Loss Aversion and the Uniform Pricing Puzzle for Media and Entertainment Products

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The uniform pricing puzzle for vertically differentiated media and entertainment products (movies, books, music, mobile apps, etc.) is that a firm with market power sells high- and low-quality products at the same price even though quality is perfectly observable and price adjustments are not costly. We resolve this puzzle by assuming that consumers have an uncertain taste for quality and accounting for consumer loss aversion in monetary and consumption utilities. The novelty of our approach is that the so-called reference transaction is endogenously set as part of a "personal equilibrium" and is based only on past purchases of same-quality products.

Key words: uniform pricing puzzle, vertically differentiated products, expectations-based loss aversion, personal equilibrium

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1. Introduction

In media and entertainment markets, popular products sometimes sell at prices that are similar to, or sometimes the same as, those of less popular ones. For instance, the price of a movie ticket does not depend on observable signals that predict gross revenue, such as the movie's ratings or popularity, or on whether the movie is a sequel or a new release (*The Atlantic* 2012, Orbach and Einav 2007). Attempts at taking these dimensions of quality into account have failed (*The Guardian* 2006). Similar pricing patterns for vertically differentiated products exist also for books (where prices do not depend on sales forecasts, (Clerides 2002)) and theater season tickets (Chu et al. 2011) as well as in other media markets such as music, including CDs or downloadable tracks (Shiller and Waldfogel 2011), and mobile apps. A considerable amount of work addresses price uniformity, but the puzzle remains for vertically differentiated products in media and entertainment. Under standard demand specifications, the profit-maximizing price should increase with quality. The puzzle occurs when observed prices either do not respond to quality (price uniformity) or respond insufficiently (price compression).

We provide a behavioral explanation that is based on consumer loss aversion and whereby a consumer has a *reference transaction* or benchmark, which accounts for both consumption utility and monetary payment, and reacts more strongly when an outcome falls short of this benchmark than otherwise. If we assume that there is a single reference transaction for all purchases, then loss aversion could explain price uniformity in certain contexts where prices do not respond to cost (Heidhues and Kőszegi 2008) or demand shocks (Chen and Cui 2013). Yet, in the case of vertically differentiated products, this assumption is unrealistic because consumers observe quality: "All consumers are familiar with the concept of different prices for different products. [...] Thus, charging premiums or giving discounts for unique categories of movies is unlikely to be perceived as unfair. For example, given the unique characteristics and highly publicized production budgets of event movies, charging premiums for such movies probably would not violate fairness perceptions" (Orbach and Einav 2007: 145-46). Thus consumers should have different reference points corresponding to different product quality classes, in which case the simple argument in Kahneman et al. (1986)—which is based on perceived fairness with respect to a single reference point—does not apply. The critical component of our model is its conceptualization of a separate reference transaction for each quality class, which enables us to analyze rigourously the effect of loss aversion on consumers' purchase decisions.

Following Mussa and Rosen (1978), we assume that the consumer's valuation for a product of quality q is $v = v_0 + q\theta$, where v_0 is the baseline value and $\theta \in [\theta_0, \theta_1]$ is an idiosyncratic random preference shock that captures the consumer's taste uncertainty. The uncertainty is associated with the unique content specific to each media and entertainment product. In the movie context, for example, all consumers prefer movies that are popular which is captured by the quality dimension q. But a consumer also has an idiosyncratic preference level θ for each movie, which reflects the movie's "match value" uncertainty and is not resolved until the time of purchase. Leslie and Sorensen (2014) adopt the same formulation—in which the taste shock and product quality are complements—to study the demand for concerts. The match value component is of more consequence for higher-quality products.

We approach the pricing problem at two levels. First we study the problem of pricing a given quality class. This is possible because, in our model, the consumer has a specific reference transaction for each quality class. Given the price and taste realization, the consumer decides whether to consume by applying the notion of *personal equilibrium* (hereafter PE) developed in Kőszegi and Rabin (2006, 2009). In other words, a loss-averse consumer compares current purchases with a reference point which is formalized as her recent expectations about past purchase outcomes in that quality class. A series of experimental and empirical papers support the notion of reference points as expectations held under uncertainty.[1]

The case *without* uncertainty in match value is trivial because the consumer faces no risk: she consumes if the valuation is above the price and otherwise does not. This means that the seller, when setting the price, need not account for consumer loss aversion. The case *without* loss aversion is standard: the consumer consumes when the taste draw is above a quality-dependent threshold. Yet if uncertainty and loss aversion are both present, then there are two new costs associated with consuming only when the taste draw exceeds a given threshold, which we illustrate with the movie example. For movies that are poor matches to a consumer's taste (low θ), the consumer suffers a loss when she compares the choice of not consuming with what she would receive from a movie with a better match (higher θ). The opposite holds for monetary comparisons because the consumer perceives paying the price for a well-matched movie as a loss when compared to not consuming. We derive a set of sufficient conditions such that full consumption—understood as consuming for all taste draws—is the preferred personal equilibrium (hereafter PPE), defined as the PE that yields the highest expected utility (Kőszegi and Rabin 2006).

To explicate the intuition underlying full-consumption outcome, we look at the consumer's utility from consuming only when her valuation is above a given consumption threshold. Although expected utility is an inverse U–shaped function of the consumption threshold under loss neutrality, even small amounts of loss aversion transform it into a U–shaped function—for which extreme consumption plans (always or never consume) dominate intermediate ones. The dominance follows because the net effect of loss aversion is significant losses associated with comparing consumption and expenditure outcomes for intermediate thresholds. If there are multiple PEs and the expected utility is U–shaped, then PEs with intermediate consumption thresholds will be dominated by those with extreme thresholds for which the consumer consumes most or least often.

Given these patterns of consumer behavior, we next investigate the firm's pricing decision for the given quality level. We establish that for moderate gain–loss parameter values in consumption and when quality is not too high, full consumption solves the firm's revenue maximization problem. Full consumption is a local minimum under risk neutrality but is a local maximum for sufficiently high monetary loss aversion—and this holds for any distribution of the taste draw. Under additional conditions, full consumption is the global maximum. Full consumption accords with the casual observation that some consumers often watch movies and read books that are not necessarily good matches.

[1] Examples include the effort provision experiments of Abeler et al. (2011), the exchange experiments of Ericson and Fuster (2011), and research by Pope and Schweitzer (2011) in the professional golf context.

As a second level of analysis, we use the results on a single quality class to compare prices across quality classes and thereby address the uniform pricing puzzle. Because quality is multiplied by the lowest taste draw θ_0 in the characterization of the optimal price, it has a limited influence on the optimal price. Contrast this with the cases where the consumer is either loss neutral or certain about her taste. The price for a product of quality q sold to a loss-neutral consumer is $v_0 + q\theta^{\text{LN}}$ (where θ^{LN} is the optimal loss-neutral consumption threshold) and is $v_0 + q\mathbb{E}\theta$ when taste is certain and normalized to $\mathbb{E}\theta$. For a consumer who faces considerable taste uncertainty, θ_0 will be much lower than either θ^{LN} or $\mathbb{E}\theta$; hence the optimal price is less responsive to changes in quality under loss aversion and taste uncertainty than it is when either of those conditions is absent. Price compression occurs when both loss aversion and taste uncertainty are present; uniform pricing obtains when $\theta_0 = 0$.

Price compression happens when full consumption is optimal, which is the case only for product classes below a threshold level of quality and for moderate levels of aversion to consumption loss. In this case, the consumer's surplus is increasing in quality, a feature consistent with the applications discussed previously. For product qualities above that threshold, full consumption is no longer optimal, and the model prescribes that firms charge more for better quality products. These findings clarify the role (in the specification of Mussa and Rosen (1978)) played by the complementarity between quality and the taste draw. Additive quality, for example, does not interact with the consumption threshold and cannot generate price compression.

The rest of the paper is organized as follows. In Section 2, we review the related literature and position our work accordingly. Section 3 presents the model and notation. Section 4 solves the consumer's decision-making problem and also the firm's revenue maximization problem. Section 5 presents our main results on the uniform pricing puzzle and discusses an application to the case of uniformly distributed consumer valuations. We provide a discussion and conclude in Section 6.

2. Literature Review: The Puzzle and Candidate Explanations

An extensive literature studies the pricing of vertically differentiated products in a monopoly (see e.g., Mussa and Rosen 1978, Anderson and Dana 2009) or a competitive setting (Liu and Zhang 2013). This literature has explained why prices increase with service quality, printer speed (Deneckere and McAfee 1996), car attributes (Verboven 1999), airfare classes (Talluri and van Ryzin 2004), and numerous other quality dimensions. We contribute to this literature by studying the rather unusual case in which a monopolist does *not* price, or *under*-prices, quality.

Practitioners have commented on the use of uniform pricing for media and entertainment products (*The Atlantic* 2012, *ArtsJournal* 2013), and a few empirical studies have looked at specific markets. Table 1 summarizes the academic literature on the use of uniform pricing. The studies we

Table 1 Uniform Pricing for Entertainment Products

	Literature	Pricing Patterns	Explanations
Movies	Orbach and Einav (2007), *The Atlantic* (2012), *The Guardian* (2006)	Star factor or costs do not influence prices. Screening technology and show times influence prices.	Menu cost, Uncertainty, Legal Constraints
Books	Clerides (2002)	Predicted sales do not influence price. Book length and hardcopy/softcopy format influence price.	Constant demand elasticity
Music	Shiller and Waldfogel (2011), Richardson and Stähler (2016)	Artist popularity has little influence on price. New versus old release matters.	Prices signal quality under asymmetric information.
Concerts	Chu et al. (2011), Courty and Pagliero (2014), Leslie and Sorensen (2014)	Artist popularity does not always influence prices. Ticket prices often vary according to seat maps and local market.	Fairness

found cover four entertainment markets.[2] The puzzle does not apply indiscriminately. Prices are less likely to respond to quality dimensions that have to do with the *subjective* and *creative* nature of entertainment products (e.g., talent). Prices however respond to changes in the *medium* used to share the entertainment product. For example, Orbach and Einav (2007) point out that show times and other movie features such as 2D vs. 3D technology typically influence prices. Clerides (2002) shows that book prices depend on the page count and the print format (softcopy vs. hardcopy). Our model is the first one that can explain these observations.

Table 1 also reviews some explanations offered for uniform pricing; see also Orbach and Einav (2007) and Eckert and West (2013). Rationales based on asymmetric information or supply-side constraints—such as menu cost (McMillan 2007) and legal constraints on the relations between distributors and movie theatres (Orbach and Einav 2007)—do help to explain the puzzle in some cases. However, we believe a gap exists for an explanation that encompasses the broad range of media and entertainment contexts which exhibit price compression. This paper presents a formal model that incorporates several features of media and entertainment markets (e.g., market power, uncertain taste for quality, loss aversion) and explains how uniform pricing and price compression arise in these markets.

An explanation based on loss aversion is compelling for the applications considered in this paper because loss aversion affects decisions of consumers who intend to consume (Novemsky and Kahneman 2005) and are emotionally attached to the product (Ariely et al. 2005). A random taste draw is plausible for the subjective dimension of quality that is associated with the creative content of each new product (and less so for for objective and predictable dimensions of quality). In formalizing a non-trivial behavioral rationale for price uniformity, we exclude from the model such

[2] A different puzzle looks at"branded variant". Prices are remarkably uniform irrespective of brand popularity (Chen and Cui 2013). Eckert and West (2013) investigate why more popular brands of beer cost the same as less popular ones, and McMillan (2007) does likewise for soft drinks. These studies differ from ours in that branded variants are *horizontally* (not *vertically*) differentiated products. More broadly, our work contributes to behavioral research in economics and management that relates price uniformity to loss aversion. (Heidhues and Kőszegi 2005, Heidhues and Kőszegi 2008, Nasiry and Popescu 2011). We follow these papers in assuming endogenous reference points, but we focus on vertically differentiated products with negligible marginal cost.

considerations as cost and competition; these factors help explain some pricing practices but are less relevant in the case of media and entertainment goods, where fixed costs and market power prevail.

3. Model: Preliminaries

Consider a firm that sells a product of quality q to a representative loss-averse consumer. In this section and Section 4, we characterize the consumer behavior as well as the firm's optimal pricing policy for the quality q. In Section 5, we let the firm offer multiple quality classes and show how uniform pricing emerges when consumers are loss averse.

The consumer's valuation of a product has two components. First, the consumer is willing to pay $v_0 \geq 0$ regardless of product quality. In the case of movies, one can view v_0 as the inherent value of a night out with family and friends to watch a movie. Second, the consumer has a random marginal willingness to pay for quality that we model as follows. Let $\theta \in \mathbf{\Theta} = [\theta_0, \theta_1]$ denote the consumer's private taste draw for quality. We will sometimes refer to θ as the "state of nature" and further assume $\theta_0 \geq 0$; that is, the marginal effect of quality on a consumer's valuation is nonnegative. Now we can write the consumer's intrinsic utility for a product of quality q in state θ as $v = v_0 + q\theta$. Our characterization is similar to Mussa and Rosen (1978) and implies that quality q and the state of nature θ are complements. The same structure of consumer preferences is applied by Leslie and Sorensen (2014) in an empirical application to live concerts. Herweg and Mierendorff (2013) consider a similar structure in the loss aversion model that they use to study flat-rate tariffs. The taste draw θ has density $g(\theta)$, cumulative distribution $G(\theta)$, and survival function $\bar{G}(\theta) = 1 - G(\theta)$. Finally, in line with the applications discussed in the Introduction, we assume that there is no marginal cost associated with serving the consumer.[3]

Assumption 1. *The density function $g(\theta)$ is increasing for $\theta < \mathbb{E}\theta$, and $g(\mathbb{E}\theta - x) = g(\mathbb{E}\theta + x)$ for $x \in [0, \mathbb{E}\theta]$.*

Assumption 1 means that $g(\cdot)$ is single peaked and symmetric. It holds for many distributions, including the truncated normal, the uniform, and any tent-shaped distribution. When there are multiple PEs, this assumption is used to rank PEs (by level of expected utility) and rule out PEs with low consumption levels.[4]

[3] The analysis naturally follows when each product is produced at a fixed cost that increases with the product's quality.

