Roth, W.-M. (2012). Mathematical learning: the unseen and unforeseen. For the Learning of Mathematics, 32(3), 15–21.

MATHEMATICAL LEARNING, THE UNSEEN AND THE UNFORESEEN

WOLFF-MICHAEL ROTH

> In painting, as elsewhere, the invisible *is received, but not produced*. (Marion, 1996, p. 46, emphasis added)

The purpose of this paper is to work towards a phenomenological theory of learning that makes a radical commitment to the fact that students cannot see or comprehend what is knowledge until *after* they have learned. A fragment from a fourth-grade mathematics lesson in which students were intended to arrive at a generalization of the type $y = 3 \cdot n + 6$, is used to think about learning given that the students cannot aim at this learning outcome precisely because they do not know (a) the generalization, (b) that they are supposed to generalize, or (c) which aspects of their experience to generalize. Because students do not know what they will know until *after* the learning event, I suggest that this future knowledge may be better thought of as the (initially) foreign or strange, which affects students *before* they can grasp what is happening to them or what/that they have learned. *Pathos* names this originary affectivity that precedes, and is the condition of, any comprehension (Henry, 2000). What students learn—that which is really new in their knowledge and therefore cannot be derived from what they already know and see—is accessible to learners only *a posteriori*. This new knowledge may then be thought of as the result of a donation, a process that presupposes an originary

affectability whereby the subject is affected prior to any understanding of the why, what, and how of this affectation. As a consequence of this approach, learning is not only the product of agency (construction) but also the result of donation that presupposes an originary affectability (*i.e.*, pathos). We can then speak of the birth of mathematical sense in and from pathos.

In *A Cultural-historical Perspective on Mathematics Teaching and Learning* (Roth & Radford, 2011), Luis Radford and I present an extended description and analysis of a student (Mario) confronted with a *learning paradox*: because Mario does not know beforehand what he is going to know after having completed the learning task, he cannot know the *motive* of and for the learning activity. We suggest that this motive has to reveal itself in and as a result of the learning activity. Because he does not know the motive, Mario cannot directly and intentionally aim at "constructing" the intended knowledge. He finds himself understanding rather than "constructing" understanding or "mental representations" in the way constructivist mathematics educators tend to describe learning. In Roth and Radford (2011), therefore, we note an inherent contradiction whereby mathematics students have to engage in activity without knowing its motive (unknown and yet-to-be-known outcome), and have to hope that this motive will somehow appear to them together with the (to the students) invisible knowledge to be acquired. That is, just as stated in the opening quotation, the invisible future understanding appears to be *received* rather than actively, intentionally "constructed" on the part of the learner.

In recognizing that the learner does not grasp or interpret or construct what is *happening* (to them), out of which new knowledge emerges, the phenomenological

position espoused here radically differs from all constructivist approaches to mathematics education. Here, the things, processes, and phenomena that appear in currently used theories come into existence for the experiencing subject only after the unfolding event has come to a closure, that is, until after some critical instant that has led to the emergence of something completely unseen, unforeseeable, and unforeseen from the perspective of the agential subject.

Fragments from an episode of algebra learning in fourth grade

In this article, I analyze fragments from the lesson mentioned above. The lesson was intended to introduce fourth-grade students to algebraic forms of thinking. Because it attempts to be consistent with the actors' perspectives on activity, the analysis is grounded in conversation analysis and ethnomethodology, which make it illegitimate to draw on information other than what the members to the setting make available to each other. The excerpts are taken from the complete French and English transcripts that Radford and I provide in the appendix of our book. In a class of 22 students attending a school in Northern Ontario, the camera is focused on one group of students, Mario, Thérèse, and Aurélie, whose desks are arranged into a large rectangle. Radford, together with the teacher (Jeannie) and the research assistants designed an experimental curriculum that nevertheless met the requirements of the provincial curriculum. In this lesson, the students worked a problem that included two main tasks about modeling the process of saving money in a piggybank.

The students have been provided with a worksheet that includes, as the problem statement, the description:

> For her birthday, Marianne receives a piggybank containing $6. She decides to save $3 each week. At the end of the first week she says to herself, 'I have $9!'

The first two questions are:

(a) Model the problem until the sixth week using goblets and chips; and

(b) Fill the following table of values (see Figure 1).

