that one can be reduced to the other or the whole. Grounded in the works of Châtelet, Badiou, and others we exhibit the movement of thinking in a case study of graphing. In our account, there is a primacy of the Saying and drawing over their traces, the Said and the graph.

Keywords: Movement; thinking; communicating; flow; unit analysis

> The to-be-thought turns away from man. It withdraws itself from him. . . . But—self-withdrawal is not nothing. Withdrawal is happening. (Heidegger, 1954, p. 5)

The lived, kinaesthetic dimensions of mathematical thinking have received growing attention in mathematics education (e.g., Roth, 2011; Sinclair, 2007). Whereas mathematical thinking is a topic many readers might consider well explored in our discipline, its nature is rarely examined through a lens of the *living* praxis of

mathematical thinking (Barwell, 2009). As critics have pointed out, the classical view on thinking *misses* the essence of knowing and learning as living processes (Lave, 1993); and it misses that not only the content of thinking changes in the course of learning, as Piaget believed, but the very process of thinking (Merleau-Ponty, 1996). The purpose of this contribution is to overcome the current fixation on fixed mathematical schema, conceptions, and constructions by describing and theorizing the movement in/of mathematical thinking, which requires us to exhibit mathematical thinking in movement. This task is not easy because, as Heidegger points out in the opening quotation, what is to be thought withdraws itself. But this withdrawal is (a) happening, which gives hope that the thinking of movement leads to thinking in movement. With this focus, we continue developing a theory of the living curriculum*-in-the-making (Roth, 2013), a notion that is opposed to the more-or-less flawless enactment of pre-existing conceptions (e.g., Moschkovich, 1998) and schemas (e.g., Núñez, 2009), and where the asterisk is used to mark its unfinished, continuously evolving nature.

The classical approach takes time out of thinking, learning, and knowing, thereby detemporalizing these phenomena, by theorizing what normally is alive by means of the difference between dead signs (thought, knowledge) (e.g., Hegel, 1807). There are recent calls to address the detemporalisation of thinking and learning in mathematics education research (Pimm, 2007). Detemporalization occurs when the flow of learning is reduced to the difference between two knowledge states: before and after. It also occurs when learning and movement are reduced to schemas, a theoretical concept frequently employed in the embodiment literature on mathematics education (e.g., Núñez, 2009); it is the very essence "of thought of the constructivist type" (Badiou, 1988, p. 317). Learning then is pictured as the *result* of processes that operate *on* the states of knowledge. Results also replace the movement of learning in the way still pictures can be used to give the illusion of motion (Fig. 1). Our general cultural experiences with image sequences makes it easy not only to recognize movement, but also to see what was slow

and hesitating at first, the acceleration in the center part, and the slowing of the movement towards the end of the line (Fig. 1). But this is not the moving of movement itself; the movement is neither in any one image nor in the set of images. In other words, our ability to see the drawing of the line and the dynamics of its unfolding is linked to the perceptual arrangement, that is, the way in which the drawing is authenticated by the *movement* of the eyes. The series of images is thus to be understood as a set of instructions for how to gaze so that the drawing is seen as drawing (Livingston, 1995). The movement of the drawing "is 'seen' with the help of the suggested order of the evidence (like the art of montage), as 'seeing' what is missing in the evidence" (Bielić, 1992, p. 230), that is, the movement of drawing itself, which cannot be seen in any other way.

«««««« Insert Fig. 1 about here »»»»»»

Whereas a series of stills certainly affords many possibilities to study movement, it does so in a way that clearly does not consider movement *in its own terms*: flow as flow. The movement of learning tends to be treated in terms of states (Lave, 1993), its *traces* taking priority over the flow that leaves them behind like footsteps in the sand. In this article, then, we do not attempt to "get a grasp" *on* the flow in mathematical thinking, for this grasping would reduce the flow to a com*prehension*, and, therefore, fixation, but to *grant* it importance and contribute to describing and theorizing flow in educational phenomena, here mathematical thinking. That is, in respect to the dynamic nature of thinking, we approach mathematical thinking dynamically, and thus make the case that *the movement of thinking is thinking in movement*. This study expands the developing conceptualization in which mathematical knowledge is movement that is also codified in mathematical forms (Radford, 2013). In the present consideration of movement and

thinking, making present requires a focus on the actual being-there (Heidegger, 2006) and being alive (Ingold, 2011), as opposed to attending to the symbolic relation of "representing," even in the form of actualization, incarnation, materialization, or manifestation (Radford, 2009).

To accomplish the purpose of the paper of describing and theorizing the movement of thinking in the context of the mathematical knowing and learning, we turn to the work of Châtelet (1993) on the association of gestures and diagrams. In this work, *movement* plays a decisive role that has been of interest to mathematics educators (e.g., de Freitas & Sinclair, 2012, 2013). Whereas fully appreciating these authors' analyses, their descriptions and examples miss the *dynamics* and the *temporality* required to think mathematical thinking as movement. This implies that mathematical diagrams are not just "capturing devices": it allows us turning to the flow that *eventually* brings forth an idea (as much as the subject). We follow suggestions that the movement of thinking cannot be grasped but has to be lived in and as *Ereignis* (happening) that is apprehended (*er-äugen* [seen]*, aneignen*) only after the fact (Heidegger, 1954). To this effect, we engage for the remainder of this essay in a careful, protracted dissection of an episode of diagramming by arguing for analytical concepts that emphasize movement (flow). We focus on an experienced research scientist because his doings cannot be easily reduced to cognitive shortcomings, as this has been done in the literature on graphing of schoolchildren and students (for a critique see Roth & McGinn, 1998). To be consistent with our argument we keep (our readers', our own) thinking in movement, so that its experience may reflect the experience of the movement of mathematical thinking. The transcriptions provided, as the descriptions and analyses, cannot be but instructions intended to start readers' thinking to move (e.g., Greiffenhagen, 2008; Roth & Bautista, 2011).

THINKING IN MOVEMENT, MOVEMENT IN THINKING: A CASE STUDY

The video fragment analyzed below derives from the first meeting in which a research team discusses the quality of its data. In the course of the events, the team leader, Greg (pseudonym), produces a graph in which one quality of the graphs observed in his laboratory, the width of approximately bell-shaped curves at maximum height (i.e., "half-maximum bandwidth," HBW) is depicted as a function of the composition of the light-absorbing retinal cells in the fish eyes that the laboratory investigates. The two chemicals responsible for the absorption of light, and therefore vision, are rhodopsin and porphyropsin. At the time, Greg had over 30 years of experience in doing this type of research. He has been a very successful scientist, currently holding a coveted, nationally funded research chair at another university. In the following subsection, we begin with a description of the event in two parts and then analyze it.