[4] In equation (13) the monetary gain-loss component, $(\lambda_p - \beta_p)pG(\theta)\bar{G}(\theta)$, is easy to compare for a pair of PEs under Assumption 1, because the function $G(\theta)\bar{G}(\theta)$ is single peaked and symmetric. The consumption gain-loss component it such that PEs with large consumption dominate those with small consumption in a sense to be made precise in the proof of Proposition 1.

According to prospect theory, the consumer experiences feelings of gain and loss when comparing her consumption outcome with a reference transaction (Tversky and Kahneman 1991). We assume that the reference transaction has a consumption component along with a monetary component and also that the consumer experiences gains and losses in both components. There is evidence that both components matter (see, for example, discussion in Carbajal and Ely (2012)), and we will show that either one is sufficient to explain the uniform pricing puzzle. In line with the extant literature, we assume that the gain–loss utility is piecewise linear (see e.g., Heidhues and Kőszegi 2008, Herweg and Mierendorff 2013, Kőszegi and Rabin 2006). The consumer experiences a loss in consumption utility equal to $\lambda_c(v'-v)$ when her valuation v is lower than her reference valuation v', and she experiences a loss in monetary utility equal to $\lambda_p(p-p')$ if she spends more than the reference amount p'. Likewise, the consumer experiences a gain in consumption utility equal to $\beta_c(v-v')$ when she consumes more than expected (i.e., $v>v'$) and a gain in monetary utility equal to $\beta_p(p'-p)$ when she spends less than expected (i.e., $p<p'$). Under prospect theory, consumers are loss averse; this means they dislike losses more than they like equal-sized gains. We assume $Min(\lambda_c,\lambda_p)\geq Max(\beta_c,\beta_p)$. This includes Köszegi and Rabin's (2006) setup where gain and loss coefficients are the same for consumption and money ($\lambda_c=\lambda_p$ and $\beta_c=\beta_p$) and the case where the gain part of the value function is flat ($\beta_c=\beta_p=0$).

Although various approaches have been proposed for modeling the reference point (Carbajal and Ely 2012, Eliaz and Spiegler 2015, Zhou 2011), we adopt the framework proposed in Kőszegi and Rabin (2006) because in our context with fairly frequent repeat purchases, it is natural to assume that consumers form lagged expectations using only past purchases of products of the same quality class.[5]

Kőszegi and Rabin (2006) introduce the personal equilibrium (PE) concept to characterize the reference transaction in the uncertain decision-making environment. In a PE, the reference transaction is a consumption plan such that the decision in each state is optimal given the reference transaction. Formally, we denote a consumption plan $\bar{\pi}=\{\pi(\theta)\}_{\theta\in\boldsymbol{\Theta}}$ in which $\pi(\theta)$ is the probability that the consumer consumes in state θ. The full-consumption plan is defined as $\pi(\theta)=1$ for all θ. According to Kőszegi and Rabin (2006), $\bar{\pi}$ is a PE if and only if

$$u(\pi(\theta)|\bar{\pi},\theta)\geq u(x|\bar{\pi},\theta)\quad \forall x\in[0,1],\ \forall\theta\in\boldsymbol{\Theta}; \tag{1}$$

where $u(\pi(\theta)|\bar{\pi},\theta)$ is the ex post realized utility of the consumer given the consumption plan $\pi(\theta)$, the reference transaction $\bar{\pi}$, and the taste draw θ. The condition expressed by (1) means that, after

[5] In Heidhues and Köszegi's (2008) model of horizontally differentiated products, all consumers share the same reference point which corresponds to a random purchase. Although this approach is reasonable for horizontally differentiated products that are hedonistically substitutable, it is less reasonable for vertically differentiated products because quality draws natural boundaries between product classes.

taste uncertainty has been resolved, the consumer has no incentive to deviate from the consumption plan given the reference point $\bar{\pi}$. The set of PEs is not necessarily a singleton, so Kőszegi and Rabin (2006) define a preferred personal equilibrium (PPE) as a PE that yields the highest ex ante consumer expected utility defined as

$$\mathrm{EU}(\bar{\pi}) = \int_{\mathbf{\Theta}} u(\pi(\theta)|\bar{\pi}, \theta)\, dG(\theta). \tag{2}$$

To summarize, the timing of events is as follows. The firm sets the price p for a product of quality q. We rule out random prices (Heidhues and Kőszegi 2014, Eliaz and Spiegler 2015) because they are not relevant in the applications we have in mind. Then, the consumer makes a consumption decision $\pi(\theta)$ based on her reference transaction $\bar{\pi}$. In solving the model, we impose two conditions as follows.

(i) *PE requirement:* The consumer follows through on her expectations; that is, the expectation $\bar{\pi}$ coincides with the actual consumption $\pi(\theta)$.

(ii) *PPE requirement:* The consumer selects the PE that yields her the most utility.

The firm maximizes its revenue, $p\int_{\mathbf{\Theta}} \pi(\theta)\, dG(\theta)$, subject to PE and PPE. Table 4 (in Appendix A) summarizes our notation. Throughout the paper, we illustrate the results assuming θ is uniformly distributed over $\mathbf{\Theta} = [0, 1]$ and that $\beta_c = \beta_p = 0$.

We conclude this section by pointing out that a similar structure for reference points has been applied to study other problems in the literature. For example, Herweg (2013) studies newsvendor ordering decisions, Yang et al. (2014) investigate queuing behavior, and Lindsey (2011) examines congestion pricing. Our model is related also to those of Herweg and Mierendorff (2013) and Hahn et al. (2015). The former paper assesses the optimality of two-part tariffs under multi-unit demand for a single product. In contrast, we restrict our analysis to a unit-demand model so that we can tackle the case of vertically differentiated products. Hahn et al. assumes endogenous product design and introduces screening, which is ruled out in our analysis.

4. Model: Analysis

The analysis for a general distribution of states of nature is complex. Therefore, in Section 4.1 we analyze a two-state case $\theta \in \{\theta_l, \theta_h\}$ where $\theta_l < \theta_h$ and $\beta_p = \beta_c = 0$. The consumer draws taste θ_l with probability γ. This model uses a simple setup to showcase the consumer's decision making problem and also the firm's pricing problem. In Section 4.2 we consider the general case. All proofs are in Appendix B.

4.1. Benchmark: Two-State Case

In this section we use a simple model to demonstrate the consumer behavior model and to derive a set of sufficient conditions such that uniform pricing and price compression are optimal for the firm. We proceed by first deriving sufficient conditions such that consuming in both states is a PE, and then a PPE, and finally such that the firm's maximum revenue in this PPE (i.e., always consume) exceeds the revenue in any other PPE. The set of sufficient conditions derived here naturally extend to the general case.

In the benchmark case we ignore mixed strategies ($\pi \in (0,1)$),[6] which means that the consumer may either consume (C) or not (N). Hence there are four possible consumption plans (a "plan" determines consumer behavior under each possible taste realization). The four candidate PEs are then $\{(C,C),(C,N),(N,C),(N,N)\}$, where the first (resp. second) element in each pair is the consumption plan in the low (resp. high) state. The plan (C,N)—which stipulates that the consumer consumes only in the low state—cannot be a PE.[7] Thus the only possible PEs involving some consumption are (C,C), when the consumer always consumes; and (N,C), when the consumer consumes only in the high state.

Table 2 reports the ex post state-dependent utilities when the consumer consumes (C) and when she does not (N) conditional on the reference point ($\bar{\pi} \in \{(C,C),(N,C)\}$). As an illustration, we discuss the consumer's utility when she consumes in the low-taste realization and when the reference plan is always to consume—that is, we derive $u(C|(C,C),l)$. The consumer receives an intrinsic utility equal to $v_0 + q\theta_l$ and pays p. She also experiences a loss equal to $(1-\gamma)\lambda_c q(\theta_h - \theta_l)$ in consumption utility because, with probability $1-\gamma$, her valuation would have been higher in the high state. In other words, she compares what she does consume ($v_0 + q\theta_l$) with what she could have consumed had the high state realized ($v_0 + q\theta_h$).

	Payoff when reference point is: $\bar{\pi} = (C,C)$	$\bar{\pi} = (N,C)$
$u(C\|\bar{\pi},l)$	$v_0 + q\theta_l - \lambda_c(1-\gamma)q(\theta_h-\theta_l) - p$	$v_0 + q\theta_l - \lambda_c(1-\gamma)q(\theta_h-\theta_l) - (1+\lambda_p\gamma)p$
$u(N\|\bar{\pi},l)$	$-\lambda_c(v_0 + q\mathbb{E}\theta)$	$-\lambda_c(1-\gamma)(v_0+q\theta_h)$
$u(C\|\bar{\pi},h)$	$v_0 + q\theta_h - p$	$v_0 + q\theta_h - (1+\lambda_p\gamma)p$
$u(N\|\bar{\pi},h)$	$-\lambda_c(v_0 + q\mathbb{E}\theta)$	$-\lambda_c(1-\gamma)(v_0+q\theta_h)$
$\mathrm{EU}(\bar{\pi})$	$\gamma u(C\|\bar{\pi},l) + (1-\gamma)u(C\|\bar{\pi},h)$ $= v_0 + q\mathbb{E}\theta - \lambda_c\gamma(1-\gamma)q(\theta_h-\theta_l) - p$	$\gamma u(N\|\bar{\pi},l) + (1-\gamma)u(C\|\bar{\pi},h)$ $= (1-\gamma)\left((v_0+q\theta_h)(1-\lambda_c\gamma) - (1+\lambda_p\gamma)p\right)$

Table 2 Reference point, state utility, and expected utility

[6] In Section 4.2, we show that this restriction is without loss of generality.

[7] We prove in Section 4.2 that consumption is increasing in θ; that is, if the consumer consumes in a state then she will consume in all higher states, too.

The consumption plan (C,C) is a PE if $u(C|(C,C),s) \geq u(N|(C,C),s)$ for $s \in \{l,h\}$. Observe that $p^{\mathrm{LA}}_{CC} \triangleq (1+\lambda_c)(v_0+q\theta_l)$ is the highest price such that both conditions hold. Recall that we intend to derive sufficient conditions under which the full-consumption plan, (C,C), is the PPE. We are therefore interested in equilibrium prices that are lower than p^{LA}_{CC}, since any higher price would violate the PE conditions.

Other candidate PEs (N,C) and (N,N) are dominated by (C,C) if $\mathrm{EU}(C,C) \geq \mathrm{EU}(N,C)$ and $\mathrm{EU}(C,C) \geq \mathrm{EU}(N,N)$. These two inequalities, when using the expressions in Table 2 for $p = p^{\mathrm{LA}}_{CC}$, yield

$$\frac{\lambda_p(1+\lambda_c)}{\lambda_c} \geq \frac{\gamma}{1-\gamma}, \tag{3}$$

and

$$\frac{q(\mathbb{E}\theta-\theta_l)}{v_0+q\theta_l} \geq \frac{\lambda_c}{1-\lambda_c\gamma}. \tag{4}$$

In short, if the firm charges p^{LA}_{CC} then full consumption is the PPE when conditions (3) and (4) hold. Next we turn to the firm's revenue maximization problem. Under (N,C), the maximum price the firm can charge is constrained by $u(C|(N,C),h) \geq u(N|(N,C),h)$[8] which yields the upper bound $p = \frac{1+(1-\gamma)\lambda_c}{1+\gamma\lambda_p}(v_0+q\theta_h)$. The associated revenue $(1-\gamma)p$ is smaller than the revenue under (C,C) (i.e., p^{LA}_{CC}) if

$$\frac{\gamma}{1-\gamma}\left(1+\frac{\lambda_c+\lambda_p+\lambda_c\lambda_p}{1+(1-\gamma)\lambda_c}\right) \geq \frac{q(\theta_h-\theta_l)}{v_0+q\theta_l}. \tag{5}$$

We conclude that, when inequalities (3)–(5) hold, the optimal price is p^{LA}_{CC} and full consumption is the PPE.

When there are multiple products of different qualities, we obtain uniform pricing; that is, $\frac{\partial p^{\mathrm{LA}}_{CC}}{\partial q} = 0$, when (3)–(5) hold and $\theta_l = 0$.[9] When $\theta_l > 0$, we say that price compression occurs if the price under loss aversion and taste uncertainty responds less to quality than when either of these conditions is absent. So conditional on (3)–(5) we have $\frac{\partial p^{\mathrm{LA}}_{CC}}{\partial q} = (1+\lambda_c)\theta_l$, which we compare next with the corresponding expressions in the certain-taste and loss-neutral cases.

Consider the case where a certain taste for quality is normalized to $\mathbb{E}\theta$. Observe that the optimal price is $p^C \triangleq v_0 + q\mathbb{E}\theta$ and that $\frac{\partial p^C}{\partial q} = \mathbb{E}\theta$. We therefore have price compression, defined as $\frac{\partial p^{\mathrm{LA}}_{CC}}{\partial q} < \frac{\partial p^C}{\partial q}$, as long as $\mathbb{E}\theta > (1+\lambda_c)\theta_l$—which holds if there is sufficient taste uncertainty (i.e., if $\mathbb{E}\theta \gg \theta_l$).[10] Similarly, in the loss-neutral case the optimal price is $p^{\mathrm{LN}} \triangleq v_0 + q\theta_h$ provided $(1-\gamma)(v_0+q\theta_h) > v_0 + q\theta_l$; in other words, it is optimal to sell to the consumer only when the taste draw is h. This condition is satisfied whenever the taste differential, defined as $q(\theta_h-\theta_l)$, is greater than

[8] This constraint guarantees that the consumer consumes when the high state realizes.

[9] Products are priced independently, an approach that we motivate (in Section 5) in the context of the applications relevant to the uniform pricing puzzle.

