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ABSTRACT Data analysis is constitutive of the discovery sciences. Few studies in mathematics education, however, investigate how people deal with (statistical) variability and statistical variance in the data to be interpreted. And even fewer, if any, focus on the uncertainties with which scientists wrestle before they are confident in the data they produce. The purpose of this study is to exhibit the work of coping with variability in one advanced research laboratory, as exemplified in a typical data analysis session. The study shows that when the scientists are confronted with novel data, their understanding of the variability does not arise in straightforward fashion and a lot of normally invisible (interactional) work is required to constitute understanding. Tentative conclusions are provided for the implication to mathematics education.

KEY WORDS: variability; data interpretation; discovery sciences; data analysis; mathematical practices

There is a “huge need” for statistical competencies in the modern workplace and, therefore, a need for statistical education (Scheaffer, 2011). Mathematics educators are interested in the way students interpret data and in their statistical practices and competencies related to data analysis (Garfield & Ben-Zvi, 2007; NCTM, 2000; Roth & McGinn, 1998). Mathematics educators often ask students to collect data that exhibit functional relations (e.g., Confrey, 1995; Gerovsky, 2010; Hardy, 2009; Kaput, 1988) and interpret “well-defined” data and samples (Konold & Pollatsek, 2002); and school science courses and textbooks tend to emphasize function-like relations between variables (Roth, Bowen, & McGinn, 1999; Roth, Pozzer-Ardenghi, & Han, 2005). Unsurprisingly perhaps, when asked to find trends in actual data, even preservice science teachers with BSc and MSc degrees had trouble dealing with variability (Roth, McGinn, &

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5 Bowen, 1998), whereas eighth-grade students following a curricular unit where they designed
6 investigations, transformed data, interpreted data, wrote reports, and argued over and about their
7 findings often used sophisticated statistical techniques to deal with variability (Roth, 1996).
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9 Conversely, scientists asked to interpret graphs from introductory college courses pointed out,
10 however, that function-like data and graphs are abnormal in science (Roth, 2003). Allowing
11 students to be able to interpret the variability of data values from empirical inquiry generally and
12 to separate chance variability from systematic variation specifically are important goals of
13 mathematics education not only in primary (English, 2012) and secondary schooling (Cobb &
14 Tzou, 2009; NCTM, 2000) but also at the tertiary level (Garfield, delMas, & Zieffler, 2012; Wild
15 & Pfannkuch, 1999). Comparing models and empirical data, and dealing with variability, are
16 integral parts of learning statistical practice (Garfield & Ben-Zvi, 2008; Pfannkuch, 2005). In the
17 development of statistical literacy, understanding the relationship between models and data is an
18 important dimension (Garfield, 2011; Garfield & Ben-Zvi, 2007). This is so because there might
19 be patterns even though data exhibit considerable variability; extracting such patterns in the face
20 of inevitable variability was, until recently, not a competency that mathematics education would
21 have allowed students to develop (Konold & Pollatsek, 2002). One of the difficulties people of
22 all ages have arises from everyday confusions between randomness and excessive alterations
23 (Falk & Konold, 1994). Similar to other studies investigating mathematics on the job for the
24 purpose of thinking about how mathematics curricula might be organized (Gainsburg, 2007; Hall
25 & Stevens, 1995; Noss, Bakker, Hoyles, & Kent, 2007; Williams, Wake, & Boreham, 2001), this
26 study investigates what do scientists do when they are uncertain about their own graphs and the
27 variability (and statistical variance) displayed therein. The study presents for the interpretation of
28 mathematical culture suitable “thick [ethnographic] description” (Geertz, 1973) of data
29 investigation practices in a situation where research scientists found themselves in a highly
30 uncertain situation about the source of variability in their data.
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60 **Research design**

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5 This study was designed to investigate what scientists do with graphs when they are in
6 situations of considerable uncertainty and, therefore, are not in a situation to provide some
7 routine explanation to something that they are highly familiar with (as an economist with a
8 supply-demand graph). The sources of the data are the records from a 5-year ethnographic
9 project in one experimental biology laboratory, where we were members and participated in the
10 design of studies and publication of research results. The particular research project from which
11 the present recording stems concerned the absorption of light in the rod-shaped photoreceptors of
12 coho salmon eyes. There are two absorbing chemicals, one based on vitamin A₁ (rhodopsin), the
13 other one based on vitamin A₂ (porphyropsin). Our research attempted to answer the question of
14 how the relative amounts of porphyropsin and rhodopsin in the photoreceptors changed in the
15 course of the coho salmon's life cycle. In this study, we focus on parts of one data interpretation
16 meeting that the team periodically organized.
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30 Five team members were present: the laboratory head (Craig), a research associate with
31 background in physics (Theo), a postdoctoral fellow (Elmar), the second author who was a
32 doctoral student in biology at the time (Shelby), and the lead author (Michael), a physicist and
33 applied cognitive scientist by training. Shelby operated the computer and projected data onto a
34 screen; a chalkboard behind his back was also used for drawing graphs. At the time, Craig
35 already has had more than 30 years of research and publication experience in this field; he now
36 holds an endowed research chair at another university. Shelby, too, has had many years of
37 experience doing research on fishes around the world, had worked as a biologist in various jobs
38 prior to completing an MSc and was working on his PhD. Theo had done undergraduate studies
39 in physics and work experience in developing analytic and visualization software.
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51 The present study treats results “not as specifications of practice but rather as guides for
52 locating the field properties that make the practices recognizable for what they are” (Koschmann
53 & Zemel, 2009, p. 234). It can be read as a micro-genetic study of scientists' methods of dealing
54 with variability while using data and graphs, here in an advanced laboratory on the
55 neuroethology of fish vision. We are concerned with providing an ethnographically *adequate*
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5 description, which means that the “ethnographer must articulate the same hesitant and
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7 momentary contexts that the natives are displaying to each other and using to organize their
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9 concerted behavior” (McDermott, Gospodinoff, & Aron, 1978, p. 246). Thus, we treat the data
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11 “as an occasion for construing the work done by participants in interpreting the document as it
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13 unfolds before them” (Woolgar, 1990, p. 127) and before they know how it will ultimately be
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15 understood. In the same way other research concerning the natural sciences are conceived, as the
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17 present is a study *of* rather than *about* scientists’ work (Garfinkel et al., 1981). Ethnographic
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19 adequacy also implies that the “analyst must be *vulgarly* competent in the local production and
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21 reflexively natural accountability of the phenomenon of order* he is ‘studying’” (Garfinkel &
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23 Wieder, 1992, p. 182). This “vulgar competence” allows “the analyst to recognize, or identify, or
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25 follow the development of, or describe phenomena of order* in local production of coherent
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27 detail” (p. 182). In our situation, these requirements are met, as we have been active members of
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29 the laboratory.
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32 33 34 35 **Variability and ways of handling it in a data analysis session of an advanced biology** 36 37 **research laboratory**

