Interpreting Unfamiliar Graphs: A Generative, Activity Theoretic Model

Wolff-Michael Roth, Yew Jin Lee, & Dan Peebles

ABSTRACT: A dialectical model is presented that accounts for the situated nature of graph interpretation at three levels—collectively motivated activity, goal-directed actions, and unmediated operations. The contradictions inherent in the dialectical units at each level are understood as the forces that drive situated graph interpretations. Materials from graph interpretation sessions with scientists are used as exemplary data.

KEYWORDS: activity theory, dialectic model, graphs, interpretation

1. Introduction

The invention of graphs and graphing were central to the historical evolution of the natural sciences (Edgerton, 1985), which take graphs as mathematical forms that are isomorphic with the fundamental structure of some natural phenomenon (Lynch, 1991). Accordingly, mathematics educators have approached the interpretation of graphs as a translation between graphs and (verbal descriptions of) situations (Greeno, 1989; Janvier, 1987). However, recent work showed that in everyday work practice, graphs do not stand in an indicating (referring) relationship to the things they are said to express; instead, familiar graphs at work have become part of the situation, they are tools that are used without drawing attention to themselves much like spectacles that are no longer noticed by their bearers (Roth, 2003a). There is further evidence that individuals interpreting graphs do not make inferences about the content of graphs but that graphs constitute occasions where these individuals articulate their existing understanding of how some aspect of the world works (Roth, 2003b). There are many unresolved questions pertaining to the interpretation of graphs including, “How do people asked to interpret unfamiliar graphs generate the requested interpretation, in real time, from moment to moment?,” “At what point do they stop?,” and, “What is
the relation between the processes and products of interpretation and the research context in which these are produced?” To take us into answering these and related questions, consider the following episodes from graph interpretation sessions during which 37 scientists (M.Sc., Ph.D.), mostly ecologists and physicists, talked, among others, about graphs from introductory level theoretical ecology.

1.1. Interpreting graphs: an introduction

Research concerning the interpretation of graphs generally considers neither setting effects, such as participants’ attunement to the research situation, nor the temporally unfolding character of graphs to participants’ consciousness.

1.1.1. Setting effects

Seldom discussed in the literature on graphing is the fact that interviews do not simply sample the contents of mind, but are collectively constructed events that arise from the interaction between interviewer and interviewee in the particular setting that they mutually agreed upon (Suchman and Jordan, 1990). During a research interview (talk-aloud session) there is a definitive sense among participants about the roles that they mutually take, the types of questions each may ask, what the transcripts of the sessions are used for, and so forth. Having agreed to participate and having shown up at the agreed-upon site, the research participant expects the researcher to introduce the tasks, whereas the researcher expects the participant to begin with the task according to tacit and explicit rules. Thus, the researcher in the following episode not only explained the source of the graphs but also asked the participant—university ecology professor Dan—to describe what he was doing as he was doing it.

Researcher: What I have is three graphs that come from a second year ecology course, either from a lecture or a textbook or a seminar, one from each of those. And we’ve got 70 or 80 or 90 people doing an interpretation of it now.

Dan: OK.

Researcher: So if you can describe what you’re doing as you’re doing it, what you’re looking at and what you’re thinking about.

Dan: OK, I can handle that. So, the first thing I’m gonna do is look over here-

Researcher: At the caption?

Dan: The text, right. (Begins reading the caption of the graph [Figure 1].)
In the logistic model, birthrate and death rate are linear. Let’s assume that the birthrate follows a quadratic function (e.g., \( b = b_0 + (k_b)N - (k_d)N^2 \)), such that the birthrate and death rate look like in the figure. Such a function is biologically realistic if, for example, individuals have trouble finding mates when they are at very low density. Discuss the implication of the birth- and death rates in the figure, as regards conservation of such a species. Focus on the birth and death rates at the two intersection points of the lines, and on what happens to population sizes in the zones of population size below, between, and above the intersection points.

**Figure 1.** This graphing task, typical for an introductory course in ecology, was presented in this study to 21 physicists.

As Dan provided a description of his first action to “look over here,” the researcher asked for clarification of the referent of the utterance (“at the caption?”). The participant’s verbal productions therefore were also a function of the interaction. As a consequence of these interactions, participants may become aware of aspects that up to that point were only implicit. Even the innocuous naming of the text as “caption” rather than as “instruction” has the potential of changing the orientation that participants have to the task. Participants also implicitly assume that the researcher asks questions whereas the interviewees respond to them; the researcher encourages the participant to talk aloud, whereas the reverse is not the case. Although it is understood that in such situations, the researcher generally knows the standard answer, it would be inappropriate for interviewees to ask for it, unless it was part of a move to terminate the task. In the following episode, Rudi (a Ph.D. ecologist more than ten years after graduating) attempted to finish working on a population graph (see Figure 1), evidently without having the sense that he has done justice to the problem. He twice refers to the fact that he did not have the knowledge or experience of a population ecologist, which can be interpreted as a discursive move that legitimized the end of the session pertaining to this graph.

Rudi: It demands more knowledge of population ecology than I have… That’s it for me! I’m not a population ecologist.
Researcher: A theoretical ecologist that we worked with described this as a saddle and cup thing.

Rudi:  With the population curve...?

Researcher: Yeah. This one, because if you hit this point, you said the population goes down, it’s like this is like falling off a type the equilibrium.

Rudi: Right.

Researcher: And this one here is like falling into a type of equilibrium-

The researcher responded to this need by beginning to give the explanation that the course instructor from whose course we culled the graph would have accepted, but he does so in a way that allowed Rudi to save face. Here, too, the participant shows that he was attuned to the situation in the sense. Simply stopping after the short amount of time that he had spent might have been interpreted as inappropriate. Thus, announcing that he was not a population ecologist would explain why he did not have more to say, and therefore constituted a way to end this task.

