Elaborating the later Vygotsky's radical initiative on the nature and function of language with implications for mathematics education

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Abstract   Scholars interested in the function of language in mathematical learning often draw on Vygotsky, whose early work on word meaning has shaped many research studies. However, near the end of a rather short life, Vygotsky heavily critiqued his own previous work and began to sketch a radical theory revision, which overturns much of what he had done and is famous for. The purpose of the present study is to elaborate a possible avenue of such a theoretical revision. This study develops the new theory in the course of an exemplary analysis. The data derive from a scientific laboratory, where three scientists discuss a graph as it evolves in real time before their eyes and as a result of transformations designed to recover the real signal from the noise that is apparently present. Implications of the emerging theory for mathematics education are discussed.

Keywords   Vygotsky; language; mediation; sense-giving field

1   Introduction

The interest of educators in the role of language in mathematical learning is a recent but, as this special issue testifies, important development in the field. At least five lines of research interests may be identified, including (a) communication and interaction, (b) representational systems, (c) language-use in mathematical practices, (d) codes or registers of mathematical communication, and (e) function of discourse in mathematical thinking (Planas et al. 2015). The importance of language in mathematical learning widely has been approached through the lens of the works of the Russian psychologist L. S. Vygotsky. Whereas some scholars claim that Vygotsky's approach is limited, when compared for example to the dialogical approach of Bakhtin (Barwell 2016; Matusov 2011; Wegerif 2008), others show that the so-called limitations are due to a misreading of Vygotsky (e.g. Roth 2014a).

Frequently attributed to sociocultural theory and Vygotsky is the role of language as mediator (e.g. Radford 2016; Sfard 2008; Venkat and Askew 2017). Importantly, he is said to have "maintained that the use of these cultural tools is first developed as an interpsychological process *between* people and later internalized into an intrapsychological process within a person" (Takeuchi 2017, p. 3, emphasis added). Barwell (2016) rightly criticises this aspect, which he refers to as instrumental and systemic. This critique is correct with respect to the interpretations of Vygotsky; but it does not do justice to Vygotsky himself, especially not to the theoretical framework that he was beginning to develop near the end of his life. Already five years before his death, he stated: "every higher psychological function … *was* the social relation between two people" (Vygotsky 1989, p. 3). The higher functions are social relations of people not processes and tools that somehow appear between them (Roth 2016). The very idea of a language as a mediator reproduces Cartesian dualism in psychology that is antithetical to Vygotsky's non-dualist Spinozist thinking, in which "the sign is by no means a mediator" (Mikhailov 2001, p. 17). The later Vygotsky does not present an instrumental view of language. Instead, the word is the relation, and it is common to the interlocutors.

Language and the things of the world are given to the senses of any individual as subjectively his/her own and thus subjectively belong to everyone (Mead 1932). The commonness of the word makes for the basis of *common sense*, and thus for the inherently "*social* character of the universe" (p. 49, emphasis added).

Barwell (2016) also is justified in his critique of Vygotsky interpreters with respect to an emphasis on dialectics versus the dialogical approach of Bakhtin. But Barwell does not discuss the later Vygotsky, who, among others, states: "the relation between thinking and speech, in which the latter is its materialization, its objectivization, its embodiment, [is] a <u>continuous</u> transition of the external to the internal and the internal to the external" (Vygotsky 2010, p. 94). There is therefore a coming and going that is central to the dialogical approach. Thus, "meaning of the word is not the object that it substitutes, but dialogue" (Vygotsky, in Zavershneva 2014, p. 69). There is an orientation toward communities and the "the real–ideal intersubjective field of their common '*co-spirituality,*' the universal forms of their shared culture" (Mikhailov 2001, p. 20). His work thereby coincided "with revolutionary breakthroughs in the theory of language … and in the theory of verbal creativity (M. M. Bakhtin)" (p. 15).

Another problem in the scholarly uptake of Vygotsky is the focus on the social in a weak sense, that is, the importance of the dyad, small group, or class as a context for learning to occur (e.g. Blanton et al. 2017). This interpretation fails to recognize that for Vygotsky, any higher psychological function and associated behaviour is social through and through. Even writing into a personal diary is social (Vygotsky 1989). Associated with the issue of the social in a weak sense is the distinction between inside and outside, the mental and the material (e.g. Coutat et al. 1916). As the analyses below show, it makes little sense to attribute thinking, for example, to the inside of the person. For Vygotsky (1987), this aspect of his work eventually became problematic. In thinking through these issue, he realized that the word is a manifestation of consciousness for the other as much as for the self.

Contrasting currently dominant receptions of Vygotsky's position of language, recent re-evaluations show that during the last two years of his life the scholar was disavowing his earlier work (Zavershneva 2010). Some scholars characterize the currently dominant representation of Vygotsky's work as misleading and false (Mikhailov 2001; Yasnitsky and van der Veer 2016). Unhappy with the latent Cartesianism in his earlier work, characterized among others by an excessive concern with the ideal aspect of language use (i.e. "meaning") and its reliance on the concept of mediation, Vygotsky contextualized the problem of language in the much larger and more important problem of consciousness (Zavershneva 2014). Vygotsky (1987) sharply critiques traditional psychological approaches for their focus on supersensual-ideal aspects of words at the expense of their material-sensual (thus affective) aspects. <u>The two most important changes relevant to this paper are:</u> Vygotsky turned away from "meanings" and ~~the role of~~ language as mediator and ~~was~~ <u>began</u> moving toward ~~an emphasis on consciousness and~~ sense~~.~~ <u>and L</u>~~l~~anguage was to be a "sense-giving field [Rus. *smyslovoe pole*]" (El'konin 1994; Mikhailov 2001), inherently common to and constitutive of the interlocutors. The concept is consistent with the notion of sense-giving consciousness [Ger. *sinngebendes Bewußtsein*], where language, like all other forms of acts, is the public manifestation of consciousness; any linguistic or practical act is part of multiple nested sense-giving contextures [Ger. *Sinnzusammenhänge*] that allow an act to make

sense (Schütz 1932). The "sense-giving field" and the "sense-giving contexture" thus are connate concepts.