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39 This study was designed to investigate how scientists deal with uncertainty in data and
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41 graphs while the latter are produced and interpreted *in the course of* discovery work, that is, prior
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43 to the scientists’ realization that they have found something of interest. In the following, we
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45 analyze an extended, 15-minute episode from a laboratory meeting. The episode was occasioned
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47 by two problems that Theo and Shelby identified in preparation for this meeting: (a) there was
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49 considerable variability in their data (which expressed itself in large standard deviations the team
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51 calculated) and (b) their compiled data differed from those published 8 years earlier by another
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53 research group, which was consistent with the current dominant scientific theory. This part of the
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55 meeting was designed to understand the first of the two problems: rather than having a lot of A_2
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57 values more sharply distributed around some mean, their A_2 values were spread across the entire
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59 spectrum of values (from 0–100%) (Fig. 1a). To get at the levels of porphyrpsin (A_2) in the
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5 appropriate reading of the graph that is now projected on the screen (Fig. 2b) needs to be
6 articulated in and for the group. This background is part of the whole research procedure that one
7 needs to know to make sense of any graph that issues from it (e.g., Noss et al., 2007; Roth,
8 2003). This is what is happening in the current subsection. The problem our team faces is that we
9 do not know what the source of the variability is: it could be within the fish eyes, due to
10 systematic error from the instrumentation, an artifact arising from the curve-fitting and extraction
11 of HBW, or random error.
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22 *Articulating the procedure for extracting the data* Shelby describes that he “can present” what
23 “is in [his] database.” Theo formulates the existence of a problem (“the problem is”), and begins
24 with a description of the curves as being “fairly long.” As Craig before, Theo gets up and walks
25 to the chalkboard. Upon arriving there, he produces a curve that we recognize to be an idealized
26 but widened absorption spectrum (Fig. 3a). Theo marks three areas where he “can look.” He
27 refers to a rule in the field, which states that the important region is defined by the right part of
28 the curve above the 50% of its height. To determine the total height, Theo states needing to know
29 where the base lies (left and right circle) and what the maximum height of the absorption curve
30 (λ_{\max}) is. First for the right side of the curve and then for the right-hand “foot,” Theo draws the
31 kinds of features that he observes—and willingly demonstrates and explains to visitors to his
32 office—while analyzing the data that he receives from the wet laboratory (where he participates
33 in some of the data collection sessions operating the software). These features, sudden drop offs
34 in otherwise “clean spectra,” make the determination of the baseline points difficult and,
35 therefore, the precise determination of the height of the absorption curve and introduces
36 variability in its location along the wavelength axis. Although an “otherwise clean” graph does
37 contain such problems, Theo explains that he does not reject the data at this point “as long as [he
38 has] enough peak in here, under, above the 50% line.” Such a curve still allows him to determine
39 λ_{\max} , and, therefore, the amount of porphyropsin present. This is how he sees the situation, and
40 based on it he keeps the data.
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In this sequence of turns, the participants (and the analyst) therefore find out that Shelby has included in the plot currently displayed all the data that Theo has provided him with. These, in turn, are based on what Theo has received from Shelby, who collected the curves from the retina in the dark laboratory. Theo explains that there indeed are curves that exhibit sudden drop offs at the left and right end of the otherwise clean absorption spectra. He does not throw these data out, at least he does not as long as he has “enough peak in here,” that is, between the half-maximum point and the top of the peak at λ_{\max} . Although not specifically articulated as such, what is available *in* and *as* their relation is a possible reason for their HBW data to be noisier than they should be in some ideal case. Theo’s presentation actually anticipates possible problems in the interpretation of the data, as he explains possible variability in the data that might wash out the true trends that possibly exist or that make interpretation possible.

Explicating the possible source of the variability When there is considerable variability in the data, the identification of “deep structure” may be difficult. When scientists articulate trouble reading or interpreting data because of variability that may not be understood, observers are provided with opportunities to see what they do when the deep structures visible in idealized graphs—such as those that Craig and Theo have drawn on the chalkboard—are not immediately available in real, experimental data. At this point in the meeting, the team is confronted with a plot of the half-maximum bandwidth of the spectra it collected (i.e., HBW) against porphyropsin levels (%A₂). This plot (Fig. 2b) reflects the real data; it constitutes a stark contrast to the idealized curve Craig has sketched on the adjoining wall. This idealized graph presents the expected (“preconceived”) deep structure whereas the graph that has arisen from the exchanges between Shelby and Theo presents the real case, the deep structure of which is at issue in this subsection. What will scientists do to come to grips with the apparent variability in the currently displayed data plot? In the sciences, the exclusion of outliers is a common practice. However, if a “point” represents many measurements rather than a single one, excluding it becomes more

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5 problematic (unless, perhaps, if scientists recognize some systematic error in the many
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7 measurements underlying the single point). In this part of the episode, the possible exclusion of
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9 points is the topic of the talk.