Research on knowing and learning to graph and about graphs often neglects to articulate the situation that shapes, and that is shaped by, situated activity. How these biologists talked in the presence of graphs was a function of the activity system in which they currently are part. Talking to their colleagues about graphs or drawing on graphs as resources are fundamentally different actions than talking about graphs during a session organized by a psychologist interested in graph interpretation. The organization of activity is not just a macro context that one can forget during data analysis but actually shapes the moment-by-moment unfolding, organization of situated cognition (Hutchins, 1995). In this article, we articulate situations in which graphs are used or are topics of talk in terms of cultural historical activity theory, which articulates subject (researcher, participant) and object of activity (graph, interpretation) in terms of a dialectical unit. Prior to the end of a particular interpretation, there is a dialectical tension between the graph as material object and the vision of a completed interpretation (outcome). This dialectical tension is the “engine” that drives an interpretation (Ricœur, 1985) and it is (temporarily) removed when research participants have the sense that what they have said is a complete interpretation corresponding to the graph.

1.1.2. Real-time production of interpretive statements

The interpretations are the results of collective motives. Researchers want participants to talk aloud, which gives rise to verbal text that subsequently can be transcribed and analyzed; participants subscribe to the motive and talk aloud thereby producing the text. Although the collective motive drives the production of graph interpretations, the moment-to-moment endeavors are not focused on a
collective motive but on accomplishing the task at hand, which the biologists in the above episodes achieved by means goal-directed actions. Although actions are subordinated to activity, they too arise from the dialectic between subject and object, which now exists in the form of material entities and goals. It has been noted that in communication, there are visual (perception, gesture) and verbal components that together form a dialectical unit (McNeill, 2002). McNeill theorized situations in which people narrated some distant events and objects; when the events and objects that are topics of talk are present the dialectical unit of communicative actions includes perception, gestures, and speech (Roth and Lawless, 2002). In the following episode, where Marianne, an experienced radio astronomer (35 years after Ph.D.), talked about the same graphs as the biologists in the previous episode, the necessity to account for the three modalities in understanding the communication becomes evident. This episode was taken from the early moments into her interpretation of the population graph.

Here, as Marianne moved to describing the birthrate graph (“and the birth...”), her pencil was lying on the paper pointing to the death rate curve that she had talked about immediately prior to this instant. In the process of grabbing the pencil (frame 2), she already rotated it counterclockwise, then after getting a hold of it moved its tip to the left base of the parabolic birthrate curve, rotating the pencil so that it was approximately vertical to the slope at every point along the trajectory. In fact, the two frames following the one displayed at 0.68 seconds showed the hand rotating the pencil into a position that it was perpendicular to the slope of the curve before following the curve while uttering the word “increasing.” It is immediately evident that we require image, gesture, and utterance to understand what Marianne communicated (i.e., the topic). In her production, there was an implicit understanding that her interlocutor (the researcher) had access to all three modes, that is, the graph, gesture, and talk.

In uttering the word “and” Marianne had committed to the next part of a sentence, though it is likely that she did not yet know its exact shape and content (McNeill, 2002), but at this time, her hand was off the pencil that she used as a pointer. At the end of “birth,” she had a hold of the pencil again, and then
moved toward the left end of the birthrate graph. The beginning of the verb “increasing” coincided with a gesture that followed the initial part of the graph. Here, the utterance “increasing” appeared to “wait” until the hand was in position so that utterance and gesture could emerge, that is, could be brought forth simultaneously.

We understand an interpretation to emerge from the chaining of many such elements that emerge from two dialectical tensions: (a) before a perceived element is described, the contradiction arising from a sense of incompleteness drives the interpreting individual to produce more communicative elements; and (b) in production, the unresolved differences between perceptual, gestural, and verbal elements create a tension that moves production forward. The two tensions occur at different levels: the first arises in the context of the conscious goal to produce a verbal description of the content said to be expressed by the graph whereas the second is a tension that arises from the copresence of three expressive modalities produced in real time as a matter of course and without prior deliberation or conscious control (McNeill, 1992).

1.2. Graphs and graphing

Graphs are central to the processes and products of science (Latour, 1987) and are preeminent features in scientific publications (Roth, Bowen and McGinn 1999). Graphs draw their strength from (a) their capacity to summarize a lot data and (b) from their integration of typological, including words and symbols, and topological features, including variously shaped lines and curves (Lemke, 1998). In the past, the performances related to graphs and graphing have generally been attributed to conceptions, mental structures, and cognitive capacities (e.g., Pinker, 1990); non-standard answers on graph-related tasks were attributed to mental deficiencies and misconceptions (e.g., Berg and Phillips, 1994; Clement, 1989).

Graph-related practices include the interpretation of graphs (Leinhardt, Zaslavsky and Stein, 1990). In fact, “interpretation of graph” is shorthand for a translation of a visual representation into a verbal description of a situation that the graph is said to be about (Janvier, 1987). Greene (1989) proposed a semantic model relationship between mathematical symbols (signs) and the lived world experienced in situated cognition (Figure 2). In this model, symbolic domain and lived experience exist at two levels: a lower level consisting of the totality of elements (notations versus objects, events) and an upper level, representing the structures in which the raw elements are involved. Symbolic structures can be operated upon, denoted in the model by the symbol [ ]; these operations correspond to operations in the lived world, denoted by the symbol [ ]. The relation between notations and the way they are organized into structures is denoted by
Objects and events are organized in lived praxis, which is denoted by \( \Omega_{\text{l.e.}} \); because the organization of objects and events depends on the particular activity, \( \Omega_{\text{l.e.}} \) is function of the activity system. Mathematical structures map onto the lived world, expressed by the function \( \Phi \).