[10] In the two-state case, this condition is equivalent to $\theta_h > (1+2\lambda_c)\theta_l$.

$\gamma(v_0 + q\theta_h)$. Then $\frac{\partial p^{\mathrm{LN}}}{\partial q} = \theta_h$ and price compression obtains if $\theta_h > (1+\lambda_c)\theta_l$, which holds for high taste differentials.

To sum up, our second main result is that if conditions (3)–(5) hold, $\theta_l > 0$, and there is enough taste uncertainty or taste differential (as defined previously), then the loss-averse price responds less to quality than does the price for a certain-taste or for a loss-neutral consumer. Thus taste uncertainty and loss aversion are both necessary for price compression to occur. Furthermore, conditions (3)–(5) are more likely to hold when the consumer faces taste uncertainty (γ away from 0 or 1), monetary loss aversion λ_p is large, consumption loss aversion λ_c is not too large, and θ_h is not too high.

For the general case with loss aversion and continuous taste draws, we proceed as above and derive sufficient conditions such that uniform pricing and price compression occur. We have to compromise on generality because the firm's revenue maximization problem is not well-behaved. Table 3 highlights the key differences between the firm's maximization problems with loss-neutral and loss-averse consumers. For a given price, the consumer consumes for all valuation draws that are above a threshold in both of these problems (Lemma 1). However, in the problem with loss-averse consumers, not all consumption thresholds can be implemented as a PPE. The bijective property of the correspondence between price and consumption under loss neutrality is lost with loss-averse consumers. The consumer's demand, $\overline{G}(V^{-1}(p))$, may be discontinuous and not downward slopping. The firm's profit is not a well-behaved function of the consumption threshold because its support is not always convex (row (e) Table 3) and there is no simple condition that makes the profit function locally concave. As a result, one cannot derive a general characterization of the optimal solution.[11]

Instead, we follow a similar approach as in the two-state example and concentrate on the case where the consumer always consumes (i.e., when θ_0 is the PPE). Technically, full consumption is the firm's preferred consumption threshold when (i) the consumer's expected utility is maximized for θ_0 (row (d) in Table 3 and Proposition 2) and (ii) the firm's profits are increasing in consumption (Proposition 3). We generalize the conditions given by inequalities (3)–(5) (see Assumptions 2 and 3) and also discuss what happens when these assumptions are violated (see also Appendix C).

4.2. The Consumer's Problem

Under loss neutrality, a consumer will consume if, when the taste uncertainty is resolved, her valuation is greater than the price. This decision procedure translates into a threshold consumption rule: the consumer consumes if the taste draw is above the threshold $\theta = \frac{p - v_0}{q}$ provided that

[11] Even in the case with uniform distributions, we do not obtain a general characterization. For a given price, there may exist at most two interior PEs but the expected utility function has cubique expressions of θ. Evaluating and comparing expected utility across PEs is done numerically.

Table 3 Consumer Demand and Producer Revenue under Loss Neutrality and Loss Aversion

	Loss Neutrality	Loss Aversion
	Inverse Demand	
(a) Threshold consumption rule	Yes	Yes (Lemma 1)
(b) All $\theta \in [\theta_0, \theta_1]$ can be PPE	Yes	No
(c) Price implementing threshold θ as a PPE (if implementable)	$V^{\mathrm{LN}}(\theta) = v_0 + q\theta$ is increasing in θ	$V(\theta) = V^{\mathrm{LN}}(\theta)L(\theta)$ (eq. 8) may not be increasing (Figure 1)
(d) Expected Utility, $U(\theta, p)$, as a function of threshold θ	Single peaked (Figure 2a)	May be decreasing or U-shaped (Figures 2b and 2c)
	Firm's Profits	
(e) Support	$\theta \in [\theta_0, \theta_1]$	Set of θ's that are PPE
(f) Profit function	$V^{\mathrm{LN}}(\theta)\overline{G}(\theta)$	$V(\theta)\overline{G}(\theta)$

$\frac{p-v_0}{q} \in \boldsymbol{\Theta}$. Otherwise, the consumer either always consumes (when $p \le v_0 + q\theta_0$) or never consumes (when $p \ge v_0 + q\theta_1$). We first show that, under loss aversion, the consumer still adopts a threshold rule in equilibrium.

Lemma 1. *In a PE, $\pi(\theta) \in \{0,1\}$ almost everywhere and $\pi(\theta)$ is nondecreasing in θ.*

Nonrandomization follows because $u(\pi(\theta)|\overline{\pi}, \theta)$ is linear in $\pi(\theta)$. Lemma 1 implies that the optimal consumption plan takes a threshold form; that is, for some $\theta^* \in \boldsymbol{\Theta}$, consume if $\theta \ge \theta^*$ but not if $\theta < \theta^*$. Define $u^1(\theta, \theta^*)$ as the ex post utility of consuming and $u^0(\theta, \theta^*)$ as the ex post utility of not consuming when the consumer's taste draw is θ and the threshold is θ^*. Following the same reasoning as in Table 2 (see Table 5 in Appendix B for full derivations), we have:

$$u^0(\theta, \theta^*) = -\lambda_c \int_{\theta^*}^{\theta_1} (v_0 + q\theta')\, dG(\theta') + \beta_p p \bar{G}(\theta^*) \quad \text{for } \theta \le \theta^*; \tag{6}$$

$$\begin{aligned} u^1(\theta, \theta^*) = & v_0 + q\theta - p - \lambda_c q \int_{\theta}^{\theta_1} (\theta' - \theta)\, dG(\theta') + \beta_c \left((v_0 + q\theta) G(\theta^*) + q \int_{\theta^*}^{\theta} (\theta - \theta')\, dG(\theta') \right) \\ & - \lambda_p G(\theta^*) p \quad \text{for } \theta \ge \theta^*. \end{aligned} \tag{7}$$

Because the utility from not consuming is independent of the taste draw, we denote it simply by $u^0(\theta^*)$. From (6) and (7) it follows that the net utility of consuming over not consuming, $u^1(\theta, \theta^*) - u^0(\theta^*)$, is increasing in θ. In other words, the higher the taste realization for quality, the more inclined the consumer is to consume. Any interior equilibrium must solve $u^1(\theta, \theta^*) = u^0(\theta^*)$. (For ease of exposition, hereafter we omit the asterisk when no confusion could result). It follows that $\theta \in (\theta_0, \theta_1)$ is an interior PE if and only if

$$V(\theta) \triangleq (v_0 + q\theta) L(\theta) = p, \tag{8}$$

where the function

$$L(\theta) \triangleq \frac{1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta)}{1 + \beta_p + (\lambda_p - \beta_p) G(\theta)} \tag{9}$$

is positive and decreasing in θ with $L(\theta_0) = \frac{1+\lambda_c}{1+\beta_p} > 1$ and $L(\theta_1) = \frac{1+\beta_c}{1+\lambda_p} < 1$. The numerator of $L(\theta)$ increases the consumer's willingness to pay and is known as the *attachment effect*. A consumer who consumes in states greater than θ suffers an "attachment" in states lower than θ for which she does not consume. Consumption loss aversion alone pushes toward greater consumption relative to the loss-neutral case. The denominator reduces the consumer's willingness to pay and is called the *comparison effect*. The consumer receives a net monetary benefit in states lower than θ because she saves p relative to the states above θ in which she consumes. Price loss aversion alone pushes toward less consumption relative to the loss-neutral case. We define $\tilde{\theta} = G^{-1}\left(\frac{\lambda_c - \beta_p}{\lambda_c - \beta_c + \lambda_p - \beta_p}\right)$ as the state in which the attachment and comparison effects are equal; thus $L(\tilde{\theta}) = 1$.

There may also be corner equilibria. A corner PE at $\theta = \theta_0$ exists if $u^1(\theta_0, \theta_0) \geq u^0(\theta_0)$ or (equivalently) if

$$p \leq \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0); \tag{10}$$

similarly, a corner PE at $\theta = \theta_1$ exists if $u^1(\theta_1, \theta_1) \leq u^0(\theta_1)$ or (equivalently) if

$$p \geq \frac{1+\beta_c}{1+\lambda_p}(v_0 + q\theta_1). \tag{11}$$

From (10), define the maximum price the firm can charge and still induce θ_0 as a PE $p_0^{\mathrm{LA}} = \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0)$.

We use $\boldsymbol{\Theta}^{\mathrm{PE}}(p)$ to denote the set of PEs associated with price p. This set may include interior and corner PEs. Equilibrium multiplicity arises when a corner PE exists simultaneously with another corner or an interior PE. Multiple interior PEs may also exist because equation (8) may admit multiple solutions. In the absence of loss aversion, there is a unique solution to $v_0 + q\theta = p$ that determines the consumption rule. With loss aversion, however, there is no standard restriction on $g(\cdot)$ to impose regular behavior on the function $V(\theta)$. Hence a PE exists every time $V(\theta)$ crosses p, which may occur multiple times. We prove that a PPE always exists.

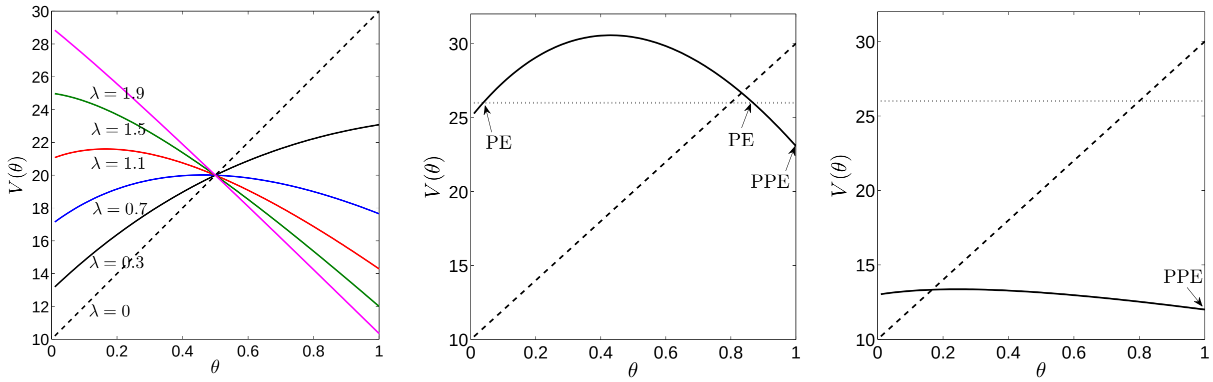
Lemma 2. *A PPE always exists.*

When θ is uniformly distributed over $[\theta_0, \theta_1]$, there are at most two interior PEs in addition to two possible corner PEs. The solid curves in Figure 1 plot $V(\theta) = (v_0 + q\theta)L(\theta)$ against the loss aversion coefficient; the dashed diagonal lines plot $V(\theta) = v_0 + q\theta$ for a loss-neutral consumer (we have $L(\theta) = 1$ under loss neutrality). The horizontal dotted lines on Figures 1(b) and 1(c) illustrate the interior PEs corresponding to hypothetical price $p = \$26$. For sufficiently large loss aversion coefficients there are either multiple interior PEs, as in Figure 1(b), or no interior PEs, as in Figure 1(c). In the latter case, only the corner θ_1 is a PE.

A consumer who consumes when her taste draw is above threshold θ derives the expected utility

$$\mathrm{EU}(\theta, p) = u^0(\theta)G(\theta) + \int_{\theta}^{\theta_1} u^1(\theta', \theta)\, dG(\theta'). \tag{12}$$

Our next result simplifies $\mathrm{EU}(\theta, p)$ and helps distinguish its components.



(a) $\lambda_c = \lambda_p = \lambda$ (b) $\lambda_c = 1.5, \lambda_p = 0.3$ (c) $\lambda_c = 0.3, \lambda_p = 1.5$

Figure 1 **The solid curves plot $V(\theta) = (v_0 + q\theta)L(\theta)$. The dashed line plots $v_0 + q\theta$, the loss-neutral consumer's willingness to pay. The parameters used for this figure are $v_0 = 10$, $q = 20$, $\beta_c = \beta_p = 0$, and $\theta \sim U[0,1]$.**

LEMMA 3. *The consumer's expected utility from consuming in accordance with the threshold consumption rule θ is*

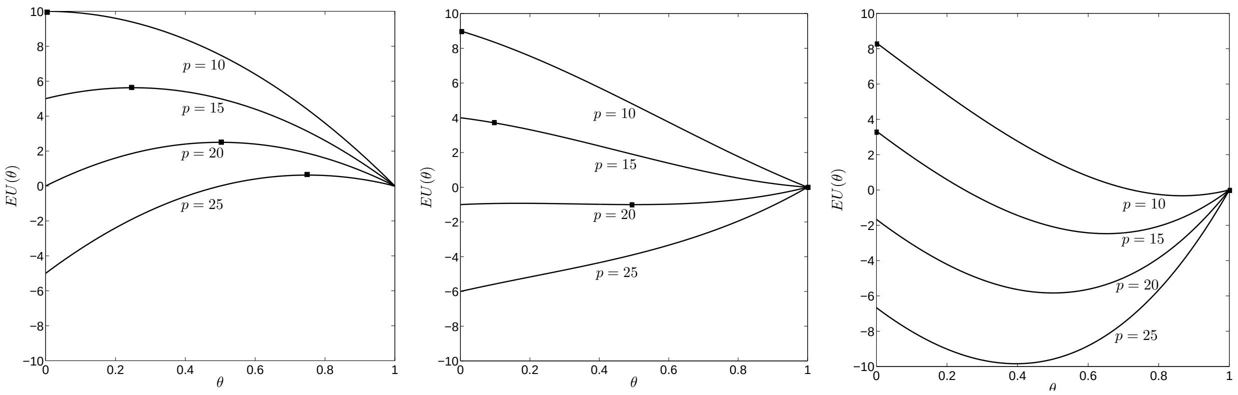
$$\mathrm{EU}(\theta, p) = \int_\theta^{\theta_1} (v_0 + q\theta' - p)\, dG(\theta') - (\lambda_c - \beta_c) \int_\theta^{\theta_1} (G(\theta') - \bar{G}(\theta'))(v_0 + q\theta')\, dG(\theta') \\ - (\lambda_p - \beta_p) p G(\theta) \bar{G}(\theta). \tag{13}$$

The expected utility in (13) has three components. The first term is the standard expected utility without loss aversion. The second term captures consumption loss aversion and is negative. The third term is the monetary loss aversion and is also negative. Although the firm's price is deterministic, the consumer still experiences monetary loss aversion for any interior PE because the consumption rule is random. Although monetary loss aversion is zero under full consumption, the consumption loss aversion is not because the consumer compares consumption utility across taste draws. Figure 2 illustrates consumer expected utility under a uniform taste distribution for three (λ_c, λ_p) pairs. The black dots represent the PPEs. In Figure 2(a), the-loss neutral consumer $(\lambda_p = \lambda_c = 0)$ has a concave expected utility and the θ that maximizes expected utility is also the threshold θ (from Figure 1) such that $v_0 + q\theta = p$. Low levels of loss aversion eliminate the curvature of the expected utility as shown in Figure 2(b). For sufficiently large values of loss aversion coefficients, the expected utility is convex (see Figure 2(c)) and the PPE is achieved at a corner.