This contribution is designed to elaborate a radical theory revision on language that Vygotsky began to bring about during the last year of his life but that his early death prevented him to articulate. Although Vygotsky's early works are widely known, his radically revised theory has yet to be recognized and applied in mathematics education (e.g. Roth 2017). Using an episode from a scientific research laboratory where data, their transformations, and their modeling are at stake, a radically different take of the late Vygotsky is worked out. The analyses also exemplify the methods appropriate for realizing Vygotsky's program.

2   Research background

2.1   Data sources and participants

The data were collected as part of a five-year ethnographic study of a scientific research group investigating the absorption of light in the eyes of salmonid fishes, which changes with a number of factors, including life history-related ones such as season (light), temperature, and salinity of the water (cf. Roth 2014b). The two full professors who feature in the episode below had pooled their independently obtained funding to realize the project. Professor 1 (P1) is a biologist who had done more than 30 years of research on fish vision; and professor 2 (P2) is a trained physicist and statistician with relevant expertise in light transmission and data analysis. The third individual in the episode is a research associate (Ra), with an undergraduate background in physics and work-related competencies in software development and data analysis.

P1 had developed a new way of collecting absorption data from a single pulse of light that was refracted and recorded in a charge-coupled device (CCD), which permits recording the intensities of the different wavelengths (frequencies) of a beam of light after it has passed the specimen and a diffraction grating. The episode took place during a phase of experimenting designed to report the performance of the new method in a scientific journal.

Besides the notes of the ethnographer as participant observer, there are many videotapes from the wet laboratory where the data were collected (shot in the infrared mode), the offices where the data were analyzed, and the regular team meetings in which progress was discussed, further experiments planned, and data interpreted in an ongoing fashion.

2.2   Context of the episode

The episode was recorded during a regular day in the wet laboratory. The day begins with a 1-hour adjustment period required for the scientists to be able to work under low-intensity deep red light. After sacrificing a fish, extracting and preserving the retina, and mounting a retinal piece on a microscope light, the data collection begins. The scientists record the absorption of light by means of two measurements. The first determines the amount of light that goes through the slide next to a conical photoreceptor (control), whereas the second beam traverses the photoreceptor. The difference between the two

recordings yields a light intensity (A) versus wavelength (λ) graph that stands for the amount of light absorbed in the receptor. The episode begins after a second recording for a particular photoreceptor has been made, when P1 turns from the microscope to the data display that Ra operates. P1 conjectures that they have made a measurement on a UV cone-shaped receptor, though knowing whether this is the case requires the graph, and what the graph displays requires knowing what has been under the microscope (Roth 2009).

2.3   The mathematics in the episode

Photoreceptors of a single conical shape maximally absorb (have a λ_{max} [lambda max] at) in the UV (~370 nm) and blue part of the visible spectrum (~ 430 nm), whereas double cones have a green (~520) and a seasonally changing red maximum (~575 or ~620 nm). Any absorption curve, which is assumed to have an approximately Gaussian shape, may be associated with background noise of roughly linear shape; the actually observed curve thus is the simple sum of a line function and a Gaussian (Fig. 1). To get the real absorption curve requires removing the line function ("detrending"). The term *bleaching* refers to the act of irradiating a photoreceptor until the absorbing molecules have been changed so that no more light can be absorbed. The two recordings are not linear functions but histograms of the amount of light falling into one of the 1340-pixel columns ("bins") of the CCD. A measurement thus produces a histogram of light intensities. Two consecutive measurements may lead to slight shifts in the histograms recorded. The expression "pixel shift" refers to aligning them.
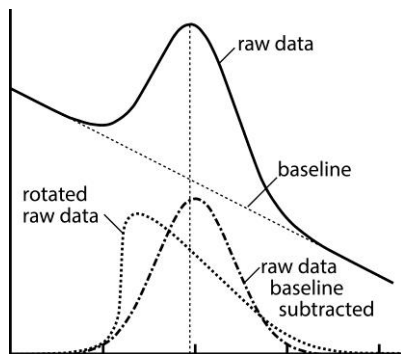

Fig. 1   Simulation of relation between peak maximum position for raw data, rotated raw data (left shift), and raw data with "baseline" subtracted (right shift).

The episode occurs in the context of collecting data, extracting relationships between two variables (amount of light absorbed vs. wavelength), relating graphical features to natural phenomena, identifying a point of interest (e.g. abscissa value of peak or "lambda max"), removing two different forms of noise from the actually observed histograms ("shifts" created by the recording technology removed by "pixel shifts," and other background noise of unknown origin removed by "detrending"), treating histograms as discrete manifestations of underlying functions, and modeling observed data by theory-based mathematical functions. The analysis below refers to a misconception concerning the removal of noise that appears in the data in the form of a linear function. When the noise is removed, that is, when a linear function is removed from the measured curve, the

maximum of the peak moves to the right rather than to the left that would be the result of a rotation of the raw data curve (Fig. 1).

