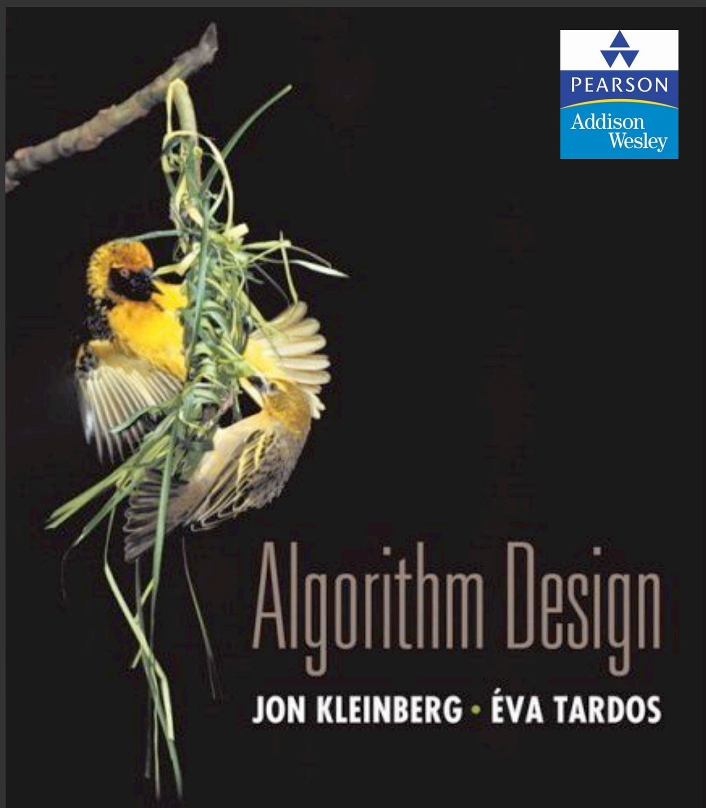


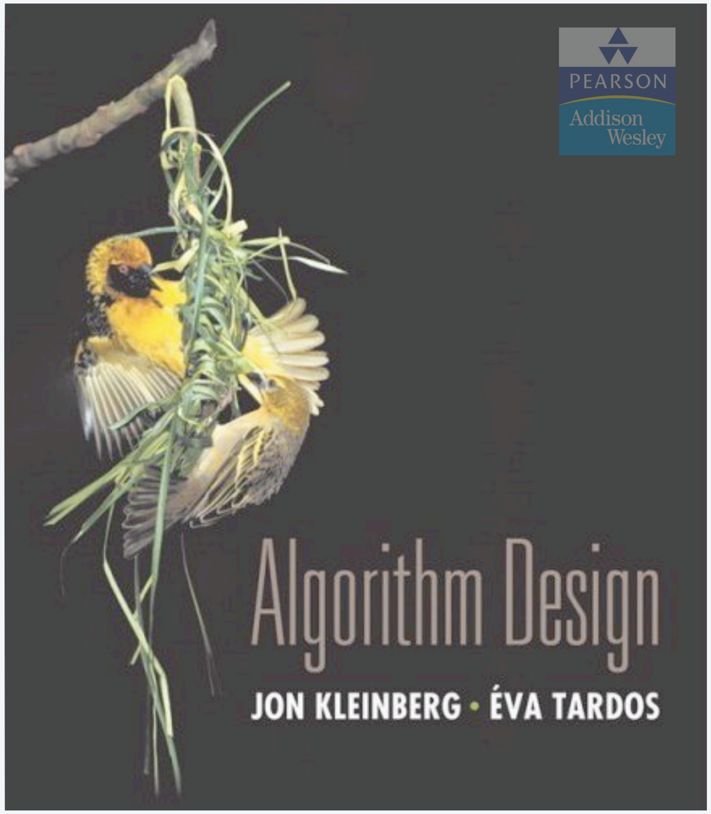
7. NETWORK FLOW I

- *max-flow and min-cut problems*
- *Ford-Fulkerson algorithm*
- *max-flow min-cut theorem*
- *capacity-scaling algorithm*
- *shortest augmenting paths*
- *blocking-flow algorithm*
- *unit-capacity simple networks*