The set of PPEs associated with price p is

$$\mathbf{\Theta}^{\mathrm{PPE}}(p) = \left\{ \theta \in \mathbf{\Theta}^{\mathrm{PE}}(p) \mid \mathrm{EU}(\theta, p) \geq \mathrm{EU}(\theta', p)\, \forall\, \theta' \in \mathbf{\Theta}^{\mathrm{PE}}(p) \right\}.$$

The set $\mathbf{\Theta}^{\mathrm{PPE}}(p)$ is nonempty (by Lemma 2) and it could have multiple elements if the consumer receives the same expected utility in multiple nondominated PEs. In this case we use the tie-breaking rule that the consumer selects the lowest PPE with nonnegative utility, which is the PPE



(a) $\lambda_c = 0, \lambda_p = 0$ (b) $\lambda_c = 0.3, \lambda_p = 0.3$ (c) $\lambda_c = 0.5, \lambda_p = 1$

Figure 2 Consumer's expected utility under loss neutrality and loss aversion. The PPEs are shown on the graphs with black rectangles. In this figure, $v_0 = 10$, $q = 20$, $\beta_c = \beta_p = 0$ and $\theta \sim U[0,1]$. The black dots represent the PPE θ corresponding to price $p = 10, 15, 20, 25$.

preferred by the firm. With this convention, $\Theta^{\text{PPE}}(p)$ has a unique element. Although a PPE exists for any price, the reverse is not true: there may exist thresholds that are not a PPE for any price. The set of implementable consumption thresholds is $\bigcup_{p>0} \boldsymbol{\Theta}^{\text{PPE}}(p)$.

Characterizing $\boldsymbol{\Theta}^{\text{PPE}}(p)$ for a given p consists of ranking PEs according to the expected utility criteria, so this ranking will depend on the shape of $\text{EU}(\theta, p)$. The three panels of Figure 2 together suggest that there may not be any simple and general ranking rules. Take, for instance, the case $\lambda_p = \lambda_c = 0.3$ (Figure 2(b)) and $p = 20$; the expected utility is almost flat. If there were multiple interior PEs, then the PPE selection could change for arbitrarily small changes in p or λ. That being said, an interesting pattern appears in Figure 2(c): extreme consumption thresholds ($\theta_0 = 0$ or $\theta_1 = 1$) dominate intermediate ones. In fact, we can derive fairly general conditions such that if the full-consumption corner θ_0 is a PE then it is also the PPE.

Proposition 1. *Suppose Assumption 1 holds and assume θ_0 is a PE such that* $\text{EU}(\theta_0) \geq 0$. *Then, θ_0 is a PPE.*

The intuition for Propostion 1 is that the expected utility decreases at $\theta = \theta_0$ and never returns to that level for $\theta > \theta_0$. To see this clearly, consider the three terms in equation (13); the first term is inverse U–shaped with a peak at $\theta = \frac{p - v_0}{q}$; the last two terms are negative and have a unique minimum at $\mathbb{E}\theta$. If these last two terms are sufficiently large, then the expected utility is initially decreasing and has a U-like shape; see Figure 2(c).

We turn to condition $\text{EU}(\theta) \geq 0$ in Lemma 6 and focus on the full consumption PE for price p_0^{LA}. The consumer's expected utility from full consumption is

$$\text{EU}(\theta_0, p_0^{\text{LA}}) = v_0 + q\mathbb{E}\theta - p_0^{\text{LA}} - (\lambda_c - \beta_c) q \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' \, dG(\theta'). \tag{14}$$

Assumption 2 gives a sufficient condition such that $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$.

Assumption 2. $q(\mathbb{E}\theta - \theta_0)\left(1 - \frac{\lambda_c - \beta_c}{2}\right) \geq \frac{\lambda_c - \beta_p}{1+\beta_p}(v_0 + q\theta_0)$.

Assumption 2 is independent of $G(\cdot)$, but clearly $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$ holds more generally. The expected consumption utility has to dominate the net loss from comparing consumption utility across θs, which it will whenever there is no loss aversion in consumption ($\lambda_c = \beta_c = 0$). More generally, Assumption 2 is more likely to hold for $\mathbb{E}\theta - \theta_0$ large and λ_c small. We can now present the main result of this section: characterizing a set of sufficient conditions under which full consumption solves the consumer's problem when the firm charges p_0^{LA}.

Proposition 2. *Suppose Assumptions 1 and 2 hold and the firm charges p_0^{LA}. Then θ_0 is a PPE.*

The firm can earn p_0^{LA} by implementing full consumption. But the firm may also charge more in interior PEs. We turn next to the firm's revenue maximization problem.

4.3. The Firm's Problem

The firm's revenue can be written as a function of the consumption threshold θ. Consider the interior values of $\theta \in (\theta_0, \theta_1)$. The threshold θ is a PE for price $p = (v_0 + q\theta)L(\theta)$. However, the consumer will buy at price p with probability $\bar{G}(\theta)$ only if θ is also a PPE for that price. This is the case if $\theta \in \mathbf{\Theta}^{\mathrm{PPE}}(p)$ and after replacement we obtain the PPE constraint $\theta \in \mathbf{\Theta}^{\mathrm{PPE}}((v_0 + q\theta)L(\theta))$. Thus, the firm maximizes revenues

$$R^{\mathrm{LA}}(\theta) = (v_0 + q\theta)L(\theta)\bar{G}(\theta) \tag{15}$$

subject to the PPE constraint. In addition to choosing interior thresholds, the firm can choose the full-consumption corner θ_0. In that case, the firm's revenue is $R^{\mathrm{LA}}(\theta_0) = \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0) = p_0^{\mathrm{LA}}$. We ignore the corner θ_1 because the firm earns zero revenue there. Denote by θ^{LA} the threshold that maximizes the firm's revenue.

Loss aversion changes the firm's objective function relative to the loss-neutral case in two ways: first, the objective function is weighted by $L(\theta)$; and second, not all consumption thresholds are feasible. The function $R^{\mathrm{LA}}(\theta)$ is not necessarily concave, and the set of thresholds that satisfy the PPE constraint is not necessarily convex. We could not find general conditions to characterize the optimal solution for all parameter values of loss aversion. Our main result rests on the observation that the revenue function reaches a maximum at θ_0 for a general subset of parameter values. In particular, this will be the case when $R^{\mathrm{LA}}(\theta)$ is decreasing in θ or, equivalently, when

$$\frac{\frac{\partial}{\partial\theta}[(v_0 + q\theta)\bar{G}(\theta)]}{(v_0 + q\theta)\bar{G}(\theta)} \leq -\frac{L_\theta(\theta)}{L(\theta)}. \tag{16}$$

We make the standard assumption that the firm revenue under loss neutrality, $R^{\mathrm{LN}}(\theta) = (v_0 + q\theta)\bar{G}(\theta)$, is single peaked with a maximum at θ^{LN}. Because $L(\theta)$ is decreasing, inequality (16) always holds for $\theta \geq \theta^{\mathrm{LN}}$. We derive a sufficient condition such that $R^{\mathrm{LA}}(\theta)$ is decreasing for any θ. Define $\varepsilon_0 = (v_0 + q\theta_0)\frac{g(\theta_0)}{q}$ as the price elasticity of a loss-neutral consumer at the corner $p = v_0 + q\theta_0$.

Assumption 3. $1 + \bar{G}(\theta^{\mathrm{LN}})\left(1 - \frac{1+\beta_c\beta_p}{(1+\lambda_c)(1+\lambda_p)}\right) \geq \varepsilon_0^{-1}$.

Since ε_0^{-1} increases with $\frac{q}{v_0}$, it follows that Assumption 3 is less likely to hold for high-quality products.

Lemma 4. *Assumption 3 implies that $R^{\mathrm{LA}}(\theta)$ is decreasing in θ. For uniformly distributed taste uncertainty, a necessary and sufficient condition for $R^{\mathrm{LA}}(\theta)$ to be decreasing in θ is that $\frac{q}{v_0} < 1 + \lambda_p + \frac{\lambda_c}{1+\lambda_c}$.*

Assumption 3 gives only a sufficient condition, and $R^{\mathrm{LA}}(\theta)$ can also be decreasing more generally. In particular, $R^{\mathrm{LA}}(\theta)$ is decreasing at θ_0 if the price elasticity of demand at θ_0, denoted $\varepsilon_0^{\mathrm{LA}}$, is greater than 1. We have $\varepsilon_0^{\mathrm{LA}} = \frac{\varepsilon_0}{1-\kappa\varepsilon_0}$, where $\kappa = \frac{\lambda_c+\lambda_p+\lambda_c\lambda_p-\beta_c\beta_p}{(1+\beta_p)(1+\lambda_c)}$. Although loss aversion increases the price elasticity of demand, monetary and consumption loss aversion do not have the same effects: $\kappa \to \infty$ with λ_p whereas $\lim_{\lambda_c\to\infty}\kappa = \frac{1+\lambda_p}{1+\beta_p}$. There is always a sufficiently large λ_p that $R^{\mathrm{LA}}(\theta)$ is decreasing at θ_0. Hence loss aversion transforms a standard inverse U–shaped revenue function into a function that is decreasing at θ_0 when $1 + \kappa > \varepsilon_0^{-1}$ and decreasing everywhere when Assumption 3 holds. For a uniform taste distribution and $\beta_c = \beta_p = 0$, the inequality $1 + \kappa > \varepsilon_0^{-1}$ is equivalent to the condition in Lemma 4 and so $R^{\mathrm{LA}}(\theta)$ decreasing at θ_0 implies that it is decreasing everywhere. This will be the case when monetary loss aversion is high or when product quality is low.

Proposition 3. *Suppose that Assumptions 1–3 hold. Then the full-consumption corner with associated price p_0^{LA} solves the firm's revenue maximization problem; that is, $\theta^{\mathrm{LA}} = \theta_0$.*

The intuition behind this proposition is as follows. Under loss neutrality, the firm must lower its price to $p_0^{\mathrm{LN}} = v_0 + \theta_0 q$ to induce the consumer to always buy. Such a low price is not necessary under loss aversion because the firm can charge $p_0^{\mathrm{LA}} > p_0^{\mathrm{LN}}$. That statement holds as long as the firm does not want to increase the price above p_0^{LA}. This will be the case if demand elasticity is not too large at θ_0 (Assumption 3).

5. The Uniform Pricing Puzzle

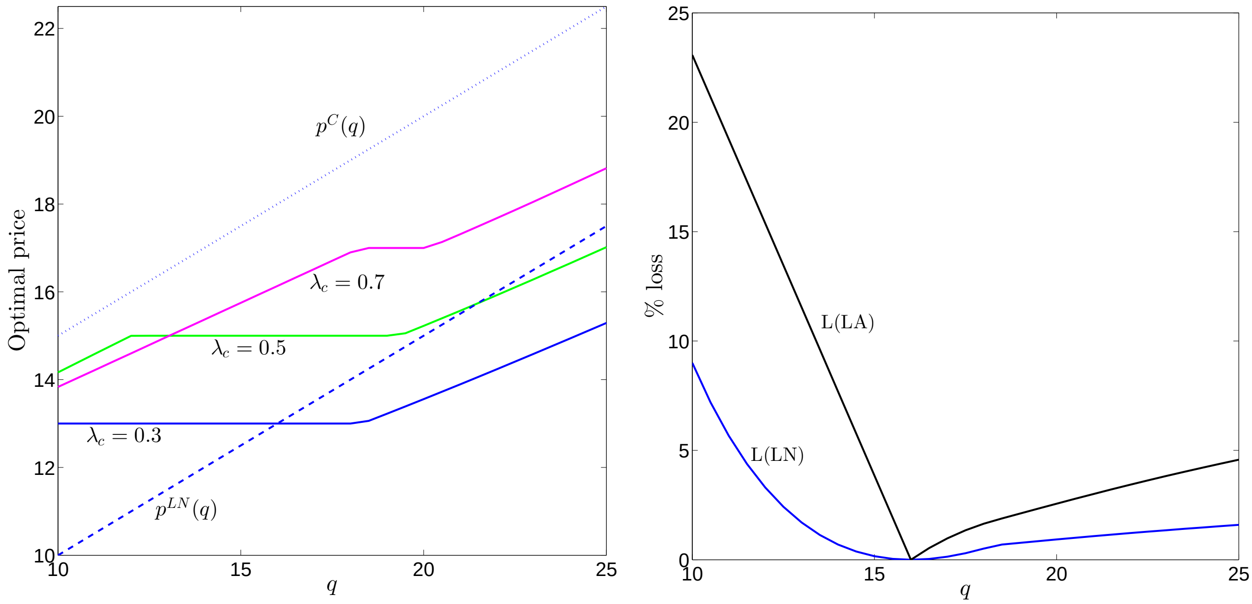
In this section, we assume that the firm sells multiple quality classes and study how the optimal price depends on quality. We follow Shiller and Waldfogel (2011) in ignoring demand interactions across quality classes. They argue that this approach is appropriate when the products' demands are "independent" and provide evidence that this condition is satisfied in their application to the pricing of music songs. In our model, demand interactions could happen through the reference point and/or through consumer substitution. Making the reference point depend on past purchases of multiple quality classes favors price compression across these quality classes, but this is not a compelling explanation for uniform pricing as we argued in the introduction. Turning to substitution across quality classes, the natural model for quality pricing with self-selection is Mussa and Rosen (1978). Consumer substitution introduces price compression relative to the case with independent demand (no private information) but uniform pricing is typically not optimal. By eliminating these two sources of demand interaction, we focus on the impact of gain-loss aversion and taste uncertainty.