2.4   The episode

The conversation unfolds while all three participants are watching the computer monitor, the display of which is shown to the right. The transcription is designed to show how talk occurs in real time rather than reducing it to its dictionary sense. (See appendix for transcription conventions.)

```
 1  P1:  looks like a single cone, and looks like a u:=v: cone.
         i do::n't kno:w;
 2       (0.6)
 3  Ra:  could be just someone
 4  P1:  could be wro:: ong
 5  Ra:             here
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 6       (17.5)
 7       now we're getting a step function
         °we shift it one pixel to the right. (1.3) no, wrong
         way (0.8) one to the left°
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 8       (2.1)
 9  P1:  okay, now; (0.7) detrend that (.) pleas::e.
10  Ra:  okay. ((Begins to detrend))
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11       (1.3) ((P1 hand gesture
         doing "rotation" on the
         graph))
12  P2:  <we're about at fou:r>
13       (1.5)
14  Ra:  four something
15  P2:  four-ten (0.3) (ichty?)
16       (1.7)
17  Ra:  °o:kay°
18       (7.1)
19  P1:  it may shift into the u-v as it is detrend'd=
         i think th is is  baseline here right here
20  P2:             uh hm
21  Ra:  YEA, i would also take this as the baseline present.
22       (11.2)
```
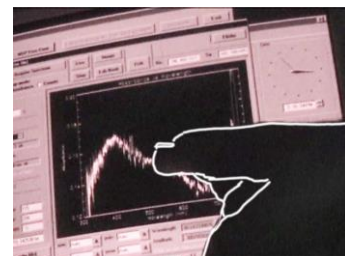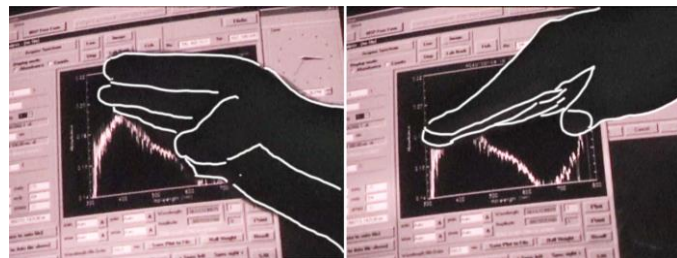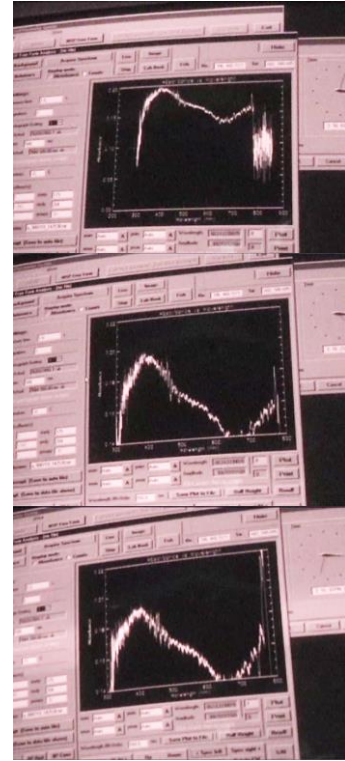
23 P2: <fou::r ten> ((*Ra continues to detrend*))
24 P1: okay, put up three-*eig*hty=UP. (0.9) w*ell*, okay.
25   (1.0)
26 P2: °is that°=
27 P1: =wait. wait. (1.0) >keep on d*etre*nding.<
28   (1.9)

29 Ra: °°yea. okay a bit more.°° °still a little bit more°
30   (3.7)
31 P2: °it's still a little⌈ bit° ⌉
32 P1:   ⌊okay,⌋(1.2) GO to lambda max th*ere*.
33   (7.1) ((*Ra puts cursor to peak*))
34 P2: still four-ten.
35   (0.6)
36 Ra: four-ten.
37   (0.8)
38 P1: °four ten?°
39 P2: wha=what *do* you expect?
40   (1.0)
41 P1: u::::h hn (0.2) i'd say <about three-eighty> but. uh:
          hn
42   (31.6)
43 P2: °hm°
44   (0.2)
45 Ra: it is getting four-ten, (1.6) that would'd be a bl*ue*
          then. (1.4) ((*Ra saves data*)) OR A V*IO*LET
          MAYBE .HHihk⌈HAHA HA HA⌉((*laughs*))
46 P1:   ⌊ or it could ⌋ be:: (0.8)
          theretheres=u::m
          >°theres=a°< tran*siti*onal phase where (0.4) the photoreceptor sh*if*ts *an*::d=uh
          (1.3) something tha:t (.) we h:v:e=da address:: (.) sooner or later.
47 Ra: yea, b'that's what i=
48 P1: =>i wanna<, i wanna keep this one.
49 Ra: yes, i have saved it just now.
50 P1: and i wanna bl*ea*ch da;
51   (0.5)
52 Ra: ok*ay*
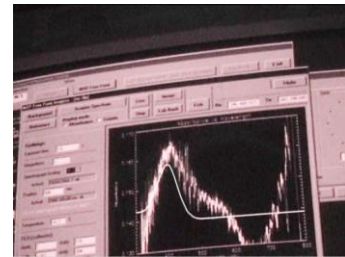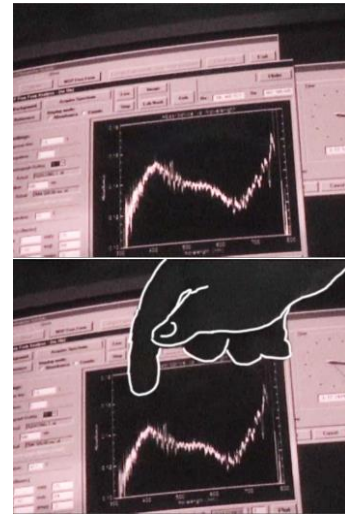
3   From meaning to sense-giving field and nested contextures

In Vygotsky's radical theory revision, the sense-giving field became the main idea
(El'konin 1994; Mikhailov 2001). The sense-giving field emerging from the social
psychology of the later Vygotsky may be thought as consisting of the six nested sense-
giving contextures that had been developed in sociology (Schütz 1932). A contexture
consists of the sign (word, phrase, language, gesture), functioning as a figure, against

everything else that makes up the ground. Figure and ground are mutually constitutive, one implying the other, for changing the ground changes the figure, and vice versa. This theoretical approach is laid out and explicated in the following analysis of the episode from the scientific research laboratory. The six subsections are organized from the most restricted contexture, "(word, sign) meaning," to the most inclusive contexture, the *in-order-to motive* [Um-zu-Motiv] of the specific communicative act. Everything said has its in-order-to motive, to make sense under present conditions, which are transformed so that any next doing or saying also can make sense. Participants not only make the situation but also make relevant aspects of the world visible—by means of the sense-giving field generally and language specifically.