Lecture slides by Kevin Wayne

http://www.cs.princeton.edu/~wayne/kleinberg-tardos

Last updated on Sep 8, 2013 6:40 AM



SECTION 7.1

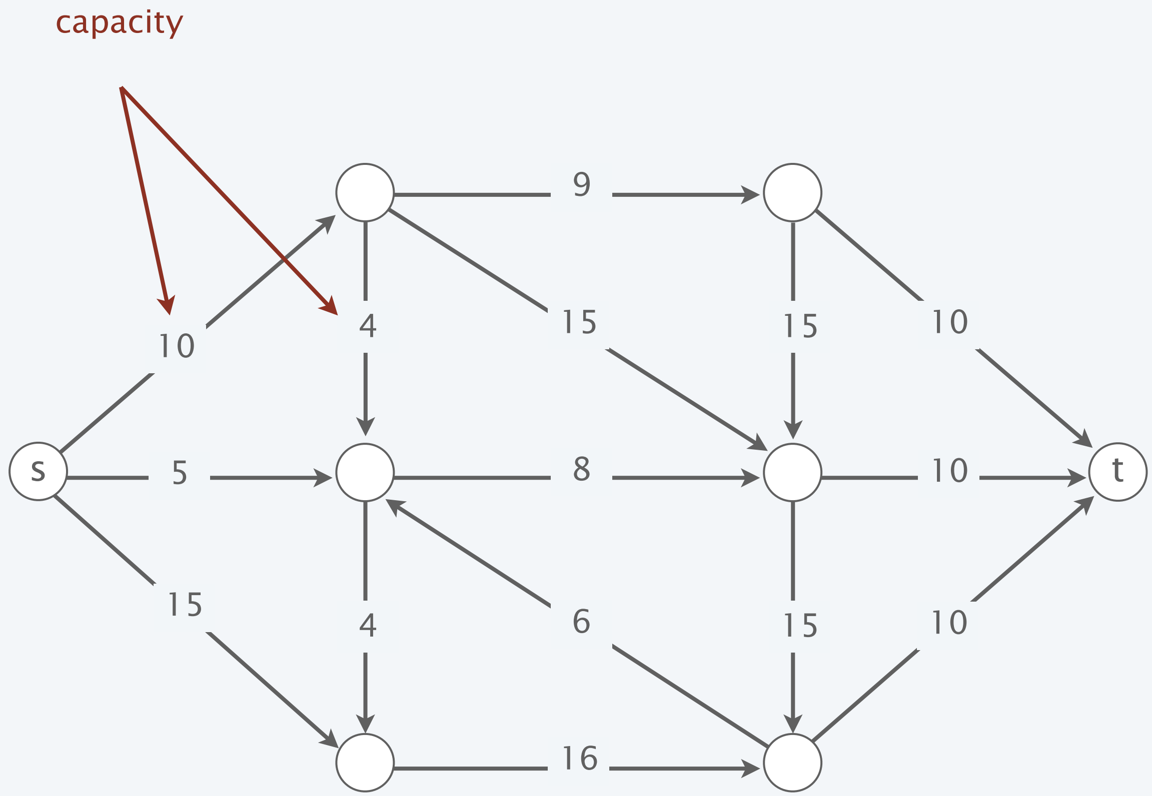
7. NETWORK FLOW I

- *max-flow and min-cut problems*
- *Ford-Fulkerson algorithm*
- *max-flow min-cut theorem*
- *capacity-scaling algorithm*
- *shortest augmenting paths*
- *blocking-flow algorithm*
- *unit-capacity simple networks*

Flow network

- Abstraction for material flowing through the edges.
- Digraph $G = (V, E)$ with source $s \in V$ and sink $t \in V$.
- Nonnegative integer capacity $c(e)$ for each $e \in E$.

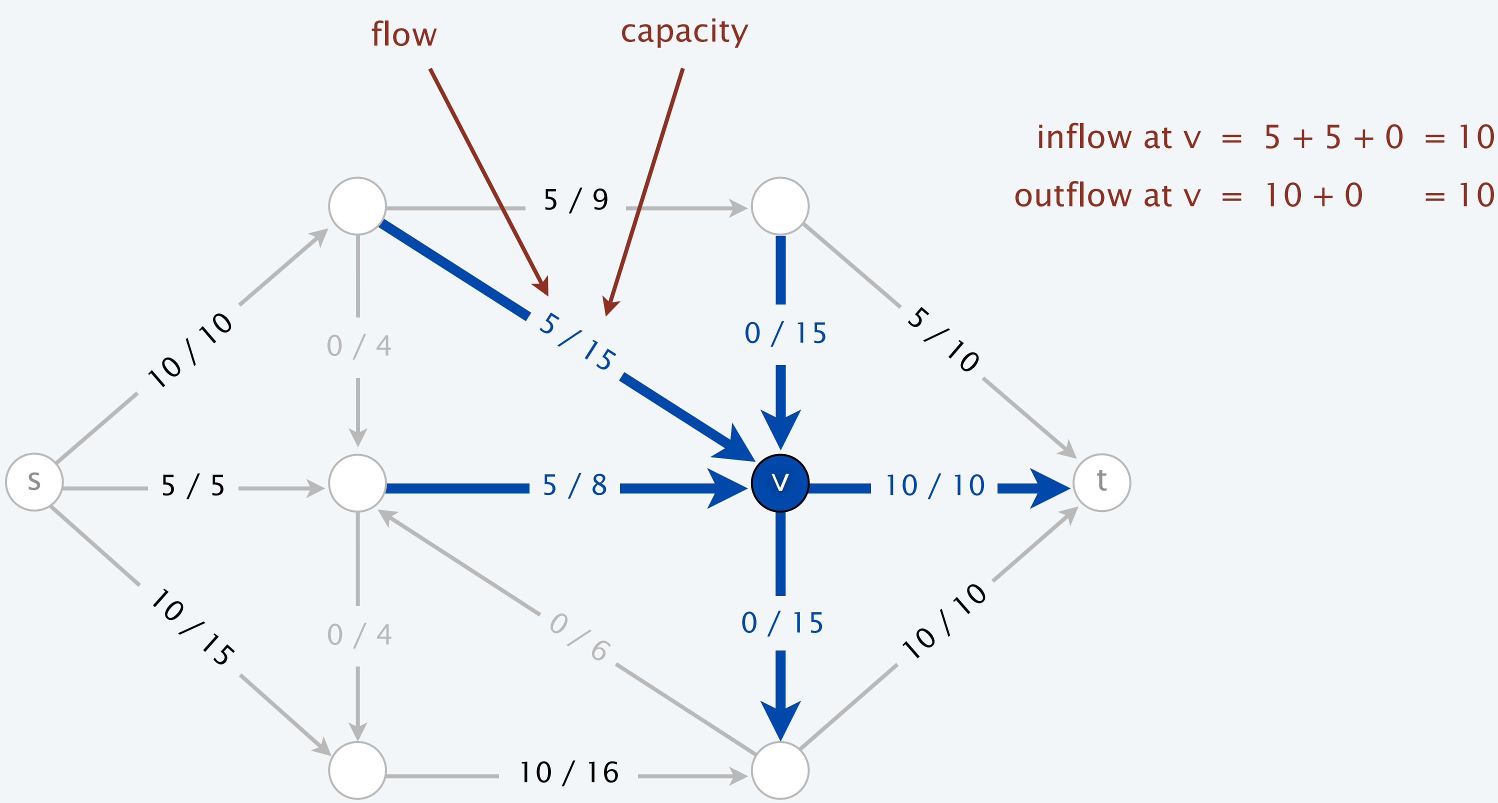
no parallel edges
no edge enters s
no edge leaves t