For a product of quality q, we denote by $p^{\mathrm{LA}}(q)$ the firm's optimal price when selling to a loss-averse consumer. Similarly, let $p^{\mathrm{LN}}(q) = v_0 + q\theta^{\mathrm{LN}}(q)$ be the firm's optimal price when selling to a loss-neutral consumer, where $\theta^{\mathrm{LN}}(q)$ maximizes the loss-neutral revenue generated by a product of quality q. We make the standard assumption that there is a unique interior solution to the firm revenue maximization problem when the consumer is loss neutral, which implies that the firm's optimal price satisfies the comparative static $p_q^{\mathrm{LN}}(q) > 0$.[12] When taste is certain, we normalize it to $\mathbb{E}\theta$; then the optimal price, denoted $p^{\mathrm{C}}(q)$, is equal to $v_0 + q\mathbb{E}\theta$ (see Section 4.1).

Figure 3(a) plots $p^{\mathrm{LA}}(q)$ for $\lambda_p = 0.6$ and for different values of λ_c. The figure also shows the optimal price when consumers are loss neutral, $p^{\mathrm{LN}}(q)$, and when there is no taste uncertainty, $p^{\mathrm{C}}(q)$. The pricing schedules $p^{\mathrm{LA}}(q)$ have at most two kinks. The flat segments correspond to the case discussed previously: full consumption is optimal and $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$. The parts to the right of these flat segments correspond to interior equilibria (i.e., where Assumption 3 is violated); the parts to the left correspond to the case where θ_1 is a PE and $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) < \mathrm{EU}(\theta_1, p_0^{\mathrm{LA}}) = 0$ (i.e., where Assumption 2 is violated). The firm has to lower the price below p_0^{LA} to make the consumer choose the consumption threshold θ_0 instead of θ_1.

Clearly, the pricing schedules under loss aversion are less steep than under loss neutrality and taste certainty. To formalize this observation, we need workable definitions of uniform pricing and price compression that are applicable to our setting. A strict interpretation of the uniform pricing puzzle is that the price does not respond to quality, $p_q^{\mathrm{LA}}(q) = 0$; we denote this property (P1). (Loss aversion may also decrease the responsiveness of price to quality more generally, which we refer to as price compression.) A second property, (P2), is that $p_q^{\mathrm{LA}}(q) < p_q^{\mathrm{i}}(q)$ for $i \in \{\mathrm{LN}, \mathrm{C}\}$. This property holds everywhere in Figure 3(a): for a given q, both the loss-neutral and no-uncertainty

[12] The unimodality of the loss-neutral revenue function $(v_0 + q\theta)\bar{G}(\theta)$ is implied, for example, by the log-concavity of $g(\cdot)$. Sufficient conditions for an interior solution are $\varepsilon_0 < 1$ (which is equivalent, for a uniform $\mathbf{\Theta} = [0,1]$ distribution, to $q > v_0$), and $g(\theta_1) > 0$. We have $p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q)$ and $\theta_q^{\mathrm{LN}} > 0$.



(a) (b)

Figure 3 In both panels we have $\lambda_p = 0.6$, $v_0 = 10$, and $\theta \sim U[0,1]$. Panel (a) plots the optimal price as a function of q for different values of λ_c. In Panel (b) the top curve $L(LA)$ plots the percentage profit loss from wrongly assuming that the consumer is loss neutral when she is in fact loss averse (with $\lambda_c = 0.3$) and the lower curve $L(LN)$ plots the same loss from wrongly assuming that the consumer is loss averse when she is in fact loss neutral.

price schedules (the dashed and dotted lines, respectively) are steeper than any loss-averse price schedule.

Here we address the case corresponding to Assumptions 1–3 (the flat portion of the pricing schedules); additional results can be derived when these conditions do not hold (see Appendix C). We next state the paper's main result on price compression and price uniformity.

Proposition 4. *Suppose Assumptions 1–3 hold. (a) If $\theta_0 = 0$, then (P1) and (P2) hold. (b) If $\theta_0 > 0$, then (P2) holds provided* $\min(\mathbb{E}\theta, \theta^{\mathrm{LN}}) > \frac{1+\lambda_c}{1+\beta_p}\theta_0$.

Proposition 4(a) explains why vertically differentiated products with a wide range of quality may sell at the same price: we have $p_q^{\mathrm{LA}}(q) = 0$ and so (P1) holds; because $p_q^{\mathrm{LN}}(q) > 0$ and $p_q^{\mathrm{C}}(q) > 0$, we conclude that (P2) holds as well. Thus price is less responsive to quality under loss aversion than under loss neutrality. We now illustrate the relevance of Proposition 4(a) when the consumer's taste is uniformly distributed.

Corollary 1. *Assume that taste is distributed* Uniform$[0,1]$ *and that $\lambda_c < 3$. If $\frac{6\lambda_c}{3-\lambda_c} \le \frac{q}{v_0} < 1 + \lambda_p + \frac{\lambda_c}{1+\lambda_c}$ then $p^{\mathrm{LA}}(q) = (1+\lambda_c)v_0$ and consumption plan $\theta = 0$ maximizes the firm's profits.*

The conditions stated in Corollary 1 are tight bounds for $p_q^{\mathrm{LA}} = 0$ (the flat segments on Figure 3(a)). Uniform pricing can be optimal with consumption loss aversion alone or with monetary loss aversion alone. Even so, the two sources of loss aversion play asymmetric roles. *Monetary loss aversion* can only make it more likely that uniform pricing is optimal, and for high quality products ($q > 2v_0$) only monetary loss aversion can make uniform pricing optimal. *Consumption loss aversion* cannot be too strong if uniform pricing is to be optimal. An increase in consumption loss aversion lowers expected utility and the firm may have to lower the price below p_0^{LA} in order to rule out θ_1 as the PPE. This is what happens in the uniform case when $\frac{6\lambda_c}{3-\lambda_c} > \frac{q}{v_0}$: the price must be set below $p_0^{\mathrm{LA}} = (1+\lambda_c)v_0$ in order to make the consumer weakly prefer the consumption threshold θ_0 over θ_1.

Proposition 4(b) shows that (P2) holds for $\theta_0 > 0$ when both θ^{LN} and $\mathbb{E}\theta$ are sufficiently large relative to θ_0. What matters is any taste uncertainty that creates a gap between the lowest possible taste draw and the rest of the distribution. The intuition here is that, under loss aversion, $p_q^{\mathrm{LA}}(q) = \theta^{\mathrm{LA}}(q)L(\theta^{\mathrm{LA}}(q)) = \frac{1+\lambda_c}{1+\beta_p}\theta_0$ because $\theta_q^{\mathrm{LA}}(q) = 0$. With taste certainty we have $p_q^{\mathrm{C}}(q) = \mathbb{E}\theta$ whereas under loss neutrality, a lower bound for $p_q^{\mathrm{LN}}(q)$ is $\theta^{\mathrm{LN}}(q)$.[13]

We remark that it is essential to assume the complementarity between the taste draw and product quality in the consumer valuation. To see why, assume to the contrary that the taste draw and product quality are additive; that is, the consumer valuation is $\tilde{q} + \theta$ (instead of $v_0 + q\theta$), where $\tilde{q}$ is an additive quality component. The analysis proceeds as before once we put $v_0 = \tilde{q}$ and $q = 1$. The optimal price under loss aversion is $p^{\mathrm{LA}}(\tilde{q}) = (\tilde{q} + \theta_0)\frac{1+\lambda_c}{1+\beta_p}$, and so $p_{\tilde{q}}^{\mathrm{LA}}(\tilde{q}) = \frac{1+\lambda_c}{1+\beta_p}$. Under loss neutrality, the price schedule is such that $p_{\tilde{q}}^{\mathrm{LN}}(\tilde{q}) < 1$.[14] The price schedule is steeper under loss aversion than under loss neutrality: $p_{\tilde{q}}^{\mathrm{LA}}(\tilde{q}) > p_{\tilde{q}}^{\mathrm{LN}}(\tilde{q})$. In both the additive and multiplicative cases, loss aversion increases consumption; that is, the consumer buys the product for all taste draws. But this alone is not sufficient to make the price schedule flatter; in addition, product quality and the taste draw must be complements. If that is the case then the firm's price, which is equal to the lower bound of the valuation support, is relatively unresponsive to a change in quality.

If the consumer is loss averse, then a firm that deviates from uniform pricing will see its profits decline substantially. The reason is that the firm's optimal profits are achieved at the full-consumption corner and the firm's profits are not flat at the optimal price. Any deviation from the optimal price imposes a first-order loss as illustrated by Figure 3(b). For $\lambda_c = 0.3$, the top curve (L(LA)) plots the percentage profit loss from wrongly assuming that the consumer is loss neutral (when she is in fact loss averse) and hence charging $p^{\mathrm{LN}}(q)$ instead of $p^{\mathrm{LA}}(q)$; the bottom curve (L(LN)) plots the loss from making the opposite mistake. When $\lambda_c = 0.3$ we have $p^{\mathrm{LN}}(16) = p^{\mathrm{LA}}(16)$ and so $L(\mathrm{LA}) = L(\mathrm{LN}) = 0$ for $q = 16$. This establishes a benchmark case for which there is no cost

[13] We have $p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q)$, where $q\theta_q^{\mathrm{LN}}(q) > 0$ is the indirect effect due to re-optimizing price.

[14] We have $p^{\mathrm{LN}}(\tilde{q}) = \tilde{q} + \theta^{\mathrm{LN}}(\tilde{q})$ and because $\theta_{\tilde{q}}^{\mathrm{LN}}(\tilde{q}) < 0$, it follows that $p_{\tilde{q}}^{\mathrm{LN}}(\tilde{q}) = 1 + \theta_{\tilde{q}}^{\mathrm{LN}}(\tilde{q}) < 1$.

of not knowing whether the consumer is loss neutral or loss averse. We see that $L(\text{LN})$ stays close to zero for small deviations from $q = 16$ because small pricing mistakes have only a second-order effect on profits under loss neutrality. Yet this is not the case for $L(\text{LA})$: when the consumer is loss averse, even small mistakes can have a large negative effect on profits.

Under general conditions, the consumer's expected utility increases with product quality. We use $\widetilde{\text{EU}}(q) = \text{EU}(\theta_0, p_0^{\text{LA}}(q))$ to denote the consumer's expected utility from product q when the price is $p_0^{\text{LA}}(q)$. Then

$$\frac{\partial \widetilde{\text{EU}}(q)}{\partial q} = \mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta').$$

LEMMA 5. *Let $\lambda_c \geq \beta_p$ and assume that Assumption 2 holds. Then* $\frac{\partial \widetilde{\text{EU}}(q)}{\partial q} \geq (\mathbb{E}\theta - \theta_0)(1 - (\lambda_c - \beta_c)(\mathbb{E}\theta - \theta_0)) - \frac{\lambda_c - \beta_p}{1+\beta_p}\theta_0 \geq 0.$

Lemma 5 states that although consumers suffer a greater consumption gain–loss utility from higher-quality products, the overall effect (after accounting for direct consumption utility) is that utility increases with quality. This establishes a key feature of the uniform pricing puzzle: price compression is observed even though consumers receive a strictly larger surplus from better products.

6. Discussion and Conclusions

We present a model of monopoly pricing with vertically differentiated products, consumer loss aversion, and taste uncertainty. In this model the consumer compares current purchases using a lagged expectation of transactions involving products of the same quality. Thus loss aversion applies within a class of products of the same quality but *not* across quality classes. We show that uniform pricing can be optimal across quality classes up to a quality threshold. This will be the case if the consumer is sufficiently loss averse in monetary utility (but not too loss averse in consumption utility), if taste is sufficiently uncertain, and if product quality and the consumer's idiosyncratic taste draw are complements. If these conditions are satisfied, then the consumer consumes for all taste draws and the price is a function of the consumer's lowest possible valuation. Price compression occurs because that lowest valuation responds little to quality, and price uniformity is optimal when the lowest valuation does not depend on quality. In both cases, the price differences across quality classes is smaller under loss aversion than under loss neutrality, and consumer surplus increases with quality (which is consistent with casual observation).

Our model emphasizes the role of taste uncertainty in explaining price compression. In markets for entertainment goods, new creative products are regularly offered and displace older ones. The reason price compression occurs in these markets is that consumers have a taste preference that is product specific and associated with the unique creative content of each new product. Our model shows that a firm does not price quality associated with creative content such as talent when the consumer has a taste valuation shock that is complement with quality. In contrast, the model predicts that the firm should price product attributes when consumers have certain preferences. This is consistent with the observation that price vary for different show-times, theater amenities, and screening technologies (e.g., 2D vs. 3D); see Orbach and Einav (2007). This also explains why book prices depend on length (number of pages) and hardcover/softcover choice, but rarely on an authors' past success which is a predictor of sales (Clerides 2002).