3.1 The lived world indicated by the sign

3.1.1 Of words and indicated things

Signs generally and words specifically are associated with real things or ideal things, themselves manifested in material ways. "Ideas" and "meanings" do not exist as otherworldly and non-material things but, to have any relational relevance, require some material presence (representation) so that they can be available to all individuals involved—a point also important in the *theory of knowledge objectification* (e.g. Radford 2016). Most analyses tend to focus on the different "meanings" that participants "construct" when mathematical entities are concerned; words substitute for these "meanings." Analyses might focus, for example, on the "meaning" that P1 has constructed with respect to what happens when the observed absorption curve is "detrended." First, P1 makes a hand movement, as if he were rotating the base of the Gaussian curve to make it horizontal (turns 11–12), and then says that the peak (λ_{max}) might shift toward the UV region, that is, toward the left on the graph. After the detrending has been done, he furthermore asks to plot a reference curve with a peak at 380 nm (turn 24), as if he expected the detrended curve to have moved to the left of where it was, somewhere between 400 and 410 nm (turns 12, 14, and 15). The analyst might then conclude that P1 has a misconception, which in fact can be identified as the result of an "iconic confusion," where a mathematical operation is confused with a real thing. This is so because a correction for the baseline is achieved by subtracting a linear function, which makes the peak move to the right rather than to the left (Fig. 1). As a video recording made in the office of the research associate shows, both Ra and P2 know about the actual behavior. The fact that both have a background in physics and applied mathematics (statistics) whereas P1 is a biologist may be mobilized by some analysts to produce some form of explanation—e.g. my research among field ecologists revealed that even within biology, there is a supposition that biologists are less mathematically inclined than other natural scientists. The researcher may thus conclude that P1 makes one kind of meaning in the situation whereas Ra and P2 make another one. The transcription shows, however, that none of this matters in the exchange—P2 does indeed ask P1 what he expected but does not take up what is made known in the reply (see pause, turn 42). No negotiation of differences occurs. On the other hand, for example, when P1 requests a detrending, Ra removes the linear background upon which the curve seems to sit (rather than rotating the graph). In this case, following an instruction, the concept of meaning is not required and even interferes in an appropriate understanding of

how language works (Wittgenstein 1997). The sound-word "detrending" solicits an action that is de facto accepted as appropriate (see below).

A focus on "meaning" as the real or ideal object to which the word refers makes invisible to complex relation between words and the world, or, indeed, between any sign and a corresponding feature in the world. Thus, the finger in turn 32 is treated as a sign pointing to a visible peak (Ra puts cursor to peak, turn 33), but the finger can be seen as pointing only because something already is visible in its extension. That is, there is a mutually constitutive relation: the finger is pointing because there is a visible peak, and the peak becomes accented visible because there is a pointing finger. As a perceptual feature, this peak is not constructed but given in perception. The relation between sound-words and features of the world is similar. By using the word "baseline," the scientists allow a feature of the graph to enter the accented visible for the purposes of the exchange. Possible misidentifications of the precise feature in question are decreased by the hand position that parallels a hypothetical line below the Gaussian-type curve (turn 11, left) and by the finger touching the screen where the plot exhibits a linear relation between absorption and wavelength (turns 19–21). Similarly, lambda max is paired with a finger directed toward the peak of the absorption curve while the word "lambda max" is articulated (turn 32). Sound-words, as material things, go with observable features in the world—or are metaphoric extensions thereof in the case of ideas (Johnson 1987); and, crucially, sound-words are manifestations of the relations between people. As a result, "the sense of a word depends on one's understanding of the world" (Vygotsky 1987, p. 276). This leads us to the concept of a "sense-giving field, different from the visible," which constitutes "an accented visible" (El'konin 1994, p. 23). This concept is not unlike that of the pragmatists' *language-game*, which constitutes the ensemble of the ongoing activity together with the pertinent language (Wittgenstein 1997). Indeed, in pragmatic philosophy, any distinction becomes impossible between knowing one's way around the world generally and knowing a language specifically (Davidson 1986). In other words, language "comes into existence" and thus is situationally relevant "only through the fact that the individual assumes the attitude common to all those involved in the common undertaking" (Mead 1932, p. 142).

A challenge to the exclusive focus on word meaning is the non-literal use of language where the dictionary sense may be entirely irrelevant. Thus, the research associate articulates what one might hear as the offer of a joke (turn 45), where hearing the improbable "or a violet [cone] may be" as a joke is further solicited by Ra's own, unusually loud laughter that immediately follows. The phrase thus is not to be heard as making known something, though there is always the possibility that a subsequently occurring scientific discovery is prefigured. Thus, elsewhere the research associate also ventured, in a joking manner, the possible correlation of a variable with temperature, which was implausible at the time, but which turned out to be one of the key findings from the laboratory and part of a significant conceptual change.

3.1.2 Words are transactional

The later Vygotsky was moving from individual "meanings" to the inherently shared consciousness made available in the word. The interpretive approach that underlies the pursuit of different "meanings" fits with the common way of presenting episodes, whereby only the words and actions of the current speaker are transcribed (see above).