Number of week	**1**	**2**	**3**	**4**	**5**	**6**
Amount saved ($)	+ 6	+ 6				
Or	6	2x **+ 6**	3x **+6**	x **+6**	x **+6**	x +

Figure 1. This table of values provided with the story about saving money in a piggy bank is partially filled up to aid students in the process of arriving at the generalization of the type $y = 3 \cdot x + 6$.

Radford and the teacher (Jeannie) had created the task because they wanted students to arrive at a generalization of the kind "*amount in the piggybank* = *number of weeks* · $3 + $6." But students inherently cannot aim at this generalization, precisely because it is unknown to them. Only after the generalization has *come* to them, in a moment of (sudden) insight that presupposes their practical comprehension of the situation, can they then make sense of what they have done

and why. This is so because intuition (from the Latin *in-* + *tuērī*, to look), that which becomes visible in perception, is *always* in excess of intention, current knowledge (Marion, 2005) and action (Nietzsche, 1954).

Incomprehension: the unseen and foreign

The students begin with the first part of the task, as Mario counts out the red and yellow chips and places them into the goblets. Once all the goblets have received the chips, the students begin filling up the cells in their worksheet. Soon after beginning the worksheet, Aurélie throws herself against the back of her seat in apparent frustration.

Fragment 1 <level 2 header>

```
→029 A: <<plaintive>i dont understAND; and I will
        nEVer understand.> ((Stares at her hands
        placed on the worksheet)) (3:38)
  ---
  037 A: ((pounds on the table))
→038 T: <<p>kay we are all mi[::xed up>        ]
→039 A:                      [i dont understand]
        ((points to her page))
  040    (2.46)
→041 T: <<confidently>uh hu:::; uh huh. >
  042    (25.56) ((M drops his hand))
```

((*Aurélie pounds desk again, throws herself back against back of seat*))

((*Mario gets back to the task, A leans back*))
→043 J: <<f>yes.> (0.52) whAT is the ques[tion.]
044 M: [its]
^this ::: (0.38) <f>um[::>] *
045 T: [auré]lie sit properly (55:00)
→046 M: look this is (.) dUMb, <<p>i dont understAND.> ((487>217Hz))

((*hands move downward, restrains not to pound on table, gazes at sheet*))

In this situation, Aurélie and Mario hearably state that they do not understand; and they produce unmistakable expressions of frustration. They do not know what the task demands of them and cannot plan what to do next or whether what they have done so far is what they are asked to do. Because they cannot know what they are supposed to know until *after* they have learned their intended lesson, Aurélie and Mario are in no position to assess whether what they have done so far is what they are supposed to have done. This exposure to a task the intent of which they *cannot* know quite apparently *affects* them, giving rise to frustration, a form of negative emotion.

Mario does not understand. His actions do not make sense; what he has to do is not intelligible. Yet to have any hope of eventually understanding, he has to act and do something. If he does not act, he will never know, as Aurélie states. This requires Mario to transcend what currently makes sense and is intelligible to him. Mario responds to something that is unknown (therefore foreign) to him, neither different nor indifferent, because it is invisible. The required action is in response to the

foreign, which, *qua* foreign, does not make or have sense, nor is subject to an existing order or rule. In such a situation, "*what* I answer owes its sense to the challenge of that to *which* I answer" (Waldenfels, 2006, p. 58). Following the answer, however, the difference between how we answer and what we answer-to tends to be leveled in favor an intention–effect relation. That is, after the fact, we give reason to dimensions of actions that we did not have reasons for prior to acting because we could not anticipate the effects that were brought about.

For students in situations like that in which Mario and Aurélie find themselves, there is no way that they can evaluate whether some specific action gets them toward the outcome intended by the task, as they do not know which outcome is intended until the point at which they have come to know the outcome that the teacher wanted them to learn. The student *has to* act in the absence of a reason so that the intention (reason) for action becomes available together with the (by the teacher intended) learning outcome. I cannot say after the fact that I could have made a better decision if I had known more prior to it because it is precisely the decision alone that allowed me to know more (von Weizsäcker, 1973). Mario is not in a position to say that this or that action is getting him closer to understanding. It is only after the fact that Mario will be able to say, "Me, I now understand"; the understanding *follows* the action, and therefore cannot be theorized as having had an intention (goal) that would have motivated it. In acting, the students open up to step across a boundary from the known into the unknown (foreign). But in that same step, the unknown (foreign) already withdraws so that what Mario has learned already is within the horizon of his possibilities.