Maximum flow problem

Def. An *st*-flow (flow) f is a function that satisfies:

- For each $e \in E$: $0 \leq f(e) \leq c(e)$ [capacity]
- For each $v \in V - \{s, t\}$: $\sum_{e \text{ in to } v} f(e) = \sum_{e \text{ out of } v} f(e)$ [flow conservation]

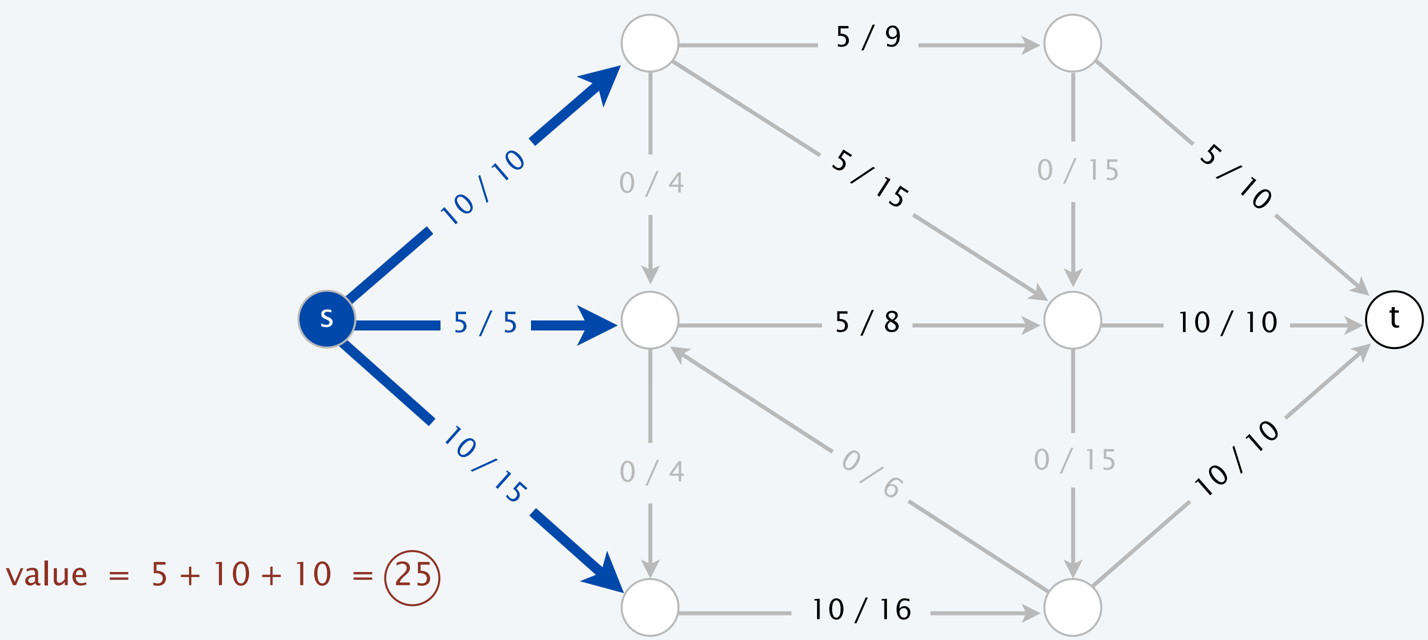


Maximum flow problem

Def. An *st*-flow (flow) f is a function that satisfies:

- For each $e \in E$: $0 \leq f(e) \leq c(e)$ [capacity]
- For each $v \in V - \{s, t\}$: $\sum_{e \text{ in to } v} f(e) = \sum_{e \text{ out of } v} f(e)$ [flow conservation]

Def. The value of a flow f is: $val(f) = \sum_{e \text{ out of } s} f(e)$.