But at any instant, the current speaker is not the only active subject. Others can *reply* only when they have actively *attended to* and *received* what someone has said. That is, while someone speaks, the listener is attending to and receiving what generally is not already known (Fig. 2). The representation of this situation makes apparent that any word and the phrase as a whole is common to the two participants, sounding in the mouth of one and ringing in the ear of the other. This is a physical relation between participants, as Vygotsky (1989) suggests but denied by others who claim that in a verbal exchange there is no "direct physical contact between interlocutors" (Sfard 2008, p. 90). There is *not* the act of an individual but a social act, *corresponding*, involving at least two individuals (Fig. 2).

Fig. 2   Revised transcription with superimposed analytical concepts

The term corresponding is used in its multiple senses: communicating and being congruous or in harmony with someone else. This is precisely what emerges from the ideas of the later Vygotsky, who takes a *trans*actional approach communication. Vygotsky's radically revised approach begins with the recognition that "in consciousness, the word … is absolutely impossible for one person but possible for two" (Vygotsky 1987, p. 285); and "language is consciousness that exists in practice for other people and therefore for myself" (p. 285).[1] It might be pointed out that the participants' perspectives differ. But in a transactional perspective, the "double view *is* the relationship," "giving a binocular view in depth" (Bateson 1979, p. 133). In the *trans*actional approach, there are not sequences of individual actions, some of which are replies so that *inter*actions result; instead, transactions involve different actors and thus only exist as joint, inherently *social* behavior.

The revised transcription (Fig. 2) highlights something else: replying arises out of, and thus is essentially shaped in and by, attending and receiving (cf. Vološinov 1930). We thus have a phenomenon of *responding*, which is spread out over *attending, receiving*, and *replying*. Responding begins before the phrase in its entirety is available to be interpreted; that is, responding begins before the person can grasp what the other will have said when talking has ended. This is so because the recipient does not know what is coming at him until the saying has been completed. The recipient is thus affected before being able to know by what.

3.1.3 Words and thinking

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[1] This actually is a quotation of Marx and Engels (1978), which the English translations, having removed the quotation marks that appear in the original, do not acknowledge as such.

The outcome of this analysis has implications for theorizing the thinking that accompanies communication, where *thinking* refers to any event that might occur consciously or subconsciously. The revised transcription makes apparent that thinking is spread across person and environment where it has its start. Thus, for example, the research associate is attending to, and receiving, sound-words that are not his own. These words constitute the stimulus and beginning point for whatever is produced (S, Fig. 2). The actual reply constitutes an act (A, Fig. 2). But the research associate cannot know precisely what he has done until he has access to the effect that his speaking has brought about, which is revealed by or indicated in the environment, here the next turn (E, Fig. 2). Thus, the thinking associated with the reply begins in the environment and ends in the environment—just as this would be the case in the approach that Vygotsky was aspiring to develop. Thinking begins and ends in the world, and thus clearly has a physical in addition to the traditionally accepted psychological (mental, ideal) dimension.

3.2 The system of signs

Any of the words in the transcription is a constitutive part of language; more generally, any sign is part of a sign system. The sense of the sign is its place in the system. But any sign legitimately may refer to multiple things (have multiple specific "word-meanings"). To understand what someone says—i.e. to be able to say, "this makes sense"— *presupposes* a sign system. This is the case even for words that otherwise do not exist, such as the verb "to detrend" (turns 9, 19, 27) which does not appear in the Merriam Webster or in the Oxford English Dictionary but has currency in this laboratory and in the larger community of researchers doing this kind of research. In the research paper resulting from this research, the team members cite another study where the term is used and defined. That paper states: "the data are 'detrended'; this is done by averaging the last three point at both ends of the range, computing a linear trend between them, and subtracting an appropriate value from every ordinate at each abscissa" (Hárosi 1987, p. 732). Here, the definition constitutes a set of other words making a number of statements that therefore give "detrending" a place in the English language; the verb makes sense because of the English language of which it has become constitutive part. At the same time, the statements describe a situation also recognizable on the computer monitor in front of the three scientists in this meeting. In turn 11, P1 produces what can be seen as an iconic and indexical sign. It is iconic in that the direction of the fingers forms a line that is parallel to a line that can be found in the image behind; it is indexical in that the hand, oriented as it is, *points to* the linear part of the spectrum. A second non-iconic sign is produced and confirmed across the pair turn 19 | turn 21. As the iconic hand movement in turn 11 shows, the verb "to detrend" corresponds to an action in the world whereby the sloped line is removed so that the curve sits above "baseline." Thus, the position of the sign in the sign system, here provided in terms of the definition of the word, is coordinated with the position of a feature in the world; the feature may solicit the use of the word and the use of the word solicits attention to the feature (see 3.1). Here, the scientists already are familiar with the form of the absorption graphs, which tend to "sit on" linear background noise. The term "baseline" orients the scientists to the *linear* form of the *background* (base).

Definitions thus are used to give a new word a place in language in terms of other, already existing words. Language is a system of words. The sense of any particular word is given by its place in language as a whole. Thus, language is a sense-giving contexture. The use of the word *presupposes* the system as a whole—at least to the extent that it is relevant and required in the situation at hand.

Within the system, any word may have multiple legitimate uses. The word "bleach" may be heard as a transitive or intransitive verb, with different dictionary senses of whitening by washing or exposing to sunlight, removing a stain, removing the silver from a light sensitive photographic film, to become white or pale. It may also be heard as a noun, referring to the act of bleaching or to the substance involved in the process of bleaching. In which specific way a word is to be heard itself is a function of the surrounding language. Thus, "I wanna bleach da" (turn 50), which others in the lab know is to be heard as a transformation of "I want to bleach that (one)," provides constraints that make hearing the verb more probable than hearing the noun; and among the possibilities of literal uses of the verb, the demonstrative "that" is treated as referring to the current photoreceptor. Additional contextures and related competencies are required for distinguishing between the different word-uses. Further restrictions occur at the higher levels of sense-giving contextures.