Recent work among scientists and technicians who used graph as part of their work suggested that in the workplace, graphs are not interpreted but are tools that are employed in a transparent way and therefore do not have sign function at all (Roth, 2003b). That is, the symbolic elements (Figure 2) are part of structured objects and events and no distinctions are made between the two: territory and map have become the same (Bateson, 1972). This suggests that instead of seeking explanations for particular graph interpretations in mental capacities, we might more fruitfully look at graphing as a practice, which, as all practices, is shaped by the relevant community that defines activity in which graph users are involved (Roth and McGinn, 1998). In fact, evidence is mounting in many fields that cognition cannot be understood apart from material practice even if at particular moments, physical materials are not involved; the roots of the cognition enacted at the moment always lie in practical, material activity (Lektorsky, 1977).

---

**Figure 2.** A model of the relationship between situations (lived experience) and symbolic structures representing them (after Greeno, 1989). The bottom layers on each side contain the raw elements of the domain, whereas the top layers represent the structures. Arrows represent the transformations between layers (\( \Theta_s, \Theta_{\text{l.e.}} \)) and domains (\( \Psi \)) or within layers (\( \Psi, \Omega \)).
In this introduction, we have already alluded to the fact that the context in which a graph is used or interpreted shapes what people will say about them and how they use them as part of their talk. We pointed out that the relevant division of labor, rules of interaction, and tools available in the situation also mediate the relation between individual and graph, and therefore the outcome of their productions. All these entities and the way they mediate one another are aspects of cultural historical activity theory (Leont’ev, 1978). Several recent studies proposed theorizing graphs and graphing in terms of cultural historical activity theory (Hershkowitz, Schwarz and Dreyfus, 2001; Roth, 2003b; Williams, Wake and Boreham, 2001), though they have done so for different reasons. In the present situation, we attempt to provide a comprehensive theory of graph interpretation that integrates macro-level (institutional, societal context) and micro-level events (moment-to-moment actions). We provide here a general introduction to activity theory and will elaborate the particular aspect of our theory in a subsequent section.

1.3. Cultural historical activity theory: a general introduction

The fundamental premise of cultural historical activity theory is that human cognition can only be understood when all relevant aspects of practical activity is taken into account (Engeström, 1987). Thus, what individuals say during interviews are not just contents of their minds expressed in words, duly recorded on audio- or videotape; rather, the entire (verbal, gestural, diagrammatic) production is a function of the situation, interviewer-interviewee interactions (e.g., Edwards and Potter, 1993), the particular task-related artifacts and language, and so on.

Consistent with current theoretical developments in domains such as cultural sociology (e.g., Sewell, 1992), activity theory conceives the relation between the acting subject (individual or collective) and the object of the activity as a dialectical one (Leont’ev, 1978). The relation is dialectical because in practical activity, the object (motive) appears twice: as a material object and as an object of thought, which orients the subject’s (individual, group) practical and cognitive activity (Lektorsky, 1977). For example, in our interviews, the graphs exist both as objectively present, material entities, black lines on white paper, and as entities perceived differently within and across individuals—some scientists focused on the relative slopes of the two curves in Figure 1 others on the relative heights. In the present situation, the subject of the activity is a group consisting of interviewee and researcher; this framing takes into account of the collective character of human interaction (Edwards and Potter, 1993).
The subject-object relation is not direct but mediated by the tools and instruments available to the subject as the means of productive activity (Figure 3). There are further relevant entities that enter the basic unit of analysis of cultural historical activity theory—the relevant, activity-determining community (of practice), the division of labor, and the rules that describe the relations between subject and community. For example, we already discussed how the division of labor attributes different roles to researcher and interviewee with respect to the object and the production of the interpretative text—the division of labor mediates the subject-object relation (Figure 3). It is also immediately evident that the community mediates the productive activity towards a graph—a physicist will do different things when she talks/writes about (in presence of) a graph when the audience (“consumers”) are her physicist peers than when she talks about (in presence of) a graph in the context of an interview to be used as data in a paper on graph interpretation. Each pair of entities in Figure 3 is mediated by other entities.

Each element and therefore the activity system as a whole undergo continuous (historical) change and therefore can be understood only through historical study. For example, as our participant scientists looked at graphs presented dur-
ing our research, what were relevant figure/ground relations emerged during their engagement in the task. That is, the graph was not given once and for all, but different aspects became salient and changed in their level of relevance over the course of the interview; similarly, the individual changed as they were likely to learn something by engaging in one task, which they used during a subsequent task.

Cultural historical activity theory distinguishes activity from actions and operations. Activity exists at the collective level (mediation by community) and is oriented towards object/motive. In the present situation, motive was the production of graph interpretations for the purpose of increasing knowledge about mathematical cognition. Actions are carried out by the subject(s) of the activity and are oriented towards goals. An example of an action is a researcher’s request to “think aloud” when he realized that the latter did not speak for a few seconds. Operations are routinely executed, an automatic response to existing conditions—the perception of height or slope of a graph is automatic.

Important elements in activity systems are the contradictions (breakdowns, resistances, dilemmas, disturbances, paradoxes, and antinomies). A contradiction scientists often experience expresses itself in their fear to become objects themselves in the social science research rather than participants, which explains why they are often reluctant to participate. Another evident contradiction is the unfamiliar nature not only of the graph, the object of the activity, but also the unfamiliar community for which the interpretation is produced, and the particular division of labor enacted. In cultural historical activity theory, contradictions are understood as the driving forces for actions and change (Il’enkov, 1977).