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11 Theo, who is still standing next to the diagram that he has earlier drawn immediately follows
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13 Shelby by saying that there is “a little noise on here” while pointing to the left side of his
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15 absorption graph. There is, as all laboratory members know, the familiar “ringing”—oscillations
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17 overlaying the left branch of the approximately Gaussian absorption curve—the origin of which
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19 was not known (Fig. 3b). Because of this variability, the half-maximum bandwidth is measured
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21 to the point where the “ring” first crosses half of the maximum height, so that its measurement
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23 ends up being smaller than it should be. The algorithm that Theo has designed at that instant in
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25 the research does not project where the Gaussian *would* lie but takes the half-maximum
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27 bandwidth at the location along the wavelength axis where the absorption curve first falls below
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29 half of maximum height.
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33 In this situation, Theo modifies the original graph for the purposes at hand. That is, the
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35 original smooth graph was to show one feature and was produced for the purposes of
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37 communicating it: the places that are of importance in determining the height of the curve and,
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39 with it, the half-maximum bandwidth. Now that the data exhibit a lot of variability, he modifies
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41 the *ideal* and idealized graph to introduce one of the aspects that can produce such variability.
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43 That is, the variability visible in Shelby’s graph does not arise from the source, the retinal cells,
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45 but from the transformations that the measured absorption curves undergo towards producing the
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47 porphyrin content (determined by means of λ_{\max}) and the half-maximum bandwidth, which
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49 the team hopes to turn into a second reliable measure of A_2 levels. It allows Theo to articulate
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51 where variability comes into the determination of the half-maximum bandwidth as existing in the
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53 database at the present point in time. Theo proposes something, though the speech volume
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55 becomes so low that it is no longer possible to retrieve what he is saying while Craig overlaps
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57 uttering the tentative proposal to throw those data points out (“huck those points”). He proposes
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59 this elimination because Shelby’s curves did not fit this template (curve), due to large variability,
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5 even though each point plotted represents the calculated mean of a series of measurements, and
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7 therefore are not simple outliers (singularities).
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10 In this situation, the graphs on the chalkboard have a different function from the graphs
11 produced from the data: these are tools to think, as research on diagrammatic reasoning has
12 shown in school settings where students discuss variability in data (Bakker & Hoffmann, 2005)
13 or in contexts where scientists are asked to interpret unfamiliar data (Hoffmann & Roth, 2005).
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15 In such situations, graphs offer opportunities for the “hypostatic abstraction,” that is, “the
16 formation of objects (what can be talked about)” (Bakker & Hoffmann, 2005, p. 350). In the
17 present instance, the “objects” to be talked about are possible sources that generate the variability
18 in the data to be interpreted.
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28 *Call for disattending to variability*
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31 Once scientists know that they can legitimately ignore some data points, for example,
32 because these can be justified to be outliers, then they are in a position to read the data in new
33 ways. They may, as happens here, *provisionally* ignore some data points to see what it yields,
34 much in the way painters do after placing a stroke and then stepping back to evaluate the effect
35 of their move and then to correct or continue with what they have done. That is, it would be
36 typical to articulate a move and thereby to objectify it to be in a better position for evaluating its
37 implications. The advantage of such moves is that new insights thereby arise from an external,
38 epistemic action that reveals its possibilities by inspection and further possibilities for theorizing
39 through reflection.
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51 *Interpretive difficulties* Following Theo’s explanation of a possible source of the variability in
52 the data, Craig provides a first reading—which we may hear as a first attempt in articulating the
53 deep structure in the data. He points to the right-most peak ([i], Fig. 4a) and marks it as
54 something that “he does not know what’s going on” and, prior to articulating a trend, he suggests
55 “ignoring it for the time being.” He also points to the two center peaks upward ([ii] & [iii], Fig.
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5 4a) and to the two downward peaks ([iv] & [v], Fig. 4a), marks them verbally as “extremes,” and
6 suggests “ignoring them.” He finally proposes “running a regression on this” (Fig. 4a). At this
7 point, Shelby brings out a contrast by suggesting that “this is not just a couple [of] points,” but in
8 fact “this is the whole shebang.” That is, we see and hear a proposal for “hucking [throwing
9 out]” or “ignoring” certain “extremes,” but this proposal is not accepted but rather confronted
10 with the statement that there are 3,000 data points. Shelby contrasts this with the description of
11 not just being “a couple of points.” This large number, as he elaborates, implies that there are
12 *some* extremes. But because this is a large number, any trend would be rather stable with respect
13 to a small number (“some”) extreme points.
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24 «««« Insert Fig. 4 about here »»»»