3.3   The selection and use of the sign, sign-use as act of expression

Every sign is part of a sense-giving contexture that arises from the fact that the production of a sign is a form of action. Even a "uh" (turn 40) or "hm" (turn 43) may be significant—e.g. depending on intonation, letting others know that the speaker is listening, does not understand, is about to speak, and so on. An action is a form of expression, which occurs against the background of the absence of apparent expressions—thought not the presence of nothing. In the analysis of conversations, it has been shown that the standard maximum silence between two speakers is about one second; and research on the amount of time teachers allow for students to respond is of the order of 0.8 seconds. In the present episode, there are many pauses that are longer than either of these measures. Some pauses are very long: 7.1 (turn 18, 33, 11.2 (turn 22), 17.5 (turn 6), and 31.6 seconds (turn 42). But this does not mean that nothing is communicated. Instead, talk is not required because there is a common sense about what is currently happening and an agreement about it (Roth 2004). Here, for example, the pauses occur while Ra is doing something to the graphs. Because of the darkness, the other two members do not see what Ra is doing (e.g. which key he is hitting); but they can see the result on the monitor. They only speak up when something unexpected occurs (as per the talk that ensues).

Against the (long) silences, speaking stands out. It is an expression that is notable and to be noted. Thus, for example, the very fact that Ra is speaking in turn 7 stands against the 17.5-second silence that with the speaking has come to an end. The speaking thus stands out as a form of action; and it is so even before we may ask questions about the content of the action. In turn 7, something is made to be salient. Everyone in the room can actually see that there is a step near the center of the absorption curve. Everyone knows that this possibly is the result of a shift in the two histograms that are subtracted and, thus, that a pixel shift is required. Therefore, the phrase in turn 7 states what

everyone can already see. The phrase does not have the function of making something known (see 3.4), for what it says already is known. But it communicates that Ra is nearly done displaying the raw absorption graph. So the literal content of the phrase is irrelevant in the present case, for it is evident and *goes without saying*. However, it does say something of the order of, "We are close to moving on to the next step." This next step is formulated after the two measurements have been aligned: "okay now, detrend that please" (turn 9).

3.4   Sign-use as communicative act (making known)

Some expressions make something known and thus have communicative function. For example, in the phrase "looks like a single cone, and looks like a UV cone," something is made known that others present do not have access to because they have not seen what is on the microscopic slide. Turn 41 states that the expected the maximum of the curve (lambda max) lies at about 380 nm. One sense-giving function of sign-use therefore arises from the contexture of *making something known* [*kundgeben*] (Schütz 1932). In the present episode, most talk does not have the function of making something known because it might have been unavailable to the other participants in the laboratory session. Some readers may be tempted to object and say, for example, that in saying "I wanna keep this one," P1 makes known *his* wish. But the situation as a whole is not about the wishes or intentions of the individual. Instead, this is a research project pursuing a specific object/motive (i.e. deriving absorption spectra from fish specimen to validate their instrumentation); and this object/motive is the same for all participants because it defines the activity as such. Unsurprisingly, Ra replies by stating that he already has saved the data.

   One important aspect of "making known" consists in *formulating*. In formulating, social actors exhibit what they currently do. Consider Fragment 1.

Fragment 1
38   P1: °four ten?°
39   P2: wha=what *do* you expect?
40   (1.0)
41   P1: u::::h hn (0.2) i'd say <about three-eighty> but. uh: hn

   Prior to the fragment, P2 and Ra have read from the graph that lambda max is about 410 nm. P1 also articulates the number but with rising intonation. The emphasis on the verb "do," treats the preceding phrase as if it had said something like, "Is it really 410 nm?" In saying "what *do* you expect?," turn 39 treats turn 38 as an expression that notes a deviation from what P1 had *expected*. Thus, turn 39 names and thus makes visible a psychological process: violation of an expectation. That something like this has happened is seen from the fact that a reply is provided. It exists in the naming of an expected lambda max of about 380 nm (turn 41), which in fact can be heard as consistent with the earlier call to plot a reference curve with peak at that wavelength (turn 24). The question about the expected value takes on particular relevance given that P2 and Ra in other parts of the data exhibit an understanding of the actual shift of a peak when the linear detrending process is brought about. In this context, we can hear P2 asking in fact

something like, "Why would you expect the peak to be different?" and especially, "Why would you expect it to be lower when, if anything, it should be higher?"

3.5   The communicative act as solicitation of a behavior

An even more encompassing sense-giving context exists when the recipient of the communicative act is taken into account in soliciting a specific behavior. Thus, the speaker not only "designs" the message to be intelligible, and thereby already takes into account the other (making this a *social* act through and through), but also anticipates stimulating a particular type of action. For this to happen, the speaker, implicitly or explicitly, orients to and addresses a specific recipient in such a way that the preferred type of action likely will be forthcoming. For example, a phrase may be such as to request another person to do something, as seen in Fragment 2.

Fragment 2
  9 P1: ok*ay*, n*ow*; (0.7) *de*trend that (.) pl*eas*::e.
10 Ra: okay. ((*Begins to detrend*))