It is proposed here that we need to move toward understand learning as a dialectical process of stepping into the unknown in such a way that when we land, the unknown has withdrawn and things begin to look more like the familiar. Learning occurs at this boundary between order and the unknown yet-to-be subject(ed) to order *(i.e.*, exceeds the categorization into order and disorder). This is so because the "radically foreign is precisely that which cannot be anticipated by subjective expectation or trans-subjective possibility conditions" (Waldenfels, 2006, p. 30). In the process, learners step into the unknown where they are exposed and vulnerable, in any case affected "by *something* in such a way that the Whereby is founded neither in a preceding What nor in an *a posteriori* achieved What-for" (p. 43, original emphasis). Being affected expresses itself in, as, and through affect.

Spreading of affect

In the preceding section, Mario and Aurélie display what others in the same culture can see and feel to be tonalities of affect. Although Jeannie (the teacher) at first suggests that Mario works with the others, she nevertheless stays with this group. Physically oriented toward Mario, she particularly works with and talks to him but, in so doing, also speaks for the benefit of the others in the group. Despite the earlier admonishment to sit properly (which might also be heard as an attempt to orient her to the task), Aurélie does not overtly participate. Thérèse fills up her table of values apparently independently. We might think that working like this will get them on the right track in their task. But, as Fragment 2 shows, this interaction, as in the interaction intervening between Fragments 1 and 2, does not get Mario back on

track; and Jeannie, too, begins to exhibit a negative emotional tone. Jeannie asks, oriented toward Mario and pointing to the table of values, why the three is in yellow (turn 069).

Fragment 2 <level 2 header>

```
  069 J: it EQuals to nine the first week.
         (0.78) wHY is the thrEE in yellow?
         whydyou think? ((Index on table))
  070    (0.19)
  071 M: um um, um ((Shrugs shoulders, shakes
         head 'no', questioning look))072
            (0.20)
→ 073 T: <<all>i don[no]>
→ 074 M:            [be]cause we are supposed
         to write it?
  075    (0.44)
  076 J: WHEREe does the thREE come from?
  077 T: donno?
→ 078 M: <<f>a:=u:> (0.24) u:: (0.17) u:
         dududu: wedding thing there?
  079    (0.76)
→ 080 J: <<exasperated>but ((turns head away
         from Mario)) (0.14) the three
         dO:LLas? is wHAT exACtly?> *
```

((*Mario, who has looked at her, grimaces in desperation, brings his hands up and covers face*))

As this fragment shows, the interactions with the teacher have not changed the tenor of the situation. Mario continues to express what culturally competent witnesses sense to be frustration, apparently not knowing what Jeannie is asking of him. Not being in a position where he could know what to do or say, he is forced to act to see what this action yields: "because we are supposed to write it" (turn 074) and "ah: uh: uh: the wedding thing there" (turn 078). That is, Mario has to act in the absence of a reason to *find* what the action yields; he has to find an object that would give reason to the action *a posteriori*. Evidently, here, his action has not been

appropriate, as Mario may witness in Jeannie's apparent exasperation. He responds to this expression by what we may gloss as grimacing and covering his face in a movement of apparent despair. Thérèse, who continues filling up the table of values on her own, nevertheless says, in reply to the teacher's questions, that she does not know. Jeannie in turn expresses exasperation, as she apparently does not know what or why the children do not know. That is, although she has begun an intervention, she does not know what the children do not know and where the problem of lacking understanding originates.

It is not just that Jeannie responds in despair, but her response actually begins by listening to what Mario has to say. She is affected by what he says, with apparent affective mediation: she is exasperated. Similarly, Mario does not just respond after Jeannie has offered up what can be heard as questions (turn 069): to be able to answer, he had to listen to Jeannie and allow himself to be affected by what she utters. By shrugging his shoulders, shaking his head, and making what we may gloss as a quizzical face, he publicly makes available his cluelessness about how to answer. Everyone can observe the affective dimension of his response by grimacing in apparent desperation and by covering his face. Jeannie will respond in making another attempt to assist Mario.

