The Value of Online Scarcity Signals *

Pascal Courty† Sinan Ozel‡

July 28, 2017

Abstract

Online retailers use scarcity cues to increase sales. Many fear that these pressure tactics are meant to manipulate behavioral biases by creating a sense of urgency. At the same time, scarcity cues could also convey valuable information. We measure the value of the scarcity messages posted by Expedia to a Bayesian rational consumer. A signal reveals information on the number of seats available at the posted price. Consumers can use this information to optimally time when they purchase a ticket. The maximum increase in expected utility for a naive consumer, who does not use publicly available information, is 8 percent. For a sophisticated consumer, the increase is between 4-7 percent. Scarcity signals have a negligible impact on seller revenue and consumption.

Keywords: Scarcity, Persuasion, Online Recommendations, Price Discrimination, Airline Ticket.

JEL Classification: L1.

*We thank seminar audiences at LUISS, EIEF and University of Victoria.
†University of Victoria and CEPR; pcourty@uvic.ca.
‡University of Victoria; sozel@uvic.ca.

1 Introduction

Scarcity cues and pressure tactics are widely used by online retailers to increase sales (Nagpal, 2014).[1] According to marketers and some social scientists, scarcity creates a sense of urgency, it increases desirability and gives a perceived benefit of acting quickly (Worchel et al., 1975; Lynn, 1991; Verhallen and Robben, 1994; Mullainathan and Shafir, 2013). Although many fear that sellers manipulate the psychology of consumers, scarcity messages can also deliver information that is not available otherwise. A Bayesian consumer could benefit from this information even if messages are meant to manipulate behavioral consumers subject to decision biases.

This paper measures the informational value of scarcity messages in the context of air travel. Airfares can vary dramatically from day to day. Many travelers have to choose whether to book a non-refundable ticket without knowing future fares and whether it would be wise to postpone purchase. Airlines try to influence travelers by presenting scarcity signals next to airfares. For example, fares displayed on Expedia sometimes mention that there are few seats left at the posted price.[2] We develop a Bayesian rational framework to evaluate the value of signals. This approach is supported by evidence, from similar contexts as ours, that shows that some consumers behave as rational optimizers (Li et al., 2014; Cui et al., 2016). Clearly, the rationality assumption may not apply to all travelers, but for our purpose, it is a starting point to derive empirical predictions that can be tested using only information on prices and signals. Our approach offers a relevant benchmark for the consumers who respond to messages as expected utility maximizers. Although we cannot test this assumption (because we do not observe consumer bookings), our measure delivers an upper bound for the value of signals.

For our application, we collect an original dataset using a web scraping script that submits queries to the Online Travel Agent (OTA) Expedia (Edelman, 2012). As a descriptive step, we compare the distributions of price changes conditional on the signal realization; whether or not 'few seats left' is posted next to the fare. The posterior under the scarcity signal first order stochastically dominates the posterior without a scarcity signal. A scarcity signal lowers the chances that the posted fare will decrease and that it will remains constant, two pieces of information consumers care about. Showing that Expedia signals are informative is a contribution in itself and it establishes the basis for the rest of the paper.

[1] For example, many Online retailers post limited-quantity messages (remaining stock left) next to selected listings. Other pressure cures include limited time offers (expiry sales) and demand interest (number of consumers browsing an item) (Aggarwal et al., 2011; Gierl and Huettl, 2010).

[2] The Expedia link explains "According to the data that we receive from the airline, there are very few tickets currently available at this price. While limited availability can be an indicator that he price for this flight may increase, this is not always the case."

In the core of the analysis, we consider a simple one-off purchase-delay decision. The traveler can buy a ticket now or postpone her purchase decision by one week and she can do so *only once*. This stylized scenario is consistent with recent related works (Li et al., 2014). We also present results where the consumer can delay her purchase twice and the main insights do not change. If the consumer believes that the price increases in expectation, which is often the case for airfares, her purchase decision depends on how much she values traveling relative to the current price. We obtain a threshold rule: the consumer buys at the current price if and only if her relative surplus, denoted $\frac{v-p}{p}$ where v is her valuation and p the current price, is above a fixed threshold. A week later, the consumer who waited buys if and only if the updated fare is below her valuation.

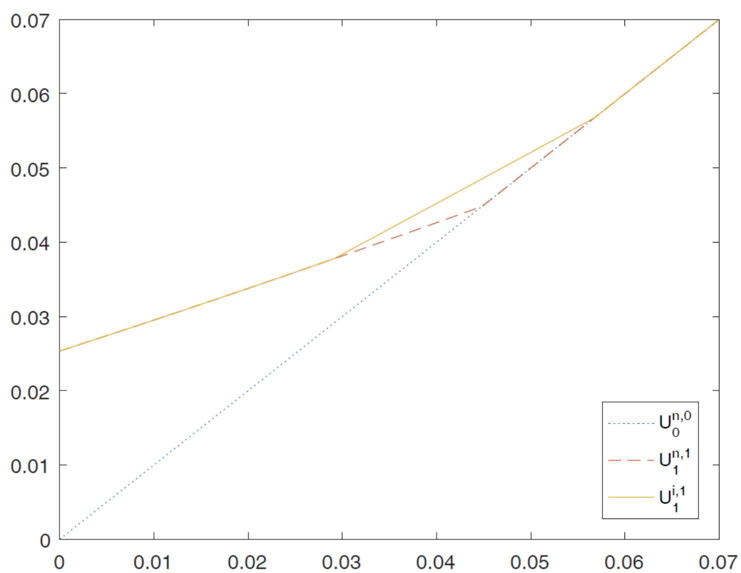

Figure 1: Expected utilities (a) without waiting option (blue dashes), (b) with the option of delaying purchase (red dashes), (c) with the option of delaying purchase conditional on the signal (orange line).

Figure 1 plots the consumer surplus under three scenarios. In line with the model, the variable on the horizontal axis is the relative consumer surplus. A higher value means that the consumer receives more surplus from buying at the current price. The blue dashed line plots the consumer surplus if she cannot delay. The consumer does not care about the signal realization. Her surplus is zero for valuations that are lower than the price (outside the range of the plot) and follows the 45 degree line for valuations above the current price (positive values on the horizontal axis). The dashed red curve plots the expected surplus if the consumer is uninformed and has the option to delay purchase. An uninformed consumer does not update her decision based on the signal's realization. The consumer with relative consumer surplus 4.48% (located at the kink of the dashed red curve) is indifferent between buying at the current price and delaying. The orange curve plots the same surplus for the informed consumer. The two kinks in the orange curve correspond to the indifferent consumers conditional on the signal realization. Consumers with valuations far from the indifferent uninformed consumer do not benefit from the signal (the dashed red and orange lines coincide) and the increase in surplus from the signal is maximum for the indifferent consumer. Figure 1 also reveals that for most travelers the gains from waiting (the difference between the red and blue curves) is significantly higher than the gains from the signal (the difference between the blue and orange curve).

A signal may be informative in a statistical sense (it helps predict future prices) although this information has no value to any consumer. This is not the case for our sample of Expedia signals. The average signal increases the expected utility of an unsophisticated traveler, who does not condition her decision on any publicly observable information, by at most 8 percent. We also compute the value of information for a sophisticated consumer, who also uses public information in addition to the signal to predict future fares (Mantin and Gillen, 2011). For a traveler who conditions her decision on the number of days remaining till departure, the increase in expected utility is between 4 and 7 percent. Finally, we compute the impact of scarcity signals on seller revenue and tickets sold. To do so, we assume that there is a uniform distribution of consumer valuation in the neighborhood of the indifferent consumer. Scarcity signals have a small negative impact on seller revenue for some subsamples of the data and little impact on the number of tickets sold.

This work is related to several strands of literature. The model touches upon the literatures on price discrimination with information revelation (Lewis and Sappington, 1994) and Bayesian persuasion (Gentzkow and Kamenica, 2011).[3] The literature on scarcity signals is mixed. Scarcity theory in psychology and marketing argues that signals are largely used to exploit consumer biases (Brock, 1968; Aggarwal et al., 2011; Aguirre-Rodriguez, 2013; DellaVigna and Gentzkow, 2009; Mullainathan and Shafir, 2013). At the same time, Cui et al. (2016) offer convincing evidence that consumers respond, as assumed in this work, rationally and strategically to real-time Online information.[4] Finally, the empirical application is related to the airline literature which is reviewed in the next section.

[3]Strategic information revelation has been studied empirically in the context of Buy/Sell recommendations by financial analysts (Stickel, 1995).

[4]Using Amazon data from a natural experiment with lightning deals, they conclude that "customers not only learn from real-time availability information, but also rationally use observable product attributes to moderate their inferences about the deal's quality. This finding supports the fundamental assumption of consumers' strategical and rational reaction to inventory information in the literature."

The rest of this paper is organized as follows. Section 3 presents a model of consumer decision making under price uncertainty, derives a measure of the value of information and computes the impact of the signal on seller revenues and consumption. The following section presents the data and descriptive statistics. Section 5 presents our main results and the last section concludes.

2 Online Travel Booking and Scarcity Signals

The broader context for this study is travel booking and air travel demand. There is a large literature on revenue management but the models used in empirical studies make simplistic assumptions about demand (McAfee and Te Velde, 2006; Escobari, 2012; Sweeting and Sweeney, 2015).[5] Consumers are largely myopic: they arrive randomly and do not anticipate future fares. Such simplicity, which is necessary to manage the complexity inherent with the inter-temporal trade-offs associated with the allocation of a fixed and perishable capacity, leave the demand side largely unexplored. Demand studies of consumer bookings are rare due to the absence of publicly available data. A notable exception that had access to a proprietary booking dataset is Li et al. (2014). They report that up to 19 percent of consumers strategically delay booking based on expectations about future prices.

In practice, most travelers make many decisions to book a plane ticket. They decide when to start a search, whether to search directly at the airline's branded site or at an OTA, what queries to make, what flights to look at and whether to visit a meta-search engine (e.g. Kayak) that relies on big data analytics to show a variety of price comparisons. It is widely accepted that finding a cheap fare for a given itinerary has a lot to do with timing. Delaying purchasing a ticket can be profitable, especially 3 weeks prior to departure and earlier because fare drops are not uncommon (Bilotkach and Rupp, 2011). Fare drops could be caused by slow sales (Escobari, 2012), temporary promotions, competitive pressure (Gerardi and Shapiro, 2009), or other reasons... Anecdotal evidence suggests that some consumers actively search for low fares. They compare prices across sellers, sign up for fare alerts, and experiment with diverse searches on multiple days.[6] Hopper.com reports that most customers purchase a ticket within two weeks of their initial search. Beyond these casual observations, we are not aware of systematic empirical research on how consumers search for airfares.

This paper makes a step toward understanding the benefits from delaying purchase and the role played by scarcity signals in that decision. We take a normative approach in that we

[5]Theoretical models can account for some consumer strategizing (Dana, 1998; Deneckere and Peck, 2012).

[6]*Fare alerts* are email notices sent to subscribers when ticket prices plunges or when it is a good time to purchase a ticket.

leverage the power of the rationality assumption to compute the value of delaying purchase and the value of scarcity signals. With the exception of Li et al. (2014), we are not aware of any work that has looked at the option value of delay in the context of airfares.[7] This is surprising given the importance of non-refundable bookings in revenue management. Our normative benchmark should be complemented with analysis of booking data to assess the cost borne by the consumers who deviate from the rationality assumption.

To keep matter simple, we model here only the gains associated with purchasing a ticket at a lower fare. Some consumers also delay because they are uncertain about their traveling plans. As time goes by, they become more confident about their traveling needs. For these consumers, there is an additional option value of waiting. We leave this issue aside and assume that consumers have a given valuation, keeping in mind that valuation uncertainty can easily be added to the analysis. We consider simple scenarios where the consumer can delay by one week once or twice because this is realistic in our context. Again the analysis could be extended to multiple delays.

3 A Model of Consumer Response to Scarcity Signals

An airline sells a ticket to a traveler. The traveler knows her willingness to pay v at the time of purchase. The current price is p_0 and the price next period changes according to $p_1 = (1+r)p_0$ where the growth rate, $r \in [-1, \infty)$, is a random variable distributed with c.d.f. $F^n()$. The ticket is not available next period with probability $1 - \lim_\infty F^n(r)$. The firm sends a scarcity signal that takes realization (b)ad or (g)ood. The bad realization occurs with probability τ_b. The posteriors about the growth rate conditional on the signal realization are $F^b(r)$ and $F^g(r)$. Bayes rule imposes

$$F^n(r) = (1-\tau_b)F^g(r) + \tau_b F^b(r). \tag{1}$$

For example, the good state could imply a better distribution of price in the sense of first order stochastic dominance, $F^g(r) \geq F^b(r)$. Note, however, that this assumption is not required in the analysis. Let $\overline{r}^s$, for $s \in \{n, g, b\}$ denote the mean expected growth rates under the prior and posteriors. The consumer is risk neutral and Bayesian. The model addresses the following issues: (a) For which value of v does the consumer buy early? (b) How does the signal realization influence this decision? (c) What is the utility increase associated with the signal? (d) What is the change in supplier revenues and units sold?

[7] There is much theoretical research on the real option of delaying purchase with fluctuating prices Ho et al. (1998).