Contrasting their written productions, which are carefully edited, scientists may make statements inconsistent with the current canon of the field while they speak in laboratory meetings, lectures, or think-aloud protocols (Roth, 2003). Such differences, allusive of thinking and movement, can be considered a result of the living work of *presenting* ideas. Repetition, is reiteration, is alteration, is invention. Therefore, anything that might be termed inconsistency is relevant in the work of presenting in and as it is made an element of conversations or thinking more generally (in which oneself is an addressee). These inconsistencies arise as markers of moving thinking. This happens in this meeting, too, when Greg articulates some mathematical relations relating HBW and the relative amounts of porphyropsin and rhodopsin in fish eyes.

In the following, we begin by presenting the meeting fragment, engage in a first reading of it, and then exemplify different ways of thinking about and theorizing mathematical thinking in terms of (a) flow, (b) the theoretical movement from thinking to thought, and (c) the analogy with *Saying* as distinct to the *Said*. Our analytic approach is exhibited in and through the analyses, which keep thinking in movement; no further specification of method is required (Garfinkel, 1967).

Presenting "What You Would Expect"

In this 1:42-minute fragment concerning uncertainty in the data, Greg articulates the relationship between the half-maximum bandwidth of porphyropsin, rhodopsin, and mixtures thereof that "a priori . . . you would expect to see." (The transcription is provided in the Appendix.) He says that the porphyropsin bandwidth to be wider than the rhodopsin bandwidth and that for the mixtures of two the bandwidth of the absorption curve would be even higher. This description is accompanied by two gestures with his hand/arm/pencil (Fig. 2). He mentions a "sort of transition" and then gets up and walks to the chalkboard.

«««««« Insert Fig. 2 about here »»»»»»

As he arrives at the chalkboard, Greg stares at it for 3.0 seconds before beginning to write "1/3," which he quickly and with a lot of pressure overwrites to yield "1/2 m" (Fig. 3a). He pauses, produces an interjection, then erases what he has written (Fig. 3b) and then writes "HMB" (Fig. 3c), each letter corresponding to the first letter of the word he utters "half" "max" "bandwidth." He then produces first the line corresponding to the ordinate next to "HMB" and then draws what will be the abscissa (Fig. 3d). He first writes "A_1/A" near the origin of the Cartesian grid (Fig. 3e) and then erases it (Fig. 3f). He writes A_2/A_1 in the same place (Fig. 3g) only to erase it again (Fig. 3h). Greg then writes "$\%A_2$" in the center of the abscissa (Fig. 3i). There is a pause in the talking. Greg looks right, left, right, and then to the left (origin) of the axes system and writes "100%" (Fig. 3j); he turns to the right and writes "0%" while uttering the corresponding numbers. Just as he says "you'd expect to see" he places the chalk, then moves the hand lower; he

pauses, goes on to draw the inverse parabolic line, pauses, gazes to the left toward the beginning of the line, and then adds a little to make the end lower (Fig. 3k). He then draws a cross at the endpoint and adds one to the beginning of the curve. There is no reaction at first (1.59 seconds), and he then utters “”Right?” with an increasing pitch.

«««««« Insert Fig. 3 about here »»»»»»

At this point the meeting moves again into exchanges between the participants. The doctoral student suggests that the situation is “more or less” in the way stated, “maybe a bit higher at one end,” but it is the “same idea.” Greg does not change the drawn graph, but turns to the right part of the chalkboard and produces a second Cartesian graph. After he finishes, Greg sits down and copies into his research notebook what he had just noted on the chalkboard, as if in itself, the presentation of the HBW versus amount of porphyropsin in these “new” graphs also made him “learn,” discover, and create something worth safe-keeping.

A First Reading of the Fragment

How can we approach and enlighten this instant of mathematical thinking with respect to its flowing nature? In all classical, currently dominant approaches to mathematical knowing, individual speakers put into words what already is in their minds. This includes, for example, the enactment of pre-existing mathematical (body) schemas (de Freitas & Sinclair, 2013; Núñez, 2009; Piaget, 1980) or knowledge structures (e.g., Garfield, Le, Zieffler, & Ben-Zvi, 2014). Typically, researchers in that line of work take enunciations (in speech or in writing) as indications of their understanding of a situation or mathematical concept. However, philosophers and psychologists alike have suggested

that in speaking, thought does not externalize itself but *becomes* itself (Merleau-Ponty, 1945; Vygotskij, 2005). It is in the talking that a dim stirring develops into a fully developed thinking, the development being the result of a coming and going between verbally articulating and thinking. From this perspective, when Greg is beginning to talk, participants and observers have no evidence that a full thought already exists. In fact, his hesitating, erasing, pausing, and so on all speak for the converse situation: a thought is in the process of developing itself in and through communicating. Thinking and communicating develop like this even in a "mature consciousness [razvitom soznanii]" (Vygotskij, 2005, p. 962). At the point of the beginning of the Saying, what will have been said once there is nothing else to add—the *Said*—still is only a dim possibility of the future and beyond the current horizon of intelligibility.

In the two parts of our fragment, the "object" of the talk *evolves* through its presentation from (a) a speech and gesture invoking expected change in bandwidth for two chromophores and their mixtures to (b) the production of a graph representing those relationships. The discourse on thinking can be disobjectified when we consider the object of communication as orienting the recipient's attention (Sfard, 2008). Whereas this line of work brings a contrasting light with regard to perspectives in which the understanding of some fact preceded its articulation in words, it does not, however, emphasize the *flowing* nature of thinking, its actually *lived* presentation in communicating, participating, and so on. With discourse as its object of attention, the research questions concern the elements or properties characterizing mathematical discourse rather than its actual flow, that which makes life alive and keeps it in motion.