In this fragment, the tone of affect has been spreading and contaminating others: exasperation now is observable in and by all participants. The phenomenological position suggests that there is affect and affectability that necessarily *precedes* and makes possible languaging (Henry, 2000). We are affected prior to any understanding of what is happening to us and prior to any language we may have in

talking about it. How is such spreading possible? The answer is that we are affected in the initial part of responding that consists in opening up to be affected by something (*e.g*., the saying of another) that we do not yet know (*i.e*., we only know what has been said when the saying has finished). This is so because pathos (being affected by the utterances and actions of another) and answer are not two events: they are manifestations of one and the same event that is shifted with respect to itself: listening to another person (without knowing what she or he will have said when finished) and answering are but two sides of *responding*.

Solution and resolution

Over the course of nearly 4 minutes, with further signs of frustration on both Jeannie's and Mario's part, the two engage each other in filling one cell after another.

Fragment 3 <level 2 header>

```
→215 J: <<excited>yES::.> ((Makes the same rH movement to right, opens
        palm toward ceiling)) (1.21) its just on the bottom its a
        [shortcut]
 216 A: [madAMe: ]
 217    (0.42)
 218 J: your fourth week; (.) how mANY three dollars do you have.
 219    (1.00)
 220 M: u:m:::  (1.73) fo. ((Fills up table, Therese makes noises))
 221    (9.48) ((Writes 4 '3s'))
 222 J: <<pp>kay> (0.97) instead of doing three plus three plus three
        plus thrEE whAT are you going to wrITE here? ((Points to the
        row on the bottom of the table of values))
```

```
  223     (0.66)
→ 224 M: <<tentative>uh:m:: (1.36) four
         times thrEE?>
→ 225 J: *
  226     (3.83)
→ 227     i=think you understand now. uh?
  228     (50.93) ((Mario slightly nods,
         writes, after 26 seconds looks
         at Therese's worksheet, back at
         his own))
→ 229 M: <<confident>ME i now
         understand.>
```

((*Turn 225: 2-handed gesture sideward, opening palm upward: 'you got it'*))

In this third and final fragment from the lesson episode, the emotional tide is turning. Mario produces responses to Jeannie's questions that she evaluates positively both gesturally and intonationally, also expressing satisfaction on her part. She tells us to be thinking that Mario understands. Mario, with a comportment that we may gloss as newfound confidence and therefore positive affect, states that he understands, the statement and the affective expression merely being two aspects of the same utterance (action). From his tentative answers prior and right up to turn 224, to his confident statement that he now understands (turn 229), there is a change: the tentativeness of the responses suggest an emerging but not yet firm understanding, whereas the confidence in the later utterance publicly exhibits a sense of understanding. He now sees and understands the situation in a way that gives him confidence to have understood what he is to learn and why he is writing 4 x 3 + 6 in the column corresponding to week 4 and below the cell in which he has written 3 + 3 + 3 + 3 + 6. As his subsequent cell entry shows (*i.e.*, 5 x 3 + 6), he *appears* to have generalized (from the concrete instances of chips in goblets) the

relationship between number of weeks and the number of chips in a goblet (or, equivalently, the amount saved in the piggybank). After conducting a book-length analysis of the entire episode, Radford and I conclude "that the entire multiplicative structure […] now is observable as a psychological function" (Roth & Radford, 2011, pp. 86–87).

Mario's generalization (if Radford and I are right in our suggestion that he had arrived at one) would be something radically new, unseen, and therefore unforeseen. It cannot have been the transitive *object* (*i.e.*, goal) of his actions. He therefore could not have worked *towards* it. Even if Mario had done the mental equivalent of doodling (non-intentional construction), he would not have been confronted with something that not only is new but also is recognized as such. As new, this order (generalization) prior to be known is the Foreign: Mario offers up his answers with a questioning intonation that signals his uncertainty. Therefore, anything that appears *as* something to us in our perception, as seen, heard, felt, and understood, cannot simply be described as "something that *receives or has its sense/signification* but as something that provokes sense without already having a sense, as something that we bump up against, that affects, irritates, surprises, and in a certain way hurts us" (Waldenfels, 2006, p. 73, emphasis added). That which eventually becomes visible, before the visible even exists, must "itself provoke the intention that will render it accessible and bearable for the eyes previously half-closed. The visible precedes the aim: *It [visible] is what must render itself to be visable by us*, since we did not expect it" (Marion, 1996, p. 62, original emphasis). Mario's understanding has emerged from a dark opening that expels the unseen and

unforeseen learning object: “in a debacle of the unseen as much as the foreseen” (p. 73).