We take the price and signal policies as given. Modeling these supply side policies is beyond the scope of this paper. What is important for this work is for these policies to be stable. Thus, a rational consumer correctly update her prior F^n to posterior F^g or F^b depending on the signal realization. We illustrate some of the results with two examples that are used for a didactic end and also to deliver benchmark close form solutions. In the first example, the signal delivers information on the probability that the price remains constant. The distribution of price growth rate has a mass probability at zero and this mass is smaller under the bad realization than under the good one. The distributions of growth rates are such that $F_1^n(r) = F_1^b(r)$ for $r < 0$ and $F_1^b(r) = F_1^n(r) - (1-\tau_b)x$ for $r \geq 0$. In the second example, the signal shifts the cumulative distribution function by a constant. The conditional posterior are horizontal shifts of the prior: $F_2^b(r) = F_2^n(r-x)$. The expected growth rate under the bad signal is equal to the expected in growth rate under the prior plus x.[8]

3.1 Informational Value of Scarcity Signals

The consumer's expected utility from buying in current period is $U_0(v) = Max(v - p_0, 0)$. Next period, the consumer purchases when $v \geq p_1$, that is, for price returns $r \leq \frac{v}{p_0} - 1$. Define the functions $\mathbf{r}(v) \triangleq \frac{v}{p_0} - 1$ and $\mathbf{v}(r) \triangleq p_0(1+r)$ to facilitate going back and forth from valuation to equivalent return. The expected utility from waiting given belief F^s is $E(Max(v-p_1,0)|s) = p_0 \int_{-1}^{\mathbf{r}(v)} F^s(r)dr$.[9] If $\overline{r}^s < 0$, the price decreases in expectation, and we have $E(Max(v-p_1,0)|s) > v - E(p_1|s) > v - p_0$ for all v. The consumer waits independently of her valuation. Otherwise, there exists a solution v^s to the indifference condition

$$E(Max(v^s - p_1, 0)|s) = v^s - p_0.$$

This is the valuation of the consumer indifferent between buying and waiting. We also call that consumer the indifferent consumer. The indifference condition is rewritten $\int_{-1}^{\mathbf{r}(v^s)} F^s(r)dr = \mathbf{r}(v^s)$. We define $\rho^s = \infty$ if $\overline{r}^s < 0$. Otherwise, ρ^s is the solution to

$$\rho^s = \int_{-1}^{\rho^s} F^s(r)dr \tag{2}$$

and note that ρ^s is independent of the initial price p_0.

Lemma 1. *There exist a unique a triplet (ρ^b, ρ^n, ρ^g) such that $Min(\rho^g, \rho^b) \leq \rho^n \leq Max(\rho^g, \rho^b)$. When consumer v has belief $F^s()$, she waits if $v \in [0, \mathbf{v}(\rho^s))$ and buys early if $v \in (\mathbf{v}(\rho^s), \infty)$.*

[8] Using equation (1), we obtain $F_1^g(r) = F_1^n(r) + \tau_b r$ for $r \geq 0$ and $F_2^g(r) = \frac{F_2^n(r) - \tau_b F_2^n(r-x)}{1-\tau_b}$.

[9] We have $E(Max(v-p_1,0)|s) = vF^s(\mathbf{r}(v)) - p_0 \int_{-1}^{\mathbf{r}(v)} (1+r)dF^s(r) = p_0 \int_{-1}^{\mathbf{r}(v)} F^s(r)dr$.

Her expected utility is:

$$U_1^s(v) = \begin{cases} p_0 \int_{-1}^{\mathbf{r}(v)} F^s(r)dr, & \textit{if } v \in [0, \mathbf{v}(\rho^s)] \\ v - p_0, & \textit{if } v \in [\mathbf{v}(\rho^s), \infty]. \end{cases} \tag{3}$$

Lemma 1 says that (ρ^b, ρ^g) lie on each side of ρ^n. For example, a sufficient condition for $\rho^b \leq \rho^g$ is first order stochastic dominance (FOSD): $F^g(r) \geq F^b(r)$ for all r. In the rest of the analysis, we label the two states such that $\rho^b \leq \rho^g$, which is a matter of convention. Under this usage, we obtain the intuitive outcome that the bad signal triggers some consumers to change their decision to wait and the good signal triggers some consumers to change their decision to buy, in a sense that is formally defined next:

Proposition 1. *(a) Consumer $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$ waits without signals. With scarcity signals, she switches to buy when the signal is bad. (b) Consumer $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$ buys early without a signal. With scarcity signals, she switches to wait when the signal is good. (c) Consumer $v \notin [\mathbf{v}(\rho^b), \mathbf{v}(\rho^g)]$ does the same with and without a signal.*

The introduction of scarcity signals changes both the decision to wait (timing of purchase) and the decision to purchase (a consumer who waits may not buy in period one). Consumer v's expected value from waiting is

$$U_1^i(v) = \tau_b U_1^b(v) + (1 - \tau_b) U_1^g(v).$$

Consumer v's gain from the signal is $\Delta U(v) = U_1^n(v) - U_1^i(v)$. We show in the Appendix that

$$\Delta U(v) = \begin{cases} \tau_b \left(v - p_0 - p_0 \int_{-1}^{\mathbf{r}(v)} F^b(r)dr \right), & \text{if } v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)] \\ (1 - \tau_b) \left(p_0 \int_{-1}^{\mathbf{r}(v)} F^g(r)dr - (v - p_0) \right), & \text{if } v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)] \end{cases} \tag{4}$$

with $\Delta U(v) = 0$ for $v \notin (\mathbf{v}(\rho^b), \mathbf{v}(\rho^g))$. We say that a signal has no value if it does not improve the consumer's decision independently of her valuation v.

Corollary 1. *A signal has no value if and only if $\rho^g = \rho^n = \rho^b$.*

The signal has no value when the indifferent consumer is the same independently of the signal realization. A random signal, for example, has no value: We have $F^g() = F^b()$ and the Corollary applies. Another special case where the signal has no value to any consumer happens when $\rho^g = \rho^b = \infty$ which is equivalent to $\overline{r}^b \leq 0$ (this implies the condition in the

Corollary). The consumer always wait. The signal is worthless but could still be statistically informative (for example, if $\overline{r}^b \neq \overline{r}^g$). Interestingly, a signal that helps predict availability ($\lim_\infty F^g(r) > \lim_\infty F^b(r)$) is not valuable if the condition in Corollary 1 holds. On the contrary, and somewhat counter to intuition, the signal can be valuable even when prices decrease on average ($\overline{r}^n < 0$).

Corollary 2. *The consumer with value $\mathbf{v}(\rho^n)$ receives the highest utility gain from the signal.*

Using identity (2), we obtain $\Delta U(\mathbf{v}(\rho^n)) = p_0\tau_b(1-\tau_b)H(\rho^n)$ where $H(r) \triangleq \int_{-1}^{r} \left(F^g(y) - F^b(y)\right) dy$. We define the value of the signal, $I \triangleq \frac{\Delta U(\mathbf{v}(\rho^n))}{U^n(\mathbf{v}(\rho^n))}$ as the relative utility change to consumer $\mathbf{v}(\rho^n)$. As explained above, only a consumer with a valuation $v \in (\mathbf{v}(\rho^b), \mathbf{v}(\rho^g))$ benefits (in expectation) from the signal. Expression I is the value of information to the indifferent consumer. It is an upper bound on the value of information across all consumers. After replacement, we have

$$I = \frac{\tau_b(1-\tau_b)}{\rho^n} H(\rho^n). \tag{5}$$

As expected, we have $I = 0$ when the condition in Corollary 1 holds ($\rho^b = \rho^g$ implies $H(\rho^n) = 0$). The value of information has the following properties: It is independent of p_0. It increases, ceteris paribus, as there is more uncertainty about the signal realization ($\tau_b(1-\tau_b)$ large), as the consumer has lower threshold (ρ^n small) and as the signal shifts the posterior further apart ($H()$ large).

Take example 1. Since $H_1(r) = rx$ for $r > 0$, the value of information simplifies to $I_1 = (1-\tau_b)\tau_b x$. The value of information is independent of the prior $F_1^n()$ and of threshold ρ^n. It increases as the signal shifts the probability of no price change by a larger amount (x large). For example 2 we have $H_2(r) = \frac{1}{1-\tau_b}\int_{r-x}^{r} F^n(y)dy \approx \frac{x}{1-\tau_b}F_2^n(r)$, where the approximation holds for x small, and $I \approx \tau_b x \frac{F_2(\rho^n)}{\rho^n}$. The value of information increases as the signal shifts the distributions of growth rate further appart (x large). It is proportional to $\tau_b x$: the consumer $\mathbf{v}(\rho^n)$ cares only about the product of the probability that the bad realization be drawn and the impact of the bad realization on the posterior distribution.[10]

[10] Note that this holds only for consumer $\mathbf{v}(\rho^n)$. Holding constant $\tau_b x$, the consumers with valuation below $\mathbf{v}(\rho^n)$ prefers a signal with low τ_b. The opposite holds for consumer $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$. Take the case of consumer $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$. Rewrite $\Delta U(v) = \tau_b\left(v - p_0 - p_0\int_{-1}^{\mathbf{r}(v)} F^n(r)dr\right) + \tau_b p_0 \int_{-1}^{\mathbf{r}(v)} \left(F^g(r) - F^n(r)\right) dr$. The second term is approximated by $\tau_b p_0 x F(\mathbf{r}(v))$ which is proportional to the product $\tau_b \delta_g$. The first term, however, decreases with τ_b since $v - p_0 - p_0\int_{-1}^{\mathbf{r}(v)} F^n(r)dr < 0$ for $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$.

3.2 Supplier Revenues and Consumption

The model has only two periods and a single consumer. Within this restricted framework, one can look at changes in expected (static) consumer revenue and consumption. A bigger picture would include profits and welfare. But investigating the impact of scarcity signals on these outcomes requires modeling dynamic trade-offs that are beyond the scope of this model.[11] Denote by $\Delta R(v)$ the difference in revenue, received from the consumer with valuation v, with and without a signal. $\Delta R(v) = 0$ for $v \notin [\mathbf{v}(\rho^b), \mathbf{v}(\rho^g)]$ and

$$\Delta R(v) = \begin{cases} p_0 \tau_b \left(1 - \int_{-1}^{\mathbf{r}(v)} (1+r) dF^b(r)\right), & \text{if } v \in (\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)) \\ -p_0(1-\tau_b)\left(1 - \int_{-1}^{\mathbf{r}(v)} (1+r) dF^g(r)\right), & \text{if } v \in (\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)). \end{cases} \tag{6}$$

Take the top line in the above equation. Traveler $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$ waits without the signal. The supplier earns $p_0 \int_{-1}^{\mathbf{r}(v)} (1+r) dF^n(r)$. With a signal the traveler buys early when the realization is bad and waits otherwise. The supplier earns p_0 in the former case and $p_0 \int_{-1}^{\mathbf{r}(v)} (1+r) dF^g(r)$ in the latter one. The expected supplier revenues with a signal are $\tau_b p_0 + (1-\tau_b) p_0 \int_{-1}^{\mathbf{r}(v)} (1+r) dF^g(r)$. Taking the difference between the two revenues gives the top expression in equation (6). The revenues for $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$ are computed similarly.

Lemma 2 in the Appendix shows that the function $\Delta R(v)$ is equal to zero up to $\mathbf{v}(\rho^b)$, at which point it jumps to a positive value, decreases up to $\mathbf{v}(\rho^n)$ where it drops to a negative value, then increases up to $\mathbf{v}(\rho^g)$ where it is still negative and where it finally jumps back to zero. The supplier loses from travelers with valuation $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$. She gains from travelers with valuation $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$ under a condition that holds in our application.

The signal also changes the expected probability of purchase. This is important to the seller because inventory is central to revenue management.

$$\Delta C(v) = \begin{cases} \tau_b(1 - F^b(\mathbf{r}(v)), & \text{if } v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)] \\ -(1-\tau_b)(1 - F^g(\mathbf{r}(v)), & \text{if } v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]. \end{cases} \tag{7}$$

The signal changes the composition of consumers who end up travelling: A consumer with valuation $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$ is more likely to consume and the opposite holds for a consumer with valuation above $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$. Stated differently, the signal has a positive impact on the set of consumers who wait in the sense that the average consumer in that pool has a higher

[11] Under the assumption of zero cost of capacity, change in welfare is equal to change in consumer surplus $v\Delta C(v)$. However, there is an opportunity cost of capacity under dynamic revenue management. The supplier sells capacity to consumers who arrive continuously till the departure date.

valuation.

Averaging across all consumer valuations, the expected change in supplier revenues is $\Delta\overline{R} = \int_{\mathbf{v}(\rho^b)}^{\mathbf{v}(\rho^g)} \Delta R(v)dG(v)$ where $G(v)$ denotes the CDF of v. The expected change in consumption (quantity sold) is $\Delta\overline{C} = \int_{\mathbf{v}(\rho^b)}^{\mathbf{v}(\rho^g)} C(v)dG(v)$. The sign and magnitude of $\Delta\overline{R}$ and $\Delta\overline{C}$ depend on the four primitives $(\tau_b, F^n(), F^b(), G())$. It is not possible to evaluate these expressions in the absence of information about $G()$. One can make progress, however, by looking at 'small changes' in signals. Takes the family of binary signals that are generated through a linear combination of the prior F^n and a perturbation S, as in, $F(r,x) = F^n(r) + xS(r)$. The prior's cumulative distribution is $F^n(r) = F(r,0)$. The bad signal occurs with probability τ_b and the associated posterior cumulative distribution is $F^b(r|x) = F(r, -x(1-\tau_b))$. Equation (1) says that the good signal's posterior is $F^g(r|x) = F(r, x\tau_b)$. We assume that the posteriors are well defined cumulative distribution function.[12] A signal with $x = 0$ does not convey any information. As the value of x increases so does the weight put on S. We denote $\Delta\overline{R}(x) = R(x) - R(0)$ the changes in expected revenues associated to signal x. $\Delta\overline{C}(x)$ is similarly defined.

Proposition 2. *For small x, the signal has no first order impact on revenue and consumption ($\Delta\overline{R}'(0) = \Delta\overline{C}'(0) = 0$). Revenue increases ($\Delta\overline{R}''(0) > 0$) if and only if $\frac{f(\rho^n)}{1-F(\rho^n)} - \frac{g'(v^n)}{g(v^n)}p_0 > \frac{2}{1+\rho^n}$. Consumption increases ($\Delta\overline{C}''(0) > 0$) if and only if $\frac{f(\rho^n)}{1-F(\rho^n)} - \frac{g'(v^n)}{g(v^n)}p_0 > 0$.*

Proposition 2 is related to the Bayesian persuasion literature. Gentzkow and Kamenica (2011) consider a sender who can inform a single receiver. They characterize the optimal multi-dimensional signal that satisfies Bayes plausibility (equation 1). Our application has multiple receivers. Each receiver's private type is her valuation v. The optimal signal depends on the type distribution $G(v)$ which is unknown. To get around this problem, we consider small changes in posteriors that influence only the receivers in a small neighborhood of the indifferent consumer (type v^n).[13] Proposition 2 delivers testable conditions that rest on minimum structure. When $g'(v^n) = 0$, the signal increases consumption. It also increases revenue if and only if $\frac{f(\rho^n)}{1-F(\rho^n)} > \frac{2}{1+\rho^n}$. In general, the signal is more likely to increase revenue and consumption when there are more consumers to the left of the indifferent consumer than to the right ($g' < 0$).[14] This is simply because the seller always looses from the consumers on the right of the indifferent consumer and benefits (at least for some) of the consumers on the left. Note that the characteristics of the

[12] Take Example 1 as an illustration. We have $S_1(r) = 1$ if $r \geq 0$ and we obtain $F_1^b(r|x) = F^n(r) - (1-\tau_b)x$ if $r \geq 0$ and $F^b(r|x) = F^n(r)$ otherwise. The parameter x measures the change in the probability that the price remains constant.