2. The Study

This study is part of a research program investigating the graphs and graphing among scientists and technicians. Here we are concerned with understanding the processes and products of interpreting graphs selected by the researchers from materials used in undergraduate courses.

2.1. Participants

In a previous study, we had interviewed 16 scientists, most of them from ecology and related areas. In this study, twenty-one scientists from a variety of departments related to physics (e.g., geophysics, astronomy, physical chemistry) agreed to participate. Four individuals had previously obtained a masters degree and were currently enrolled in a Ph.D. program. The remaining 17 individuals had obtained a Ph.D. between 0 and 42 years prior to our interviews ($X = 21.9$, $SD = 14.7$ years). All but two research associates were involved in teaching
graduate, undergraduate, or laboratory courses. For the purposes of exemplifying the model of graph interpretation, we randomly selected for detailed analysis two individuals. Marie is a radio astronomer with 37 years since her Ph.D., publishing about one to two co-authored pieces per year. Karl is a high-energy particle physicist with research interests in the experimental exploration of the laws of nature at the energy frontier; over the 13 years since his Ph.D., he has had more than 40 (co-authored) journal articles.

2.2. Tasks

In a previous study, we investigated the responses of scientists to graphs from their own field. In this study, we were interested in developing a better understanding of interpretations of graphs from an unfamiliar domain in addition to interpreting graphs from their own field. For the graphs from an unfamiliar domain, we used the same set of ecology-related graphing tasks but asked physicists and scientists from related fields to interpret them. These graphs were representative of those found in a popular 800-page introductory textbook (Ricklefs, 1990): we counted 120 graphs of the type used in our research. These included (a) a plate showing the distributions of three types of plants (differentiated by their photosynthetic pathways) as function of three covarying independent variables (elevation, temperature, moisture); (b) a plate displaying birthrate and death rate as functions of population size (density) and intersecting twice (Figure 1); and a plate featuring three isographs representing the functional relation between a dependent variable (growth) and two independent variables. We briefly discuss the graph that constitutes the context for all interview excerpts in this article.

The task featured here (Figure 1) constitutes a graphical model of the type that ecologists began to use during the 1950s and 1960s (Kingsland, 1995). Such population graphs are prominent features of and appear early in textbooks on ecology and biomathematics (e.g., Murray, 1989). There were 60 such models in the textbook we consulted. In essence, the combination of the two curves represents a stable (right intersection) and an unstable equilibrium (left intersection) of the population size. Below the left intersection, because the birthrate is always below the death rate, the population will decrease in size until it disappears. For populations between the two intersections, the size (density) will increase, whereas above the right intersection, the population will decrease in size until birthrate equals death rate. Because the graphs show rates, the maximum increase in individuals will occur where the function (birthrate – death rate) * N is maximized. Past research showed that even experienced (non-university) ecologists may (a) incorrectly suggest that the population crashes (disappears,
vanishes, is extinguished) for population sizes to the right of the right intersection (b) likely and incorrectly suggest that the maximum increase in individuals occurs where the birthrate or birthrate – death rate are maximized (Roth & Bowen, 2001).

2.3. Data and analyses

All videotapes were transcribed in an ongoing manner so that the text was available in written form during our analysis. For those moments where the interviewees pointed or made gestural reference to a graph feature, we saved the frame and imported it as an image into the transcription. For salient and theoretically interesting episodes, the transcription was improved to contain multiple video images, the length of the pauses, or the overlap between speech and gestures.

Our analyses are based on the assumption that reasoning is observable because it is a practical, socially structured, and embodied activity (Garfinkel, 1991). In our analyses, the videotapes, transcripts, and artifacts produced by the observed individuals are natural protocols of their efforts in making sense of and imposing structure on the task at hand. These protocols constituted the materials that we structured, elaborated, and theorized.

We independently read the transcripts and viewed videotapes before meeting for collaborative analysis. We conducted collaborative analyses of lectures and interviews (Jordan & Henderson, 1995), stopping the replay whenever one of us thought a significant event had occurred. This person then stated an assertion before the event was reviewed as often as necessary for a full exploration. We subsequently reviewed other episodes to check the degree to which they confirmed or disconfirmed the assertion. On the basis of these checks, we reformulated initial assertions until they were representative of the data. We then discussed personal constructions, subjected them to critique and analysis, and tested them against the entire data set to evaluate fit and plausibility. Our model therefore emerged as a grounded theory after several iterations.

3. Model of Graph Interpretation

The model of graph interpretation presented here is grounded in activity theory more broadly and in dialectical approaches to mathematical cognition more specifically (e.g., Walkerdine, 1988). Fundamental to these approaches is the dialectical constitution of human subjects and their settings, which mutually constitute the nature and shape of problems and solutions. Rather than ignoring the interactions between interviewees and research setting, dialectically informed analysts take them into account (Lave, Murtaugh, & de la Rocha, 1984).
The motive of activity is the production of a verbal description (and explanation) that corresponds to, or expresses, what the visual representation contains. At the level of the activity, there exists a dialectic tension between the graph and what is envisioned as the completed product, the finished interpretation (though it only exists as a generalized idea). At the level of actions, there is a dialectic of the holistic visual image and the sequential nature that the talk materializing the description will take. That is, there is a tension between global meaning of the graph as a whole, presented synchronically and the analytic meaning that unfolds in producing the description, presented diachronically, that will subsequently count as “the interpretation” (product). Finally, at the level of operation, the production of (interpretive) statements, there exists a dialectic tension between the visual image, on the one hand, and the convention-laden form of the language used in speech on the other. In dialectical theory, these tensions seek resolution and therefore drive events (activity, action, operation) until resolutions are sensed.