   Here, P1 uses a verb that describes an action that Ra then executes. Indeed, what we observe here is the relation between a plan (instruction) and an action. Applying the analytic approach shown in Fig. 2 immediately reveals that this relation is a social relation because the verb exists for P1 and Ra (i.e. it is a transaction, corresponding); and this transaction exists for Ra in the form of *responding*. Such social relations are the origin of all higher psychological functions in human development (Mead 1972; Vygotsky 1989), here the relation between plan (thought, instruction) and action. In an instructional situation, an evaluative turn would normally occur next, which would exhibit that the action corresponds to the instruction (Roth 2017). Similarly, when P1 says, "Put up the three-eighty UP" (turn 24), Ra makes a reference graph with a peak at 380 nm show up; when P1 says, "keep on detrending," Ra continues to detrend; or when P1 says, "Keep this one" (turn 48), Ra saves the data. "Go to lambda max" is followed by a move that places the cursor at the peak of the absorption curve. The statement "I wanna bleach da," incomprehensible in this way outside of the particular situation, actually solicits a whole sequence of actions, including the irradiation of the same cone with white light to transform all of the photosensitive material (not regenerated in vitro) and subjecting the irradiated cell to another measurement. When the absorption curve that had showed up the first time no longer exists in the second measurement pair, then this is taken to be evidence that the first curve is attributable to a real phenomenon (a cone absorbing in the region) rather than a spurious effect (artifact). This, as everyone present knows, is the step required the credibility of the fact.
   In other instances, we observe what readers immediately recognize as {query | reply} pairs: a query turn solicits a reply turn, such as in the pairing of "what do you expect?" (turn 39) and "I'd say about three-eighty" (turn 41). The first phrase begins with an interrogative and the intonation rises toward the end, both of which are markers that the phrase embodied a query that the addressee is invited to answer, which is the form the next turn takes here.

A solicitation may fail. An example of this may be seen when the research associate says that there might be a violet cone and then laughs. Neither P1 nor P2 laugh or take up what might be heard as an attempt to make a joke. It might be treated as a joke because everyone knows that there are no known cones with absorption peaks in the violet part of the spectrum. Indeed, the take up is serious in tone (intonation), for P1 makes a statement that describes the existence of a transitional phase as a possibility that requires their attention. The transitional phase would then have a peak between the cones maximally absorbing in the UV and the blue regions, which is precisely where the violet lies in the spectrum. In transactional terms, what might have been intended as a joke de facto is treated as a serious matter requiring further action. This further action is initiated by saving the data and by submitting the cone to further investigation, a bleaching.

3.6   The *now, here*, and *thus* of the communicative act and the in-order-to motive

Any act of speaking also makes sense within an even more encompassing contexture that arises from the *here, now,* and *thus* of the speaking (Schütz 1932). This here, now, and thus of speaking is part of a more encompassing *in-order-to motive* that makes for a relevant sense-giving contexture. For example, in the present context, it makes sense to hear "three-eighty" and "four-ten" as alternatives to "380 nm" and "410 nm." If these individuals used the same expressions in the supermarket while considering some purchase, these same words, to make sense, have to be heard as alternatives to "$3.80" and "$4.10."

This most-encompassing layer of sense-giving contexture is important in distinguishing literal from non-literal senses of the word, phrase, or non-verbal expression. The possibility of the phrase "that would be a blue [cone] or a violet maybe" as encompassing a joke exists in the fact that there are no known photoreceptors with a maximum absorption in the violet. The lambda max in the blue actually lies closer to 430 nm. Thus, the observed lambda max of about 410 nm lies between the known UV (about 380 nm) and blue lambda max. In the statement, a *conceptual monster* is thereby created with the creation of a new category of the unexplained, just as it has been observed among scientists in very different contexts (e.g. Roth 2005). It is precisely the monstrosity of the suggestion that constitutes the possible humor. That joke has to be grasped; and this grasp goes beyond a mere knowledge of words and language. It is a way of dealing with something unknown and apparently monstrous, something that does not have a place in the science known at the time. But there is no rigorous boundary separating the obvious from the non-obvious or the literal from the monstrous. Thus, for example, the phrase "I think that is baseline here right here" (turn 19) actually contains the modalizing expression "I think," which makes the phrase content ("that is baseline") a possible rather than self-evident fact. This aspect of sense was already intimated above in the analysis of the phrase in turn 7, which followed a long, 17.5-second silence. When a phrase apparently states the obvious, when it goes without saying, then the question is why it has been made now, here, and thus and therefore what its in-order-to motive is.

It was noted that something going without saying normally is not verbalized. If it is said nevertheless, there is some perhaps unknown in-order-to motive. A special case of this phenomenon exists in the repetition of the same words. Thus, for example, each of the three participants articulates the words "four ten" (turns 34, 36, 38). In the first

instance, we can hear "still four ten" as making to things stand out: (a) the peak of the absorption curve lies at 410 nm and (b) nothing or little has changed, for it "still" is 410 nm, which is what it previously has been said to be. Although the peak is visible to everyone and thus goes without saying, the phrase states more in that it also states that the detrending has not led to new information. But then Ra says the same words, "four ten" (turn 36). The transcription features a period, which means that there was a falling intonation typical of constative phrases. The specific function here is that in repeating what has been said it confirms it and the associated reading. When turn 38 repeats the same word, the pitch moves upward in a way typical for a question. In these two instances, therefore, the intonation makes the sound-word different. But human speakers generally are not conscious of their intonations, which are determined by the situation and manifest a person's affective orientation or attitude (Vološinov 1930; Vygotsky 1987). Intonations are bodily, that is, they are of the same nature as affect (emotion). Intonations thus manifest a particular disposition in and to the situation that cannot be derived or reduced to the mental.

The specific sense-giving function of the *here, now,* and *thus* of speech has been discussed in situations where the same word or phrase is produced repeatedly. Thus, why would six drunken workmen have a conversation that exists of the same obscenity repeated six times (Vološinov 1930; Vygotsky 1987)? Why would physics students have a conversation consisting of the 10-fold repetition of the word "penis"? (Roth 2015). These authors suggest that precisely because the dictionary sense of the word in use is the same, repeated use achieves something other than "meaning."

4   Discussion and implications for mathematics education

Scholars using a Vygotskian approach to language tend to exhibit a predominant concern for "word-meaning," Vygotsky, however, was moving away from this concern because he realized it to be overly intellectualist (Zavershneva 2010). Instead, he articulated the role of language ("the word") in consciousness as a whole. The present study shows that there are at least five sense-giving contexture other than the one related to "word meaning," which actually is revised in the new take on language. These six contextures, which are *presupposed* for the word to make sense, include (a) the signified entity as part of the world, (b) language as a system of words, (c) word-use as act among acts, (d) word-use as a communicative act, (e) word-use as solicitation of behavior, and (f) the *here, now,* and *thus* of the speech situation and its in-order-to motive.