[13] We also restrict to binary signals. This is not restrictive when signals are coarse which holds with our application.

[14] The seller's revenue in the absence of signal, $R(v) = p_0(1 - G(p_0))$, is concave when $-\frac{g'(v^n)}{g(v^n)}p_0 < 2$, which does not exclude the possibility that $g'(v^n) < 0$.

signal ($\tau_b, S()$) do not influence whether the signal is profitable or not. It influences the scale of the impact.[15] A signal is more profitable if $\tau_b = .5$ and if $S()$ puts more weight on low growth rate realizations.

4 Data and Descriptive Statistics

The Expedia signals are framed in term of the number of available seats at the posted price.[16] The model takes the signal, and posterior distributions, as given. This is reasonable since the focus here is on the consumer decision problem.

4.1 Data Collection

We use a web-scrapping script to collect data on airfares and signals. Many sellers send on a daily basis scarcity signals for a large number of travel itineraries. We select a small subset of sellers, routes, and travel dates. As with past research the sampling is constrained by the time horizon and restrictions on query processing (Edelman, 2012). We end up running daily queries for travels plans that take place at most 100 days in the future. Our dataset is similar to past studies using Internet airfares (Bilotkach and Rupp, 2011; Escobari, 2012; McAfee and Te Velde, 2006) with the shared caveat that what will be learned is sample-specific. Following Escobari (2010) and Bilotkach et al. (2010), we use Expedia which is one of the largest OTA worldwide. We conduct a number of specific searches, or *travel queries*, for one-way trips. A query comprises a route and departure date. The methodology used to collect the data is described in the appendix (Section 10).

The travel queries span 10 routes (city pairs) and 22 departure dates. We selected routes with a single non-stop carrier and significant gains from purchase timing according to hopper.com. The selected routes rank low on the FAA measures of competition. For these routes, we expect sellers to have more information about future fare changes because price randomness associated with competitive dynamics is less important. Many of our routes are the same as the sample of monopoly routes used by Bilotkach and Rupp (2011).

[15] The Appendix, shows that $\Delta\overline{R}''(0)$ and $\Delta\overline{C}''(0)$ are proportional to $\tau_b(1-\tau_b)\left(\int_{-1}^{\rho^n} S(r)dr\right)^2$.

[16] This is due to the way airlines revenue management systems work: A fixed number of seats is made available at a given fare and the system updates the fare when few seats remain (Lazarev, 2013). That being said, airlines and/or Expedia may also strategically manipulate the information sent to consumers. These strategic issues are not relevant here.

A travel query may return a large number of *fight* options. We collect the prices and signals for each option displayed. The signal is a dummy variable that is equal to one if a scarcity message is posted. We conduct travel queries each day starting between the 19th of July and the 26th of October, 2015.[17] We started with a given set of departure date and added new ones as existing ones expire. Denote *day-in-advance* (DiA) the number of days between the booking day and the departure day. Due to the sampling methodology, DiA is about evenly distributed between 1 to 100 days. Figure 2 presents the basic nature of the data. The figure plots the price and signal realization as a function of DiA for a selected set of flights corresponding to a given query (the panels correspond to different flights). Although the signal rarely varies from day to day for a given query, we see much variation in the signal value across flight options for the same query.

4.2 Descriptive Statistics

Table 1 presents summary statistics on the main variables. The price increases on average by 6 percent over the next 7 days. As expected, the average price growth increases with the length of the window used to compute changes (1 versus 7 or 14 days). There is a small chance of scarcity signal ($\tau_b = .33$) and the signal shifts the posteriors to 12 percent under the bad realization and to 4 percent under the good one. The top panel on Figure 3 plots the distribution of price change conditional on the signal. The signal shifts the posterior distribution by significant amounts: The good signal CDF first-order stochastically dominates the bad signal CDF. Moreover, the probability that the price stays the same (jump at $r_7 = 0$) is higher with the good signal. There is also a jump at $r_7 = \infty$ and this is because a flight option may not be available (which is coded as an infinite price increase). The probability of non-availability is higher with the bad signal. All the evidence point to the same conclusion that the signal is informative. That being said, the two conditional distributions have the same support. A large price (in)decrease is consistent with a (good) bad realization.

The distributions presented on the top panel on Figures 3 are averages over all DiA. One would like to make sure that the patterns observed on this figure remain for subsamples of DiA where airfares are stable. The concern is that low DiA could be associated with more frequent bad signals and higher price growth. The bottom panel (Figure 3) reproduces the top panel but only for DiA greater than 56 days (more than 8 weeks prior to departure). The main patterns found on the top panel remain although slightly attenuated. We conclude that the

[17] Fare sales typically last for a few days. Daily price collection minimizes the probability of 'missing' such a fare sale which are available to travelers who check fares on a daily basis.

signal contains information that is not solely about the changes that take place in the last few weeks before departure.

Table 2 reports key quantiles of the distributions of price returns broken down by week. Recall that the signal has no value when $\bar{r}^b < 0$. This is never the case. The two distributions are ordered by FOSD with some exception (in weeks 2-7, the first decile is weakly smaller in the bad state). Thus, Figure 3 conceals heterogeneity that could be important when we compute the value of information. This is relevant because a violation of FOSD can imply negative value of information (which means that the consumer should do the opposite from what the signal advices).

Figure 3 reveals that the main difference between two CDFs is the size of the jump at $r = 0$: Under the good signal there is a greater probability of no price change. Thus, Example 1 may not be a bad approximation of what the signal does to the two posteriors. Using the formula for I specific to Example 1, $I = (1 - \tau_b)\tau_b x$, we plug the values $\tau_b = .33$ from Table 1 and approximation $x = .2$ from Figure 3, to obtain the value $I = .042$. This rough approximation says that the signal increases the consumer utility by 4.2 percent.

5 Results

Given the close connection between the model and data, the empirical analysis uses simple statistics: The empirical prior and the two posteriors $(F^n(), F^b(), F^g())$ are computed to estimate non-parametrically the values of ρ^g, ρ^n, ρ^g and I. We estimate these values using the empirical distributions and report equal-tail confidence intervals that are computed using case resampling bootstrapping.[18] In the computations presented in the Tables, we normalize all expressions assuming $p_0 = 1$ which implies $\mathbf{v}(r) = 1 + r$. The value of v is expressed in units of p_0. Since the average ticket price in the sample is close to \$300, we sometimes multiply the results by that value to present monetary figures. As Li et al. (2014), we initially assume that a traveler postpones her decision only once and by one week. For robustness sake, we subsequently consider sequential delays. We compute the gains from conditioning this decision on the Expedia signal. We initially treat all flight options returned for a query as unique products. Then, we assume that the traveler cares only about the cheapest option.

[18]We re-compute all statistics reported for each bootstrap sample. The bootstrap samples depend on the original data subsample used to compute the statistic. The data subsamples are: entire sample, decomposition by DiA, DiA28-56XCarrier, DiA28-56XRoute.

5.1 Baseline Value of Information

We compute $(\rho^n, \rho^b, \rho^g, I)$ for an unsophisticated consumer who does not condition her decision to buy/wait on public information. In order to eyeball ρ^s, Figure 4 plots the function $f_1(r) = \int_{-1}^{r} F^s(x)dx$ and the forty-five degree line. According to equation (2), ρ^s is found where these two intercept. Repeating this for the two posteriors (conditional on the signal realization), the top line of Table 3 reports the values of (ρ^n, ρ^b, ρ^g) for the entire sample.

The consumers who value traveling between 2.9 and 4.5 percent more than the price of the ticket would wait without a signal but prefer to buy early when the signal is bad. Instead, consumers in the range 4.5 to 5.7 percent would buy early without a signal and change their decision to wait when the signal is good. Stated in dollar amounts, the consumers with a valuation in the interval $[\$308.7, \$317.1]$ respond the signal and the remaining consumers do not respond. The consumers with a valuation below \$308.7 always wait and those with a valuation above \$317.1 always buy.

We use equation (5) to compute $I = \frac{\tau_b(1-\tau_b)}{\rho^n} H(\rho^n)$, which is reported in column 6 of Table 3. The signal increases the utility of the consumer with valuation $\mathbf{v}(\rho^n)$ by about 8.3 percent. This is not a negligible amount. It is higher than the approximation presented in the previous Section. That approximation took into account only the change in probability that the price remain constant. The higher figure presented in Table 3 says that the signal does not only help the traveler to predict events when prices remain constant ($r = 0$) but also when prices are more likely to decrease (on Figure 3, the good posterior lays above the bad one for $r < 0$).

Figure 5 plots the percentage utility change for the consumers who respond to the signal. The consumer who benefits the most correspond to the indifferent consumer used to compute the value of I. The value of information is positive, peaks at ρ^n, and has a tent shape. The average gain amongst the consumers who respond to the signal, $\frac{\Delta \overline{U}}{\rho^g - \rho^b}$, is reported in Table 3 as 0.00186 corresponding to a dollar value of \$.56. The average percentage utility increase amongst the consumers who respond to the signal is $\frac{\Delta \overline{U}}{\overline{U}} = 4\%$. The percentage utility increase is large relative to the absolute utility increase because the consumers who respond to the signal receive a small surplus in the absence of the signal.

One may argue that the positive value of information we found could have just happened by chance. To make sure that this is not the case we go back to Corollary 1 to construct a *placebo test*. The Corollary says that a random signal should have no value. We draw a thousand replications of a random signal (a vector of binary draws with the same τ_b as in the Expedia sample). Corollary 1 predicts that $\rho^g = \rho^n = \rho^b$. This is indeed the case. The averages are exactly equal (.0448) with very small 95 percent confidence intervals (range at most .001). To make sure, we also compute the value of information. It is very small on average ($I = .0001\%$) and the 95 percent confidence interval over the thousand draws is also small, $[-.002, .002]$. The values of information computed from the Expedia sample (reported in Table 3) fall outside this interval.

5.2 Consumer Sophistication, Synthetic Signals, and Sequential Delays

Consumer sopphistication. The signal is correlated with public information that is also correlated with price changes. For example, Expedia scarcity signal are more common close to the departure date (low DiA) which is when prices are also more likely to increase. A sophisticated consumer, who conditions her decision on DiA, may not benefit from the signal if DiA is a sufficient statistics for the signal. Similarly, a sophisticated consumer may condition her posterior on route, airline, or other publicly observable variables. In order to investigate whether a sophisticated consumer still benefits from the information in the signal, we report the value of I after controlling for a set of conditioning variables to document the influence of traveler sophistication on the value of information.

Table 3 reports the value of I for three subsets of DiA: less than 28 days, 28 to 56 days, and more than 56 days. The value are respectively 3.7, 7.4 and 6.7. The value of the signal is of the same magnitude and still significant. The traveller benefits most from signals sent 5 to 8 weeks in advance. The value of information is lowest within four weeks of departure. This is because it is more difficult to predict prices close to the departure date (the three distributions $F^s()$ are closer to one another). The information value of the signal also remains positive and significant after conditioning on airline or routes. The value of I varies across airlines in the range $6.3 - 11.6$. The variation across routes is in the range $4.4 - 8.5$. These differences could be because airfare are more difficult to predict in some routes, because of differences in airline policies, or because competitive conditions vary across routes.

Synthetic signal. To put the reported values of I into perspective, we conduct the following thought experiment. Take a world without signal and consider two travellers: one use DiA to condition her purchasing decision and the other doesn't. We compute how much the sophisticated traveler, who understands that the distribution of price returns depends on DiA, gains relative to an unsophisticated one, who does not base her decision on DiA. This is like creating a *synthetic* DiA signal. Take the case where the synthetic signal conditions the distribution of returns on $DiA \leq 28$ and $DiA > 28$. The motivation is that a consumer who delay purchasing a ticket a month in advance knows that prices are more likely to increase than if she would do so more than a month in advance. The utility gain from becoming sophisticated (learn about the distribution of returns conditional on DiA) is $I = \frac{\tau_b(1-\tau_b)}{\rho^n} H(\rho^n)$ where ρ^n is the indifferent unsophisticated traveler (.448), τ_b is the fraction of observations in the sample with $DiA \leq 28$ (29 percent), $F^n()$ is the distribution of price return in the entire sample, and $H()$ is computed using for $F^g()$ the distribution of price return for $DiA > 28$. In this counterfactual thought experiment, we obtain $I = 10.4\%$ which is a little more than the baseline value of the Expedia signal (8.3%). It is important to note, however, that the Expedia signal contains information that is not contained in the synthetic signal. This is because a consumer who conditions her decision on DiA still benefits from the signal (recall the figures $3.7 - 6.7\%$ reported above). Thus, a sophisticated traveller still benefits from using the signal.

Signal definition. An Expedia signal reveals that there is a limited number, typically between 1 and 5, of seats left at the posted price (see Table 10 in the Appendix). Since the model is based on binary signals, we have defined the two states as no signal versus any number of seats left. For the sake of robustness, we discuss how the results change when we define the binary states differently: a 'bad' signal occurs only when there is exactly one seat left. For this new definition of the signal, we find that the value of information in the entire sample drops to 3.5% and varies between $2.4 - 3.3\%$ across the three DiA windows. The value of information is smaller when scarcity is defined more narrowly. Although consumers gain from knowing that there is only one seat left, they gain even more when they know that there is between one and five seats left.