Interested in realizing Bakhtin's (1993) proposal to theorize what is alive in life, we examine instead the *continuous, moving movement* necessary for communication to appear as *mediating* or *coordinating* one and others' activities, which includes thinking. From this perspective, presenting (making-present) "the expected relationship," Greg is not transparently embodying a discourse on chromophore mixtures and bandwidth

personalized in voluntary decisions. The object of the talk transforms. A central distinction can be found in the assertion that the mathematical ideas which come into shape through this presentation remain unknown until thinking moves to something else. At the beginning of the episode, we do not yet know that there will be a relationship between the movements of Greg's hand (Fig. 2) and what subsequently will have been drawn on the chalkboard (Fig. 3k). Relating the two and considering them as fixed states that are "close enough" to be counted as significant bits of *one* unfolding thought might in fact lead the hasty researcher to the conclusion that some idea, some image of the graph was already finished in Greg's mind when he begins talking about the relation of the half-maximum bandwidth and relative concentrations of porphyropsin and rhodopsin in the make up of the photoreceptor. It would be hasty given what we actually observe in the production of the coordinate system and the graph. The hand movements (Fig. 2a.ii, iii and Fig. 2bii, iii) may in fact be just the transition between the two endpoints (Fig. 2a.i and Fig. 2a.iv; Fig. 2b.i and Fig. 2b.iv), corresponding to the bandwidths of the two substances, respectively, without any signifying function of the positions in between. For such a perspective speaks the fact that Greg subsequently points specifically to the half-maximum bandwidth of a mixture of the two substances to articulating the two endpoints. This is so because even in the instant prior to drawing the curve itself on the chalkboard (Fig. 3), his gaze is moving across the entire coordinate system, to the right, and returns to the left where the hand is closing in on the chalkboard. But then, rather than beginning a trace, the hand is slowly moving downward, stopping, moving further down, stopping again, and then, drawing the curve in one swoop, extending the sound-words such that these coincide with the making of the trace.

This detailed analysis thus shows that potential "states" in the movement of thinking can in fact hardly be isolated, let alone be taken as the signifier of *pre*formed mathematical ideas. Representations can be ascribed to gestures and other traces (such as words or curves) only after the fact, while the living mathematical thinking is busy with

relating such traces to one another, *making* present, from what *has been* present, and what *will be* present in such a way that what appears as a signifying mathematical idea finally falls out of movement, left as trace behind. This evolution, however, cannot be predicted (Châtelet, 1993), something the episode renders particularity accessible since one can see and hear that he is apparently "looking for" what he is going to say and do about the relationships he is presenting. This does not mean that just about anything might arise—there are inherent constraints existing in Greg's biography, the social situation, or the nature of language and graphical representation. The point is that the *possible* does not capture the creative dimension of thinking, for what is to come is already prefigured as one of the possible futures. Instead, Châtelet (1993) orients us to the virtual in movement.[1]

Moving from States to Flow: The Movement of/as Movement in the Becoming of Mathematics

The problem then is not merely one of the existence of the mathematician (e.g., Pimm 2007) or of the interagency to the bodies of objects (graphs), mathematical concepts, and of learners (e.g., de Freitas & Sinclair, 2013). Instead, we are interested in the *living* mathematician in the actual work of writing or reading a graph. Time and temporality are important dimensions in this approach: They reveal, in fact, the work that normally goes lost and becomes invisible in smooth, highly practiced performance and in representations thereof. From an objective (scientific) perspective, there are *long* pauses, in any event, pauses that are much longer than those generally observed in the course of a conversation: 1.0-second pauses in face-to-face talk, and 0.8-seconds in teacher-centered

[1] Mathematical physicists concretely use the idea of virtual displacements of a system to develop its mathematical description. They may take an arbitrary trajectory and then stipulate that the system will be in such a state that the virtual forces orthogonal to the movement disappear.

classroom talk). For example, there are 9 pauses between around 1 and 2 seconds in length (lines 2, 3, 4, 7, 13, 14, 16, 18, 25); and there are 7 pauses longer than 2 seconds (lines 17, 18, 21, 23, 27) up to about 14 seconds (line 19) (see arrows). We also observe the lengthening, shortening, and running together of syllables and elisions, changes that are experienced as the slowing down and speeding up of talk. However, we need an approach that enables us to consider time and temporality as *inner* features of *any* instantiation of mathematical thinking, and not only those in which pauses and changes in speech development are as clearly observable as in this example. Châtelet (1993) therefore insists on cultivating patience in our relation to graphs to engage in a similar way with any other trace of thinking we might encounter.

In the geometrical diagram as an account of mathematical work, mathematics is a dead Platonic object (Husserl, 1976; Livingston, 1986; Pimm, 2007); in the diagramming movement, as a form of labor, mathematics is continuously becoming. This is why in the ethnomethodological literature on mathematical thinking, a clear distinction is made between (living) *work* and the (dead) *accounts* thereof (Greiffenhagen, 2008; Livingston, 2000). Irregularities in the presentation on the chalkboard substantiate the becoming presence of the work of mathematical thinking, as turbulence in the flow renders its directionality more visible. In Greg's production of a graph, there are long pauses, repeated writing and erasure, and re-writing (Fragment 1, Fig. 3). All of these index the emergence of thinking shaped by the real-time graphing on the board. In pausing, time is being made as much as being taken; in slowing down and speeding up, the abscissa-making gesture (Fig. 1) takes the time it actually makes. Drawing, as writing, is generative, even though thinking might disagree, post-*par*tum, with what has been birthing. Greg is not just marking up the chalkboard but rather, in a punctuated and punctuating way, he is marking something, as seen in the example of the abscissa label (Fig. 4a), stepping back to gaze at it (Fig. 4b) as if to see what his markings have given, and then steps forward again only to erase what was there (Fig. 4c). Greg is receding only

for the purpose of returning and erasing what he has been writing. That is, taking on the open-ended nature of the presence of a sign thanks to which it is only when we "move on" after "everything is said" that there can be something like a finished idea. Here it is being rendered as a chalk trace on the board. In this case, apparently something inappropriate, for in the next movement, the previously written now is being erased. A further indication that this instant reflects thinking that is emerging can be recognized in the fact that Greg subsequently copies what he produced on the chalkboard into his own research notebook even though this topic appears in his publications decades before.

«««««« Insert Fig. 4 about here »»»»»»

From Thinking to Thought

There is research in mathematics education that attempts to return to thinking movement (de Freitas & Sinclair, 2012, 2013). The focus of this research, however, is on the diagrams left behind into which life is to be breathed. But diagrams hide the *actual* mathematical thinking, which disappears for the benefit of ideal mathematical objects: the actual *drawing* of a circle is receding and vanishing, being replaced by the pure idea of a circle that only comes to be when the circling has ended and when the trace it has left behind becomes an object for thinking. Not only has circling ended, but the trace is theorized as an outward expression of a pre-existing inner thought, conception, or schema. But this analytical, mostly deductive enframing of circling contrasts the coming-into-being of the circular trace in which we are interested. Despite the Greek (and Renaissance) attempts to negate traces, signs of the process of making, and situate genesis in a secondary position in relation to mathematical thinking, the very necessity to

speak or write or draw mathematics for presenting mathematical thinking to (it/one)self infuses it with temporality, the before and after of traces: of movement.