Coming up against the unseen and thus unforeseen: saturated phenomena and the excess of intuition over intention

The statement that we learn in terms of what we already know has an aphoristic quality in educational circles. However, the aphorism recognizes only one part of the dialectical process of learning as stepping from the known into the unknown. How can those in the process of learning know *what* they are doing, that they are doing the right thing, and why it is the right thing? Radford and I already propose that such knowledge requires knowing the motive of activity, and that this motive *inherently* is not available to the person who is in the process of learning the learning object (Roth & Radford, 2011). In the present, the relationship between the new and the previously existing knowledge is different. Learning is like stepping into the unknown, where we do not know what to expect; but once we land, we encounter already apparently familiar things. The new has already been subjected to the colonization by the familiar and known. It is precisely because of this colonization that the new can be recognized *as* new, that is, that we have actually learned and changed.

In French, the verb “viser” means to aim at. Marion (1996) creates the neological adjective “visable” (a word English translators of his texts use as is) to characterize something that can be aimed at. Paraphrasing Marion (1996) for the purpose of the

present context, we might say that *coming among its own*, the new knowledge had to note that its own [knowledge] did not foresee it and therefore sur/rendered itself (to being) visable by it. That is, what Mario has come to see exceeds everything and anything that he could intend prior to the task. This is a characteristic dimension of *saturated phenomena*, that is, phenomena "where intuition gives more, indeed disproportionately more, than what intention could have ever aimed at or anticipated" (Marion, 2005, p. 54). Insight, surprise, discovery, invention, and birth are typical saturated phenomena; even a conversation where we cannot anticipate what the other will say is a saturated phenomenon.

What is unseen and that which Mario cannot aim at is the generalization $y = n \cdot 3 + 6$. That is, there are no apparent conceptual elements from which Mario could have constructed the generalization. If there had been, Mario could have *derived* the new from what is already known. If he learned something new, it had to be invented *ex novo*. *Ex novo* translates to "from scratch," "from nothing": to invent *ex novo* is to produce a beginning of something rather than grounding it in a preceding order. As the etymology of the verb "to invent" suggests (from Latin *in-,* in, into, upon, against + *venīre*, to come) the process involves elements of unpredictability and surprise: essentially passive forms of experience or exposure. This surprise is in fact a sign of the excess of intuition over intention and action and, therefore, of a saturated phenomenon.

The teacher could have given Mario the formula *amount in piggybank* = 3·*number of weeks* + 6, but this would not have been the generalization that has emerged to him just prior to his announcement "Me, I now understand." Not just the

formula is important but its significance in the face of the goblets and the table of values. It (the equation and its sense) emerges as something new, totally unseen and unforeseeable—since there is no concept to be seen by Mario but only the ways in which it realizes itself concretely in the materials he has at hand. The newly seen (understood) arises for him in the ascent from the unseen to the seen, traversing the ground (table of values, goblets). Even if Jeannie had given him the formula, it would have been but another aspect of the ground from which the heretofore unknown would have to arise before making the crossing via the ground into the visible.

Some readers may want to suggest that the teacher is helping Mario out of the dilemma (contradiction) in which he currently finds himself. But the interaction between the teacher and Mario can be (and in fact needs to be) theorized in the same manner. Mario's response does not lie in his filling out a table (turn 221) or in answering "four times three" (turn 224) when the teacher says, "What are you going to write here" (turn 222). Rather, his response begins with attending and listening to the teacher. As he cannot anticipate what she is going to say, he is inherently affected by what she says and how she says it. That is, we cannot think Mario's response to the challenge of the unknown (foreign) other than in the tension with his exposure to the teacher's utterance. Pathos is as much part of this event as agency, deciding to expose himself to that which he cannot anticipate.

How does a *new* order arise?