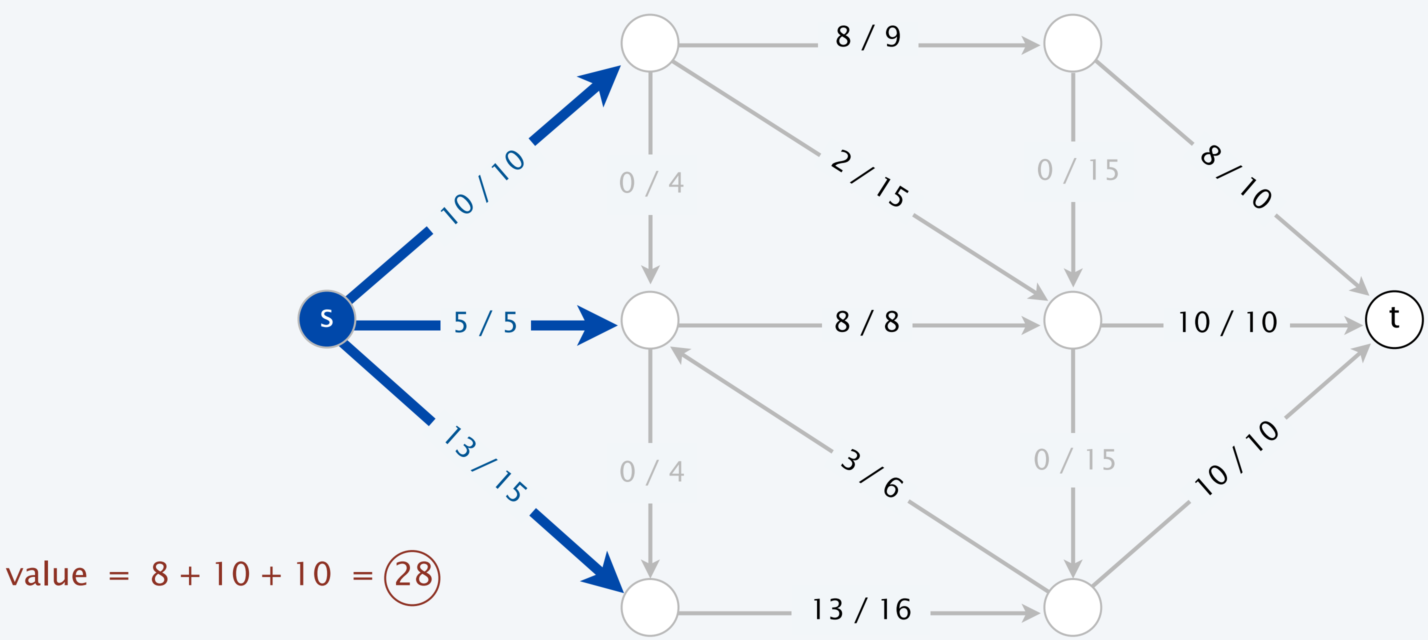
Maximum flow problem

Def. An *st*-flow (flow) f is a function that satisfies:

- For each $e \in E$: $0 \leq f(e) \leq c(e)$ [capacity]
- For each $v \in V - \{s, t\}$: $\sum_{e \text{ in to } v} f(e) = \sum_{e \text{ out of } v} f(e)$ [flow conservation]

Def. The value of a flow f is: $val(f) = \sum_{e \text{ out of } s} f(e)$.

Max-flow problem. Find a flow of maximum value.

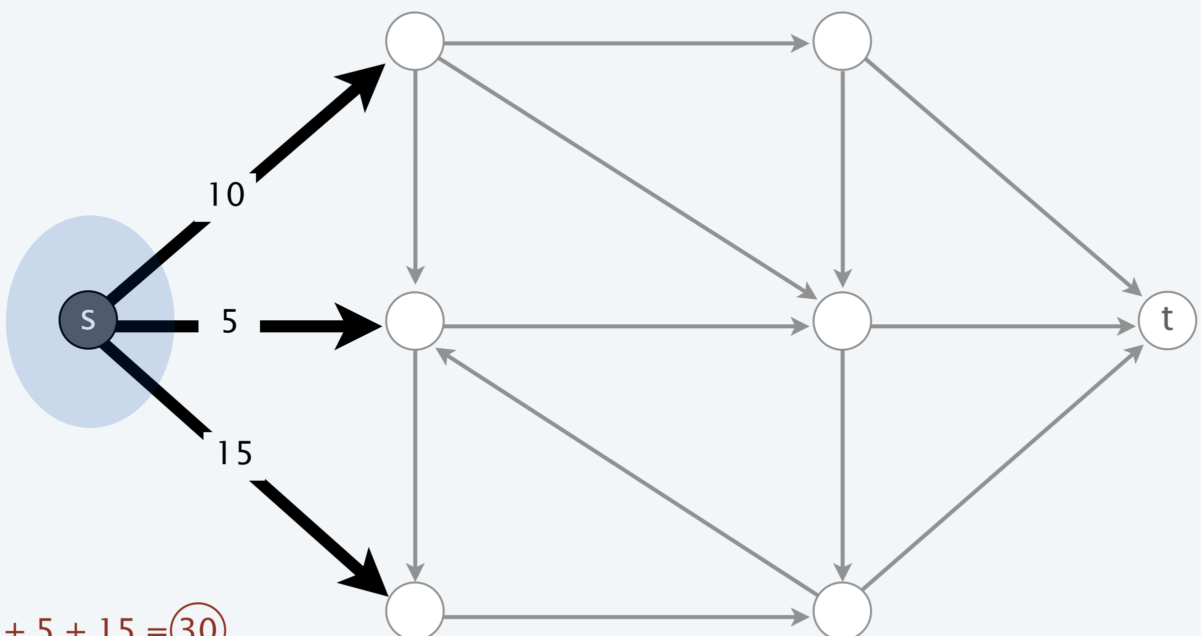


Minimum cut problem

Def. A *st*-cut (cut) is a partition (A, B) of the vertices with $s \in A$ and $t \in B$.