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26 The extreme points are not just there. To exist qua orderly object in the hand of the
27 researchers and therefore at hand, any “point” has to be demonstrably “extreme.” Here, this is
28 achieved in the sequentially produced contrast Craig produces between the point made salient by
29 the indexical gesture and the subsequent hand movement that not only bypasses the point but
30 also does so to such an extent that it becomes an instructible instance of “extreme.” Although
31 much of the work on gestures does not treat the two gestures as distinctive—other than denoting
32 them as “deictic” and “iconic”—there is an ontological difference between them. This difference
33 constitutes the heart of the dynamical understandings in physics and mathematics (Châtelet,
34 1993). This is so because the hand movement enacts a curve, represented in mathematics by a
35 relation, such as $G(x,y) = 0$, whereas the individual points on such a curve, denoted by indexical
36 gestures, represents a parametric or function of the kind $y = f(x)$ where the separation of data and
37 result is already accomplished. It is precisely in the gesture that a virtual relation becomes actual
38 and material, in and through the body of the scientist (Châtelet, 1993). That is, “the gesture
39 envelops before grasping and sketches its deployment before denoting or exemplifying” (p. 33,
40 my translation).
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59 *Hypothesizing a pattern* Craig has suggested “huck[ing] those points” that vary a lot from the
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5 apparent trend (i.e., the outliers). This outlier aspect exists in a demonstrable manner such that
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7 when required, the spiky nature of the “downward spikes” can be exhibited for anyone not yet
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9 convinced. He again suggests ignoring some of the outliers, but Shelby has responded saying that
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11 the dataset is large and that in this case it is likely to have some that reflect large variations.
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13 Craig now suggests running a “polynomial on it” to “see what sort of correlation we can come up
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15 with,” but adds that they first have to “*recursively* go back through the data” to see whether
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17 “those points [outliers]” are “due to errant spectra.”
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20 Craig further proposes that these could “have provided considerably more variance to that
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22 point in the transition,” thereby referring to data reflecting—according to the “dogma” that
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24 framed their work at the time—a changeover from high porphyropsin levels thought to be typical
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26 of salmon in freshwater to lower porphyropsin levels thought to be typical of salmon in
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28 saltwater. As he continues, Craig first acknowledges what Shelby has said (“yea”) and then
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30 provides an elaboration of the data by showing the trend that they display (Fig. 4b). Placing his
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32 pen against the projection screen, Craig gesturally “draws” a curve (Fig. 4b) that looks like the
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34 one he has earlier drawn on the chalkboard (Fig. 2a)—even though he has just prior to this
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36 instant exhibited the opposite relation. There is a high degree of similarity, even in the relative
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38 position of the HBW, which is higher for 100% A_2 than for 0% A_2 . (Shelby has drawn the
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40 abscissa from left to right, and, therefore, in the reverse of what Craig has done on the
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42 chalkboard.) In this situation, therefore, Craig explicitly notes that the trend he has just exhibited
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44 in and through his hand/arm movement, “this,” “looks like that,” the earlier graph. The same
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46 movement in the reverse that earlier had produced the line on the chalkboard now, ephemerally,
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48 exhibits the trend in the data. In this way, what we might alternatively see as a parabolic trend,
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50 visible if a regression were to be performed on the data displayed, becomes an idealized inverted
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52 parabolic trend (deep structure) represented in the trajectory of the pen over the data.
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56 In the course of his talk, Craig has actually changed what he brought out as the trend
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58 underlying the data. Initially, he gestures—indexically/iconically in his movement, iconically
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60 with the hand inclined to show the linear relation from lower left to upper right at the right end of
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5 the plot. He then turns and points to the graph on the chalkboard, which in fact runs differently
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7 from what he has shown just now. He articulates a curvilinear relationship with a maximum in
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9 the center (Fig. 4b). That is, the hand movement both exhibits what is required to demonstrate
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11 the curvilinear relationship and serves as an instruction for seeing (again) this very relationship.
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13 It therefore exists twice, in the form of “two technologies”: as orderly phenomenon that everyone
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15 present can see once he has learned how to look and as embodied practice, which, when enacted,
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17 *gives* immediate access to the so articulated phenomenon of the curvilinear relationship.
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20 In this situation, the HBW versus %A₂ graph provides an indication of the variability in the
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22 data, perhaps more sensitive than other indicators because of the reasons that Theo earlier
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24 elaborated. Craig insists on checking the data that they all meet the criteria, especially those “low
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26 points.” He is concerned with the fact that data such as those in the “downward spikes” would
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28 introduce unwanted variability in the determination of the actual proportion of A₂. Craig later
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30 brings the conversation back to the issue of what has been included as data in the plot, to which
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32 Shelby responds that he did something quick. Craig insists, “does that mean *everything*?” and
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34 Shelby affirms. But again, he insists that there are “a lot of data points,” as if he wanted to warn
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36 the participant to “huck [throw out] those points” all too quickly.
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40 *Articulating further uncertainty* Craig begins making a statement about being “pretty . . .” but
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42 he is interrupted by Michael’s question about how many data points there are in each of the
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44 spikes. This question can be heard as co-articulating uncertainty about what exactly each data
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46 point reflects: individual or collated measurements. Shelby ascertains having heard my utterance
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48 as a “good question” but says that he does “not know how to tease this apart.” He articulates
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50 being surprised about the “spiky” nature of the graph and that he may have “chosen the wrong
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52 type of graph.” He elaborates by reiterating that there are 3,000 data points, which would mean
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54 that it “takes a lot” for the curve “to go down that far.” Then he states again to be “kind of
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56 surprised.” We can hear him express surprise about the variability and, therefore, co-express his
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58 anticipation to have a much smoother graph. Craig then articulates the hypothesis that Shelby has
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5 “mix[ed] quite a broad spectrum of fish” and “that could have something to do with it.” In fact,
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7 the fish that have contributed to the data were from different hatcheries and from different river
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9 systems. This “could have something to do with [the variance].” Here, only the familiarity with
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11 the background to the data collection allows Craig to articulate this hypothesis.
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16 *Reducing complexity*