When uniform pricing is optimal, deviating from it imposes a first-order loss on profits. In Section 5, we show that the losses that result from wrongly assuming loss neutrality when the consumer is in fact loss averse are always greater than the losses from making the opposite mistake. The reason is that, under loss neutrality, the profit function is flat around the optimal price. Under loss aversion, however, the optimal profit is achieved at the full-consumption corner, and a deviation from the optimal policy imposes a first-order loss. This finding is of relevance to the empirical literature that compares the profitability of price uniformity and variable pricing (e.g., Chu et al. 2011, Shiller and Waldfogel 2011). Take Shiller and Waldfogel's study of iTunes uniform pricing at $0.99 per song. The authors argue that revenue could increase by at least a sixth and as much as a third if Apple abandoned uniform pricing. Yet, these figures do not account for loss aversion, which probably affects consumers' music purchases (Kahneman et al. 1986). Uniform pricing may be optimal once we account for the loss aversion costs associated with differential pricing of quality classes. Our analysis also suggests that uniform pricing is more likely to be optimal for less popular songs.

Our aim in this paper is to demonstrate that loss aversion together with uncertain taste for quality can explain price compression and price uniformity for vertically differentiated products. Although our approach is plausible in the contexts described here, we do not rule out other explanations based on menu cost, contractual constraints, or other rationales in the various contexts where uniform pricing applies. The main message delivered by our research is that even small values of loss aversion can have significant effect on the firm's optimal price when consumers' valuation is uncertain.

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Appendix A: Notation

Table 4 summarizes the notation used in the paper.

Table 4 Notation

q	product quality
$\theta \in \mathbf{\Theta} = [\theta_0, \theta_1]$	consumer's taste draw
$g(\theta)$, $G(\theta)$	p.d.f and c.d.f. of θ
$\hat{\theta}(\theta)$	symmetric value of $\theta \in [\theta_0, \mathbb{E}\theta]$ relative to $\mathbb{E}\theta$
λ_c, β_c	loss–gain parameters in consumption utility
λ_p, β_p	loss–gain parameters in monetary utility
$\pi(\theta)$	consumption probability in state θ
$\bar{\pi} = \{\pi(\theta)\}_{\theta \in \mathbf{\Theta}}$	consumption plan
$u(\bar{\pi}\|\bar{\pi}, \theta)$	utility from consumption plan $\bar{\pi}$ in state θ; equation (17)
$\mathrm{EU}(\bar{\pi})$	expected utility of consumption plan $\bar{\pi}$; equation (2)
$u^0(\theta^*)$	utility of not consuming for threshold θ^*; equation (6)
$u^1(\theta, \theta^*)$	utility of consuming for taste draw θ and threshold θ^*; equation (7)
$\mathrm{EU}(\theta, p)$	expected utility from threshold θ; equations (12) and (13)
$L(\theta)$	ratio of attachment effect to comparison effect; equation (9)
$\tilde{\theta}$	state θ at which $L(\theta) = 1$
$\mathbf{\Theta}^{\mathrm{PE}}(p)$	set of interior PEs associated with price p
$\mathbf{\Theta}^{\mathrm{PPE}}(p)$	PPE associated with price p
$R^{\mathrm{LA}}(\theta; q)$	firm's revenue when consumers are loss averse; equation (15)
$\theta^{\mathrm{LA}}(q)$, $p^{\mathrm{LA}}(q)$	consumption threshold and price-maximizing revenue under loss aversion
p_0^{LA}	highest price such that full consumption is a PE
$\theta^{\mathrm{LN}}(q)$, $p^{\mathrm{LN}}(q)$	consumption threshold and price-maximizing revenue under loss neutrality

Appendix B: Proofs

Proof of Lemma 1: We first establish the ex post utility $u(\pi(\theta)|\bar{\pi}, \theta)$ in its most general form and then identify some of its derivatives. Table 5 presents the utility from following a random consumption plan $\pi(\theta)$ in

Table 5 Utility $u(\pi(\theta)|\bar{\pi}, \theta)$

Consumption Utility	$\pi(\theta)(v_0 + q\theta - p)$	
	Gain–Loss from:	
	Consumption Utility	Monetary Utility
Case 1: Consume (probability $\pi(\theta)$)	$-\lambda_c q \int_\theta^{\theta_1} \pi(\theta')(\theta' - \theta)\, dG(\theta')$ $+\beta_c(v_0 + q\theta) \int_{\mathbf{\Theta}} (1 - \pi(\theta'))\, dG(\theta')$ $+\beta_c q \int_{\theta_0}^{\theta} \pi(\theta')(\theta - \theta')\, dG(\theta')$	$-\lambda_p p \int_{\mathbf{\Theta}} (1 - \pi(\theta'))\, dG(\theta')$
Case 2: Not Consume (probability $1 - \pi(\theta)$)	$-\lambda_c \int_{\mathbf{\Theta}} \pi(\theta')(v_0 + q\theta')\, dG(\theta')$	$\beta_p p \int_{\mathbf{\Theta}} \pi(\theta')\, dG(\theta')$

state θ when the the reference consumption plan is $\bar{\pi}$. The first line corresponds to the standard consumption utility, and the other terms correspond to the consumption and monetary gain–loss utilities. These gain–loss terms compare what actually happens in state θ (consume with probability $\pi(\theta)$) with what the consumer expects to happen in her reference transaction (consume with probability $\pi(\theta')$ in state θ', which occurs with density $g(\theta')$). The ex post utility simplifies to

$$u(\pi(\theta)|\bar{\pi},\theta) = \pi(\theta)(v_0+q\theta-p) - (1-\pi(\theta))\int_{\Theta}\pi(\theta')\big(\lambda_c(v_0+q\theta')-p\beta_p\big)dG(\theta')$$
$$+\pi(\theta)\int_{\Theta}\big(\pi(\theta')\big(\beta_c(\theta-\theta')^+-\lambda_c(\theta'-\theta)^+\big)q+(1-\pi(\theta'))\big(\beta_c(v_0+q\theta)-\lambda_p p\big)\big)\,dG(\theta'). \quad (17)$$

The derivative with respect to $\pi(\theta)$ is

$$\frac{\partial u(\pi(\theta)|\bar{\pi},\theta)}{\partial \pi(\theta)} = v_0+q\theta-p+\int_{\Theta}\pi(\theta')\big(\lambda_c(v_0+q\theta')-p\beta_p\big)\,dG(\theta')$$
$$+\int_{\Theta}\big(\pi(\theta')\big(\beta_c(\theta-\theta')^+-\lambda_c(\theta'-\theta)^+\big)q+(1-\pi(\theta'))\big(\beta_c(v_0+q\theta)-\lambda_p p\big)\big)\,dG(\theta'),$$

so the cross partial derivative with respect to $\pi(\theta)$ and θ is

$$\frac{\partial^2 u(\pi(\theta)|\bar{\pi},\theta)}{\partial\theta\partial\pi(\theta)} = q+\lambda_c q\int_{\theta}^{\theta_1}\pi(\theta')\,dG(\theta')+\beta_c q\int_{\theta_0}^{\theta}\pi(\theta')\,dG(\theta')+\beta_c q\int_{\Theta}(1-\pi(\theta'))\,dG(\theta')>0. \quad (18)$$

Next we show that $\pi(\theta)$ is nondecreasing in θ. The proof proceeds by way of contradiction. Assume there exist $\theta_i<\theta_j$ such that $\pi(\theta_i)>\pi(\theta_j)$. Then

$$u(\pi(\theta_i)|\bar{\pi},\theta_i) \ge u(\pi(\theta_j)|\bar{\pi},\theta_i),$$
$$u(\pi(\theta_j)|\bar{\pi},\theta_j) \ge u(\pi(\theta_i)|\bar{\pi},\theta_j).$$

Summing up these two inequalities yields

$$(u(\pi(\theta_i)|\bar{\pi},\theta_j)-u(\pi(\theta_i)|\bar{\pi},\theta_i))-(u(\pi(\theta_j)|\bar{\pi},\theta_j)-u(\pi(\theta_j)|\bar{\pi},\theta_i))\le 0,$$

which contradicts (18).

Finally, we show that $\pi(\theta)\in\{0,1\}$ almost everywhere. Assume by contradiction that this is not the case. Then there exists an interval $[\theta_a,\theta_b]$ such that $\pi(\theta)\in(0,1)$ for $\theta\in[\theta_a,\theta_b]$ and so $u(1|\bar{\pi},\theta)=u(0|\bar{\pi},\theta)$ for $\theta\in[\theta_a,\theta_b]$. However,

$$u(0|\bar{\pi},\theta) = -\lambda_c\int_{\Theta}\pi(\theta')(v_0+q\theta')\,dG(\theta')+\beta_p p\int_{\Theta}\pi(\theta')\,dG(\theta'),$$

and $\frac{\partial u(0|\bar{\pi},\theta)}{\partial\theta}=0$ for $\theta\in[\theta_a,\theta_b]$ whereas

$$\frac{\partial u(1|\bar{\pi},\theta)}{\partial\theta} = q+\lambda_c q\int_{\theta}^{\theta_1}\pi(\theta')\,dG(\theta')+\beta_c q\int_{\theta_0}^{\theta}\pi(\theta')\,dG(\theta')+\beta_c q\int_{\Theta}(1-\pi(\theta'))\,dG(\theta')>0,$$

which is a contradiction. □

Proof of Lemma 2: If $u^1(\theta_0,\theta_0)-u^0(\theta_0)>0$ then $\theta=\theta_0$ is a corner PE, and if $u^1(\theta_1,\theta_1)-u^0(\theta_1)<0$ then $\theta=\theta_1$ is a corner PE. If neither inequality holds then, by continuity of the function $u^1(x,x)-u^0(x)$, there exists an interior PE $\theta\in(\theta_0,\theta_1)$ such that $u^1(\theta,\theta)=u^0(\theta)$. Thus a PE always exists, the existence of a PPE follows as a necessary consequence. □

Proof of Lemma 3: Recall from (6) and (7) that

$$u^0(\theta) = -\lambda_c\int_{\theta}^{\theta_1}(v_0+q\theta')\,dG(\theta')+\beta_p p\bar{G}(\theta) \quad \text{and}$$

$$u^1(\theta',\theta)=v_0+q\theta'-p-\lambda_c q\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'')+\beta_c\big((v_0+q\theta')G(\theta)+q\int_{\theta}^{\theta'}(\theta'-\theta'')\,dG(\theta'')\big)-\lambda_p pG(\theta),$$

for $\theta'>\theta$. Plugging these terms into (12), we obtain

$$\begin{aligned}\mathrm{EU}(\theta,p)=&(1-(\lambda_c-\beta_c)G(\theta))\int_{\theta}^{\theta_1}(v_0+q\theta')\,dG(\theta')-(\lambda_c-\beta_c)q\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'')\,dG(\theta')\\&-p\bar{G}(\theta)(1+(\lambda_p-\beta_p)G(\theta)).\end{aligned}\tag{19}$$

Observe that

$$\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'')\,dG(\theta')=\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}\theta''\,dG(\theta'')\,dG(\theta')-\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}\theta'\,dG(\theta'')\,dG(\theta').$$

Applying integration by parts to the first term yields $-G(\theta)\int_{\theta}^{\theta_1}\theta'\,dG(\theta')+\int_{\theta}^{\theta_1}G(\theta')\theta'\,dG(\theta')$. Collecting terms, we obtain

$$\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'')\,dG(\theta')=-\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'\,dG(\theta')-G(\theta)\int_{\theta}^{\theta_1}\theta'\,dG(\theta').$$

Plugging this expression into (19), we obtain

$$\begin{aligned}\mathrm{EU}(\theta,p)=&(1-(\lambda_c-\beta_c)G(\theta))\int_{\theta}^{\theta_1}(v_0+q\theta')\,dG(\theta')\\&+(\lambda_c-\beta_c)q\Big(\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'\,dG(\theta')+G(\theta)\int_{\theta}^{\theta_1}\theta'\,dG(\theta')\Big)-p\bar{G}(\theta)(1+(\lambda_p-\beta_p)G(\theta))\\=&\int_{\theta}^{\theta_1}(v_0+q\theta'-p)\,dG(\theta')-(\lambda_c-\beta_c)v_0G(\theta)\bar{G}(\theta)+(\lambda_c-\beta_c)q\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'\,dG(\theta')\\&-(\lambda_p-\beta_p)pG(\theta)\bar{G}(\theta).\end{aligned}$$

Because $G(\theta)\bar{G}(\theta)=-\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\,dG(\theta')$, we can write $\mathrm{EU}(\theta,p)$ as

$$\mathrm{EU}(\theta,p)=\int_{\theta}^{\theta_1}(v_0+q\theta'-p)\,dG(\theta')-(\lambda_c-\beta_c)\int_{\theta}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(v_0+q\theta')\,dG(\theta')-(\lambda_p-\beta_p)pG(\theta)\bar{G}(\theta).$$

□

Proof of Proposition 1: The proof follows from the next two Lemmas.

LEMMA 6. *Suppose Assumption 1 holds, let* $p\le v_0+q\mathbb{E}\theta$*, and assume* θ *is a PE in* $[\theta_0,\mathbb{E}\theta]$ *such that* $\frac{\partial \mathrm{EU}(\theta)}{\partial\theta}<0$ *and* $\mathrm{EU}(\theta)\ge 0$*. Then* θ *dominates any PE in* $(\theta,\hat{\theta}]$*.*

Proof of Lemma 6: We state and prove two claims that together prove Lemma 6.

CLAIM 1. *Suppose* $p\le v_0+q\mathbb{E}\theta$ *and* θ^* *is a PE in* $[\theta_0,\mathbb{E}\theta]$ *such that* $\frac{d\mathrm{EU}(\theta)}{d\theta}\big|_{\theta=\theta^*}<0$ *and* $\mathrm{EU}(\theta^*)\ge 0$*. Then* $\frac{d\mathrm{EU}(\theta)}{d\theta}<0$ *for* $\theta\in[\theta^*,\mathbb{E}\theta)$*.*

Proof of Claim 1: From Lemma 3 we obtain

$$\frac{d}{d\theta}\mathrm{EU}(\theta)=g(\theta)(M(\theta)-F(\theta)),\tag{20}$$

where $M(\theta)=p-(v_0+q\theta)$ and $F(\theta)=(\bar{G}(\theta)-G(\theta))((\lambda_c-\beta_c)(v_0+q\theta)+(\lambda_p-\beta_p)p))$. Since by assumption $\frac{d\mathrm{EU}(\theta)}{d\theta}\big|_{\theta=\theta^*}<0$, it follows that $F(\theta^*)>M(\theta^*)$. Furthermore, $v_0+q\mathbb{E}\theta\ge p$ implies that $F(\mathbb{E}\theta)=0\ge p-(v_0+q\mathbb{E}\theta)=M(\mathbb{E}\theta)$.