Def. Its capacity is the sum of the capacities of the edges from A to B.

$$cap(A, B) = \sum_{e \text{ out of } A} c(e)$$



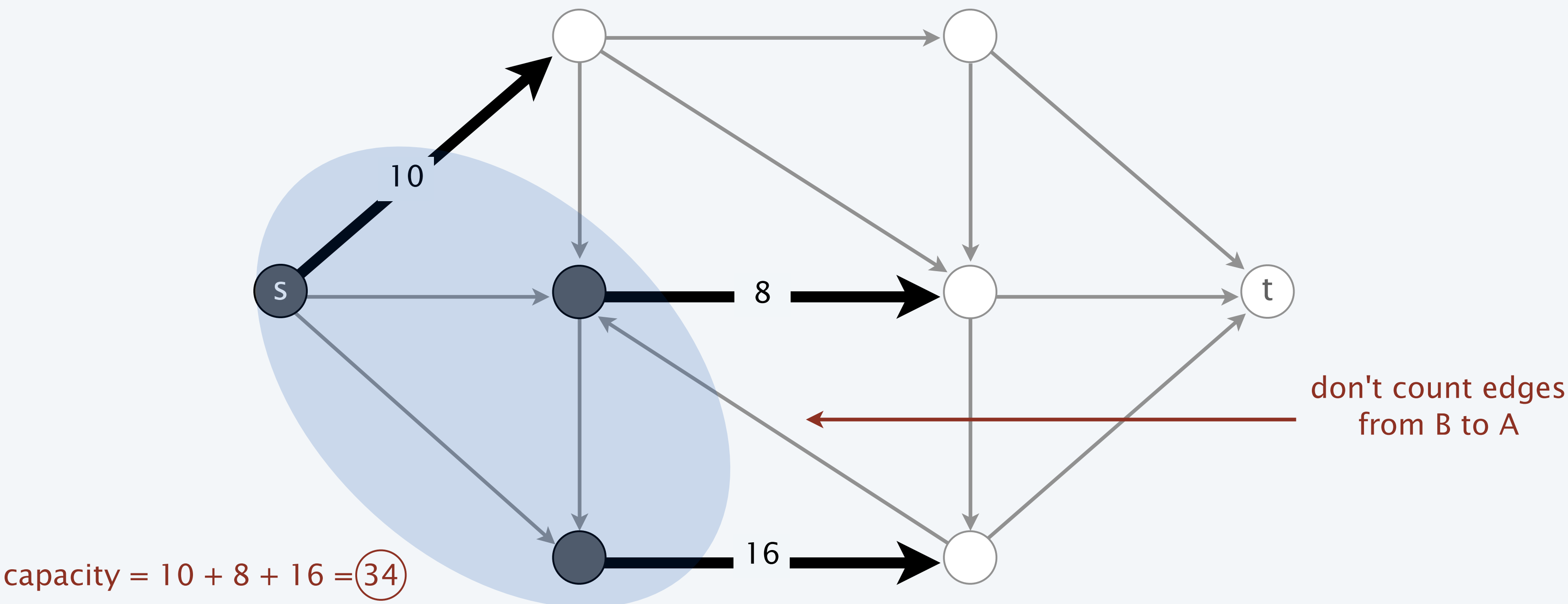
capacity = 10 + 5 + 15 = 30

Minimum cut problem

Def. A *st*-cut (cut) is a partition (A, B) of the vertices with $s \in A$ and $t \in B$.

Def. Its capacity is the sum of the capacities of the edges from A to B.