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18 Reducing the complexity in and of the data by limiting the source was one of the strategies
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20 that was repeatedly observed in the course of this study. In the present instance, this occurred
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22 immediately after Craig’s articulation of the hypothesis that the variability might be caused by
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24 pooling all the fish in their study. Shelby suggests that he “could just choose one set.” He
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26 proposes using fish that had come from the Kispiox River in northern British Columbia. Craig
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28 accepts by restating that possibility. The selection of a subset of the data, which was introduced
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30 as a way of decreasing variability actually appears to have increased it. However, a direct
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32 comparison between the former and the new plot shows that there are in fact new features that
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34 appeared and old features that had disappeared (Fig. 4c). Thus, the one peak that Craig had
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36 repeatedly pointed to and marked as a candidate for elimination has disappeared. But on the
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38 right-hand side of the plot, there not only is a new peak but the existing peaks are as high as
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40 those in the middle earlier designated for elimination. What is important to this meeting,
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42 however, is what arises in the relation between Craig and Shelby: the two lower peaks that are
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44 still present. Shelby characterizes them, therefore, as particular to the Kispiox fish. After and
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46 thereby breaking the long pause, Craig tentatively (“maybe”) proposes a plot with error bars.
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52 *Trying to get a grip on variability*

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54 *Replotting the data together with a statistical measure of variability* The preceding move has
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56 not led to the intended reduction in variability of the data. In such a situation of uncertainty,
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58 when scientists are in the dark and without the knowledge that later comes with hindsight—that
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60 is, before scientists are familiar with the domain—any action appears as good as any other (Roth,
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2004). It is only after the fact that a value “good” or “poor” can be attributed to action and its ascribed intention or, in other words, that the quality of fit can be determined between situated actions and the plans that are said to underlie them. It may not come as a surprise then that Craig proposes trying another avenue: plotting the data with error bars. The uncertainty of what such a move may bring about is expressed in the adverb “maybe,” which modalizes the verb “to try” as a possible but not necessary next move. The verb “to try” implies decision between alternatives and finding out about something that is doubtful or obscure. At first, Shelby expresses uncertainty about whether “that” is “going to work with that” and, speaking increasingly faster, states that he will give it a try.

When the scatter plot appears on the screen (Fig. 5a), Shelby notes that the two lowest data points also are those without error bars ([i] & [ii], Fig. 5a), and that they therefore represent unique points. Their salience is increased by the cursor, which he moves directly over one then the other point ([i] & [ii], Fig. 5a), moving back and forth three times. *In and as the relation* with Craig, this statement is confirmed and marked as salient, as if this fact provided a reason: “okay, so there you go.” That is, in this instance, the visibility of the two points exists in and as their relation. It is not just there, merely pointed out by Shelby. Rather, the distinction of these data points and their relation to the interpretation is a result of the relation. “They *are* just two unique points,” that can be pointed to (by means of the cursor) and that can be seen and confirmed on the part of the listener (“yea,” and “here you go”).

««««« Insert Fig. 5 about here »»»»»

A first interpretive attempt Shelby first draws a comparison to what has been: “yea, it’s quite different.” He then states an implication for the trend of the data: “you do get a general increase from zero to a hundred, very, very subtle.” As he makes the statement, one can see the cursor move from the cluster of points at the left margin of the graph to the aligned cluster of points on the right ([iii], Fig. 5a). Although the relationship is different from the one Craig has articulated before, nobody confronts the two different hypothetical deep structures in the data and the trends

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5 that they express. At this point, there are no comments to the fact that Shelby proposes a linear
6 correlation whereas Craig has made visible a curved relationship: one of these is still visible in
7 the line on the chalkboard, the ephemeral gesture perhaps present in perceptual memory.
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12 *A second, alternative interpretation* At first, Theo asks Shelby to change the scale on the “y-
13 axis.” Together they establish the new limits on the plot. Craig affirms and names what is
14 happening “a higher resolution there.” There is a 3.90-s pause, then the new graph appears (Fig.
15 5b), and then another 6-second pause unfolds. Craig is the first to speak. He gets up, walks to the
16 screen, and then moves his hand in front of it to exhibit three trends in three parts of the plot.
17 Craig points out—not unlike in his first analysis—that there is a linear trend on the right-hand
18 side of the plot ([i], Fig. 5b). He then makes salient a “component,” which, as the shape of the
19 hand trajectory shows, is approximately inverse parabolic ([ii], Fig. 5b). He repeats the gesture
20 first backward and then forward again. In the 0.68-second pause that follows, his index finger
21 first makes a slight upward movement following what might be seen as the trend of the first 11
22 data points ([iii], Fig. 5b), then moves back toward the point where the parabolic movement
23 ended and produces a second movement now slightly downward ([iv], Fig. 5b) while uttering
24 “down again.”
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40 When we compare the hand movements produced in this situation (Fig. 5b), we note the
41 similarities and differences with earlier presentations. These differences, however, are not
42 explicitly articulated as changes in the idea or assessment of what they are looking at. The first
43 part of the gesture, on the right end, is similar to what Craig produced in the context of the
44 Kispiox fish, but different from the inverse parabola that he produced even earlier. The central
45 part does have the inverse parabolic shape. Finally, on the right end, Craig first makes a
46 movement *upward* from right to left and then, almost in the same breath, changes the description
47 and hand movement to one that goes *downward* from right to left. The hand gestures here are
48 forms of epistemic movement that constitutes thinking itself rather than being a *representation*
49 thereof (Radford, 2003; Roth, 2012). Again, the trends depicted in the movement are different
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5 from the “general increase,” a “very, very subtle” one that Shelby has articulated together with a
6 nearly linear trajectory of the cursor from the left to the right. We therefore observe
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8 contradictions both within the articulations and movements that Craig produces—even within a
9 fraction of a second—and with respect to those that Shelby previously articulated. However,
10 these contradictions again are not made the topic of discussion. It is as if they did not exist or
11 were salient.
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19 *Another attempt at understanding variability as it expressed itself as variance (error bars)*