Flight substitution. Consumers may not care equally about the options returned for a given query. A query returns on average 178 different fare options in our sample. Using all flight options is reasonable if differentiation (in term of departure time or airline loyalty, for example) is important. The assumption is that different consumers are interested in different flight options. An alternative way to proceed, which we present now as a robustness check, is to assume that all flight options are perfect substitute. Under this scenario, the traveler cares only about the cheapest option. Accordingly, we compute the value of information using the much smaller sample composed of the lowest airfare per query. The value of information is 5.9% in this new sample and varies between $3.9 - 7\%$ across the three DiA windows. We conclude that the value of information presented in Table 3 is not driven by outlier fares that are rarely purchased. We find that Expedia signals are valuable even when we take the lowest fare per query.

Sequential delays. The analysis assumes that the traveler receives one signal and can delay purchase once by one week. In practice, the consumer can delay purchase multiple times, and learn information from multiple signal realizations. We investigate how these considerations change the results. Appendix 9 extends the model to three dates to evaluate: (a) the benefit of sequential delays, and (b) the value of receiving multiple signals. This extension is for the sake of robustness and also to compare the value of the signals with the value from delaying consumption. Figure 7 replicates Figures 1 when the consumer can sequentially delay twice. To be consistent with Figure 1, we use the entire sample of observations for which we have three consecutive prices. Figure 7 shows that the key findings from the one delay case carry through to two delays: (a) the gains from information relative to no information (difference between orange and dash red curves) is small relative to the gains from sequential delays relative to no delay (difference between dash red curve and blue line), (b) the indifferent consumer benefits the most from the signal, (c) consumers with high and low surpluses do not benefit or benefit little from the signal.

Some differences between the two Figures are worth mentioning. Consumers with low initial surplus now still benefit from information. To explain, take the consumer $v = p_0$. This consumer initially waits independently of the signal realization. But it is possible for the price to decrease by an amount such that this consumer becomes the indifferent consumer in the intermediate date $\frac{v-p_1}{p_1} = \rho_u$. For this second decision, the consumer cares about the second signal. By extension, the consumer values information in the intermediate period for a small interval of intermediate prices. Obviously these intermediate prices are unlikely when the consumer has a low initial surplus. This explains why the value of information decreases with the relative consumer surplus. Another difference is that the consumer surplus without information has two kinks. The new kink is due to the fact that the price does not change ($r = 0$) with a mass probability.[19]

5.3 Revenue and Consumption

The impact of the signal on revenue and consumption depends on the consumer's willingness to pay. To illustrate, Figure 6 plots the change in revenues, $\Delta R(v)$ from equation 6, computed using the entire sample. (a) Revenue increases from ρ^b till ρ^n ($\Delta R(v) > 0$) and decreases from ρ^n till ρ^g ($\Delta R(v) < 0$), with a large drop at ρ^n. (b) Although the signal has a large impact the revenue per individual type, this impact largely cancels out once averaged across all consumers. These predictions are consistent with the theoretical properties of the revenue function (see Lemma 2 in Appendix). To document the overall impact of the signal on revenue, we report the arithmetic average of revenue and consumption across all consumers (assuming a uniform

[19]Consumers with high relative surplus always buy early. This explains the second kink. The first kind happens because the first period growth rate distribution $F()$ has a unique probability mass at $r = 0$. With a fixed probability, the consumer surplus has the shape of Figure 1. The consumer surplus for the other draws of the first period growth rate also have a kink but these kinks are smoothed out.

distribution of valuation $g(v) = 1$).[20] The main findings for the change in revenue are reported in Table 4:

1. The change in revenue for individual travelers is large and significant. Column 1, for example, reports for consumer $\mathbf{v}(\rho^b)$ a positive increase in revenue, $\Delta R(\mathbf{v}(\rho^b)) = .245$, corresponding to a \$73.5 increase for a \$300 ticket. The largest decrease in revenue occurs for consumer $\mathbf{v}(\rho_u^+)$ with $\Delta R(\mathbf{v}(\rho_u^+)) = -.33$ (column 2).

2. Column 5 reports the change in per-consumer revenue (averaged across all consumers who respond to the signal and measured as a fraction of p_0), $\frac{\Delta \overline{R}}{\rho^g - \rho^b}$, while column 6 reports the percentage increase in revenues for the consumers who respond, $\frac{\Delta \overline{R}}{\overline{R}}$. The numbers are small and non-significant for most subsamples. In four subsamples, the numbers are negative and significant although the effect is small (a decrease of about 3\$ per consumer corresponding to a percentage decrease in revenue of 1%).

3. Column 4 reports the value of $\frac{f(\rho^n)}{1-F(\rho^n)} - \frac{2}{1+\rho^n}$. According to Proposition 2, profits should decrease when this expression is negative.[21] The value is indeed negative in all but two cases. As expected, the subsamples for which the changes in revenue in column 5 and 6 are statistically significant are entirely consistent with the prediction reported in column 4.

Overall, the results suggest that the signal has a small and negative impact on revenue (for some subsamples). This may appear surprising until one acknowledges that revenue is not everything that matters to the seller. To start, the signal also changes consumption. A lower consumption is good for the seller because it increases remaining inventory. Table 5, however, shows that the signal has little impact on consumption:

1. The average per consumer change in consumption, $\frac{\Delta \overline{C}}{\rho^g - \rho^b}$, is reported in column 3. It is small in magnitude and significant for only two subsamples. It is small because the mass of consumers who postpone purchase under a good signal is very close to the mass of consumer who anticipate purchase under a bad signal.

2. The percentage change in consumption, $\frac{\Delta \overline{C}}{\overline{C}}$, is reported in column 4. It is very small in magnitude and insignificant in all but one subsample.

[20] The formula for this expression is derived in the appendix.

[21] This expression has to be greater than $\frac{g'(v^n)}{g(v^n)} p_0$ for the use of signal to be profitable. This term, however, cancels under the uniform assumption ($g'(v) = 0$).

A final consideration, in addition to revenue and consumption, is that the signal also changes how consumer sort between those who purchase early and those postpone. The consumers who change from 'buy' to 'wait' have a higher valuation than those who change their decisions the other way around (valuations $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$ versus $[\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$ respectively). With a scarcity signal, a more attractive set of consumers remains potential buyers. The seller can earn more from her remaining inventory. Thus, a scarcity signal can be beneficial even if does not change revenue or the number of units sold. These dynamic considerations go beyond the scope of what can be done with these data.

6 Conclusion

This paper computes the value of scarcity signals to a Bayesian risk neutral consumer in the context of the Online travel industry. We find that scarcity signals can be valuable to both an unsophisticated traveler (who does not condition her decision on publicly available information) and to a sophisticated one. That being said, scarcity signals benefit only a small range of consumers and even for these consumers the signals have a very small impact on consumer welfare, of the order of a few dollars for a ticket that costs on average \$300. But signals can have a significant proportional impact on expected utility because it influences travelers who would not receive much surplus in the absence of signals. For most consumers the gains from delaying purchase is much larger than the additional gains from conditioning this decision on scarcity signals.

This paper takes a first step toward measuring the benefits of delaying travel booking. The bigger picture is to study the inefficiencies associated with travel booking and this includes the time wasted searching, sub-optimal choice due to pre-mature commitment, no-shows and cancellations, to name just a few issues...

We also find that scarcity signals have little impact on average revenue and average consumption. This is because the revenue increase from the consumers who anticipate a purchase (buy when the signal is bad) are balanced by the losses from the consumers who do the opposite (wait when the signal is good). The finding that scarcity signals do not affect revenue under a Bayesian benchmark is somewhat puzzling and calls for more research on how consumers book tickets. Do some consumers deviate from a pure optimizing behavior? What are the implications for the use of scarcity signals? And more generally, do airlines and search engines manipulate the information that is communicated to consumers?

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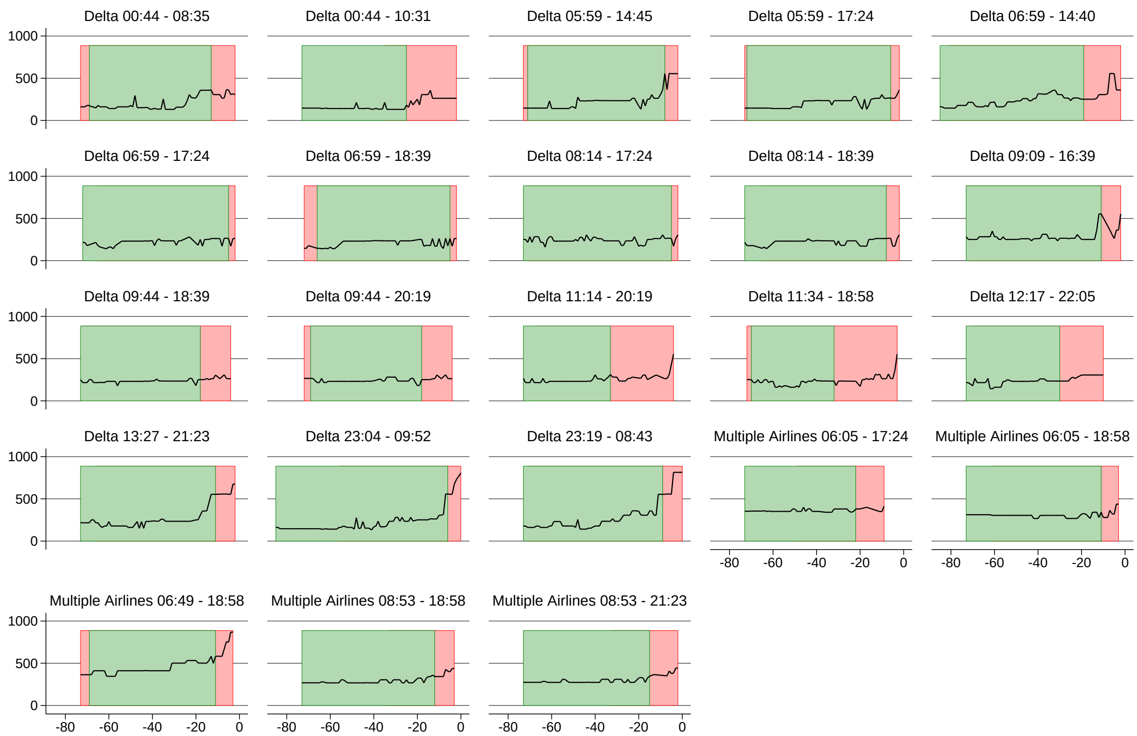
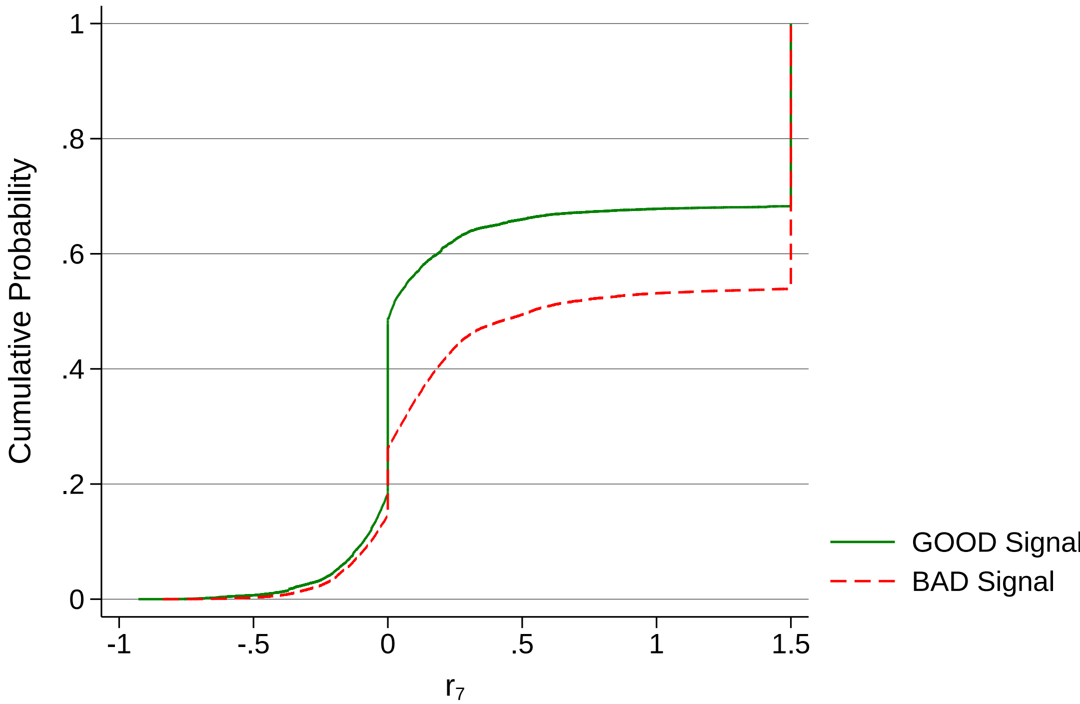
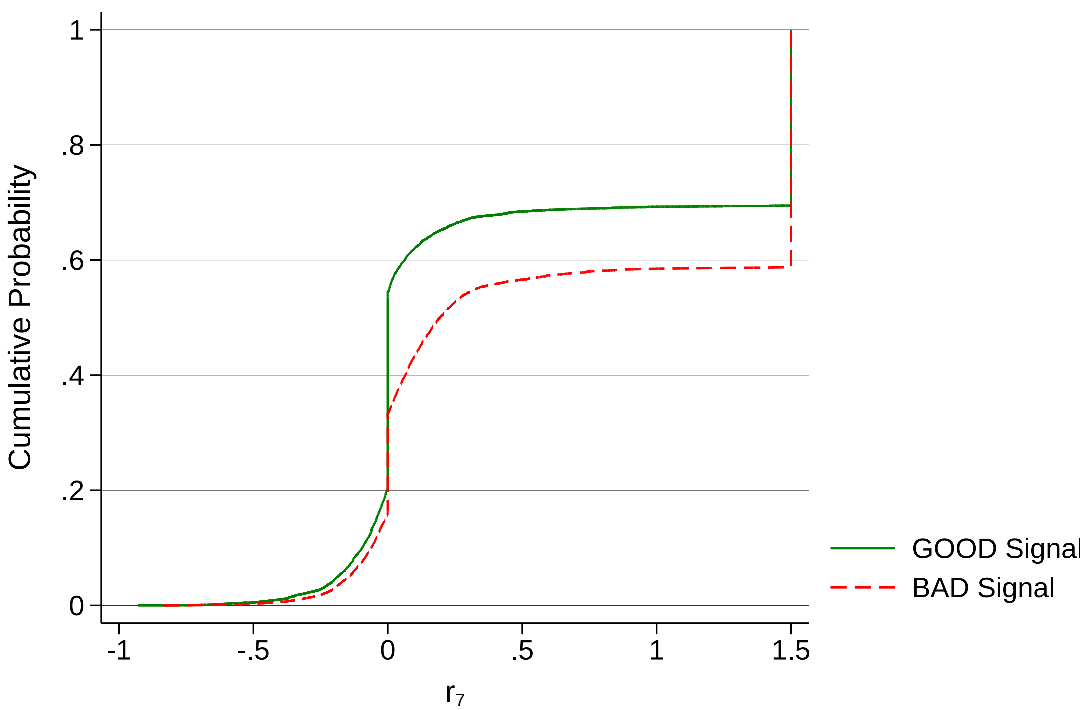


Figure 2: Each panel shows for a given Expedia querie, some of the flights available (panels), and for each flight, non-availability (blank), the value of the signal (green/red shade) and price (black line) as a function of DiA (horizontal axis).