To date, there are no satisfactory responses to the learning paradox, for how can the subject of learning know the new order required when the new *exceeds* the existing

order? (Roth, 2010). Inherently, the foundation of a new order of understanding is a critical and crisis-like event that exceeds and transcends any previously existing order; and it exceeds the order of the actions out of which new order *emerges* (Roth, 2011). Thus, an individual claiming to begin a new order of understanding within himself or herself would only repeat what already is and would therefore *not* begin something new. The new understanding cannot be (entirely) grounded in the learner, who is as much subject(ed) to as the subject of cognitive development—saturated phenomena are "irreducible to the I" (Marion, 2005, p. 70), who cannot see what is coming. This (by Mario, non-anticipatable) historical learning event "does not only happen to its witness without his ability to understand (non-constituting *I*), but encompasses him in return (constituted *I*): the *I* is understood on the basis of the event that happens to him to the same extent that he does not understand it" (p. 74). This phenomenological approach to learning something new, cognitive development, and insight learning de facto adds a dimension of learning over and above all intentionalist (generally constructivist) approaches to mathematical knowing.

Although Mario, Aurélie, and all the other fourth-grade children in the algebra class cannot anticipate what they will know, they nevertheless engage with the task environment, where they have to respond to arrive at something yet unknown. In this engagement that requires them to act without knowing why: their "responsivity transcends intentionality, because the fact of entering into something that is happening to us goes beyond the sense, intelligibility, or truth of the response" (Waldenfels, 2006, p. 45). Responsivity, therefore, also is a saturated phenomenon.

Much like a painting "does not offer any object to be seen" (Marion, 1996, p. 79), his objective activity does not exhibit the learning object to Mario. Rather, much as the painting "*impresses our gaze with its own movement* as the imprescriptible condition to be able, precisely, to follow in the gaze the ascent to itself of the unseen into the visible" (p. 79, emphasis added), the objective life activity allows the learning object *to reveal itself* to Mario. It does not reveal itself to Aurélie, precisely because she has abandoned engaging with the task. The task itself serves in the manner of the background in the painting, from which the new order *extracts itself*. Much as we do not "learn to see the painting" but "that the painting, by having given itself, teaches us to see it" (p. 76), the objective life activity brings understanding to life and makes the crossing from the strange into the familiar. Here, the learner "receives the stranger by effacing his strangeness at the threshold, it would thus never have us receive him. But the stranger insists, and breaks in" (Nancy, 2000, p. 12).

In the end, Mario announces, "Me, I understand *now*." He discovers a result from his doings after the fact, like the painter who, in stepping back, realizes after the fact what his brush strokes have allowed to emerge. And precisely this stepping back indicates that the person who has effected a stroke of the pen, placed a splotch of color onto the canvas, "did not know, at the moment of effecting it, what he did, since, in order to see its effect, he must detach himself from his work, in order to learn, afterward, what visible appears there" (Marion, 1996, p. 80). To see what an act has yielded (given rise to) is submitting oneself to donation, surrendering oneself to that which appears *from itself*. Stepping back allows the new order, the new structures to emerge in the person who has been an integral part of the event.

These new structures (*e.g.*, the generalization $y = 3 \cdot n + 6$), *ectypes*, are the stigmata of the unseen that arise from the situation itself rather than being imposed from the outside, the observer. Ectypes erupt from the background; they arise as a formerly unseen from the unknown to the point at which they appear. These ectypes then "triumph over the unseen by escaping from the background" (p. 71) and they surge "from the unseen into the visible but the unseen still shows through in the background" (p. 40).

In painting, the background is that against which the figure (type) comes to be. But the analysis of painting shows that "the background is not added to the ectypes, but the ectypes originate as from their most intimate unseen and, henceforth, the most foreign" (Marion, 1996, p. 71). The background itself shows nothing: it is from and through the background that the new types (forms) suddenly appear, "miraculous survivors of the unseen" (p. 71). Once we recognize in Mario's learning a true miracle rather than the mere result of constructive action, mathematics educators will have evolved a new form of appreciation. Future knowledge, inherently invisible and, therefore, also is *invisable*, cannot be aimed at.

Pathos and the subject of learning

In this classroom example, the children, and their teacher for that matter, are not entirely the subjects of their activity, pure agents who construct their knowledge to solve the problem at hand. They are subject and subjected to their actions, which they cannot entirely comprehend. That is, the approach to learning offered here requires us to rethink the learning subject, who is not only agent but also patient. As

saturated phenomena, learning and development exceed intention and, therefore, exceed the agential subject that constructs its knowledge and itself.