The three levels are not always cleanly separable. Although actions are part of an activity one cannot simply add up actions to get activity as its sum. Rather, the two levels are mutually constitutive. For example, specific questions (actions) may be asked during a graph-interpretation session, but content, form, and source are a function of the division of labor and rules that characterize the activity. Further, although an individual may read the instruction, “Discuss the implication of the birth- and death rates in the figure, as regards conservation of such a species” and then begin talking about implications, their speech and gesture production in doing so actually occurs at the unconscious level (operation).

3.1. Participating in research about graphing

In this section, we suggest that the particulars of the graph interpretation session bear the marks of the activity system, and therefore must not be omitted from analysis and theorization. People do not just interpret graphs but do so in a particular context to which they are oriented to, shown in the nature of their actions throughout the session.

Fundamental to cultural historical activity theory is the assumption that human behavior and cognition cannot be understood apart from the concrete activity in which they occur—Figure 3 represents the basic unit of analysis. Our model integrates cultural historical activity theory and other dialectical approaches to understanding cognition. We begin by articulating the model at its top level, where it describes interpreting graphs as part of an activity system that produces graph interpretations and their analysis for inclusion in scientific articles. Because of the transactions between researcher and interviewee, who co-produce the session, both individuals (more if there are two interviewees or a
camera person and so forth) are constitutive of the “subject” but take different parts in the division of labor towards the production of the interpretation (Figure 3). Past research on graphing has not generally attended to the effects of participants’ attunement to the situation on the results of a graph interpretation session. For example, if the present researchers had not talked to the scientist participants other than to encourage their thinking aloud or introducing the task, the latter might have felt like traditional “research subject.” Generally, natural scientists are therefore reluctant to participate in social science research (where they feel a bit like “guinea pigs” or “lab rats”) unless they have the sense that the researcher treats them as a participant and does not put them into situations that make them appear ignorant.

3.1.1. Nature of the object (graph)

What people do with mathematical representations such as graphs depends on the activity system—success rates on best-buy problems (Lave, 1988) and graph interpretations (Roth, 2003b) change significantly when we move from familiar, everyday activity to psychological (sociological) research on mathematical cognition with tasks specially selected. The reasons for such changes are not only in the different means of production, community, division of labor, and rules but also in the special nature of the object. In fact, in the workplace, scientists and technicians use graphs as tools to get the job done; graphs become objects of reflection only in the face of some contradiction (Roth, 2003a, in press).

In dialectical approaches, the subject-object relation is always dialectical in nature; in the present situation, there exists a contradiction between the material presence of the graph and the absence of the graph interpretation, the motive of the activity. The contradiction diminishes as the gap between the object and the intended outcome closes with every step in the production of an interpretation. The process of interpretation unfolds as individuals take whatever they perceive as relevant parts or as parts to be considered. It unfolds as interviewees cover, and sometimes repeat, describing and explaining the different pieces of the graph that are salient to them. The overall dynamic is only complete when all salient parts of a graph have been covered, and when those parts that enter into consideration are consistent and free of contradiction. The “interpretation” is complete when the utterances taken as a whole correspond to the graph as a whole. What is to be taken as the relevant set of utterances can only be established after the fact once it is known that there are no more changes in the description, attributions (interpretants), that is, that whatever was produced is a catchment that is being reiterated in any subsequent repeat utterances or repeats of talk about a feature.
The actions pertaining to a graph continue until the subject has a sense of completion, that is, that the outcome of the activity corresponds to the object. At this point, the expression of the graph in its synchronic entirety and the expression of the descriptive talk taken in its diachronic entirety are taken to be equivalent. The relationship between the subject of activity and outcome is itself mediated by the community, here the researcher. That is, interviewees are expected to continue until they have completed the production of what they had committed to (rules). Whether the contradiction has been removed, that is, whether the given and expected interpretations match is also subject to situated interactions. Often, we found that the interviewees engage in some process of terminating the particular interpretation process. In our introductory example, Rudi made two comments about not being a specialist in population ecology, which had both the function to get him out of the obligation to continue. That is, even if there had been a sense that the task was not completed, mentioning his limited expertise allowed him to stop while it also explained its apparently incomplete nature. Similarly, Marianne’s repeated requests for hints (“Where is your hint?,” “Give me a hint”) eventually changed the nature of the situation into teaching session.

3.1.2. Changing nature of the object

The object of activity does not remain constant but continuously changes in the course of the interpretation. First, although the graph is given materially in an instance, it is not apperceived in its detail in the same instantaneous way. The elements of the graph are not given once and for all at the beginning, but emerge into the consciousness of the participant in the course of their interpretation. Thus, the content of the interpretation (or interpretation as product) evolves, being shaped by the available resources at hand, including the language that the interviewee takes as shared with the researcher. Not only the language that the interviewee senses to be appropriate but also the particular words and other semiotic resources available in the task, as well as all other communicative elements are resources to be employed in the production of what comes to be the interpretation. For example, in the following interaction, Marianne asked the researcher about the nature of letters (which she already assumes to be variables). The researcher responded by referring to the equation (Figure 1) as a “quadratic function” and \( N \) as “population density.” Using these names rather than any others may subsequently become resources for the interviewee.

Marianne: What are these variables \( N \), \( b \)?
Interviewer: That’s just an example of a quadratic function. \( N \) is the population density.
Marianne: I see that and hum- so over here we’ve got death rate... (Continues her interpretation.)