$$cap(A, B) = \sum_{e \text{ out of } A} c(e)$$



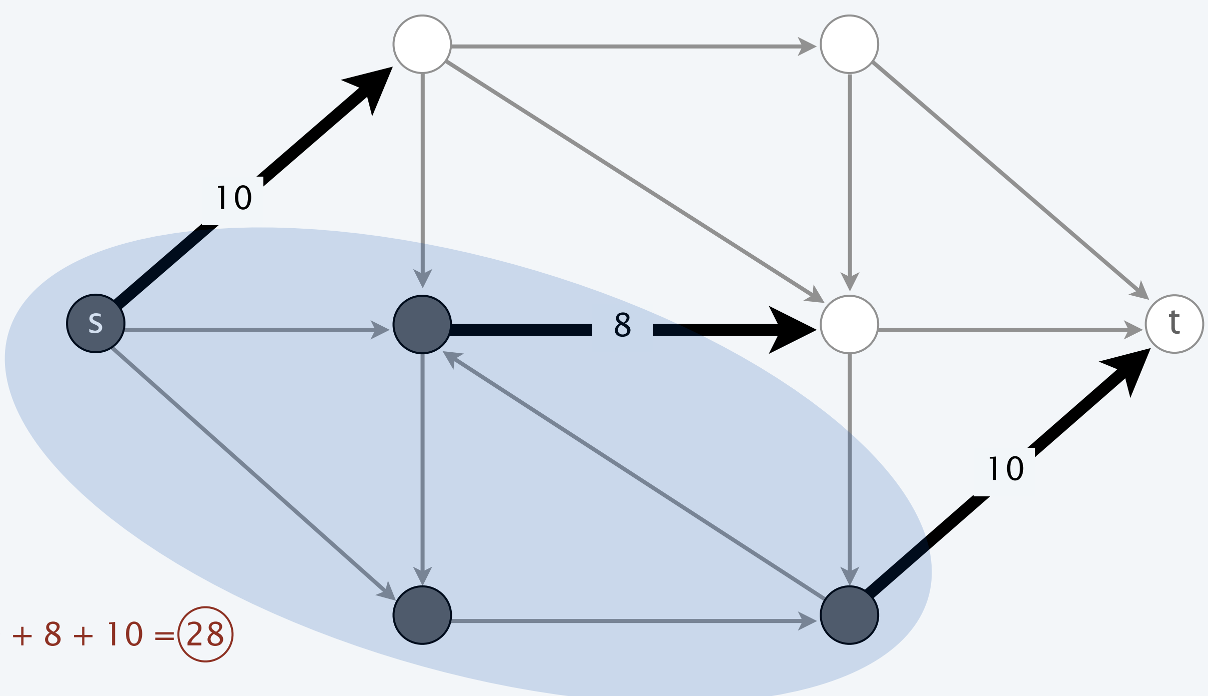
Minimum cut problem

Def. A *st*-cut (cut) is a partition (A, B) of the vertices with $s \in A$ and $t \in B$.

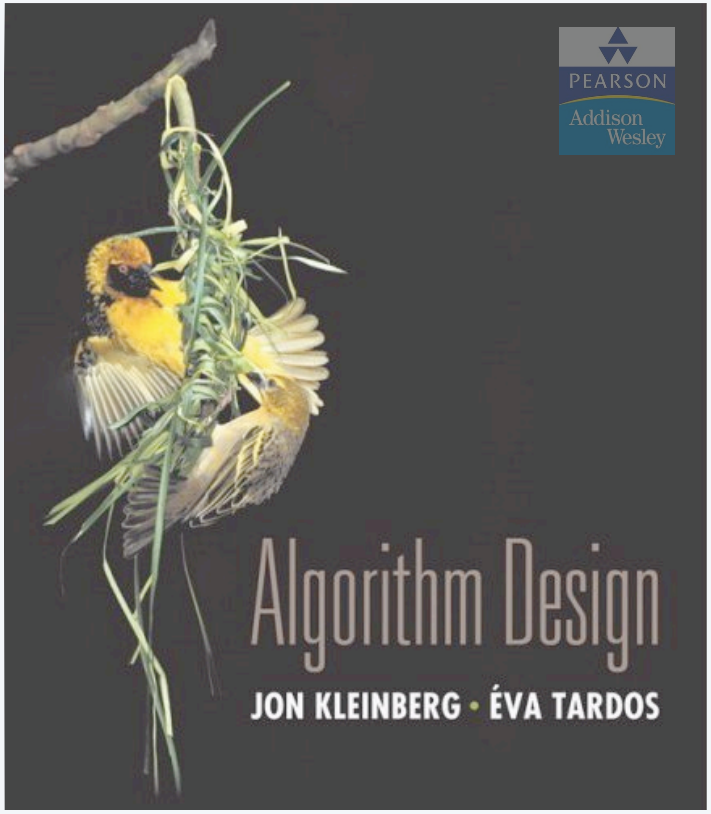
Def. Its capacity is the sum of the capacities of the edges from A to B.

$$cap(A, B) = \sum_{e \text{ out of } A} c(e)$$

Min-cut problem. Find a cut of minimum capacity.



capacity = 10 + 8 + 10 = 28



SECTION 7.1

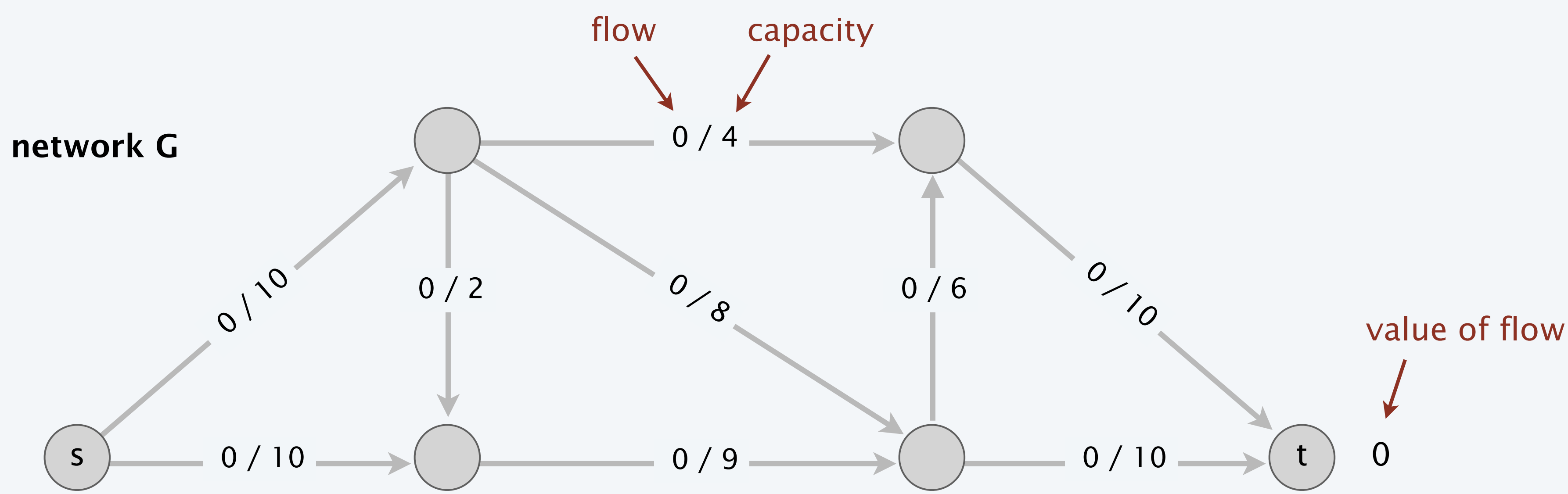
7. NETWORK FLOW I

- *max-flow and min-cut problems*
- ***Ford-Fulkerson algorithm***
- *max-flow min-cut theorem*
- *capacity-scaling algorithm*
- *shortest augmenting paths*
- *blocking-flow algorithm*
- *unit-capacity simple networks*

Towards a max-flow algorithm

Greedy algorithm.