Given Greg's repeated stepping back, considering, writing, and erasing, we make a less stringent assumption when we say that the thought (idea) is becoming rather than merely moving from inside (the brain case, the body, the mind) to outside. Not only is there a decalage of the expression and thought in time but also a dehiscence of psychological and grammatical subject and predicate. In an exact parallel of the analysis Vygotsky provides for the sentence "the clock fell" (Vygotskij, 2005, p. 966), where the verb becomes the psychological subject about which is something expressed (it was the clock that fell), the "plot" (turn 16) is the grammatical subject about which something is expressed, the predicate (what can be seen).

There is further evidence that we have witnessed more than a translation from the inside (mind) to the outside (gesture, movement, chalkboard). That is, moving thinking implies more than a mere open-endedness of the path emerging from one mathematical step (in thinking) to another. In Châtelet's spirit of conceptualizing graphs in terms of virtuality, even when the hand stretches toward the board and begins to leave a chalk mark, what the trace will "end up" looking like and *potentially* will be signifying remains uncertain and underdetermined. The going of the hand is itself an instance of this moving thinking, which is continuously becoming, linking present to past and future in an always open-ended way, whether this potentiality is observably acted upon or not.

Greg has worked in this area of research for three decades. One might think that he had constructed some conceptions, mental representations, or schema that constituted his thought. But if the thought had existed, if there had been schemas as in Núñez's (2009) analysis, if the (final) idea had been finished in the mind only to be dumped to appear on the chalkboard, then there would not have been a need to erase what has been written. The inappropriateness was recognized *after having been noted* in the step back to gaze at what has been done. It is as if the thinking of the inappropriateness of what has been

written emerged after the writing (expression) had appeared, thinking has finished and yielded a thought. We are marginally interested in the drawing left behind, but in the event of the crossing from the invisible (nothing) to the visible left behind (idea). The nature of the re-writing gesture observable here also largely makes us discard the positivist metaphor of perfecting the representation of an already formed reality, as if the scientist would merely "fix" his creation to make it close to what stands before his eyes. Replacing $\%A_2$ with A_2/A_1 (turn 20, Fig. 3i) next to the abscissa completely changes what the graph might look like: the curvature of the function would be more accentuated between 0 and 1, and then slowly decreasing with an asymptote at $y = 0$. Inappropriateness does not stand in a written representation of a fixed picture but emerges as the object being thought of itself emerges, forms, and transforms.

Pimm's work regarding the mathematics curriculum provides us with an opening for describing and theorizing the movement thinking. In such reading, each pair of photographs in Figs. 1, 2, or 3 requires a *per*formance—a term fortuitously drawing on the Proto-Indo-European root *per*, which was used in verbs that denote "to go over, fare," "to go towards a limit," "to risk," "put oneself in danger." This reading, just as writing, not only gives birth to the idea that undoes itself but also is a form of erasing, for the empty space that existed before now has disappeared, and erasing is un-writing, writing in which an existing trace comes to be undone (Derrida, 1967). Even if Greg were to have created this diagram before (and precisely for this reason), the experience would be "*lived-through [durchlebt] quasi new and actively*" (Husserl, 1976, p. 370, original emphasis). The very possibility of living through the experience for a first time also makes it possible to do so again—though in a quasi-new and active way. With this possibility of living-through the experience again, one can become increasingly experienced despite the singularity of each one of those experiences.[2] This is so precisely

[2] The sense of the Proto-Indo-European root *per-* includes "going through"; this sense is carried over into English words such as ex*per*ience, *per*il, *per*formance, and *per*egrination. Experience

because experience passes away so that it actually *can* be made present again, temporality arising precisely from the experience of passing experience (Merleau-Ponty, 1996). Temporality arises in the difference and the differing, what Derrida altogether calls différance, between what is aimed at and what is produced in any actual performance, between apparent intention and that which, within the intention itself, cannot be anticipated: the unforeseen (Roth, 2012a).

From the *Saying* to the *Said:* Accepting what Presents Itself

It is when living presenting has come to an end—i.e., when the written sign exhibits itself in its abstractness from Greg's performance—that the audience, Greg, and the readers can find an accomplished thought that no longer is alive and warm (Pimm, 2007). As long as the *Saying* unfolds, we do not know what will have been said once the Saying will have stopped (Levinas, 1978). But what the unfolding Saying leaves behind, the *Said, affects* the Saying in progress: Greg erases what his writing and saying have left behind *after* he is being confronted with it. Producing the half-maximum bandwidth graph therefore is not just a straightforward "construction," but in fact a moving back-and-forth that, in standing back and meditating upon itself, comes to transform what it has done. Châtelet (1993) uses *virtuality* to exhibit how thinking begins as a germ that is open to different trajectories to come. In concretizing itself, thinking develops. At the same time, undeveloped mathematical thinking is moving towards developed mathematical thinking so that in moving, thinking is transforming the process of thinking and not only the object of thinking.

Thinking is an encounter in which the subject of thinking is becoming while it is considering the object of antecedent thinking. Greg's presenting is not projecting

and performance, therefore, literally index processes of going through, undergoing, and being imperiled.

something from an inside (mind) to the outside (recorded and recordable speech, graph on the chalkboard). From Latin *prae* "before" + *esse* "to be," presencing begins *before being* although it also is producing what then becomes present as in Latin *present*, "being at hand." Moving in, ideas nevertheless are surprising us precisely because we cannot know from the start how they will be affecting us, and where they will be leading us (Sinclair, 2007). In this fragment Greg is working it out, constantly adjusting his speaking and writing as the presenting unfolds. It is precisely in this working out that we observe mathematics alive, whereas the completed diagram is but a dead trace left behind. What we would expect to see from close analysis of his first gesture (Fig. 2a) differs from what the second gesture offers (Fig. 2b), whose shape *resembles* the one finally left on the board (Fig. 3k) or in his notebook.