20 Shelby then evaluates Craig’s presentation saying that he finds it “not surprising” and continues
21 to articulate a reason. He talks about the data representing not individual fish but rods that are
22 grouped. He ends by saying that he is skeptical of those data near 0% and 100% (A_2). Craig
23 follows the comment about being skeptical by querying the nature of the error bar—whether they
24 represent ± 1 standard deviation. By asking the question in this way, the normative character of
25 this value over other possible values comes to the fore. Shelby responds by saying that he had
26 plotted using 95% confidence intervals. There is a long pause and then begins some elaboration
27 about wanting to use 1.96 standard deviations and just as he was beginning to suggest what this
28 would lead to “then you would get,” Theo interrupts him making salient the points “up there.”
29 (The two numbers, $SD = 1.96$ and 95% confidence intervals are equivalent.) Shelby continues,
30 “one [1 SD]” “would be a bit cleaner.” In this, he articulates a reference to Craig’s question
31 about the error bars corresponding to $SD = 1$. Theo suggests that the 95% confidence intervals
32 are a bit much, thereby adding to the preceding question about the standard deviation being 1 and
33 the assertion that “one would be a bit cleaner.” Michael is asking about the number of fish
34 corresponding to the right side of the plot: Is there a smaller number of fish? A reason for the
35 query follows immediately, “because the overall variability increases there, as you go to the
36 right?” Shelby ascertains that it makes sense to have greater variability when there is a smaller
37 number of fish; and he adds that most of their fish have been exhibiting near 0% A_2 .
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A sudden end of the discussion of HBW and its treatment in the final publication

At this point, this discussion of the half-maximum bandwidth ends suddenly. There is no clear conclusion any one member offers but a clear proposal and acceptance to move to some other topic. As in many other instances during this research project, rather than making the problem the core topic of the discussion, the scientists go on to some other point. They do not wonder why something does not work. As soon as it does, they go on with their business never wondering what it was that did not allow them to make sense or do their work (Roth, 2004). There is perhaps an underlying and unstated assumption that the problems will go away once the data set is only large enough; alternatively, any remaining problems can be dealt with should they remain to the end. In the journal publication of this work, the team noted the high within-fish variability in percent A_2 and described different ways in which it controlled this variability (Temple et al., 2006). At the experimental level, only photoreceptors from the dorsal area of the eye were taken. The measurements actually taken were subject to inclusion criteria—presence of a baseline in the absorption curve, peak absorption within the commonly accepted range, minimal absorption by other materials on the slide, and a signal-to-noise ratio greater than 5 to 1. To further control variability in the determination of A_2 , the team smoothed the curves using an averaging function and determined the location of the peak on the wavelength axis. Although half-maximum bandwidth was mentioned to broaden in the life history of fishes and although was shown in one of the figures, it was not described as having been used in the determination of A_2 .

Discussion: graphs and graphing in discovery work

This study was designed to understand how scientists deal with variability that shows up in their graphs (e.g., in the size of the error bars they plot), and the associated uncertainties that arise for them in making sense of their data. It draws on data collected during a 5-year ethnographic effort in one advanced scientific research laboratory. The results show that although there is some understanding how variability results from the production of the data and