The (*a posteriori*) description of the events allows us to see that all participants in the classroom are as much subjects of the events, bringing these events about, as they are subject to and subjected to them. In this formulation, the different dialectical moments of events, the agential and the pathic [1], express themselves simultaneously. Because of our pathic nature, we can be affected, not only physically but also emotionally: "The subjectivity of the subject is vulnerability, exposure to affection, sensibility, passivity more passive than any passivity, an irrecuperable time, un-assemblable dia-chrony of patience, exposure always to be exposed, exposure to expressing, and thus to saying, thus to giving" (Levinas, 1978/1990, p. 85). This hyperbolic expression brings to the foreground vulnerability, which comes from exposure to the unknown, foreign, and unseen. This allows us to include in our analyses of mathematical learning the issues of affect, fear of the unknown, feeling of insecurity and danger that comes when one is in an unknown situation, left to one's own, without markers of possible success.

Students and teacher cannot ever com*prehend* what is happening *while* it is happening, precisely because the event is unfinished and does not yet exist as inner-worldly fact (Romano, 1998). Yet they have to act (respond) by stepping into the unknown without knowing what will come of it and without the ability to have a clear intention oriented to the object of activity—they respond, as we can see in the episode discussed in this article, and thereby rise to the challenge. Here, responding means "answering to a non-thematizable provocation and thus non-vocation,

traumatism responding, *before* any understanding, to a debt contracted before any freedom, before any consciousness, before any present; but answering, as if the invisible that bypasses the present left a trace by the very fact that it bypasses the present" (Levinas, 1978/1990, p. 26, emphasis added). That is, thinking learning in terms of pathos—originary affectability that precedes comprehension but arises from being materially comprehended in the physical world—also requires us to think it in terms of affect, being affected by something alien (foreign) to ourselves. It means thinking about and theorizing learning in terms of exposure, vulnerability, which is an "exposure to outrage, to wounding, passivity more passive than all patience, passivity of the accusative form, trauma of accusation suffered by a hostage to the point of persecution" (Levinas, 1978/1990, p. 31).

Notes

[1] The term pathos derives from the Greek *πάθος*, suffering, feeling, emotion, or passion. In the phenomenological literature, the term is used to mean an experience that befalls a person, something adverse that is associated with suffering, and something that relates to learning through suffering (e.g., Roth, 2011). "*Pathos* or affection," is "the foundation and background against which all intentional and well-ordered behavior stands out . . . as doing" (Waldenfels, 2008, p. 132).

References

Henry, M. (2000) *Incarnation: une philosophie de la chair,* Paris, FR, Éditions du Seuil.

Levinas, E. (1978/1990) *Autrement qu'être ou au-delà de l'essence*, Paris, FR, Librarie Générale Française.

Marion, J.-L. (1996) *La croisée du visible*, Paris, FR, Presses Universitaires de France.

Marion, J.-L. (2005) *Le visible et le révélé*, Paris, FR, Les Éditions du CERF.

Nancy, J.-L. (2000) *L'intrus*, Paris, FR, Galilée.

Nietzsche, F. (1954) *Werke Band 1*, Munich, GER, Hanser.

Romano, C. (1998) *L'événement et le monde,* Paris, FR, Presses Universitaires de France.

Roth, W.-M. (2010) 'Incarnation: Radicalizing the embodiment of mathematics', *For the Learning of Mathematics* **30**(2), 2–9.

Roth, W.-M. (2011) *Passibility: at the limits of the constructivist metaphor*, Dordrecht, NL, Springer.

Roth, W.-M. and Radford, L. (2011) *A cultural-historical perspective on mathematics teaching and learning*, Rotterdam, NL, Sense Publishers.

von Weizsäcker, V. (1973) *Der Gestaltkreis*, Frankfurt/M, DE, Suhrkamp.

Waldenfels, B. (2006) *Grundmotive einer Phänomenologie des Fremden*, Frankfurt/M, DE, Suhrkamp.

Waldenfels, B. (2008) 'The role of the live-body in feeling', *Continental Philosophy Review* **41**, 127–142.