We also have $\frac{dF(\theta)}{d\theta} = -2((\lambda_p - \beta_p)p + (\lambda_c - \beta_c)(v_0 + q\theta))g(\theta) + (\lambda_c - \beta_c)q(\bar{G}(\theta) - G(\theta))$ and $\frac{d^2F(\theta)}{d\theta^2} = -4(\lambda_c - \beta_c)qg(\theta) - 2((\lambda_p - \beta_p)p + (\lambda_c - \beta_c)(v_0 + q\theta))g'(\theta)$. Both terms are negative and so we obtain $\frac{d^2F(\theta)}{d\theta^2} < 0$ for $\theta < \mathbb{E}\theta$.

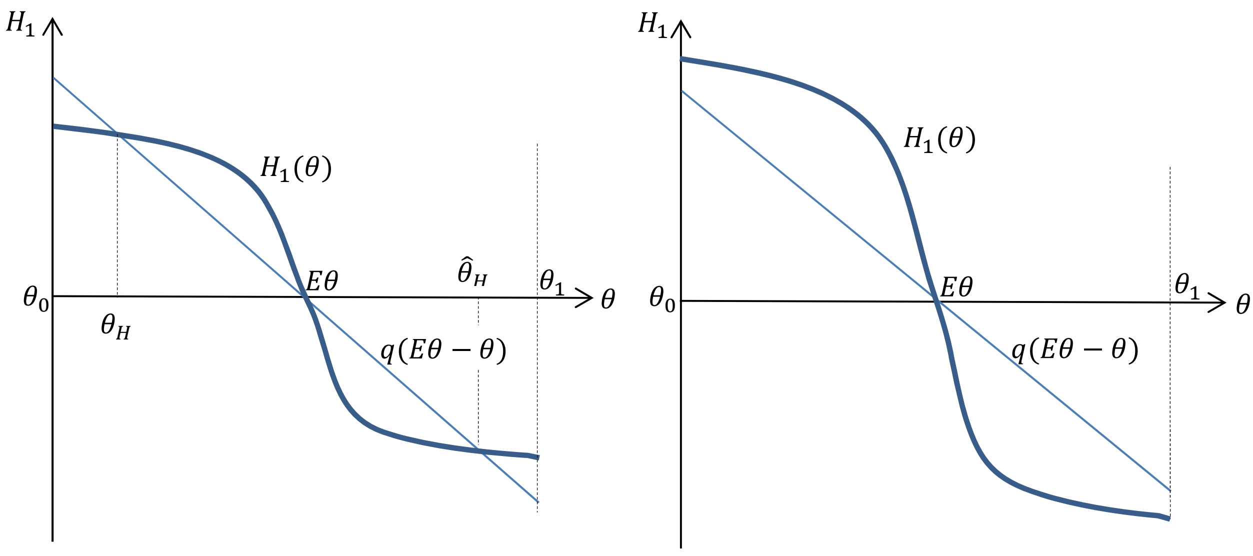
To sum up, we have $F(\theta^*) > M(\theta^*)$, $F(\cdot)$ is concave over $[\theta^*, \mathbb{E}\theta]$, and $F(\mathbb{E}\theta) \geq M(\mathbb{E}\theta)$. Hence we conclude that $F(\theta) > M(\theta)$ and $\frac{d\mathrm{EU}(\theta)}{d\theta} = g(\theta)(M(\theta) - F(\theta)) < 0$ for $\theta \in [\theta^*, \mathbb{E}\theta)$. □

Claim 2. *Suppose Assumption 1 holds, and θ^* is a PE in $[\theta_0, \mathbb{E}\theta]$ such that* $\frac{d\mathrm{EU}(\theta)}{d\theta}\Big|_{\theta=\theta^*} < 0$, *and* $\mathrm{EU}(\theta^*) \geq 0$. *Then* $\mathrm{EU}(\theta) < \mathrm{EU}(\theta^*)$ *for* $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.

Proof of Claim 2: By Lemma 3,

$$\frac{d\mathrm{EU}(\theta)}{d\theta} = g(\theta)(H(\theta) + K(\theta)), \tag{21}$$

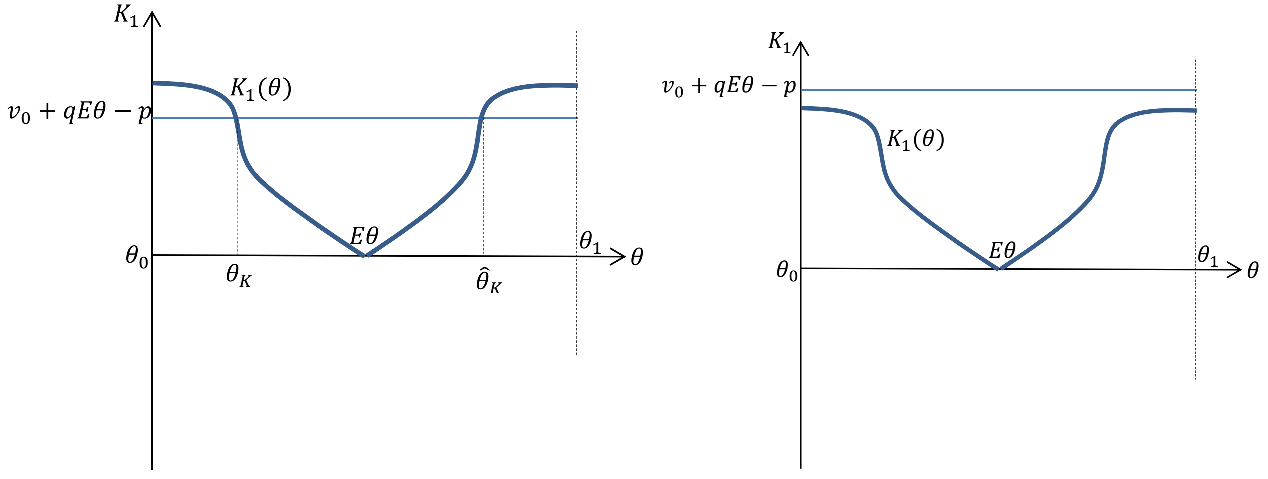
with $H(\theta) = q(\mathbb{E}\theta - \theta) - (\bar{G}(\theta) - G(\theta))((\lambda_c - \beta_c)(v_0 + q\mathbb{E}\theta) + (\lambda_p - \beta_p)p)$ and $K(\theta) = (\lambda_c - \beta_c)q(\mathbb{E}\theta - \theta)(\bar{G}(\theta) - G(\theta)) - (v_0 + q\mathbb{E}\theta - p)$. We can also write $H(\theta) = q(\mathbb{E}\theta - \theta) - H_1(\theta)$ and $K(\theta) = K_1(\theta) - (v_0 + q\mathbb{E}\theta - p)$. Given these definitions, the following properties hold (see Figures 4 and 5 for schematic representations)



(a) $H(\theta_0) \geq 0$ (b) $H(\theta_0) < 0$

Figure 4 $H(\theta) = q(\mathbb{E}\theta - \theta) - H_1(\theta)$.

($\triangle$1): $H_1(\theta)$ is positive over $[\theta_0, \mathbb{E}\theta]$, $\frac{dH_1(\theta)}{d\theta} = -2g(\theta)((\lambda_c - \beta_c)(v_0 + q\mathbb{E}\theta) + p(\lambda_p - \beta_p)) \leq 0$ for $\theta \in [\theta_0, \theta_1]$, $\frac{d^2H_1(\theta)}{d\theta^2} = -2g'(\theta)((\lambda_c - \beta_c)(v_0 + q\mathbb{E}\theta) + p(\lambda_p - \beta_p)) \leq 0$ for $\theta \in [\theta_0, \mathbb{E}\theta]$, and $H_1(\mathbb{E}\theta - x) = -H_1(\mathbb{E}\theta + x)$. Next, we claim that (i) when $H(\theta_0) < 0$, $H_1(\theta)$ never crosses $q(\theta - \mathbb{E}\theta)$ over $[\theta_0, \mathbb{E}\theta)$ (see Figure 4(b)); and (ii) when $H(\theta_0) \geq 0$, $H_1(\theta)$ crosses $q(\theta - \mathbb{E}\theta)$ exactly once over $[\theta_0, \mathbb{E}\theta)$ at a point that we denote by θ_H (see Figure 4(a)). Take the latter statement. If $H(\theta_0) \geq 0$, then $H_1(\cdot)$ is weakly lower than $q(\theta - \mathbb{E}\theta)$ at $\theta = \theta_0$ ($H_1(\theta_0) \leq q\mathbb{E}\theta$), the two are equal at $\theta = \mathbb{E}\theta$ ($H_1(\mathbb{E}\theta) = 0$), and H_1 is decreasing and concave while $q(\theta - \mathbb{E}\theta)$ is linear. Thus the two curves cross exactly once.



(a) $K(0) \geq 0$ (b) $K(0) < 0$

Figure 5 $K(\theta) = K_1(\theta) - (v_0 + q\mathbb{E}\theta - p)$.

($\triangle 2$): $K_1(\theta) \geq 0$, $\frac{dK_1(\theta)}{d\theta} = -(\lambda_c - \beta_c)q(\bar{G}(\theta) - G(\theta) + 2g(\theta)(\mathbb{E}\theta - \theta)) \leq 0$ for $\theta \in [\theta_0, \mathbb{E}\theta]$, and $K_1(\mathbb{E}\theta - x) = K_1(\mathbb{E}\theta + x)$. If $K(\theta_0) \geq 0$ (Figure 5(a)), then $K_1(\theta)$ intercepts $v_0 + p\mathbb{E}\theta - p$ exactly once in $[\theta_0, \mathbb{E}\theta]$ at a point that we denote by θ_K. When $K(\theta_0) < 0$ (Figure 5(b)), $K(\theta) < 0$ for $\theta \in [\theta_0, \theta_1]$. Because $\frac{d\mathrm{EU}(\theta_0)}{d\theta} < 0$, we have $H(\theta_0) + K(\theta_0) < 0$. Using this fact, we distinguish three cases.

Case 1: $K(\theta_0) < 0$,, $H(\theta_0) \geq 0$, and $\theta^* \leq \theta^H$; (see Figures 5(b) and 4(b)). We distinguish three intervals as follows. (1) $\theta \in [\theta^*, \theta_H]$. By Claim 1, $\mathrm{EU}(\theta)$ is decreasing in θ. (2) $\theta \in [\theta_H, 1 - \theta_H]$. We have the following properties: $H(\theta_H) = H(1 - \theta_H) = 0$; and ($\triangle 2$) implies that $\int_{\theta_H}^{\theta} H(\theta)\, dG(\theta) \leq 0$. We can now use the inequality $K(\theta) < 0$ to conclude that $\int_{\theta_H}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' < 0$. (3) $\theta \in [1 - \theta_H, \hat{\theta}^*]$. As a result, $H(\theta) \leq 0$ and $K(\theta) < 0$; hence $\frac{d\mathrm{EU}(\theta)}{d\theta} < 0$. Combining the conclusions drawn for each of these three intervals, we conclude that $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' < 0$ for any $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.

Case 2: $H(\theta_0) < 0$, $K(\theta_0) \geq 0$, and $\theta^* \leq \theta^K$ (see Figures 4(a) and 5(a)). Again we distinguish three intervals. (1) $\theta \in [\theta^*, \theta_K]$. By Claim 1, $\mathrm{EU}(\theta)$ is decreasing in θ. (2) $\theta \in [\theta_K, 1 - \theta_K]$. We have the following properties: because $\theta_K \leq \mathbb{E}\theta$, ($\triangle 2$) implies that $\int_{\theta_K}^{\theta} H(\theta)\, dG(\theta) \leq 0$ for any $\theta \in [\theta_K, 1 - \theta_K]$; since $K(\theta) \leq 0$, we conclude that $\int_{\theta_K}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' \leq 0$. (3) $\theta \in [1 - \hat{\theta}_K, \hat{\theta}^*]$. We now have $H(\theta) \geq 0$ and $K(\theta) \geq 0$ for $\theta \in [1 - \theta_K, \theta_1]$. Therefore, $\mathrm{EU}(\theta)$ increases over $[1 - \theta_K, \theta_1]$ and reaches its maximum $\mathrm{EU}(\theta_1) = 0$ at θ_1. Over that interval, we have $\mathrm{EU}(\theta) \leq \mathrm{EU}(\theta_1) = 0 \leq \mathrm{EU}(\theta^*)$. We combine the conclusions drawn for each of the intervals intervals (1)–(3) to conclude that $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' < 0$ for any $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.

Case 3: This includes all remaining cases: (3a) $K(\theta_0) < 0$, $H(\theta_0) \geq 0$, and $\theta^* > \theta^H$ (Figures 5(b) and 4(b)); (3b) $H(\theta_0) < 0$, $K(\theta_0) \geq 0$, and $\theta^* > \theta^K$ (Figures 4(a) and 5(a)); (3c) $H(\theta_0) < 0$ and $K(\theta_0) < 0$ (Figures 4(a) and 5(b)). The argument in each case is the same: $K(\theta) < 0$ and $\int_{\theta^*}^{\theta} H(\theta')\, dG(\theta') \leq 0$ for any $\theta \in [\theta^*, \hat{\theta}^*]$. Again $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' \leq 0$ for any $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.