Figure 3: Distributions of Expedia percentage price change over 7 day period as a function of Expedia signal realization, F^n and F^b (top figure is all DiA; bottom one is $DIA \geq 56$).





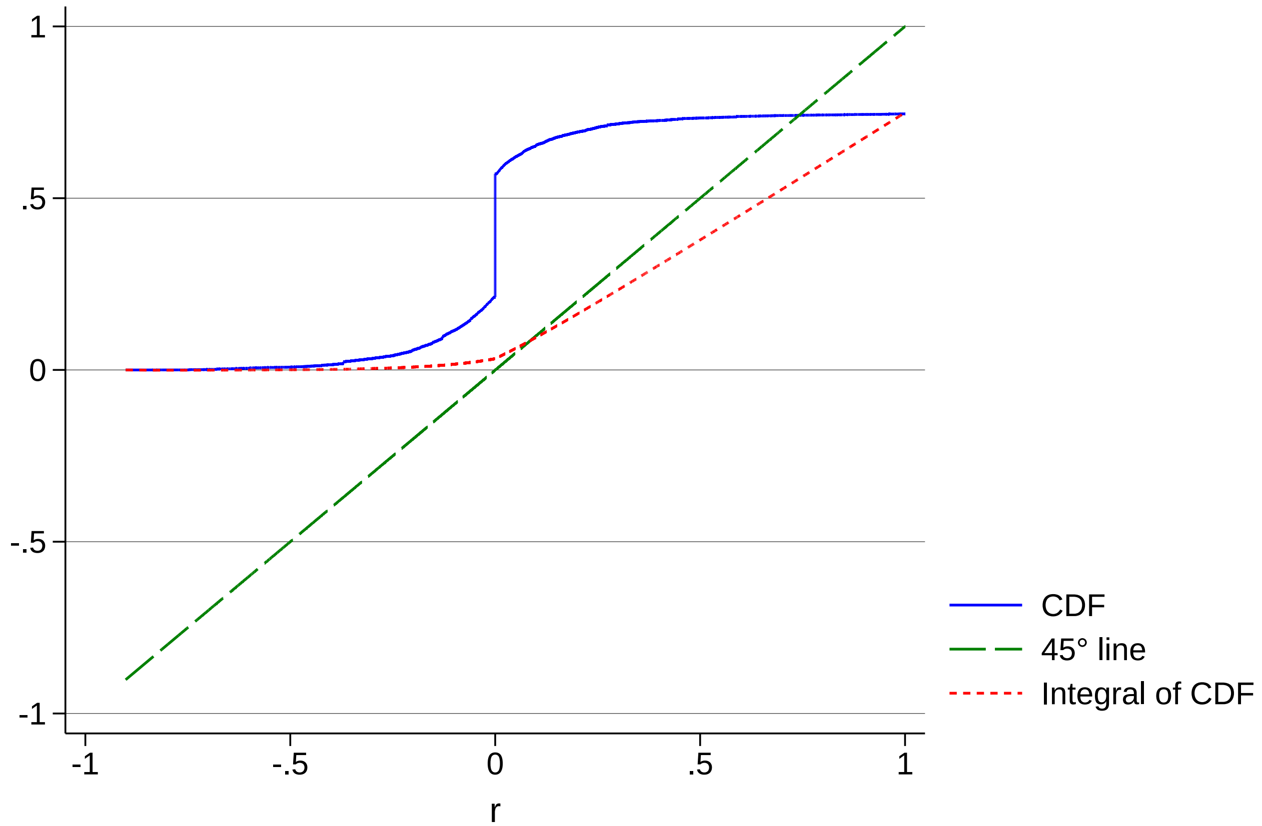


Figure 4: Computation of ρ^n: Functions $f_1(r) = \int_{-1}^{r} F^n(x)dx$ and $f_0(r) = r$ are computed using entire sample. ρ^n is such that $f_1(\rho) = f_0(\rho)$.

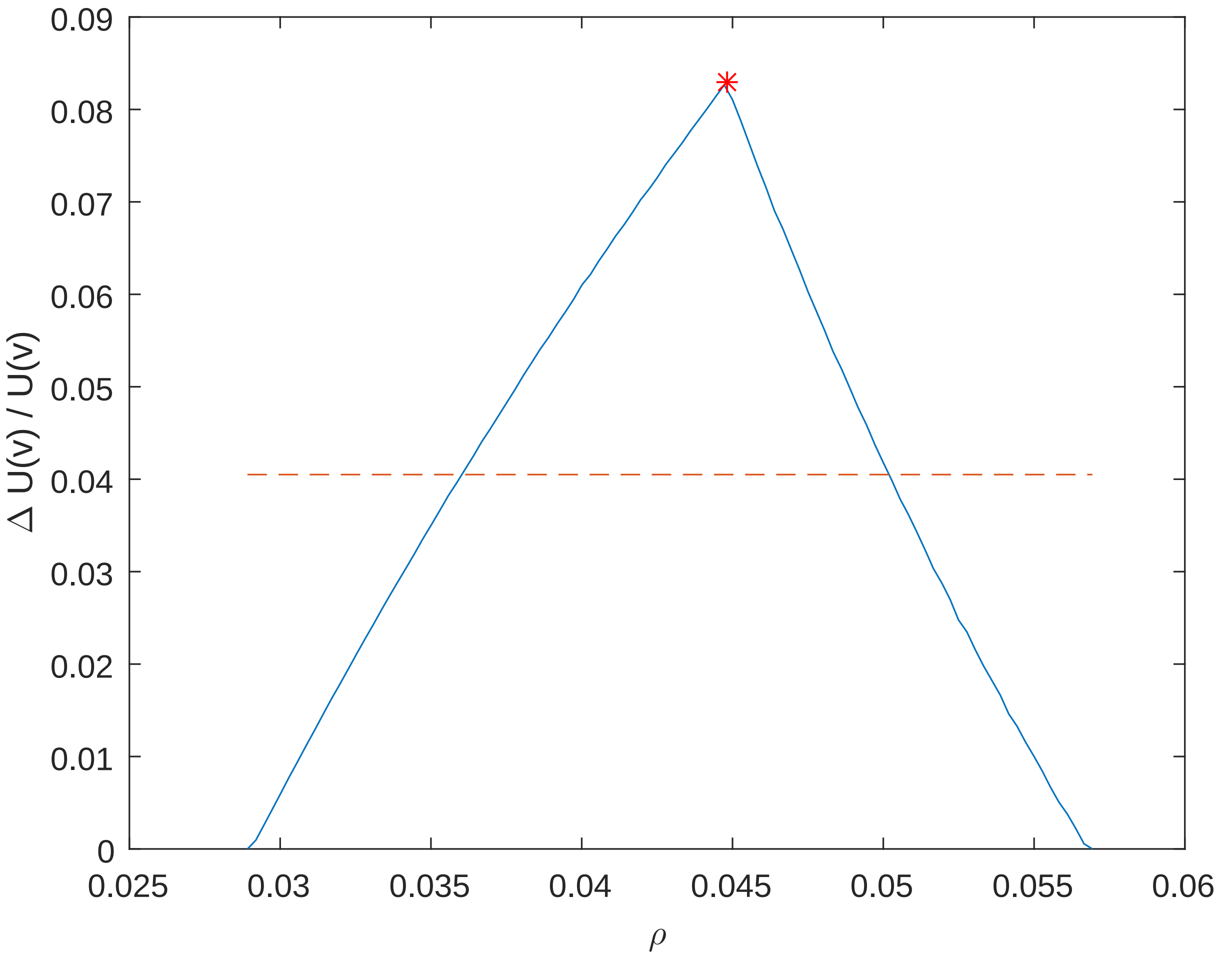


Figure 5: Solid blue line shows percentage utility increase $\frac{\Delta U(\mathbf{v}(r))}{U(\mathbf{v}(r))}$. The red dashed line plots the average across all $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^g)]$. The red star presents I, the maximum percentage utility increase.

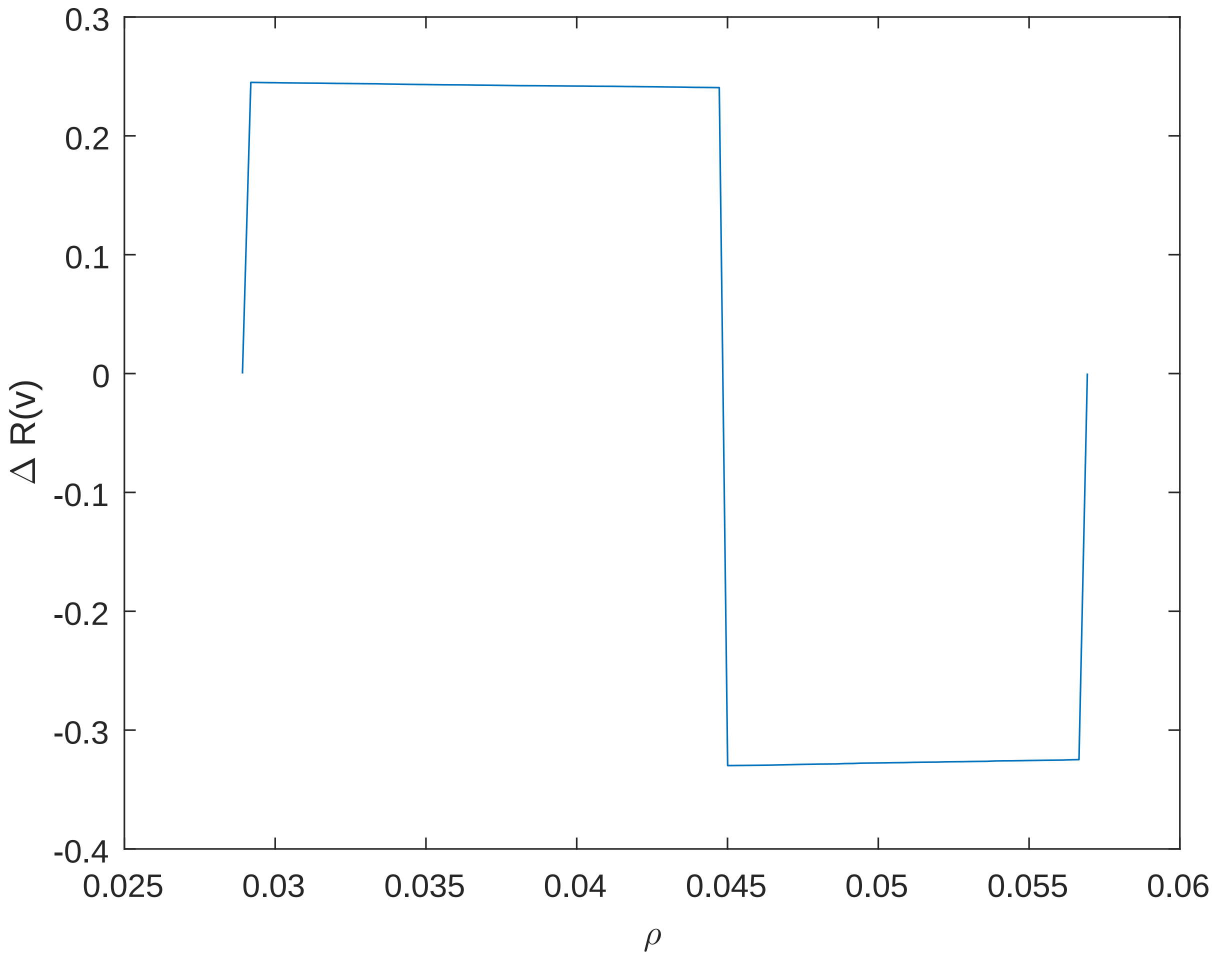


Figure 6: Change in Revenue $R(v)$ (measured as a fraction of p_0).

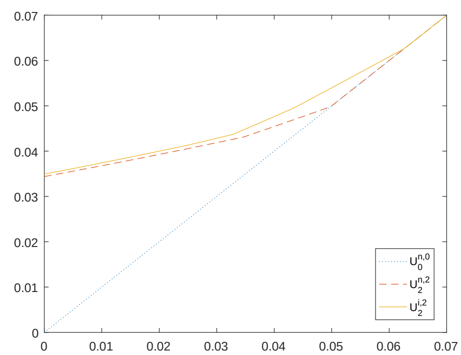


Figure 7: Expected utilities (a) without waiting option (blue dashes), (b) with the option of delaying purchase twice (red dashes), (c) with the option of delaying purchase twice and two signals (orange line).