Second, as the participant produces statements (gesturally, verbally), there now exists, in addition to the material graph, an emerging text accumulating in the past. The interpretation has its own unfolding history that constitutes a frame for what is to come. If there are statements contradictory to what has been said in the past, there is an “obligation” to settle these (rules); if something has already been said but the researcher asks a question about it, this can be taken as a statement that there is something wrong with has been said... (or any other interpretation). The simple fact that the researcher in the episode described the function as quadratic rather than as polynomial or curved or defined “N” as population density rather than as population size changes the available resources. Even a non-response on the part of the researcher would have shaped the setting and how participants related to it.

3.1.3. Roles and rules

The orientation of the interviewee to elements of the activity system (Figure 3) that generates graph interpretation is evidenced throughout the research process, including the graph interpretation session itself. An orientation to the division of labor between researcher and interviewee is continuously present, influencing who contributes in what way to the production of the recorded talk and what (tacit) rules of interaction are appropriate. Most salient in the present project were the different roles that the undergraduate student on the author team took with respect to professors in his and related departments. For example, Marianne started the task with the question, “Do you want me to read it out loud?” This statement indicates her understanding of division of labor salient in the current activity: The researcher, an undergraduate student in her department, can be asked a question of procedure though in a different context, such as an astronomy class, she would hardly ever ask this person this question. As part of the division of labor and rules, the subjects commit to producing an interpretation until not only they have the sense to have produced something suitable, but also until they have the sense that what they have done is suitable to the virtual contract that they have engaged in with their agreement to participate in the research.

The participants also adhere to unwritten, tacit rules of interaction with others to whom they have made some sort of commitment. Here, the researcher had asked them to participate in a research project on graphing. The participants will therefore attempt to engage in the production of the interpretative text until they have a sense to have if not completed the task in some absolute sense but suffi-
cient to have lived up to their commitment. Once this is the situation, they could ask, for example, for an authoritative answer to the interpretation task even before having settled all the issues that have appeared during their own attempt. For example, whenever Marianne asked whether she was on track, she was at such a point. In the following episode, she summarized what she had produced so far.

So they are both increasing but the birthrate in fact is increasing further than the death rate and presumably that means that the population is increasing. (Gesture from intersection to the point shown in the figure.) Is that right then?

At this point, then, an endpoint was reached, and Anne asked for confirmation from the researcher whether her verbal description in fact corresponded to the sought for, correct, description. She stopped because a point of completion was reached, at least a temporary one. With her question, the particular situation became clearly marked as part of an activity system on graph interpretation. The researcher is assumed to know the answer, and, in this situation, was asked whether the interpretation delivered so far was correct. Even if the preceding sentences and the summary were uttered without giving clues about the context, the final sentence “Is that right then?” provided such a link between the task and activity level. Participants are attuned to the context even if they do not make available to one another evidence of this attunement. In fact, showing attunement directly would probably be interpreted as unseemly, for the nature of a situation is taken for granted to participants present and therefore ought to go without saying.

3.2. Interpreting graphs while participating in research

3.2.1. Phenomenology of interpreting graphs

Graph interpretation, as reading more generally (Ricoeur, 1985), has a dialectical nature. The first dialectic consists in the presence of the text and the absence of an immediately intelligible configuration, which places “on the reader’s shoulders the burden of configuring the work” (p. 168). The reader expects a configuration, which is not immediately available, a contradiction that forms the core of the dialectic. This dialectic propels the process of reading, which then becomes the search for coherence, both globally, at the level of the entire graph, and locally, at the level of the individual elements. As interviewees work their way through its different parts, their viewpoint of the graph is that of a moving ob-
server involved in a continual interplay between modified expectations of what they will see in the future and changing memories of what they have seen in the past. This relation between (immediate) past and future can be seen in the following excerpt from Marianne’s session.

So I was just looking above the intersection point. Wasn’t I? At least that is what I thought I was doing just then. But then before that (Pause) the birthrate was lower than the death rate. So in this region (left end of graph), the death rate is higher than the birthrate and so it seems to me that the population is declining.

Marianne stopped her engagement and reflected on what she had done and the content of her reading, before talking about a corresponding situation in her familiar world. That is, the act of interpreting graphs is a synthetic process by means of which the interpretation as a product is constituted utterance by utterance through an interplay of modified expectations what the graph might be about and the transformed memories of what has been said so far about its content. As part of the process of achieving the goal of the activity, the subject perceptually moves to different parts of the graph, in fact, parses the graph and then verbally elaborates the different parts made salient in the process.

The first dialectic gives rise to a second dialectic, whereby a text reveals both a lack of determinacy and an excess of meaning. “Every text, even a systematically fragmentary one, is revealed to be inexhaustible in terms of reading, as though, through its unavoidable selective character, reading revealed an unwritten aspect in text” (Ricœur, 1985, p. 169). In reading, a figure for the unwritten aspect of the text is provided. The second dialectics becomes evident, for example, when Anne requested whether she is on the right track. That is, next to the interpretation that she had given up to that moment, she envisioned the possibility of other, more appropriate interpretations. When this tension becomes unbearable, the person may stop and request to be told the correct interpretation. This occurred some way through the session, at which Anne abandoned her reading and submitted to the guidance by the (undergraduate student) researcher, who assisted her in structuring in configuring the graph and her reading so that she could approximate the standard reading.

Graphs are not ready made and present to be interpreted as a whole from the beginning of the relevant session. Rather, the nature of the graph as it appears to the subject emerged from and as result of the session. A respondent may perceive the slope or height, in sequence or alternation, but which one is salient to the task at hand? In the following example, Marianne’s actions made salient the difference in the slopes of the birthrate and death rate curves.
and the birth*rate * is * increasing faster (1.00)

than the death rate.