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5 although the members of the research team knew that there was trouble, they did not know what
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7 to make of the variability (and statistical variance) in their data and what patterns these
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9 expressed. In an attempt to make interpretation easier, they only considered part of their data.
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11 The session concerning these data ended without a conclusion and despite the inconsistency of
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13 the data with the theory, and despite very different patterns proposed not only between scientists
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15 but also within a scientist. In focusing on the local practices of scientists to bring order to their
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17 data through sequentially ordered turn-taking that made for an orderly meeting, our study is
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19 oriented very differently from those that attempt to identify mental processes that affect
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21 perception of graphs or that operate in the identification of non-obvious properties (Friel, Curcio,
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23 & Bright, 2001). It also differs from studies that “heavily rel[y] on [participants’] *explanations* of
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25 their work and thinking” (Gainsburg, 2007, p. 482) when this is done *after the fact* rather than
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27 during and as integral part of the work.
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31 In the episode as a whole, one can see that because the team members did not have a ready
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33 answer, they did not and could not know whether some action will produce the anticipated
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35 changes in the data displayed. But any orderliness that after the fact was accounted for arose
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37 from the practical aspects of the work, which led to “transcendental orderliness” (Garfinkel,
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39 Lynch, & Livingston, 1981, p. 141) of the natural objects in the team’s hands. In the preceding
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41 section, it is the “transcendental” disorderliness, the properties of *this* display that the work of the
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43 scientists comes to provide. In fact, orderliness and disorderliness reflexively make each other
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45 possible as ground against which the other emerged as figure. It has been noted that inscriptions
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47 may in fact hide certain aspects of mathematics in the workplace—it becomes a black box
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49 (Williams & Wake, 2007). Work has to be done to unpack that which is hidden and invisible—
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51 such as the number of data underlying each point plotted invisible to Craig but pointed to by
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53 Shelby. In the present instance, the particular representational form Shelby had chosen exhibited
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55 and hid variability. It exhibited variability that the team wanted and needed to get under control
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57 or at least understand; it was variability that muddied the anticipated or real trend in the
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59 phenomenon. But it also hid the variability of each data point, itself the mean of many
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5 measurements, which total over 3,000.
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7 In the end, the team members present abandoned what they themselves had identified as
8 problematic. It may be just a further example of abandonment that is typical of everyday problem
9 solving, where it constitutes one possible way of going about a problem as coming up with some
10 definite answer (Lave, 1988). It stands, of course, in striking contrast to school situations, where
11 students tend to be required not only to have but one single answer in contrast to multiple
12 answers, but where they are not generally allowed to abandon what the teacher has designed as
13 the problem at hand (Roth & McGinn, 1997). In the present situation, the lead scientist works out
14 some “preconceived notions” as to the relationship between two salient variables, %A₂ and half-
15 maximum bandwidth of the absorption spectrum. This relationship is important, because the
16 scientists were hoping to use it for the determination of the relative amounts of rhodopsin (A₁)
17 and porphyropsin (A₂) in the pigment of the fish eyes, which are the relevant variables for
18 determining—so the Nobel prize winning canon at the time—the readiness for the saltwater
19 journey. According to Theo, he has not yet “checked what the bandwidth gives [them].” This is
20 what they pursue in the episode analyzed here.
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37 The present analysis exhibits the work that the scientists produce for finding “what the
38 bandwidth gives them.” Craig makes an argument for eliminating some of the data points, but
39 Shelby resists, for, as he argues, there are about 3,000 data points. This means that there will be
40 some large variations, on the one hand, and relative stability of the data, on the other hand. Theo
41 does provide an explanation of why some individual HBW measurements might be much smaller
42 than they ought to be: His algorithm cuts off one part of HBW when the “ringing” that all of
43 them are familiar with crosses the HBW line before the best-fit curve does. Craig provides a first
44 reading of the trend, which, in his gestural movement, takes on a curvilinear shape that matches
45 his “preconceived notion.” The scientists here attempt to control the variability by looking at a
46 subset of the data. But the variability that is the salient aspect in their work remains when they
47 look at the data from one of the seven sources. The next move is to establish a better
48 understanding of the variability within each data point by “trying to plot with error bars.” Here it
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5 turns out that two of the “spikes” do not have error bars and, according to the team, are “unique
6 points.” Once they have changed the scale of the plot, we observe the scientists enact two
7 readings of the data.
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11 Shelby, by means of the cursor, traced out a more-or-less linear relation, “a general increase
12 from zero to a hundred” that is “very, very subtle.” In this way, the trend was precisely of the
13 kind that a dominant researcher in the field (Hárosi) and others previously published. It had, as
14 predicted, a lower bandwidth for rhodopsin (A_1) on the left end of the graph and a higher
15 bandwidth of porphyropsin (A_2) on the right end of the graph (in sharp contrast to the
16 “preconceived notion” that Craig had articulated earlier). This is also the way one would expect
17 it to occur if one knew that the HBW values plotted here are not measured in nanometers, that is,
18 on the same scale that the absorption spectra are plotted. Rather, HBW, as plotted here according
19 to the values on the abscissa, would vary linearly with % A_2 if the widths of the spectra,
20 measured in nanometers, vary as a negative square function. This is not salient in the situation,
21 however, and therefore does not play a role in its internal dynamic of the data analysis meeting.
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35 Craig also provides a reading; in fact, he provides two different readings of the data. There is
36 a linear increase on the right-hand side of the plot similar to what Shelby has shown. In the
37 center, however, Craig has a parabolic relationship, as exhibited in his “preconceived notion”
38 and as gestured in an earlier plot. Finally, Craig first gestures a linear relation with a negative
39 slope at the left end of the plot; he then changes to a linear relation with a positive slope. In the
40 course of the data analysis, there are different trends proposed. There are also trends that have
41 been published in the research for nearly two decades. However, in this meeting, the question is
42 never posed or answered why the measured curve does not match the anticipated curve, as
43 articulated at the beginning. This even though this very issue is a stated goal for the laboratory.
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5 Ochs, Gonzales, & Jacoby, 1996). In the deictic gestures, the graph as a whole or individual
6 points are the topic, whereas the graph is the ground against the *dynamic* hand/arm movement
7 that expresses something not itself in the display. It is layered over and above the display but
8 represents something “underneath” or underlying the data, the deep structure. The erratic data
9 points are but concrete expressions of extraneous variability of some true signal that is made
10 visible in and by Craig’s hand movements. Gestures play an integral role in the search for trends
11 (Roth, 2012), which are found because gestures are generative (de Freitas & Sinclair, 2012),
12 semiotic means of objectification (Radford, 2003). These are a way of orienting visual
13 perception through the maze of the data that Craig did “not know what [was] going on here.” Of
14 course, the gesture itself was motivated by the perceptual gestalt that the data themselves
15 offered; the gesture, as words in other situations, is but a means that “lets something be seen,
16 namely what is talked about, and *for* the speaker (the medium) or for the interlocutors”
17 (Heidegger, 1977, p. 32). The trends existed in their relation with each other, where they take on
18 the *public* visibility of their characteristics; and the trends revealed themselves such that others
19 could perceive them too. Even if there are multiple accounts of possible trends, these are in any
20 case articulated and therefore rationally accountable trends. When Craig moved his hand/index
21 finger across the display, he actually directed and taught others and himself how to look and
22 make visible a possible trend. Similarly, in moving the cursor across the data display, Shelby
23 exhibited in his movement *how* others in the meeting had to look so that they “[got] a general
24 increase from zero to a hundred, very, very subtle.” The movement of the cursor exhibited the
25 material that justified the description of the general but subtle increase from the very left to the
26 very right of the display. The movements constituted the natural accountability that grounded the
27 observation of the trend.
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53 54 55 56 **Some preliminary thoughts on graphing and uncertainty in schools**

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58 Studies of how scientists deal with variability, while telling us a lot about knowing and
59 learning applied mathematics, should not be taken as indicative of strategies for school
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5 mathematics curricula without careful prior discussion and consideration. The following are but
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7 some preliminary thoughts about the relevance of this study of graphs and graphing in the
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9 discovery sciences to mathematics education.

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11 In this study, research team members apparently talked about and wrestled with variability, a
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13 topic of interest to mathematics and statistics educators (e.g., Cobb & Tzou, 2009; English, 2012;
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15 Pfannkuch, 2005). Random variations that obscure true relationships underlying data are
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17 potential trouble spots in university students' modeling activities (Doerr, 2000; Falk & Konold,
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19 1994). This is so because random errors may exhibit a linear relationship when the true
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21 relationship is exponential. This has to be particularly the case when the size of the error that
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23 comes with each data point is unknown but subsequently understood to be systematic.