Table 1: Signals, Prices and Availability

Variable	Obs	Mean	Std. Dev.	P25	P50	P75
Signal (1: BAD, 0: GOOD)	539506	.33	.47	0	0	1
Days in Advance	539506	47.66	26.91	24	47	71
Price	539506	299.91	160.07	165.1	274.6	409.1
Price | BAD	180071	293.58	148.93	179.2	266.6	366.2
Price | GOOD	359435	303.08	165.28	162.1	281.6	409.1
r_1	455910	.02	.19	0	0	0
r_1| BAD	141828	.04	.22	0	0	.03
r_1| GOOD	314082	0	.16	0	0	0
r_7	345129	.06	.32	-.01	0	.08
r_7| BAD	98109	.12	.36	-.01	.01	.2
r_7| GOOD	247020	.04	.29	-.01	0	.02
r_{14}	266187	.11	.42	-.04	0	.16
r_{14}| BAD	71179	.18	.48	-.04	.06	.27
r_{14}| GOOD	195008	.08	.39	-.04	0	.1

Table 2: r_{t+7} by Signal

Variable	Obs	Mean	Std. Dev.	P10	P25	P50	P75	P90
				Entire Sample				
r_{t+7}	357894	.06	.31	-.15	-.01	0	.08	.29
$r_{t+7}\|B$	101152	.12	.36	-.15	-.01	.01	.2	.47
$r_{t+7}\|G$	256742	.04	.29	-.15	-.01	0	.02	.23
				Week 1				
r_{t+7}	11474	.3	.56	-.13	0	.2	.49	.84
$r_{t+7}\|B$	4642	.36	.66	-.21	0	.21	.56	1.03
$r_{t+7}\|G$	6832	.25	.47	-.04	0	.19	.36	.68
				Week 2				
r_{t+7}	27221	.24	.49	-.13	0	.14	.36	.68
$r_{t+7}\|B$	11286	.26	.49	-.17	0	.16	.43	.77
$r_{t+7}\|G$	15935	.23	.49	-.05	0	.13	.32	.61
				Week 3				
r_{t+7}	29653	.15	.41	-.15	0	.02	.23	.52
$r_{t+7}\|B$	11397	.18	.39	-.17	0	.1	.29	.57
$r_{t+7}\|G$	18256	.13	.43	-.13	0	0	.18	.44
				Week 4				
r_{t+7}	30376	.08	.35	-.18	-.02	0	.12	.35
$r_{t+7}\|B$	10601	.12	.35	-.18	-.03	.03	.22	.47
$r_{t+7}\|G$	19775	.06	.35	-.17	-.01	0	.04	.27
				Week 5				
r_{t+7}	30084	.03	.29	-.19	-.04	0	.05	.24
$r_{t+7}\|B$	9393	.08	.32	-.17	-.05	0	.16	.35
$r_{t+7}\|G$	20691	.01	.27	-.2	-.04	0	0	.17
				Week 6				
r_{t+7}	29740	.03	.25	-.15	-.02	0	.04	.22
$r_{t+7}\|B$	8956	.07	.26	-.15	-.03	0	.14	.31
$r_{t+7}\|G$	20784	.01	.24	-.15	-.02	0	0	.15
				Week 7				
r_{t+7}	30133	.02	.22	-.14	-.02	0	.03	.2
$r_{t+7}\|B$	8044	.06	.24	-.15	-.02	0	.13	.28
$r_{t+7}\|G$	22089	.01	.21	-.14	-.02	0	0	.14
				Week 8				
r_{t+7}	30384	.02	.22	-.14	-.01	0	.03	.19
$r_{t+7}\|B$	7368	.06	.23	-.12	-.01	0	.12	.26
$r_{t+7}\|G$	23016	.01	.22	-.14	-.02	0	0	.14
				Week 9				
r_{t+7}	29557	.02	.23	-.14	-.03	0	.01	.18
$r_{t+7}\|B$	6813	.06	.27	-.13	-.02	0	.11	.26
$r_{t+7}\|G$	22744	0	.21	-.15	-.03	0	0	.13
				Week 10				
r_{t+7}	26336	.03	.24	-.14	-.02	0	.01	.17
$r_{t+7}\|B$	5555	.07	.29	-.12	0	0	.12	.28
$r_{t+7}\|G$	20781	.01	.23	-.14	-.02	0	0	.13

Table 3: Main Results - Utility

Utility	ρ^u	ρ^b	ρ^g	τ_b	I	$\frac{\Delta \overline{U}}{\rho^g - \rho^b}$	$\frac{\Delta \overline{U}}{\overline{U}}$	N
All	0.0448	0.0291	0.0568	0.334	0.0829	0.00186	0.0405	539506
	(0.0437, 0.0458)	(0.0281, 0.0299)	(0.0553, 0.0583)		(0.0795, 0.0877)	(0.00177, 0.00198)	(0.0389, 0.0427)	
$DiA \leq 28$	0.0234	0.0212	0.0257	0.469	0.0367	0.000436	0.0181	153842
	(0.0224, 0.0243)	(0.0202, 0.0222)	(0.0243, 0.0272)		(0.0243, 0.049)	(0.000298, 0.000589)	(0.0124, 0.0242)	
$28 \leq DiA \leq 56$	0.0672	0.0418	0.0875	0.327	0.0743	0.00249	0.0364	165325
	(0.0649, 0.0697)	(0.0396, 0.0438)	(0.0837, 0.0911)		(0.0694, 0.0793)	(0.00228, 0.00268)	(0.034, 0.0388)	
$56 \leq DiA \leq 84$	0.0569	0.0316	0.0702	0.25	0.0675	0.00192	0.0348	155047
	(0.0547, 0.0587)	(0.0295, 0.0338)	(0.0672, 0.0729)		(0.0623, 0.0736)	(0.00175, 0.0021)	(0.032, 0.038)	
Delta	0.0607	0.04	0.0793	0.362	0.0824	0.00249	0.0395	157598
	(0.0589, 0.0622)	(0.0382, 0.0416)	(0.076, 0.0824)		(0.0755, 0.0882)	(0.00228, 0.00272)	(0.0366, 0.0423)	
American Airlines	0.0544	0.0306	0.0773	0.371	0.116	0.00317	0.0548	112925
	(0.0515, 0.0573)	(0.0284, 0.0326)	(0.0726, 0.0821)		(0.107, 0.125)	(0.00286, 0.00347)	(0.0514, 0.0588)	
Multiple Airlines	0.0208	0.0156	0.0243	0.339	0.0632	0.000658	0.0311	149417
	(0.0201, 0.0215)	(0.0147, 0.0164)	(0.0233, 0.0254)		(0.054, 0.0729)	(0.000559, 0.000763)	(0.0265, 0.0358)	
US Airways	0.0509	0.0357	0.0613	0.318	0.0681	0.00171	0.0332	48562
	(0.0467, 0.0552)	(0.032, 0.0394)	(0.0553, 0.0665)		(0.0555, 0.0816)	(0.00133, 0.00208)	(0.0273, 0.0398)	
United	0.0639	0.0253	0.0853	0.258	0.107	0.00342	0.055	39220
	(0.0601, 0.0678)	(0.0225, 0.0286)	(0.0795, 0.0916)		(0.096, 0.114)	(0.00302, 0.00379)	(0.0502, 0.0594)	
MIA-BOS, $28 \leq DiA \leq 56$	0.0663	0.0475	0.0873	0.434	0.0778	0.00251	0.0357	21792
	(0.0611, 0.072)	(0.0425, 0.0525)	(0.0774, 0.0966)		(0.0624, 0.0944)	(0.00193, 0.00315)	(0.0292, 0.0428)	
MIA-DFW, $28 \leq DiA \leq 56$	0.104	0.0608	0.137	0.345	0.0842	0.00428	0.0406	20522
	(0.0952, 0.113)	(0.0516, 0.0684)	(0.123, 0.15)		(0.0682, 0.0982)	(0.00339, 0.00506)	(0.0336, 0.0474)	
SNA-DFW, $28 \leq DiA \leq 56$	0.0813	0.0289	0.111	0.199	0.0854	0.00331	0.0432	15595
	(0.0693, 0.0919)	(0.0226, 0.0336)	(0.0936, 0.127)		(0.0753, 0.0974)	(0.00273, 0.00402)	(0.0383, 0.0498)	
PHX-MDW, $28 \leq DiA \leq 56$	0.0506	0.0403	0.0585	0.362	0.0436	0.00108	0.021	13451
	(0.0449, 0.0547)	(0.0337, 0.0461)	(0.0519, 0.0641)		(0.0235, 0.0618)	(0.000547, 0.00154)	(0.0111, 0.03)	

Table 4: Main Results - Revenue

Revenue	$\Delta R(v(\rho^b))$	$\Delta R(v(\rho^{u+}))$	$\frac{f(\rho^u)}{1-F(\rho^u)} - \frac{2}{1+\rho^u}$	$\frac{\Delta \overline{R}}{\rho^g - \rho^b}$	$\frac{\Delta \overline{R}}{\overline{R}}$	N
All	0.245	-0.33	-1.91	-0.0024	-0.0035	539506
	(0.244, 0.246)	(-0.333, -0.327)	(-1.92, -1.91)	(-0.0037, 0.004)	(-0.0053, 0.0057)	
$DiA \leq 28$	0.403	-0.393	-1.95	-0.0016	-0.0026	153842
	(0.401, 0.405)	(-0.398, -0.39)	(-1.96, -1.95)	(-0.0075, 0.0074)	(-0.012, 0.012)	
$28 \leq DiA \leq 56$	0.209	-0.272	-1.87	-0.0075	-0.0097	165325
	(0.206, 0.211)	(-0.277, -0.268)	(-1.88, -1.87)	(-0.014, -0.0062)	(-0.018, -0.008)	
$56 \leq DiA \leq 84$	0.16	-0.311	-1.89	-0.0062	-0.0088	155047
	(0.157, 0.162)	(-0.316, -0.305)	(-1.9, -1.89)	(-0.012, -0.0046)	(-0.017, -0.0065)	
Delta	0.255	-0.295	-1.89	-0.0028	-0.0038	157598
	(0.253, 0.258)	(-0.299, -0.29)	(-1.89, -1.88)	(-0.0071, 0.0052)	(-0.0096, 0.0069)	
American Airlines	0.278	-0.303	-1.9	-0.0068	-0.0093	112925
	(0.275, 0.282)	(-0.311, -0.296)	(-1.9, -1.89)	(-0.015, 0.0012)	(-0.021, 0.0016)	
Multiple Airlines	0.258	-0.389	-1.96	-0.00073	-0.0012	149417
	(0.256, 0.261)	(-0.395, -0.384)	(-1.96, -1.96)	(-0.0078, 0.0069)	(-0.013, 0.011)	
US Airways	0.238	-0.358	-1.9	-0.0069	-0.01	48562
	(0.233, 0.242)	(-0.37, -0.347)	(-1.91, -1.9)	(-0.02, 0.0008)	(-0.029, 0.0011)	
United	0.186	-0.355	-1.88	-0.0091	-0.013	39220
	(0.181, 0.19)	(-0.364, -0.346)	(-1.89, -1.87)	(-0.02, -0.002)	(-0.029, -0.0032)	
MIA-BOS, $28 \leq DiA \leq 56$	0.29	-0.271	-1.88	-0.0074	-0.0096	21792
	(0.283, 0.299)	(-0.281, -0.26)	(-1.88, -1.87)	(-0.019, 0.0033)	(-0.025, 0.0042)	
MIA-DFW, $28 \leq DiA \leq 56$	0.222	-0.311	-1.81	-0.012	-0.016	20522
	(0.215, 0.229)	(-0.322, -0.301)	(-1.83, -1.8)	(-0.022, -0.0025)	(-0.03, -0.0039)	
SNA-DFW, $28 \leq DiA \leq 56$	0.139	-0.261	-1.85	-0.0064	-0.0083	15595
	(0.133, 0.144)	(-0.274, -0.247)	(-1.87, -1.83)	(-0.016, 0.0036)	(-0.021, 0.0045)	
PHX-MDW, $28 \leq DiA \leq 56$	0.225	-0.305	-1.9	-0.0035	-0.0048	13451
	(0.214, 0.235)	(-0.318, -0.291)	(-1.91, -1.9)	(-0.013, 0.01)	(-0.018, 0.014)	

Table 5: Main Results - Consumption

Consumption	$\Delta\overline{C}$	$\frac{\Delta\overline{C}}{\rho^g-\rho^b}$	$\overline{C}$	$\frac{\Delta\overline{C}}{\overline{C}}$	N
All	-2.6e-06	-9.5e-05	0.019	-0.00014	539506
	(-8.1e-05, 8.9e-05)	(-0.0029, 0.0032)	(0.018, 0.02)	(-0.0043, 0.0046)	
$DiA \leq 28$	2.8e-05	0.0063	0.0028	0.01	153842
	(1.1e-05, 0.00011)	(0.0022, 0.023)	(0.0019, 0.0038)	(0.0034, 0.038)	
$28 \leq DiA \leq 56$	-5.4e-06	-0.00012	0.034	-0.00016	165325
	(-0.00016, 0.00016)	(-0.0035, 0.0036)	(0.031, 0.037)	(-0.0047, 0.0048)	
$56 \leq DiA \leq 84$	-3.3e-05	-0.00086	0.027	-0.0012	155047
	(-0.00018, 0.00012)	(-0.0048, 0.0033)	(0.025, 0.03)	(-0.0067, 0.0047)	
Delta	2e-05	0.00051	0.028	0.0007	157598
	(-0.00014, 0.00031)	(-0.0039, 0.0077)	(0.026, 0.031)	(-0.0053, 0.011)	
American Airlines	5.8e-05	0.0012	0.033	0.0017	112925
	(-0.00025, 0.00054)	(-0.005, 0.012)	(0.03, 0.037)	(-0.007, 0.016)	
Multiple Airlines	-2.3e-07	-2.6e-05	0.0054	-4.2e-05	149417
	(-5.4e-05, 5.5e-05)	(-0.0059, 0.0062)	(0.0046, 0.0062)	(-0.0096, 0.0099)	
US Airways	0.00012	0.0045	0.017	0.0069	48562
	(-0.00019, 0.00078)	(-0.0055, 0.03)	(0.012, 0.02)	(-0.0083, 0.046)	
United	0.0003	0.0049	0.04	0.0074	39220
	(-4.9e-05, 0.0011)	(-0.00087, 0.018)	(0.035, 0.045)	(-0.0015, 0.027)	
MIA-BOS, $28 \leq DiA \leq 56$	-0.00025	-0.0063	0.03	-0.0083	21792
	(-0.0015, 0.00027)	(-0.038, 0.0047)	(0.021, 0.038)	(-0.051, 0.0061)	
MIA-DFW, $28 \leq DiA \leq 56$	0.00026	0.0034	0.055	0.0047	20522
	(-0.00034, 0.0023)	(-0.0054, 0.03)	(0.043, 0.065)	(-0.0075, 0.041)	
SNA-DFW, $28 \leq DiA \leq 56$	-0.00043	-0.0052	0.062	-0.0069	15595
	(-0.0024, 0.00097)	(-0.028, 0.012)	(0.048, 0.076)	(-0.038, 0.016)	
PHX-MDW, $28 \leq DiA \leq 56$	-4e-05	-0.0022	0.013	-0.0031	13451
	(-0.00072, 0.00036)	(-0.036, 0.025)	(0.0067, 0.018)	(-0.052, 0.035)	