This episode occurred immediately after the one featured in our introduction. As Marianne uttered “and the birthrate,” the pencil tip followed the birthrate curve a little ways up and down to the starting point (00 to 0.53 seconds). Then, while uttering “is,” the pencil rotated to be more perpendicular to the death rate curve and then moved towards this curve (frame 4) while beginning (“increasing”) the comparison between which was subsequently completed (“faster than the death rate”) with the pencil in the same place. Here, the comparison was not only communicated in speech, but also in the gesture that rotated the pencil. The episode provides evidence that the salient features here were the two slopes and their relative size.

Sometimes respondents did not perceive a particular aspect until some contradiction occurred to them, which they attempt to resolve by investigating the graph for additional aspects not yet perceived or considered. We can identify the emergence as a growth point, and its recurrence as a catchment. Subsequent ideas may be contradictory, with or without the participant noticing it. To better understand the recurrence of actions associated with perceptual elements in the course of an interpretation, we coded features of the interpretive sessions, such as “Points to Intersection I,” “Points to Intersection II,” “Gestures parabolic birthrate curve,” and “Points to birthrate curve.” Figure 4 represents Rudi’s actions in the course of his 13-minute session related to the population graph.

To see whether there are patterns in the transition between remarking on or gesturing to one feature and the succeeding feature. Most of the transitions appeared only once. But the transitions between behaviors 5 10 and 10 5 each occurred four times (5 = pointing to birthrate; 10 = gesturing difference between birthrate and death rate). Any particular element identified a priori by the researcher on graphing may not be addressed in an observable way by the interviewee and, when addressed, to different extent by different individuals. There was a considerable range in the frequency with which a particular feature is being referred to in a session. For example, birthrate and death rate were directly compared in gesture between 0 and 8 times, with a mean of $x = 2.76$ ($SD = 2.53$).
3.2.2. Language in use

The previously discussed model of graph interpretation as translation between symbolic representation and (descriptions of) lived experience (Greeno, 1989; Janvier, 1987) does not distinguish between language used as part of praxis and language used to talk (write) about praxis. Previous research provided substantial empirical evidence for the difference between language in praxis and language about praxis (Bourdieu, 1990) and recognized a epistemological need to distinguish the two languages (Heidegger, 1977). In a graph interpretation session, participants use language primarily to get the job done rather than to represent something. In part, their job is to make salient and articulate aspects of the graph at hand, in part they have to articulate real or possible situations. The interpretation consists in the coincidence of the two articulations. In the following episode, Marianne first describes an aspect of the graph ("birthrate is increasing faster than the death rate") and then, overlapping, an aspect of the world ("as long as the birthrate is increasing faster than the death rate is increasing, then the population is in good shape").

* As long as the birthrate is increasing faster than the
In this episode, the first part of the statement pertains to both worlds, the one of the graph and the familiar experiential world where birthrates and death rates (or their increases) bear particular relations. We introduce a linguistic domain existing between the two (Figure 5). This linguistic domain can be understood as a meta-level domain in which the two other domains can be described and reasoned about.

Drawing on shared language as means of production (Figure 3), the research participant describes mathematical structures and lived world in such a way that the linguistic structures map into both mathematical-symbolic domain and lived experience. The way language is used is a function of the current activity system, participating in research on graphing. Our concern here is with unfamiliar graphs, which implies that the relevant aspects of graphs (i.e., notations in Figure 5) are not given a priori but that subjects have to articulate the aspects and structure those aspects that they identified. Similarly, the interpreting subject has to identify objects and events that seem to correspond to the notational elements, identify relevance structures in the lived world. The subject arrives at a sense of completion when the linguistic descriptions of both structures appear congruent.

Our model takes into account that both symbol systems and biographical experience are embedded in and interact with natural language (Kaput, 1987; Wittgenstein, 1958), a situation from which arise consistent difficulties that individuals unfamiliar with some context experience. Thus, language is actually part of all three systems in Figure 5, but contexts and uses are very different; so are intentionality, purpose. A typical error is the reversal of structures as when subjects translate “for every six students, there is one professor” into the equation “6S = P” (rather than the correct S/6 = P), although in their seminar, they would never think that there are more processors than students even if they did not count.

3.3. Producing interpretive statements

The production of interpreting sentences is at the level of actions (activity theory), though there is not nor can there inherently be a plan for how to proceed in the text. (Even with literary texts, some readers will read different parts of the
text at random before reading it from beginning to end.) Sense emerges in the course and as product of the engagement with the graph. The concepts of growth point and catchments describe the point of emergence of an idea and its repeated occurrence—which may be a way to stabilize the idea, even confirming it after having considered additional features of the graph as a whole.

3.3.1. Interpretive statements as bodily perspectives on the world

In our model, we view interpretation absorbed engagement in the world, which implies that the subject does not simultaneously represents its own activity. Figure ground relations, that is, what counts as relevant notations against a more diffuse ground emerge from absorbed praxis. Similarly, process and product interpretation can be represented only a posteriori. Making notations and structures emerge is like moving about in a dark room, establishing the objects and events it contains by interacting with them (sensing, acting on). It is not that the interview subjects relate symbolic structures to their lived experience (professional, personal) in some abstract way; rather, doing an interpretation of a graph in a research situation is itself a form of lived praxis. We do not see reasoning about a world, but reasoning as lived, moment-to-moment praxis. It is not the

Figure 5. Our model of the relationship between symbolic (mathematical) structures, linguistic representation, and lived experience. In the interpretation of an unfamiliar graph, notations are identified, structured, and described and compared with linguistic descriptions of (a variety of) lived experiences. The interpretation is the real-time construction of a linguistic representation as a middle ground mapping both onto mathematical structures (\(\Phi_1\)) and lived experiences (\(\Phi_2\)).
manipulation of mental representation but interpretation as situated activity: In each statement the body positions itself “to become the idea or the intention that it signifies to us” (Merleau-Ponty 1945, p. 230). It even withheld the utterance until it was positioned such as to express in its gesture a content that is consistent with that articulated in speech. This can be seen in the following excerpt.