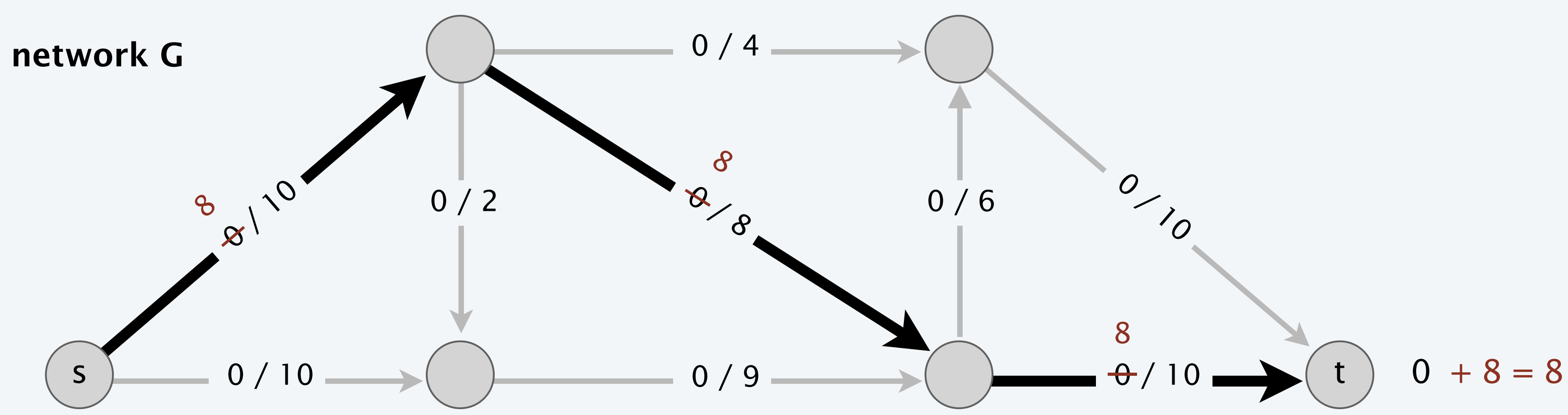
- Start with $f(e) = 0$ for all edge $e \in E$.
- Find an $s \rightsquigarrow t$ path P where each edge has $f(e) < c(e)$.
- Augment flow along path P.
- Repeat until you get stuck.



Towards a max-flow algorithm

Greedy algorithm.

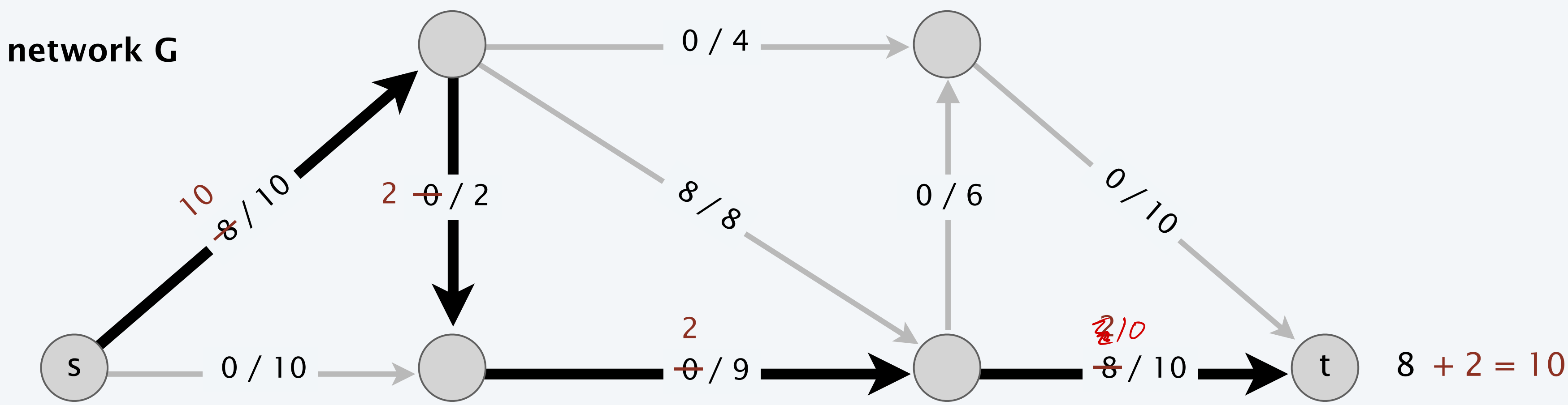
- Start with $f(e) = 0$ for all edge $e \in E$.
- Find an $s \rightsquigarrow t$ path P where each edge has $f(e) < c(e)$.
- Augment flow along path P.
- Repeat until you get stuck.



Towards a max-flow algorithm

Greedy algorithm.