Greg is finding himself confronted with what he has done, which changes what he is doing. Mathematical thinking in movement therefore also means exposing oneself to and accepting what is presenting itself (Roth, 2012a). It is not only that drawing requires motor skills that are not necessarily part of imaging an image. Readers may want to try and [a] think about a circle; [b] gesture a circle in the air; and [c] draw a circle on paper . . . and consider how different the three productions might be not only in terms of the final product, but also in the kind of work they require. Readers will then experience what Heidegger (1984) has denoted as a continuous journeying, re/departing, and arriving.

The fragment leaves us with an apparently finalized graph, accompanied however by the formulation of what can easily be heard as a demand for validation ("right?", turn 28). Here again, the episode is exemplary in that it is providing us with an excellent material to engage in reflecting on the dynamical nature of thinking and communicating. Thus, if we wish to take the drawn diagram as an end, we must account for its emerging, evolving nature, together with the being-alive of its making. Even the simplest mathematical concept or action—e.g., a 7-year-old's multiplying of numbers and striking out of decimals—can turn to be much more complex and subtle than expected (Sinclair, 2007).

Even if at some point—e.g., end of Fragment 1a, turn 16—Greg decided to direct himself to the production of a graph on the board, he cannot know exactly what he might find once he arrives, perhaps discovering something new about these relationships or about graphing altogether, realizing for example that his quadratic-like curve (visible in Fig. 3k) is, from a mathematical perspective, totally different than the bell-shaped curve, typical of normal distribution we can see. An example of such a case was described in a 35-year veteran physics professor, who, at the end of a 15-minute lecture with graph is telling his students that "there is something wrong with the picture [graph]" (Roth, 2012b, p. 215). Only *a posteriori*, once the journey is over, is it possible to step back and consider it—in the very same way that Greg or the physics professor can walk from the board and examine what he did only after traces are left. In the remove from their thinking, they may be accepting it, as a gift, which they are apprehending, comprehending, and accepting.

On the other hand, the erasing gestures we saw in the very course of the production of the graph remain as possibilities even once the drawing appears to be finished. After turn 16, someone could have questioned the curvature presented, leading Greg to go back to the board and change it once again. There was in fact a possibility for this to happen when the doctoral student in the room said in response to turn 28, "yea, more or less, maybe a bit higher on the one end (*gesture toward the chalkboard*), but same idea"; but Greg, though responding by saying "right," did not revise the graph. And then again, even without this possibility, the figure presents itself not as a given, but as a trace that requires interpretation. That is, even fixed traces in ink on paper are open-ended with regard to thinking: they are but fugitive instants in a history of thinking that is continuing beyond its factual production (Châtelet, 1993). What is seen in a diagram may then very well differ from "what you could expect," and truly, once again, awaits it *presentation*. It is because thinking is not once formed and then transferred to other situations that anyone attending needs to re-think everything that Greg has said or done. Greg's taking notes of

the final product of his earlier workings on the board shows that he is no exception to this. Our lengthy analysis of the graph and graphing taking place on that day explicates how the movement of thinking is thinking in movement, a journey.

THE STRUGGLE WITH THINKING THE UNTHINKABLE, MAKING VISIBLE WHAT IS INVISIBLE, MAKING ACTUAL WHAT IS VIRTUAL

> Virtuality is that which, in movement, permits to knot together an "already" and a "not yet" . . . it extols the latencies coiled in bodies. (Châtelet, 1993, p. 45–46)

The purpose of this article is to propose a way of thinking the *movement* of thinking, and to do so by exhibiting thinking in movement without reducing thinking or movement to something else. The problem in thinking movement arises from the received ways of theorizing in operation from the Greek to the present day, which privileges objects (Heidegger's *Seiendes,* "beings," *étant*) over the flow of Being (*Sein, Être*). As philosophers time and again have pointed out, the living and becoming is really invisible and virtual (e.g., Henry, 2000; Merleau-Ponty, 1964), which is erased at the instant we attempt to grasp it. Our own efforts would be subject to the same critique were it not for the fact that we recognize our effort as gesturing towards the invisible and virtual of (Greg's and our own) thinking in movement, something readers experience in their own thinking that follows ours. In the following, we first describe the attempt to think movement dialectically, as Vygotsky suggested, and then offer advancing the dialectical approach

From Thought to Moving Thinking

Dissatisfied with classical approaches to psychology, Vygotsky emphasizes the movement of thinking arising from the movement between thought and (verbal) articulation, itself a movement. Thus, just as this moving thinking can be in the articulated sentence, the process of the emergence of mathematical ideas is captured (seized, gelled) in the series of still images composing Figs. 1, 2, or 3. We cannot, as we point out above, understand *moving* thinking if we animate these images from the outside (e.g., by a homunculus). This recognition led Vygotsky to propose a materialist dialectical approach (Vygotskij, 2005). Doing this requires dynamical categories that embody difference *with* itself, pure difference; and, inasmuch, a unit of movement cannot be self-identical. To move our thinking in the direction of moving thinking, we may take Vygotsky, Il'enkov, and their Spinozist origins as point of departure. The Spinozist conception underlying the dialectical approach takes the "organ of thought, while remaining wholly corporeal and therefore incapable of having schemes of its present and future actions *ready-made* and *innate within it* together with its bodily-organized structure" to be "capable of actively building them [schemes] anew each time in accordance with the forms and arrangement of the 'external things'" (Il'enkov, 1977, pp. 50–51). From this follows that in drawing a circle, the living body and its reflection in conscious awareness enter a fully identical state because the living action is itself a circle. But being aware only follows the state of the body, the living action, which is taking the form of a circle. This is even more sharply articulated when saying that "the body, knowing how to be in a state of movement along the contours of a circle, in that way knows how to be in a state in common with the state and arrangement of all circles or external bodies moving in a circle" (Il'enkov, 1977, p. 69–70).

Central to a living conception of thinking and speech are (structural and historical) contradictions (Badiou, 2009; Vygotskij, 2005). For Vygotsky, the incongruence (non-congruence) of grammar and logic in the development of language does not exclude the unit of thought and language but, quite the contrary, constitutes a necessary condition for

the relationship between signification, thought, logic, and word; it is a necessary condition for realizing how thinking is moving into speaking and speaking into thinking. Development implies an inner contradiction because of the unit of thinking and its accomplishment in speaking or, to expand to communicating writ large, between thinking and any sign form. Thinking change as change requires categories (basic units) of change. These units embody inner differences, which is another way of saying that a system is undergoing self-movement (Il'enkov, 1977). If we consider change in terms of before and after states, then there is a force external to the system that brings about the change. This is elemental analysis (Vygotskij, 2005). However, if we consider the before, after, and the change process *as part of the same unit* (category), then we have a unit (category) of change. The term category is on purpose, because in Russian theater it has been used to denote the dramatic event (Vygotskij, 1983). Thinking-in-movement is part of a more encompassing moving system (Vygotskij, 2005). But the becoming of the new also means the erasing of the old—in moving, thinking not only is creating but also erasing itself because the before and the after no longer are independent but are one-sided expressions of the irreducible unit. This is the fundamental idea of unit analysis.