7 Appendix: Notations

Table 6: Notations

$g(v), G(v)$	Consumer valuation PDF and cdf
$s \in \{n, b, g\}$	State (no signal, bad, good)
$p_1 = (1 + r^s)p_0$	Prices (period 0 and 1)
$\overline{r}^s$	Expected growth rate
$F^s()$	Distributions of growth rate (eq. 1)
$H(r)$	$\int_{-1}^{r}(F^g(y) - F^b(y))dy$
$\mathbf{v}(r)$, $\mathbf{r}(v)$	$\mathbf{v}(r) = p_0(1 + r)$, $\mathbf{r}(v) = \frac{v}{p_0} - 1$
ρ^s	Marginal consumer (eq. 2)
$U_0(v)$	Utility of consumer v if she does not delay purchase
$U_1^s(v)$	Expected utility of consumer v given belief F^s (eq. 3)
$U_1^i(v)$	$\tau_b U^b(v) + (1 - \tau_b)U^g(v)$
I	Maximum value of information (eq. 5)
$R(v)$, $C(v)$	Revenue and consumption from consumer v under prior ($s = n$)
$\overline{U}$, $\overline{R}$, $\overline{C}$	$\overline{U} = \int_{\mathbf{v}(\rho^b)}^{\mathbf{v}(\rho^g)} U^n(v)dv$ and same for $\overline{R}$, $\overline{C}$
$\Delta U(v)$, $\Delta R(v)$, $\Delta C(v)$	Impact of signal on individual outcomes (see eq. 4, 6 and 7)
$\Delta\overline{U}$, $\Delta\overline{R}$, $\Delta\overline{C}$	Average impact across all v (e.g. $\Delta\overline{U} = \int_{\mathbf{v}(\rho^b)}^{\mathbf{v}(\rho^g)} \Delta U(v)dv$)
$F(r, y)$, $S(r)$, $F^s(r\|x)$	Small signal (Proposition 2)
$\Delta\overline{U}(x)$, $\Delta\overline{R}(x)$, $\Delta\overline{C}(x)$	Utility, Revenue Consumption with small signal

In the result section, we normalize all expression assuming $p_0 = 1$, $\mathbf{v}(r) = 1 + r$ and $\mathbf{r}(v) = v - 1$. The value of v is thus expressed in units of p_0. Averages are computed over the individuals who respond to the signal, $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^g)]$, and using arithmetic average ($g(v) = 1$). We obtain:[22]

Table 7: Utility, Revenue, Consumption

	$v \le \mathbf{v}(\rho^n)$	$v \ge \mathbf{v}(\rho^n)$	Average	
$U^n(v)$	$\int_{-1}^{v-1} F^n(x)dx$	$v - 1$	$\overline{U}$	$\int_{\rho^b}^{\rho^n} \left(\int_{-1}^{r} F^n(x)dx\right) dr + \int_{\rho^n}^{\rho^g} rdr$
$R(v)$	$\int_{-1}^{v-1}(1 + r)dF^n(r)$	1	$\overline{R}$	$\int_{\rho^b}^{\rho^n} \left((1 + r)F^n(r) - \int_{-1}^{r} F^n(x)dx\right) dr + \rho^g - \rho^n$
$C(v)$	$F(v - 1)$	1	$\overline{C}$	$\int_{\rho^b}^{\rho^n} F^n(r)dr + \rho^g - \rho^n$

Using equations 4, 6 and 7, we obtain $\Delta\overline{U}$, $\Delta\overline{R}$, $\Delta\overline{C}$. For example, $\Delta\overline{U} = \tau_b \int_{\rho^b}^{\rho^n} \left(r - \int_{-1}^{r} F^b(x)dx\right) dr + (1 - \tau_b) \int_{\rho^n}^{\rho^g} \left(\int_{-1}^{r} F^g(x)dx - r\right) dr$.

[22]Note that $\int_{-1}^{r}(1 + x)dF(x) = (1 + r)F(r) - \int_{-1}^{r} F(x)dx$.

8 Appendix: Proofs

Proof of Lemma 1 The function $G^s(x) = x - \int_{-1}^{x} F^s(r)dr$ is strictly increasing on the support of F^s. We have $G^s(-1) < 0$ and $\lim_\infty G^s(x) = \lim_\infty \left(x + \int_{-1}^{x} rdF^s(r) - xF^s(x)\right) = \overline{r}^s$. If $\overline{r}^s > 0$, equation (2) has a unique solution ρ^s and this solution is non-negative since $G^s(0) \leq 0$.

Next, we prove by contradiction that $Min(\rho^g, \rho^b) \leq \rho^n \leq Max(\rho^g, \rho^b)$. Assume, for example, that $Min(\rho^g, \rho^b) > \rho^n$. The following three obsevations (a) $\rho^n < \rho^g$, (b) $G^g(x)$ strictly increasing, and (c) $G^g(\rho^g) = 0$ imply that $G^g(\rho^n) < 0$. The same reasoning implies that $G^b(\rho^n) < 0$. Thus $\tau_b G^b(\rho^n) + (1 - \tau^b)G^g(\rho^n) < 0$ or $\rho^n < \int_{-1}^{\rho^n} F^n(r)dr$. A contradiction. The same logic proves the other inequality.

Finally, $v - p_0 - E(Max(v - p_1, 0)|s) = p_0 G^s(\mathbf{r}(v))$. Thus, consumer v stricly prefers to buy early when her belief is $F^s()$ if and only if $G^s(\mathbf{r}(v)) > 0$, or $v \in (\mathbf{v}(\rho^s), \infty)$. □

Proof of Proposition 1: (a) For $v \in [0, \mathbf{v}(\rho^b))$ we have $E(Max(v-p_1, 0)|s) > v-p_0$. Thus, the consumer prefers to wait with or without signal. (b) For $v \in (\mathbf{v}(\rho^b), \mathbf{v}(\rho^n))$ we have $E(Max(v - p_1, 0)|n) > v - p_0$, $E(Max(v-p_1, 0)|g) > v-p_0$ and $E(Max(v-p_1, 0)|b) < v-p_0$. Thus, the consumer buys early only if the signal is bad. (c) For $v \in (\mathbf{v}(\rho^n), \mathbf{v}(\rho^g))$ we have $v-p_0 > E(Max(v-p_1, 0)|n)$, $v-p_0 > E(Max(v-p_1, 0)|b)$ and $E(Max(v-p_1, 0)|g) > v-p_0$. Thus, the consumer waits only if the signal is good. (d) For $v > \mathbf{v}(\rho^g)$ we have $E(Max(v - p_1, 0)|s) < v - p_0$. Thus, the consumer prefers to buy early with or without signal. □

Derivation of equation (4) for $\Delta U(v)$: Consumer $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$ waits without a signal. Her expected utility is $p_0 \int_{-1}^{\mathbf{r}(v)} F^n(r)dr$. With the signal, she wait when the realization is good. Her expected utility is $p_0 \int_{-1}^{\mathbf{r}(v)} F^g(r)dr$. She buys early when the realization is bad and receive utility $v - p_0$. Taking expectation, we obtain:

$$\Delta U(v) = \tau_b(v - p_0) + (1 - \tau_b)p_0 \int_{-1}^{\mathbf{r}(v)} F^g(r)dr - p_0 \int_{-1}^{\mathbf{r}(v)} F^n(r)dr$$

for $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$. Using identity (1), we obtain the top part in equation (4). The same reasoning applies to the bottom part of equation (4) corresponding to $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$.

From Lemma 1 and the convention used for labeling states (b,g), we have $p_0 \int_{-1}^{\mathbf{r}(v)} F^n(r)dr > v - p_0 > p_0 \int_{-1}^{\mathbf{r}(v)} F^b(r)dr$ for $v \in (\mathbf{v}(\rho^b), \mathbf{v}(\rho^n))$. This implies $\Delta U(v) > 0$. Similarly, we have $p_0 \int_{-1}^{\mathbf{r}(v)} F^g(r)dr > v - p_0 > p_0 \int_{-1}^{\mathbf{r}(v)} F^n(r)dr$ for $v \in (\mathbf{v}(\rho^n), \mathbf{v}(\rho^g))$, and again $\Delta U(v) > 0$.

Proof of Corollary 2: $\frac{\partial}{\partial v}\Delta U(v) = \tau_b(1 - F^b(\mathbf{r}(v))) \geq 0$ for $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$ and $\frac{\partial}{\partial v}\Delta U(v) = (1 - \tau_b)(F^g(\mathbf{r}(v)) - 1) \leq 0$ for $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$. Thus, $\Delta U(v)$ reaches a maximum at $\mathbf{v}(\rho^n)$. □

Properties of the Revenue Function: Using the identity $\int_{-1}^{\rho}(1+r)dF(r) = (1+\rho)F(\rho) - \int_{-1}^{\rho} F(r)dr$, we can rewrite equation (6) as

$$\Delta R(v) = \begin{cases} p_0\tau_b \left(1-(1+\mathbf{r}(v))F^b(\mathbf{r}(v)) + \int_{-1}^{\mathbf{r}(v)} F^b(r)dr\right), & \text{if } v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)] \\ -p_0(1-\tau_b)\left(1-(1+\mathbf{r}(v))F^g(\mathbf{r}(v)) + \int_{-1}^{\mathbf{r}(v)} F^g(r)dr\right), & \text{if } v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]. \end{cases} \tag{8}$$

The supplier's profit function has the following properties.

Lemma 2. *(a)* $\Delta R'(v) < 0$ *for* $v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)]$.

$\Delta R'(v) > 0$ *for* $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$.

(b) $\Delta R(\mathbf{v}(\rho^b)) = p_0\tau_b(1+\rho^b)(1-F^b(\rho^b)) > 0$,

$\Delta R(\mathbf{v}(\rho^g)) = -p_0(1-\tau_b)(1+\rho^g)(1-F^g(\rho^g)) < 0$.

$\lim_{\rho\to\rho_u^+} \Delta R(\mathbf{v}((\rho)) < 0$.

$\lim_{\rho\to\rho_u^+} \Delta R(\mathbf{v}((\rho)) - \lim_{\rho\to\rho_u^-} \Delta R(\mathbf{v}((\rho)) = -p_0(1+\rho^n)(1-F^n(\rho^n)) < 0$.

$\lim_{\rho\to\rho_u^-} \Delta R(\mathbf{v}((\rho)) > 0$ *if and only if* $(1+\rho^b)(1-F^b(\rho^n)) > \int_{\rho^b}^{\rho^n} (F^b(\rho^n) - F^b(x))dx$.

(c) $\Delta R(v) < 0$ *for* $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$.

Note that the Lemma implies that $\Delta R(\mathbf{v}(\rho^b)) = Max_v \Delta R(v)$ and $\lim_{\rho\to\rho_u^+} \Delta R(\mathbf{v}((\rho)) = Min_v \Delta R(v)$. The jump at $\mathbf{v}(\rho^b)$ is equal to $\Delta R(\mathbf{v}(\rho^b)) = p_0\tau_b(1+\rho^b)(1-F^b(\rho^b))$.

Proof of Lemma 2 (a) Take derivatives with respect to v in equation (6)

$$\Delta R'(v) = \begin{cases} -p_0\tau_b(1+\mathbf{r}(v))f^b(\mathbf{r}(v)) < 0, & \text{if } v \in [\mathbf{v}(\rho^b), \mathbf{v}(\rho^n)] \\ p_0(1-\tau_b)(1+\mathbf{r}(v))f^g(\mathbf{r}(v)) > 0, & \text{if } v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]. \end{cases}$$

(b) From equation (8), we have $\Delta R(\mathbf{v}(\rho^b)) = p_0\tau_b\left(1-(1+\rho^b)F^b(\rho^b) + \int_{-1}^{\rho^b} F^b(r)dr\right)$ and since $\int_{-1}^{\rho^b} F^b(r)dr = \rho^b$, we obtain, $\Delta R(\mathbf{v}(\rho^b)) = p_0\tau_b(1+\rho^b)(1-F^b(\rho^b))$. Applying the same logic one obtains $\Delta R(\mathbf{v}(\rho^g))$.

Evaluate equation (8) at ρ_u^+, add ρ^n and substract $\int_{-1}^{\rho^n} F^n(r)dr$ (recall $\rho^n = \int_{-1}^{\rho^n} F^n(r)dr$), to obtain, $\lim_{\rho\to\rho_u^+} \Delta R(\mathbf{v}((\rho)) = -p_0(1-\tau_b)\left((1+\rho^n)(1-F^g(\rho^n)) + \int_{-1}^{\rho^n}(F^g(r) - F^n(r))dr\right) < 0$ where the inequality follows from $F^g(r) > F^n(r)$.