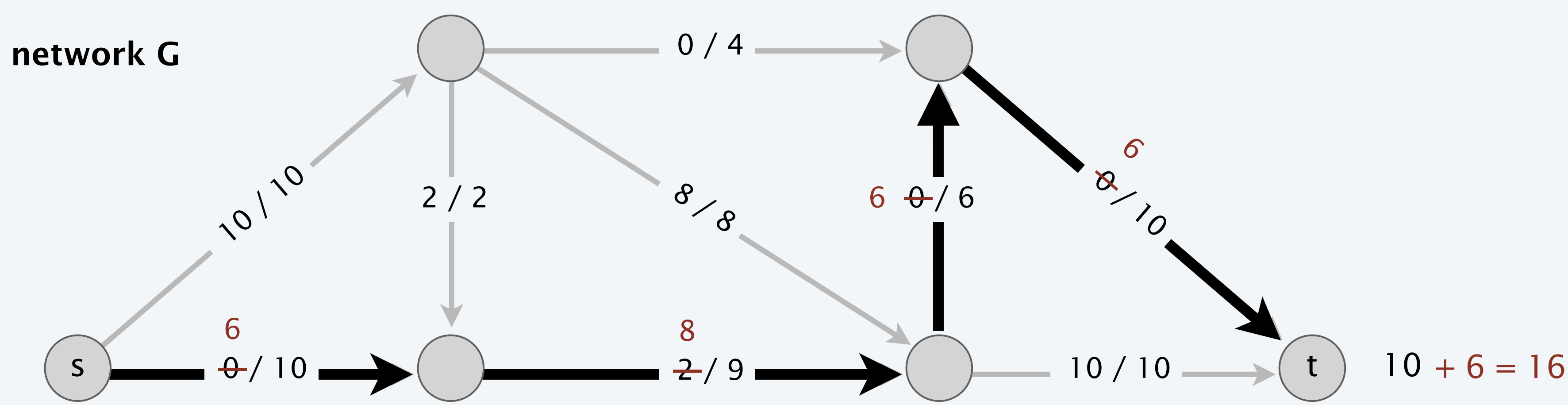
- Start with $f(e) = 0$ for all edge $e \in E$.
- Find an $s \rightsquigarrow t$ path P where each edge has $f(e) < c(e)$.
- Augment flow along path P.
- Repeat until you get stuck.



Towards a max-flow algorithm

Greedy algorithm.

- Start with $f(e) = 0$ for all edge $e \in E$.
- Find an $s \rightsquigarrow t$ path P where each edge has $f(e) < c(e)$.
- Augment flow along path P.
- Repeat until you get stuck.

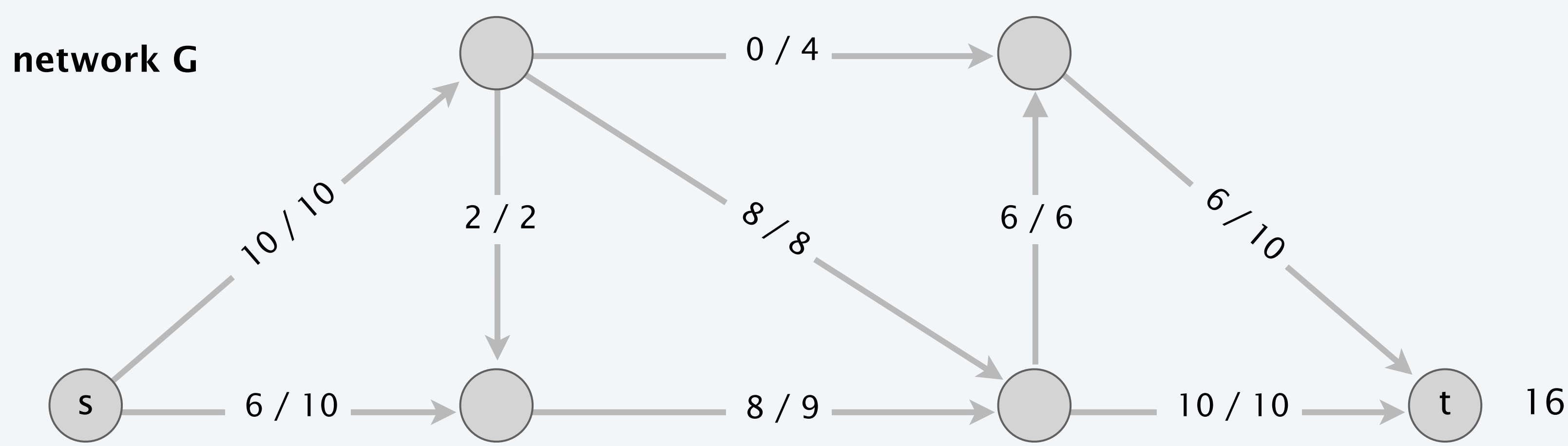


Towards a max-flow algorithm

Greedy algorithm.

- Start with $f(e) = 0$ for all edge $e \in E$.
- Find an $s \rightsquigarrow t$ path P where each edge has $f(e) < c(e)$.
- Augment flow along path P.
- Repeat until you get stuck.

ending flow value = 16



Towards a max-flow algorithm

Greedy algorithm.

- Start with $f(e) = 0$ for all edge $e \in E$.
- Find an $s \rightsquigarrow t$ path P where each edge has $f(e) < c(e)$.
- Augment flow along path P.
- Repeat until you get stuck.

but max-flow value = 19