One problem in the uptake of Vygotsky's work is that his Russian *myšlenie* is translated into English as *thought* rather than as *thinking*, which would be equivalent to the practice of rendering the Russian word in German as *Denken*. Whereas the Russian and German terms are noun forms of verbs, the English *thought* is a thing. However, Vygotsky is quite adamant about the fact that to understand the *process* of thinking, we have to "capture it alive" in the same way languaging does (Roth, 2014). This requires conducting unit rather than element analysis (Vygotskij, 2005). That is, thinking is a part of a system in self-movement, and this self-movement can only be understood in terms of difference in and for itself. Movement in/of thinking is grasped (prehended) in the episode because we rupture the flow of the ongoing presentation of "what you would expect" in fragments.

Rekindling Movement by Considering Two Stills Simultaneously

Received theories tend to approach living phenomena by means of fixed points—such as pre-tests and post-tests to suggest learning has (not) occurred. Doing so is problematic because it resembles trying to understand the flow of a river in terms of its banks and the bridges that cross it by establishing transitive relations (Ingold, 2011). But approaching the river's flux by means of fluid mechanics does not get us to the living phenomenon either, because differential calculus still is calculation, prediction, that is, not what Heidegger (1954) is engaging us to consider—especially when asking "what is thinking." Our own figures and diagrams therefore cannot be more than traces of thinking and moving, a stepping-stone on the journey of approaching them, while the true sense of this peregrination is of course not in the destination itself, thinking and moving, but in the pilgrimage, the going trough, the journeying as journeying.

We can rekindle moving thinking by considering these simultaneously, as one. If we were capturing images at a speed rate higher than the 30 frames per second of our cameras, such as an industrial high-speed camera shooting 10,000 frames per second, those fragments would be "slowed down" and would break up into a series of pictures reproducing the movement within what previously counted as unitary instants (a single photogram). At this rate, we see movement in how the presentation goes from a speech and gesture invoking (a) an expected transformation of bandwidth from one chromophore to another in which the latter is slightly wider (turns 01–06), to (b) the idea of an expected increase in bandwidth with regard to their mixture (turns 07–11), and then to (c) a combination of both in a "sort of" transition plotted on the board (turns 12–16). In the fragments reproduced above, apparent changes in the object of talk become visible from frame to frame, but only when two such frames (utterances) are considered together, irreducible parts of the same unit (whole). The signification of the verb "to see" also

seems to change, from its use in the sentence "a priori what you would expect to see" (turn 2) to "guess you're gonna see this, you know, if you plot" (turns 12–16). The actual use of the observer's eyes is increasingly present (but never explicitly mentioned) as gestures are about to be called upon to leave more permanent traces in the form of writings and drawings on the board, traces which themselves will be recorded in Greg's notebook. The same goes for Fragment 1b, where our rendering of talking already illustrates the possibility to slow the motion even more, attending to how a single sentence ("Half, half max bandwidth versus or . . . render this in percent, from one . . . say a hundred to zero . . . you would expect to see something like this, right?") can be described as moving from one yet-to-be-articulated mathematical idea (half-max bandwidth written as "H," turns 17–18) to another (a single comparison in a Cartesian graph, turn 18), to another (identifying the other variable), and so on. Examining Fig. 2 in such a perspective, the reader should now easily appreciate how this simulation of movement is created in the reproduction of a single gesture from the proximal presence of *similar*, *still* pictures, which nevertheless *seems* to move.

Doing this in the context of the reported fragment generally and to the eventing *between* Fig. 3e to Fig. 3i specifically means taking neighboring frames at a time and think these as *markers* or manifestations of minimal units. As soon as we are envisioning the transitioning from one frame to another, our thinking *is* moving. In the resulting unit there is a spatial relation that simultaneously denotes a temporal relation that is replayed when thinking transitioning; but then there is also a dehiscence between thinking and its (self-) realization (in the sign). The minimal unit integrates these two markers, before and after, as moments of the same *movement*, which means that, in pictorial form, our minimum unit does not consist of a photograph but of two or more photographs. The relation between consecutive photographs is no longer *external*—provided by the motor of a video projector or the software package running the digital movie—but internal to the movement of our thinking. We could not even see a geometrical line without the eye

moving transversally and parallel to the line (Yarbus, 1967). We therefore have to approach those still images as products, as something fixed in form, which is but a *vanishing moment* of that movement.

From movement driven by some external aspect, we advance to consider movement as *self*-movement *within* a system. In so doing, we go beyond representation to offer what Châtelet (1993) calls the "operative contemplation," exemplified by Einstein's "thought experiment" of riding a proton as a manner of really making relativity familiar. It is not "change" we pursue, not barter, but transformation and morphing. Châtelet shows us how, while attempting to capture and grasp movement, we always only offer traces, but traces to which movement is attached by the interpretation of those traces. Movement exists in/as interpreting, because interpreting is moving, while traces are static. The only way of getting immobile things moving is by re-animating them (Ingold, 2011). Actually "make it," moves it, it moves. Only our own moving can bring back life to the inanimate. *Duration*, in the movement that is thought, is pure qualitative heterogeneity (Derrida, 1967), the moving of thinking (logos) is moving temporalization itself. To put it differently: only *we* "fill the gaps" between the fixed images, and it is this filling which *makes* movement present again.

Any moment of a unit used to construe movement—e.g., the consecutive frames of a video track—is but a *marker*, one defining the before and another one the after. Between them lies an unidentified and unidentifiable gap where transformation actually occurs, where movement as movement needs to play. Movement as movement exists in the active linking, the participative thinking which, on the part of the reader, forms a whole and recreates continuity from a series of still pictures. Movement as movement—i.e., thinking as thinking—is then not captured but provoked (e.g., Livingston, 1995). Thinking and movement are not captured in Greg's traces but are present in the actual diagramming as diagramming. And this is how we can see that the mathematical relation between HBW and chromophore is not in his Said, in the content of the talk, in some

metaphysical "meaning" attributed to the words he uses, but in his Saying *as saying*, in Greg's presenting and in our reader's reading of the episode as such. One way to illustrate this in a non-grasping manner might be found in the "operative contemplation" of an image of Greg's hand (Fig. 5), which evokes Marcel Duchamp's *Nu descendant l'escalier*[3]. Here, moving and thinking are stimulated in that "hard work" of interpreting that is necessary for figuring out what is going on, toing and froing, erasing, and transforming in the signification given to a what is offered to the eye.