Evaluate equation (8) at ρ_u^-, $\lim_{\rho\to\rho_u^-} \Delta R(\mathbf{v}((\rho)) = p_0\tau_b\left(1-(1+\rho^n)F^b(\rho^n) + \int_{-1}^{\rho^n} F^b(r))dr\right)$. But since $\rho^b = \int_{-1}^{\rho^b} F^b(r))dr$, we obtain $\lim_{\rho\to\rho_u^-} \Delta R(\mathbf{v}((\rho)) = p_0\tau_b\left(1+\rho^b-(1+\rho^n)F^b(\rho^n) + \int_{\rho^b}^{\rho^n} F^b(r))dr\right)$ or $\lim_{\rho\to\rho_u^-} \Delta R(\mathbf{v}((\rho)) = p_0\tau_b\left((1+\rho^b)(1-F^b(\rho^n)) - \int_{\rho^b}^{\rho^n}(F^b(\rho^n) - F^b(r))dr\right)$. And $\lim_{\rho\to\rho_u^-} \Delta R(\mathbf{v}((\rho)) > 0$ is equivalent to $(1+\rho^b)(1-F^b(\rho^n)) > \int_{\rho^b}^{\rho^n}(F^b(\rho^n) - F^b(r))dr$. Although one can construct examples such that this condition is violated, this does not happen in our application because $\rho^n - \rho^b$ is small relative to $1 - F^b(\rho^n)$.

(c) $\Delta R(v) < 0$ for $v \in [\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$ follows from $\Delta R(v)$ monotone in $[\mathbf{v}(\rho^n), \mathbf{v}(\rho^g)]$, $\Delta R(\mathbf{v}(\rho^g)) < 0$

and $\lim_{\rho\to\rho_u^+} \Delta R(\mathbf{v}((\rho)) < 0$. $\square$

Proof of Proposition 2: Let $\rho(y)$ denote the indifferent consumer when the cumulative distribution of price returns is $F(r, y)$. Specifically, $\rho(y)$ is defined by equation (2) after replacing $F^s(r)$ with $F(r, y)$. For ease of notation, we denote $\rho^n = \rho(0)$ and $v^n = \mathbf{v}(\rho(0))$. When the cumulative distribution of price returns is $F(r, y)$, the supplier revenues are

$$R(y) = p_0 \int_0^{\mathbf{v}(\rho(y))} \left(\int_{-1}^{\mathbf{r}(v)} (1+r) dF(r, y) \right) dG(v) + p_0 \left(1 - G(\mathbf{v}(\rho(y)))\right).$$

This is because a traveler with a valuation below $\mathbf{v}(\rho(y))$ waits, and subsequently purchases if the price is below her valuation. A traveler with a valuation above $\mathbf{v}(\rho(y))$ buys early. Expected revenues with signal x are $\tau_b R(-x(1-\tau_b)) + (1-\tau_b)R(x\tau_b)$ and

$$\Delta\overline{R}(x) = \tau_b R(-x(1-\tau_b)) + (1-\tau_b)R(x\tau_b) - R(0).$$

We have $\Delta\overline{R}'(x) = \tau_b(1-\tau_b)(-R'(-x(1-\tau_b)) + R'(x\tau_b))$ and $\Delta\overline{R}'(0) = 0$. There is no first-order impact of the signal on profits.

We have $\Delta\overline{R}''(x) = \tau_b(1-\tau_b)((1-\tau_b)R''(-x(1-\tau_b)) + \tau_b R''(x\tau_b))$ and $\Delta\overline{R}''(0) = \tau_b(1-\tau_b)R''(0)$. Denote $R(y) = p_0 K(\rho(y), y)$. Differentiating twice with respect to y gives $R''(y) = p_0 \frac{d^2K}{dy^2}(y)$ with

$$\frac{dK}{dy} = K_1\rho' + K_2$$
$$\frac{d^2K}{dy^2} = K_1\rho'' + K_{11}(\rho')^2 + K_{22} + 2K_{12}\rho'$$

where K_1, for example, denotes the derivative of K with respect to its first argument. We have

$$K_1(\rho, y) = p_0 g(\mathbf{v}(\rho) \left(\int_{-1}^{\rho} (1+r) d(F(r, y)) - 1 \right)$$
$$K_2(\rho, y) = \int_0^{\mathbf{v}(\rho)} \int_{-1}^{\mathbf{r}(v)} (1+r) dS(r) dG(v)$$
$$K_{11}(\rho, y) = p_0 g(\mathbf{v}(\rho))(1+\rho) \left(f(\rho, y) - \frac{g'(\mathbf{v}(\rho))}{g(\mathbf{v}(\rho))} p_0(1 - F(\rho, y) \right)$$
$$K_{22}(\rho, y) = 0$$
$$K_{12}(\rho, y) = p_0 g(\mathbf{v}(\rho)) \left((1+\rho)S(\rho) - \int_{-1}^{\rho} S(r) dr \right)$$

where the last expression uses the identity $\int_{-1}^{\rho}(1+r)dS(r) = (1+\rho)S(\rho) - \int_{-1}^{\rho} S(r)dr$ is obtained using integration by part. Next, take $\int_{-1}^{\rho}(1+r)d(F(r, y)) - 1$ in K_1. Apply the same integration by part identity, and evaluate at $\rho = \rho(y)$ to obtain $\int_{-1}^{\rho(y)}(1+r)dF(r, y) - 1 = -(1+\rho(y))(1 - F(\rho(y), y))$ and consequently

$$K_1(\rho(y), y) = -p_0 g(\mathbf{v}(\rho(y))(1+\rho(y))(1 - F(r, y)).$$

Using equation (2), we evaluate:

$$\begin{aligned} \rho'(0) &= \frac{\int_{-1}^{\rho(y)} S(r)dr}{1 - F(\rho, 0)} \\ \rho''(0) &= \frac{2S(r)\int_{-1}^{\rho(y)} S(r)dr}{(1 - F(\rho, 0))^2} \end{aligned} \tag{9}$$

Replacing the expressions for K_1, K_{11}, K_{22}, K_{12}, ρ' and ρ'' gives:

$$\left.\frac{d^2K}{dy^2}\right|_{y=0} = p_0(1+\rho^n)g(v^n)\frac{\left(\int_{-1}^{\rho^n} S(r)dr\right)^2}{1 - F^n(\rho^n)}\left(\frac{f(\rho^n)}{1 - F^n(\rho^n)} - \frac{g'(v^n)}{g(v^n)}p_0 - \frac{2}{1+\rho^n}\right)$$

from which we conclude

$$\Delta\overline{R}''(0) = p_0^2(1+\rho^n)g(v^n)\tau_b(1-\tau_b)\frac{\left(\int_{-1}^{\rho^n} S(r)dr\right)^2}{1 - F^n(\rho^n)}\left(\frac{f(\rho^n)}{1 - F^n(\rho^n)} - \frac{g'(v^n)}{g(v^n)}p_0 - \frac{2}{1+\rho^n}\right)$$

A similar argument applies for consumption where $R(y)$ is replaced with $C(y) = \int_0^{\mathbf{v}(\rho(y))} F(\mathbf{r}(v))dG(v) + 1 - G(\mathbf{v}(\rho(y)))$. We obtain $\Delta\overline{C}''(0) = \tau_b(1-\tau_b)C''(0)$.

$$\Delta\overline{C}''(0) = p_0 g(v^n)\tau_b(1-\tau_b)\frac{\left(\int_{-1}^{\rho^n} S(r)dr\right)^2}{1 - F^n(\rho^n)}\left(\frac{f(\rho^n)}{1 - F^n(\rho^n)} - \frac{g'(v^n)}{g(v^n)}p_0\right).\square$$

9 Appendix: Sequential Booking Decisions

The traveler can buy at date zero, one or two. Without loss of generality, let $p_0 = 1$, $p_1 = 1 + r_1$ and $p_2 = (1 + r_1)(1 + r_2)$. The random variable r_1 (resp. r_2) is distributed with c.d.f. $F_1^{s_1}()$ when the first signal realization in s_1 (resp. $F_2^{s_1,s_2}(.|r_1)$ when the two signal realizations are s_1 and s_2). $F_1(r_1)$ and $F_2(r_2|r_1)$ denote the unconditional c.d.f. defined similarly as in equation (1). All expected utility are measured at date zero. $U_2^n(v)$ is the date zero expected utility if the traveler has no information and can delay purchase twice. U_2^i is the traveler's date zero utility if she can delay twice and receives a new signals each time. We have:

$$U_2^n(v) = Max\left(v - 1, \int_{-1}^{\infty} Max\left(v - (1 + r_1), \int_{-1}^{\infty} Max\,(v - (1 + r_1)(1 + r_2), 0)\,dF_2(r_2|r_1)\right) dF_1(r_1)\right).$$

This is because a traveler who has not bought in date zero or one, buys in period two if $v-(1+r_1)(1+r_2) > 0$. This corresponds to the third Max operator. If the traveler has not bought in date zero, she anticipates in period one to receive $\int_{-1}^{\infty} Max\,(v - (1 + r_1)(1 + r_2), 0)\,dF_2(r_2|r_1)$ if she waits. She buys in date one if $v - (1 + r_1) > \int_{-1}^{\infty} Max\,(v - (1 + r_1)(1 + r_2), 0)\,dF_2(r_2|r_1)$. This explains the second Max operator. In date zero, she can buy or delay (first Max operator). When the traveler receives signals, the decisions are conditional on the signals' realizations and we obtain:

$$U_2^i(v) = \sum_{s_1=b,g} Pr(s_1)$$

$$Max\left(v - 1, \sum_{s_2=b,g} Pr(s_2|s_1)\int_{-1}^{\infty} Max\left(v - (1 + r_1), \int_{-1}^{\infty} Max\,(v - (1 + r_1)(1 + r_2), 0)\,dF_2^{s_1,s_2}(r_2|r_1)\right) dF_1^{s_1}(r_1)\right).$$

$U_2^i(v) - U_2^n(v)$ is the value of information under sequential booking. It is positive and should be compared with $\Delta U(v)$ which is the value of information with one period.

10 Appendix: Expedia Dataset

We have collected prices and scarcity signals for one-way travel from the Expedia website. It may be that Expedia employs cookies to track visitors, and set an individual price for someone who returns to the site. Although our scraper accepts cookies from Expedia, it also delete the cookies after each query. Therefore, Expedia cannot identify the scraper through cookies. The price we collect is the price that Expedia would return a fresh query.

A *route* is a pair composed of an origin and a destination airport. We use the standard three-letter designations for airports. A *query* is a pair composed of a *route* and a *departure date*. Each query is submitted on different *booking dates* and returns a number of flight options. Each *flight* is identified by a query, a departure and arrival time, the number of layovers and the carrier(s). Therefore, the nesting goes from route to query to flight. For each flight, we collect the price and signal. We construct the variable days-in-advance DiA as the number of days between the departure and booking dates.

Route Selection: We selected routes on the basis of four criteria: (a) Routes with a dominant carrier. (b) Busy routes in term of passenger. (c) Routes previously selected in airline literature. (d) Routes with large potential price savings. We implemented these selection criteria as follows. To find routes operated by a dominant carrier, we use the T-100 data bank from the Bureau of Transportation Statistics web site (www.transtats.bts.gov). The T-100 data bank includes the number of airlines per route, as well as total number of passengers. All of our ten routes are operated by a single airline. Although there is a single carrier offering a direct flight, it is possible to combine two or more flights to make the city-pair travel. Within this set of routes, we selected the top 25 subset in terms of number of passengers transported. A similar approach was previously used by Bilotkach and Rupp (2011). Indeed, many of our routes are the same as theirs and also as Bilotkach and Pejcinovska (2007). Finally, 7 of our 10 routes have destinations listed as domestic destinations with largest potential savings by Hopper.com.[23] For these routes, signals are likely to have a greater value.

Sampling: We run queries daily for about 100 days, starting on July 19th and ending on October 26th, 2015. The first departure date is the 10th of August, which is approximately three weeks from our first booking date (19th of July). The following 11 departures dates are 8 days apart (18th of August, 26th of August, and so on... till November 5th). A flight expires when its departure date has past. When this happens, a new query is automatically added. Using this rolling window sampling method, we end up with 22 departure dates that cover fairly evenly the seven days of the week. Combined with 10 routes, this adds to 220 distinct queries. For some queries (those with a departure date between Octobeer 26th and November 5th), we obtain time series (price, signal) for each flight that cover 100 days. The time series are shorter for the rest of the queries.

Table 8 reports the observation count broken down by route and airline. The more busy routes offer more flight options and end up with more observations. There are 9 main airlines serving the 10

[23] Hopper.com is an aggregator website that collects price data and reports the best days in advance to purchase tickets.

Table 8: Observation Count

Route	N
AUS-DAL	12635
CLT-FLL	80099
DFW-SNA	56752
LAS-MDW	48524
MCO-MDW	54519
MDW-LAS	57053
MIA-BOS	72854
MIA-DFW	62791
PHX-MDW	43616
SNA-DFW	52539
Total	541382

Source: expedia_v1.0.dta

Airline	N
Alaska Airlines	23534
American Airline	112925
Delta	157598
Frontier Airline	1734
JetBlue Airways	4158
Multiple Airline	149417
Sun Country Airl	142
US Airways	48562
United	39220
Virgin America	4092
Total	541382

Source: expedia_v1.0.dta

routes. The flights observations have between one and 14 weeks in advance, and because of the rolling departure date sampling methodology, the number observations is roughly evenly distributed with about 40K observations per week in advance. The number of observations is about 9K for the first departure date, peaks to about 47K on the 10th departure date and then decreases to about 2K on the last one (22nd departure date). This is because the early and late departure dates are queried less frequently.

Table 9 shows that there are on average 179 flights displayed for a given query made on a given booking date. This average varies a little by route. Table 10 shows that the signal can take five positive values: 1, 2,.., 5 seats left at a given price. The frequency of sending signals with high values is slightly lower. In most of the empirical analysis we assume that the signal takes a binary value: 0 if Expedia does not report a number of seats left at the posted price) and 1 otherwise.

Table 9: Number of flights per queryXbooking date (mean/min/max)

route	mean	min	max
AUS-DAL	24.0979	9	42
CLT-FLL	187.0916	105	288
DFW-SNA	173.3658	56	304
LAS-MDW	175.6805	25	289
MCO-MDW	179.5969	45	289
MDW-LAS	179.6123	35	282
MIA-BOS	210.2532	76	372
MIA-DFW	203.59	99	349
PHX-MDW	148.1207	26	237
SNA-DFW	163.4948	66	278
Total	178.9122	9	372

Table 10: Observation count for each signal realization (number of seat left at posted price)

Signal Realization	**N**
0	359435
1	45611
2	39744
3	35083
4	31978
5	27655
Total	539506