The utterance begins with the index “here,” which usually denotes a place, but which, in the present situation, is part of the observation that the death rate is increasing. The initial hand movement is toward the origin, with respect to which the subsequent gestures are presented as “increasing.” Initially, the hand moves up toward the graph, then follows it to its right extremity, where it comes to a stop (2.27). The utterance has stopped and continuous only while the hand is on its way to the lower part of the death rate curve, where it stops and then moves upward for a short distance while uttering the attribute “increasing.” We can parse the sequence into three parts, an movement upward to the endpoint, where the word “death rate” is marked, a movement down and left to the left end of the death rate graph, and a final movement following the death rate graph to the right and up.

Here, the first and third gestures have as their content the “increase,” upward and away from the origin, whereas the second gesture constitutes its inverse. At the moment where the pencil pointed to the upper right, there was no room for a gesture consistent with the “increase.” The descriptor was uttered only after the hand had come to a position where the gesture “increase” could be
enacted consistently with the utterance. That is, at 2.27, there was a contradiction between the perceptually available end (no continuation) to the right of the hand and the “increase,” which the hand motion already enacted on the trajectory from the origin to the point.

Therefore, in the present situation, the emerging description of the death rate as increasing and the current position of the hand in 2.27 precipitated a re- 1location of the hand, which itself enacted the opposite of “increasing,” but also in the opposite direction of the normal reading (left to right), and a subsequent coincident production of the corresponding gesture and utterance. At this point, the description was well formed, and a “natural” stop- and transition point had been reached.

The relationship involving perceptual aspects, gestures, and speech is dialectic. A dialectic relation implies a conflict or opposition within the unit, which does not come from logical inconsistency, but is inherent in the thing itself. This conflict, opposition, or contradiction seeks resolution, and therefore becomes a force of change. The conflict is (temporarily) removed when there is a stopping point, a sense that an end has been achieved. The “stop-order” arises from the speakers emerging intuitions about the completeness of what is being said (McNeill 2002).

3.3.2. Content, catchment, and stability

The content of the utterance may be a description of the perceptual feature (e.g., “the death rate is increasing”) or pertain to a meaning unit in the experienced world or in a possible world, both in abstract (e.g., “population is increasing”) or concrete terms (e.g., “there are few rats [people]”). In both instances, we have dialectical oppositions. In the first, the opposition exists between the perceptual feature and the utterance describing it (e.g., a sloped line [death rate] and the sentence, “death rate is increasing”). In the second, the opposition is between the graph as object and the real or imagined world within which the subject takes position at the moment of the utterance.1

1 It has been pointed out that utterances do not come from disembodied minds, nor are they expressions of a disembodied mind; the production of language and gesture requires the consideration of the body as seat of cognition (McNeill 2002). Merleau-Ponty (1945) suggested, “It has always been remarked that gesture or speech transfigured the body, but one was content saying that they developed or manifested another power, thinking, or soul. It has not been perceived that to express these [power, thinking, soul], the body has to become, in the final analysis, the thinking or intention that it signifies to us. It is the body that shows, speaks …” (p. 230, our translation).
He revisited the stable region, defining it with numerous up/down gestures between the two curves. By these actions he seemed less unsure in stability talk compared to earlier such accounts. He did not provide specific biological examples in this short comment about the stable region that seemed to be entirely defined by the gesture actions. In fact, when he mentioned “sustainable population” his pen dropped down below the death rate curve from the birthrate curve, and moved up again, demonstrating a possible “stable” region. This catchment related to stability when birthrate was higher than the death rate was contrasted to a second catchment related to instability. Thus, in the following excerpt, Marc’s actions were directed toward the right-most section of the graph where the death rate is larger than the birthrate.

In this situation, the gesture and speech articulated the lower death rate compared to birthrate, which had as consequence the disappearance of the population. Birthrate, death rate, and their difference were stable rather than affecting the population, with an adjustment of the rates, which would lead to a dynamic model.

4. Discussion

Some research on graph interpretation takes both an information processing approach, whereby graphs offer clear pieces of information, and assumes structural
isomorphism between graph and what it is said to represent (e.g., Tabachneck-Schijf, Leonardo, & Simon, 1997). Such research generally disregards the body of literature on interpretation, that is, the science of hermeneutics—as a result, alternative interpretations of some feature are interpreted in terms of cognitive deficits and misconceptions. Conversely, those using semiotic theory to characterize interpretation more generally (Eco, 1984) and interpretation of graphs more specifically (Roth & Bowen, 2001; Williams et al., 2001) clearly addressed the inherent nature of signs to be interpreted in different ways. However, semiotic theory theorizes the relationship between signs (sign, interpretant) and other segmentations of matter (referents) that the signs are said to refer to but leaves out the interpreting person, her history, and the context within which interpretation occurs. The present, dialectical model of graph interpretation was designed to address these problems of earlier theories. Activity theory has shown helpful, for it makes thematic dialectical tensions between subject (the person interpreting) and object (graph to be interpreted) at three levels, collective activity, individual action, and unconscious operation. The model is recursive, as the interpretation session itself is part of lived experience, augments it in a way, at the same time that it represents and interprets aspects of this lived experience.

References