The shift from meaning to sense also means a shift from the metaphysical to the physical (material), which is accessible to the senses. This is consistent with recent philosophical analyses according to which the body of sense is the sense of the body (Nancy 2008). This may fly in the face of those who see mathematics as dealing with abstract concepts. But cognitive scientists have shown how abstract notions really are metaphors of image and body schemata (e.g. Johnson 1987). Non-sense is the background "against which every universal undertaking is silhouetted and by which [the undertaking] is threatened" (Merleau-Ponty 1996, p. 8). The extent of the sense in common to collaborators will differ according to the situation, which makes this group of scientists different from students in mathematics lessons. But even though students have less experience, and thus a less developed sense of how the world works, the relations of

the different contextures will be the same as are the ways in which the complex behaviors of individuals first exist in relations with other people.

In mathematics education, language specifically and signs generally are taken to be mediating tools. The analyses in section 3.1.2 show that the idea of the sense-giving field is incompatible with the notion of language as a mediator that somehow sits and operates between people. Contrasting the currently predominating view that new knowledge first exists *in* a social relation and then is internalized, Vygotsky (1989) notes that *any* higher psychological function first exists *as* relation (Fig. 2, section 3.5). This means that a particular mathematical practice or form of mathematical reasoning *is* a social relation between two or more people in which the individual already takes part (Roth 2016). Language as sense-giving field not only is common to the subjects—who are thus part of a *trans*action rather than an *inter*action—but also constitutive thereof (section 3.1). Thus it is better to say that humans *live in* language, and thus *live in the consciousness of others*. Using a language means living in the consciousness of the other, and, thus, living in, and having something like consciousness of one's own.

One important area of research concerns the problem of how mathematics students and teachers *make meaning* of and with language (e.g. Radford et al. 2011), an event normally attributed to the mind (head). The framework of the sense-giving field consisting of nested sense-giving contextures places the problem back on its feet. This is so because the sense of a word (figure) depends on the contexture (ground): the specific word relates to something in a practical world that always already makes sense. Thus, if students do indeed make something, for example, *interpret* the word, then practical understanding of how the world works initiates, accompanies, and completes the explanatory part of such an interpretation (Ricœur 1986). It is more useful to say that a word makes sense than that a word is being made sense of. What mathematics students then do is not making sense of a word but finding contextures in which the word makes sense.

An important concept that reappears in the literature is that of *negotiation*. Accordingly, students or teachers negotiate meaning, identity, etc. But the concept is problematic because negotiation presupposes that participants already have what is to be negotiated (Radford and Roth 2011); and it represents a market utilitarian lens (Radford 2012), which supports the above-noted instrumental viewpoint. Another implication lies in the field of research on classroom social and sociomathematical norms (e.g. Cobb and Tzou 2009). To construct or negotiate such norms, Radford contends, one already has to be familiar with and be competent in the practices—just as children first learn their mother tongue and then learn grammatical rules that they have been using without knowing it. The present framework concerning sense-giving contextures, their practical comprehension of which leads to the fact that a phrase *makes sense*, allows understanding why the norms and negotiation presupposes a massive amount of common sense and familiarity with the various sense-giving contextures (fields) in which actions, words, and signs and complexes thereof make sense. The analyses show that the word or phrase not only is in common but also designed for the other, to be understood and to solicit specific forms of behavior (see especially sections 3.1, 3.4, 3.5). The individual thus does not first institute meaning and then compare it with those of others; instead, the language "comes into existence only through the fact that the individual assumes the attitude common to all those involved in the undertaking" (Mead 1932, p. 142). That is, if there are differences

that become apparent in language-in-use, then this recognition as much as the resolution of differences is possible because of commonsense language, employed in a speech field constitutive not only of the mathematical issues but also of the personalities of participants (see section 3.4). If participants in a communicative exchange use a word differently, then this is due to a difference that exists within the word; indeed, the difference *is* the relation and a necessary condition of mind (Bateson 1979; Mead 1932). If there are any differences in what participants in a meeting attend to, then both the recognition of these differences and removal thereof requires a massive amount of common ground. This common ground exists in the form of common sense. Sense thus is more important and a phenomenon more massive than that of "meaning" (as a signified thing or idea); "meaning," in revised use, is but a minor aspect of sense more generally. Indeed, the questions concerning (a) which of multiple possible dictionary meanings are currently relevant or (b) whether dictionary (literal) meanings are relevant at all in the face of possible metaphorical meanings are a function of the other sense contextures— e.g. the question concerning the *now, here,* and *thus* of a phrase, sign, or physical action.

Acknowledgments

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Appendix: transcription conventions

The transcription conventions follow Jefferson (2004).

In-text appearance	Explanation
do::n't	Each colon indicates lengthening of sound by about 0.1 second
(0.6)	Speaking pause in tenth of seconds
(0.1)	Noticeable pause less than 0.1 second
be wr[ong / [there]	Overlapping speech
function	Grey highlighted words overlap with visual image to the right
*de*trend	Emphasis, achieved by higher pitch or volume or both
OR A VIOLET	Louder than normal speech
°okay°	Softer than normal speech
°°okay°°	Much softer than normal speech
wha=what	Run-in, words are not separated
;.,?	Punctuation marks the prosody of the phrase, slightly falling, strongly falling, slightly rising, and strongly rising
<about>	Slowed down
>about<	Speeded up
((*laughs*))	Transcriber's comment
(fifteen?)	Uncertain hearing
kno::w	Punched up, i.e., rising falling intonation
.hh	Inbreath