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25 Attempting to understand the source of variance or limiting the statistical variance by controlling
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27 the nature of (variability in) the data, therefore, may be an important step to understanding the
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29 true relations. Understanding and getting a grip on variability may be even more important than
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31 finding patterns. There are suggestions that students of science may learn more and better
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33 understand a phenomenon if they understand when, where, and how it can get lost (Garfinkel,
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35 2002; Roth, 2013). It may be more important for students to discuss sources of variability,
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37 changes in the phenomenon when different types of systematic sources of statistical variance
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39 modifies the signal, and the role of stochastic error (e.g., Cobb & Tzou, 2009).

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42 Some scholars have observed that we “can expect *learners* to generate multiple
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44 interpretations of inscriptions, use multiple meanings of words, and focus on different aspects of
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46 inscriptions” (Moschkovich, 2008, p. 578, emphasis added). As the present investigation shows,
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48 this state is not only characteristic of students—who are in the process of learning something
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50 they do not know and therefore cannot evaluate—but also of highly experienced and successful
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52 research scientists confronted with some data first time through when they are in the process of
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54 establishing possible deep structures of the data. The present analysis shows that there may be
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56 alternative and contradictory interpretations articulated in the same meeting *without* there being a
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58 contrasting evaluation that might lead to further learning. Yet research has shown that learning
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5 arises precisely from the opposition and confrontation of different propositions or while
6 reasoning about alternative hypotheses and, in the process, build consensus and sets of shared
7 rules about data interpretations and variation (Cobb & Tzou, 2009). In school mathematics, this
8 might be the place where teachers—in the process of observing and overhearing students—
9 decide to enter their discussion and assist students in making the differences explicit and in
10 making the resulting contradictions the topic of discussion. It is precisely in the ensuing
11 dialogical relations that learning results from inquiry. Do we have to conclude that scientists, too,
12 may need such support? Or do we have to conclude that students do not need such support
13 because scientists themselves do not use opposition? Concerning the discovery sciences, at least,
14 the very fact that scientists work at the cutting edge of their field and the fact that these meetings
15 focus on topics in real time that prevents a reflective approach may mitigate such attempts.
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28 In this study, the variability in the data arose from the scientific phenomenon itself and from
29 the ways in which measurements were gathered, inclusion criteria were defined and applied, and
30 ways in which graphs were transformed to produce the A_2 values that were presented plotted
31 against time of year. This may be used as part of an argument for interdisciplinary approaches to
32 science, technology, engineering and mathematics education (e.g., Artigue, 2012) because it
33 provides students with opportunities to understand variability and its effect on comparisons even
34 at the university level (e.g., Tra & Evans, 2010). As an above-mentioned study had shown,
35 eighth-grade students familiar with the analysis of data that they collected themselves, and
36 therefore familiar with variability and how to control it, outperformed university graduates on
37 data interpretation tasks (Roth et al., 1998). On the other hand, students in mathematics classes
38 presented with data sets were asking questions extensively to understand the provenance of the
39 data and what might possibly have affected them (Cobb & Tzou, 2009).
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54 There are many presuppositions about what students of science *ought* to do when analyzing
55 data. For example, one study provides a list of questions students are to draw on in their
56 argumentation about graphs (Chin & Osborne, 2010). The questions that the authors provide are
57 categorized according to some of the basic science process skills—for example, observing,
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5 comparing, analyzing, predicting—and include: “‘Is there anything unusual or unexpected?’
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7 ‘How are A and B similar?’ ‘How are A and B different?’ ‘What is the significance of these
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9 similarities and differences?’ . . . ‘Is there anything I am puzzled about?’ . . . ‘Which is the better
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11 graph? Why?’” (p. 242). As the analysis of the episode shows, the research scientists did not ask
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13 such questions, at least not in my presence and in the sessions we recorded. They were able to
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15 live with apparent contradictions in the descriptions various members provided for the possible
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17 trend in the data and with the apparent contradiction between the theory or a priori conceptions
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19 and the actual data. The kinds of processes educators often postulate—including disagreement,
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21 challenge, cognitive conflict, and puzzlement—are not (as) explicit in the data analysis sessions
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23 that we report on here. But this does not mean that these processes *ought not* be fostered in
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25 schools, where they might function as a means to develop competencies that are currently not
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27 generally observed in most working place contexts, including scientific discovery work.
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34
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36
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39 opinions are those of the authors.
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39 Captions

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41 Fig. 1. a. The A_2 values from 3 sites are spread across the entire spectrum of A_2 (0–100%) rather
42 than clustering more sharply around some mean value. b. A raw absorption spectrum, as it
43 comes from the instrumentation, is fitted with a 7th-order polynomial derived from the
44 literature. The wavelength where the polynomial has its maximum is called “lambda-max”
45 (λ_{\max}) and the width of the polynomial at half its height with respect to “baseline” is referred
46 to as “half-maximum bandwidth (HBW).”
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54 Fig. 2. a. Craig draws a graph representing the expected relationship between half-maximum
55 bandwidth (HBW) and amount of porphyrin (% A_2). b. Shelby plots actual HBW against
56 % A_2 .
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60 Fig. 3. a. Theo shows how lambda-max and HBW are extracted. b. He identifies the “ringing” as
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5 a problem in the raw data, which introduces variation in the determination of HBW.
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7 Fig. 4. a. Craig identifies 4 points as outliers ([ii]–[v]) and gestures ([i]) where a regression should
8 be conducted. b. Craig gestures a hypothesized best fit. c. The suggestion to plot only a
9 subset of the data does not lead to a reduction in the variation.
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13 Fig. 5. a. After plotting the error bars for each data point, Shelby points to two critical points ([i],
14 [ii]) and then gestures a best fit ([iii]). b. After Shelby rescaled the plot, Craig uses gestures
15 to show what the trends in the data are.
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