«««««««« Insert Fig. 5 about here »»»»»»»»

The overprint (Fig. 5) requires being read/seen as an instruction. When readers follow this (dead) instruction, an overlap is created in the impression, where the instants are but partially captured, already erasing and re-writing one another, explicitly requiring reading by actively moving the eyes rather than presenting themselves "ready-to-hand." Moving is provoked here through the disturbing presence of signs as signs. The perceiver is set off moving, taking off from the traces into a moving experiencing in which even the moving body instantly stands out in its handiness: as a way moving properly. Greg's *presentation* is nowhere short of this, per-forming the mathematical relation both with and beyond the traces (spoken words, viewed gestures, unerased marks on the board) left behind in the diagramming episode.

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In this paper, we are tracing a way for thinking the moving mathematical thinking

[3] See, e.g., http://en.wikipedia.org/wiki/Nude_Descending_a_Staircase,_No._2

that occurs in the mathematics curriculum—by keeping thinking moving. We thereby follow a path that over the recent 200 years philosophers such as Hegel, Nietzsche, Heidegger, Derrida, and Châtelet have taken but whose original tracing is tied to Heraclitus. Our thinking, however, is not old, but an im*per*iled *per*egrination in its move beyond current perimeters of thinking thinking. In keeping thinking moving, however, "forces" have to be thought *internal* to the units of thinking; and change requires thinking with minimum units (categories) that themselves are changing, moving, rather than as a difference between things and states. That is, we propose thinking a curricular event in mathematics *as* an eventing (Roth, 2013). If, however, we move to such a description, then also falls the distinction between agent (student, teacher) and action, which requires a complete rethinking of the subject of action and, therefore, the subject of mathematical knowing and learning. As a result of this study, our thinking moves towards considering speaking and thinking in/as moving, and, as such, constituting history; moving transcendence—i.e., of the being-alive of becoming aware—*is* history. As evoked in our title, "The stakes of movement," there is indeed a lot at stake in the moving thinking in/to the mathematics curriculum. It requires abandoning cherished modes of thinking in (mathematics) education. Such an abandonment may be a good beginning, for it gets a ball rolling again (re-animating it) that has come to a screeching halt with a fixation on construction and the concepts and (mental) structures that result from it.

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Fig. 1. A series of consecutive still pictures from video of a biologist doing a graph, played or traversed in rapid succession, gives the illusion of movement.
187x50mm (300 x 300 DPI)

Fig. 2. In the course of describing the relationship between the bandwidths of porphyropsin, rhodopsin, and mixtures thereof, Greg moves the pencil through a path while talking about (a) porphyropsin and rhodopsin bandwidth levels and (b) on a similar path on the reverse way from rhodopsin to porphyropsin bandwidths.
102x78mm (300 x 300 DPI)

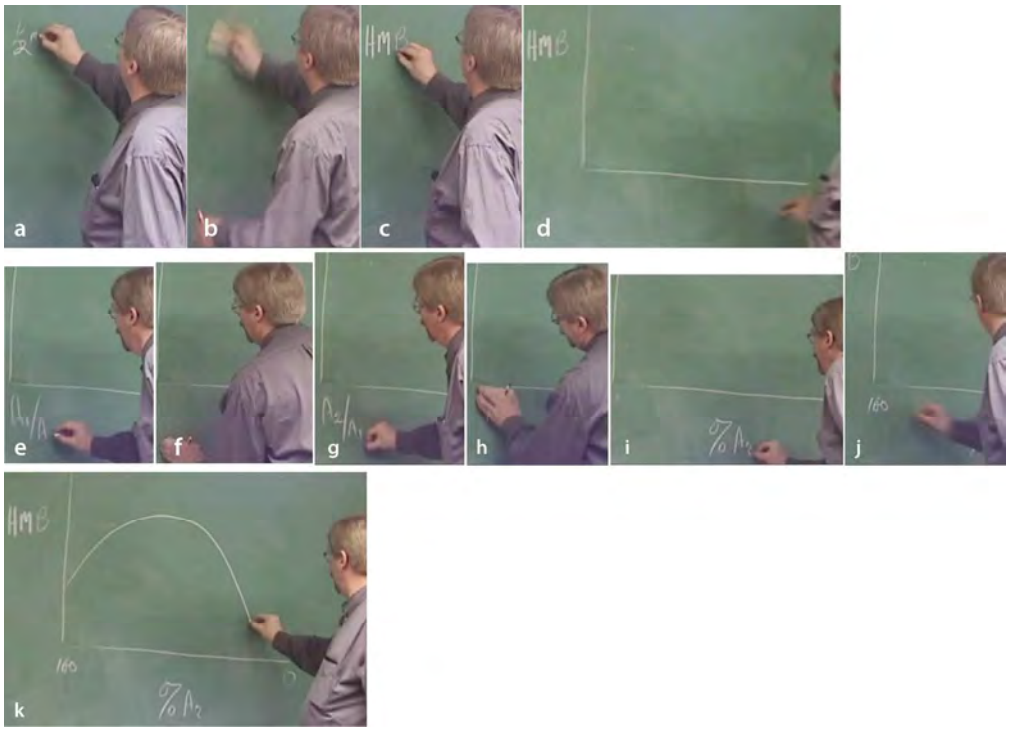

Fig. 3. The production of the graph is marked by writing, erasing, rewriting, interspersed by pausing.
169x122mm (300 x 300 DPI)

Fig. 4. In the case of the abscissa label, (a) Greg first writes A2/A1 on the left near the origin, (b) then steps back (as if to see what he has done), and (c) erases it again.
127x33mm (300 x 300 DPI)

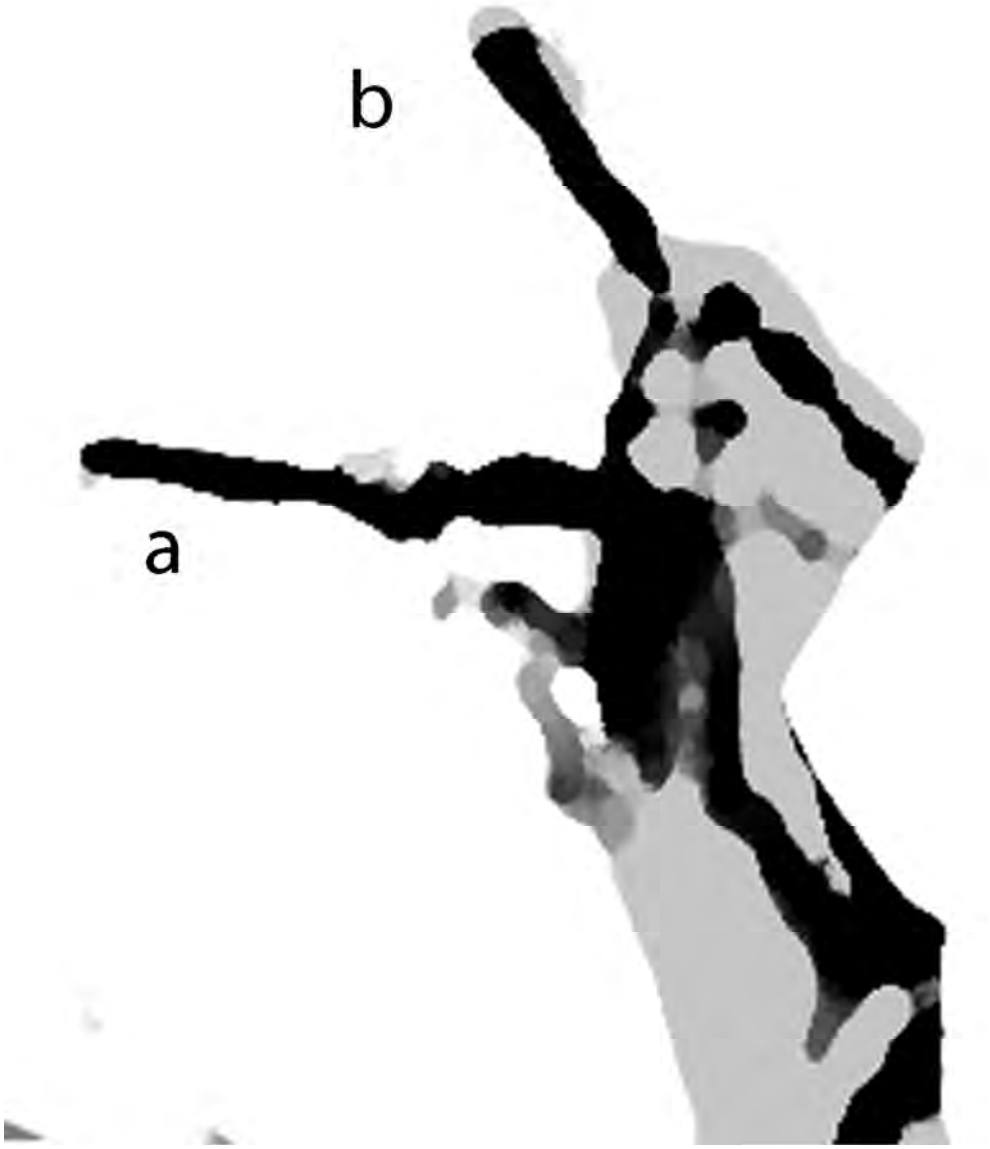

Fig. 5. The superposition of two instants in the production of a (moving) gesture evoking the relationship of chromophore and bandwidth resembles Duchamp's study of movement in Nu descendant l'escalier.
76x89mm (300 x 300 DPI)

Appendix

Fragment (00:04:38-00:06:20, 010618T1)

```
   01  C:  like forINstance, i mean what you mIGHt,
→  02      (1.33) um A: priORi what you would expECt to sEE::
           (0.66) I:s::: (0.81) the bANDwidth (0.46) to gO:o from
→  03      (1.45) u:m::. (0.38) the pORphyrOPSin
→  04      (0.99)
   05      [bandwidth which is slIGHtly wi:der than the]
           [((Gesture in Fig. 3a.))                    ]

           rhodOPsin bandwidth
   06      [    (0.49)             ]
           [((Gesture in Fig. 3b.))]

           but thEN you would expect to see an IN:crease in
           bandwidth beYON::d=the a=two
→  07      (1.22)
   08      representing the a=one=a=two mixtures
   09  T:  uh hm
   10  C:  to a varying degree;
   11  T:  yea
   12  C:  guess youre gonna sEEe
→  13      [(1.21) u::m:;                          ]
           [((pencil in air, begins "drawing" line))]

→  14      (1.61)
   15      this [(0.81) sort=o=tranSItion where
                [((gets up, walks to chalkboard))

→  16      (1.24) yknow=if=ye plOT]
           arrives at chalkboard))]

→  17      [(2.68) um (3.88)                  ]
           [(Picks up chalk, stares at board))]

   18      hA[Lf;                           ]
             [((writes "1/2 m," Fig. 4a))]

→          [(5.93)                          ]
           [((erases 1/2 m, Fig. 4b stares))]

→          [hALf (0.53)   [mAX (1.27) [bANDwidth (0.46)
   18      [((writes "H"))["M"        [(("B") Fig. 4c

           <<dim>vER[su::sssss          ]>
                    [((draws ordinate))]

→  19      [(14.01)
           [((Draws abscissa, Fig. 4d; writes "A1/A" on the left
           end, Fig. 4e, stares, then erases it, Fig. 4f; then
           writes A2/A1, Fig. 4g, then erases it, Fig. 4h))

   20      [or <<pp>(render?) this in percent>
           [((writes "%A2" in center of abscissa, Fig. 4i))

→  21      [(6.56)
           [((looks repeatedly left and right ends of abscissa))
   22      from=[one=say=a=hUNdred to zERo.           [
```

[((*Writes "100," Fig. 4j, moves right* [((*notes "0"*

→ 23 [(2.14)]
[((*Steps back, gazes at coordinates*))]

24 [you=wouldexpect=t=sEE::::]
[((*Steps forward, places chalk*))]

→ 25 [(1.61)
[((*Hand moves slowly down vertically*))

26 [something=like=thISS:::.]
[((*Draws curve, until arrives at Fig. 4k*))]

→ 27 [(7.60)
[((*Gazes at curve, then places "x" at end and beginning points of curve, steps back to gaze at graph, turns to audience*))

28 right? ((*00:06:20*))