In each of Cases 1–3, we obtain $U(\theta^*) > U(\theta)$ for $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$. Thus the PE θ^* dominates any other candidate PE $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$. □

The assumption $p \le v_0 + q\mathbb{E}\theta$ implies that the expected surplus of a loss-neutral consumer under full consumption is nonnegative, or that the price is not too high. According to Lemma 6, if the expected utility is decreasing at a PE for which consumption happens frequently ($\theta < \mathbb{E}\theta$) then that PE dominates any PE with intermediate consumption frequencies. Our next result establishes when the inequality $\frac{\partial \mathrm{EU}(\theta)}{\partial\theta} < 0$ holds.

LEMMA 7. *Assume that θ is a PE. Then $\frac{\partial \mathrm{EU}(\theta)}{\partial\theta} < 0$ if and only if $G(\theta) < \frac{\lambda_p(1+\lambda_c)-\beta_c(1+\beta_p)}{\lambda_c-\beta_c+\lambda_p-\beta_p+2(\lambda_p\lambda_c-\beta_c\beta_p)}$.*

Proof of Lemma 7: Differentiating (13) with respect to θ, we obtain

$$\frac{d}{d\theta}\mathrm{EU}(\theta) = -g(\theta)\big(v_0 + q\theta - p + (\bar{G}(\theta) - G(\theta))((\lambda_c - \beta_c)(v_0 + q\theta) + p(\lambda_p - \beta_p))\big).$$

Therefore, $\frac{d\mathrm{EU}(\theta)}{d\theta} < 0$ is equivalent to

$$1 - (\lambda_p - \beta_p)(\bar{G}(\theta) - G(\theta)) < \frac{v_0 + q\theta}{p}\big(1 + (\lambda_c - \beta_c)(\bar{G}(\theta) - G(\theta))\big).$$

By (8), at an interior PE we have $\frac{v_0+q\theta^*}{p} = \frac{1+\beta_p+(\lambda_p-\beta_p)G(\theta^*)}{1+\beta_c+(\lambda_c-\beta_c)\bar{G}(\theta^*)}$. Evaluating $\frac{d\mathrm{EU}(\theta)}{d\theta} < 0$ at $\theta = \theta^*$ now yields

$$1 - (\lambda_p - \beta_p)(\bar{G}(\theta^*) - G(\theta^*)) < \frac{1 + \beta_p + (\lambda_p - \beta_p)G(\theta^*)}{1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta^*)}\big(1 + (\lambda_c - \beta_c)(\bar{G}(\theta^*) - G(\theta^*))\big).$$

After some simplification, we obtain the inequality claimed in the lemma. □

Lemma 7 trivially applies to the corner PE at θ_0 because $\lambda_p > \beta_c$. This concludes the proof of Proposition 1. □

Proof of Proposition 2: We show that all conditions stated in Lemma 6 hold when the firm charges p_0^{LA} and the consumption threshold is θ_0: (a) θ_0 is a PE for price p_0^{LA}. (b) Assumption 2 implies $v_0 + q\mathbb{E}\theta \ge p_0^{LA}$. (c) Because $\lambda_p > \beta_c$, Lemma 7 implies that $\frac{\partial \mathrm{EU}(\theta)}{\partial\theta} < 0$ at θ_0. (d) It is shown below that Assumption 2 implies $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \ge 0$. Lemma 6 says that θ_0 is a PPE when the firm charges p_0^{LA}.

The expected utility under full consumption is given by (14). Observe that $\int_{\theta_0}^{\theta_1}(G(\theta') - \bar{G}(\theta'))\,dG(\theta') = 0$ and $\int_{\theta_0}^{\mathbb{E}\theta}(G(\theta') - \bar{G}(\theta'))(\theta' - \mathbb{E}\theta)\,dG(\theta') = \int_{\mathbb{E}\theta}^{\theta_1}(G(\theta') - \bar{G}(\theta'))(\theta' - \mathbb{E}\theta)\,dG(\theta')$. We can use these identities to rewrite the loss aversion component in (14) as

$$\int_{\theta_0}^{\theta_1}(G(\theta') - \bar{G}(\theta'))\theta'\,dG(\theta') = \int_{\theta_0}^{\theta_1}(G(\theta') - \bar{G}(\theta'))(\theta' - \mathbb{E}\theta)\,dG(\theta') = 2\int_{\theta_0}^{\mathbb{E}\theta}(G(\theta') - \bar{G}(\theta'))(\theta' - \mathbb{E}\theta)\,dG(\theta').$$

In addition, for $\theta \le \mathbb{E}\theta$ we have $-1 \le G(\theta') - \bar{G}(\theta') \le 0$. Hence

$$\int_{\theta_0}^{\mathbb{E}\theta}(G(\theta') - \bar{G}(\theta'))(\theta' - \mathbb{E}\theta)\,dG(\theta') \le \int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta - \theta')\,dG(\theta').$$

We next establish that $\int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta - \theta')\,dG(\theta') \le (\mathbb{E}\theta - \theta_0)/4$. The first step in proving this relation is to show that there must exist a $\theta_k \in [\theta_0, \mathbb{E}\theta]$ such that $g(\theta) \le \frac{1}{\theta_1-\theta_0}$ for $\theta \le \theta_k$ and $g(\theta) \ge \frac{1}{\theta_1-\theta_0}$ for $\theta \ge \theta_k$. Assume this is not the case. Then, because $g(\cdot)$ is positive and increasing on $[\theta_0, \mathbb{E}\theta]$, we must have either that $g(\theta) > \frac{1}{\theta_1-\theta_0}$ for $\theta \in [\theta_0, \mathbb{E}\theta]$, which leads to the contradiction $G(\mathbb{E}\theta) > \frac{1}{2}$, or that $g(\theta) < \frac{1}{\theta_1-\theta_0}$ for $\theta \in [\theta_0, \mathbb{E}\theta]$, which leads to the contradiction $G(\mathbb{E}\theta) < \frac{1}{2}$.

The second step is to show that

$$\int_{\theta_0}^{\mathbb{E}\theta} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' \leq 0.$$

Yet this inequality holds since

$$\int_{\theta_0}^{\mathbb{E}\theta} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' = \int_{\theta_0}^{\theta_k} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' + \int_{\theta_k}^{\mathbb{E}\theta} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta'$$

and since, moreover, $\int_{\theta_0}^{\theta_k} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' \leq (\mathbb{E}\theta - \theta_k) \int_{\theta_0}^{\theta_k} \big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta'$ (because $g(\theta') \leq \frac{1}{\theta_1 - \theta_0}$ for $\theta \leq \theta_k$) and $\int_{\theta_k}^{\mathbb{E}\theta} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' \leq (\mathbb{E}\theta - \theta_k) \int_{\theta_k}^{\mathbb{E}\theta} \big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta'$ (because $g(\theta') \geq \frac{1}{\theta_1 - \theta_0}$ for $\theta \geq \theta_k$). Taking these inequalities into account, we arrive at

$$\int_{\theta_0}^{\mathbb{E}\theta} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' \leq (\mathbb{E}\theta - \theta_k) \int_{\theta_0}^{\mathbb{E}\theta} \big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' = (\mathbb{E}\theta - \theta_k)(\frac{1}{2} - \frac{1}{2}) = 0.$$

To conclude, we remark that $\int_{\theta_0}^{\mathbb{E}\theta} (\mathbb{E}\theta - \theta')\big(g(\theta') - \frac{1}{\theta_1 - \theta_0}\big)\, d\theta' \leq 0$ can be rewritten as

$$\int_{\theta_0}^{\mathbb{E}\theta} (\mathbb{E}\theta - \theta') g(\theta')\, d\theta' \leq \int_{\theta_0}^{\mathbb{E}\theta} \frac{\mathbb{E}\theta - \theta'}{\theta_1 - \theta_0}\, d\theta' = \frac{\theta_1 - \theta_0}{8} = \frac{\mathbb{E}\theta - \theta_0}{4}.$$

Putting everything together yields an upper bound for the loss aversion component:

$$\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta') \leq \frac{\mathbb{E}\theta - \theta_0}{2}.$$

After replacing the loss aversion component in equation (14), we obtain

$$\mathrm{EU}\big(\theta_0, p_0^{\mathrm{LA}}\big) \geq v_0 + q\mathbb{E}\theta - p_0^{\mathrm{LA}} - (\lambda_c - \beta_c) q \frac{\mathbb{E}\theta - \theta_0}{2}.$$

Thus $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$ for any $G(\cdot)$ if

$$v_0 + q\mathbb{E}\theta - p_0^{\mathrm{LA}} - (\lambda_c - \beta_c) q \frac{\mathbb{E}\theta - \theta_0}{2} \geq 0.$$

This expression simplifies to Assumption 2 once we plug in the value for p_0^{LA}. □

Proof of Lemma 4: We have

$$\frac{\frac{d}{d\theta}[(v_0 + q\theta)\bar{G}(\theta)]}{(v_0 + q\theta)\bar{G}(\theta)} = \frac{q}{v_0 + q\theta} - \frac{g(\theta)}{\bar{G}(\theta)}$$

and

$$\frac{L_\theta(\theta)}{L(\theta)} = \frac{-(\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p) g(\theta)}{(1 + \beta_p + (\lambda_p - \beta_p)G(\theta))(1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta))}.$$

Therefore, $\frac{\frac{d}{d\theta}[(v_0 + q\theta)\bar{G}(\theta)]}{(v_0 + q\theta)\bar{G}(\theta)} \leq -\frac{L_\theta(\theta)}{L(\theta)}$ is equivalent to

$$1 + \bar{G}(\theta) \frac{\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p}{(1 + \beta_p + (\lambda_p - \beta_p)G(\theta))(1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta))} \geq \frac{q\bar{G}(\theta)}{(v_0 + q\theta) g(\theta)}.$$

A bound for the first fraction in this inequality is

$$\frac{\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p}{(1 + \beta_p + (\lambda_p - \beta_p)G(\theta))(1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta))} \geq \frac{\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p}{(1 + \lambda_p)(1 + \lambda_c)} = 1 - \frac{1 + \beta_c\beta_p}{(1 + \lambda_p)(1 + \lambda_c)}.$$

It follows that $R^{\mathrm{LA}}(\theta)$ is decreasing as long as

$$1+\bar{G}(\theta)\left(1-\frac{1+\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)}\right)\geq \frac{q\bar{G}(\theta)}{(v_0+q\theta)g(\theta)}.$$

For $\theta\in[\theta_0,\theta^{\mathrm{LN}})$, we have

$$1+\bar{G}(\theta)\left(1-\frac{1+\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)}\right)\geq 1+\bar{G}(\theta^{\mathrm{LN}})\left(1-\frac{1+\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)}\right)\geq \varepsilon_0^{-1}\geq \frac{q\bar{G}(\theta)}{(v_0+q\theta)g(\theta)}$$

where the middle inequality follows from Assumption 3.

In the uniform case with $\theta_0=0$, $\theta_1=1$, and $\beta_c=\beta_p=0$, equation (15) states that the firm's revenue as a function of θ is

$$R^{\mathrm{LA}}(\theta)=\frac{(1-\theta)(q\theta+v_0)(1+\lambda_c(1-\theta))}{\theta\,\lambda_p+1}. \tag{22}$$

Differentiating $R^{\mathrm{LA}}(\theta)$ now gives

$$\frac{dR^{\mathrm{LA}}}{d\theta}=\frac{2\lambda_c\lambda_p q\theta^3-(2\lambda_c\lambda_p q-\lambda_c\lambda_p v_0-3\lambda_c q+\lambda_p q)\theta^2-2(2\lambda_c q-\lambda_c v_0+q)\theta-(\lambda_c\lambda_p+2\lambda_c+\lambda_p+1)v_0+(\lambda_c+1)q}{(1+\lambda_p\theta)^2}.$$

We have $\frac{dR^{\mathrm{LA}}}{d\theta}\big|_{\theta=1}=-\frac{q\lambda_p+\lambda_p v_0+q+v_0}{(\lambda_p+1)^2}<0$ as well as $\frac{dR^{\mathrm{LA}}}{d\theta}\big|_{\theta=0}=-((1+\lambda_c)(1+\lambda_p)+\lambda_c)v_0+q(1+\lambda_c)$. The condition specified in the lemma, $\frac{q}{v_0}<1+\lambda_p+\frac{\lambda_c}{1+\lambda_c}$, is necessary for the revenue function to be decreasing at θ_0. Given that condition, it suffices to show that the derivative is negative for all $\theta\in(0,1)$. We begin by noting some properties exhibited by the derivative of the numerator of $\frac{dR^{\mathrm{LA}}}{d\theta}$: (i) it is a quadratic and convex function of θ; (ii) at $\theta=0$ it takes the value $2(q(-2\lambda_c-1)+\lambda_c v_0)$ and at $\theta=1$ the value $2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0)$, (iii) $2\ (q(-2\lambda_c-1)+\lambda_c v_0)<2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0)$.

Next we distinguish three cases. (1) $2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0)\leq 0$. In this case, the derivative is decreasing in $\theta\in(0,1)$ and, because the derivative is negative at $\theta=0$, it is negative for all $\theta\in[0,1]$. (2) $(q(-2\lambda_c-1)+\lambda_c v_0)\geq 0$. In this case, the derivative is increasing in $\theta\in(0,1)$ and, because the derivative is negative at $\theta=1$, it is negative for all $\theta\in[0,1]$. (3) $(q(-2\lambda_c-1)+\lambda_c v_0)<0<2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0)$. In this case, the derivative first decreases and then increases; because the derivative is negative at both $\theta=0$ and $\theta=1$, we conclude that it is negative over $\theta\in[0,1]$. So in all cases, the R^{LA} is decreasing over $[0,1]$. □

Proof of Proposition 4: (a) $p_0^{\mathrm{LA}}=\frac{1+\lambda_c}{1+\beta_p}v_0$ and $p_q=0$. (b) We have $p_q^{\mathrm{LN}}(q)=\theta^{\mathrm{LN}}(q)+q\theta_q^{\mathrm{LN}}(q)$ and $\theta_q^{\mathrm{LN}}(q)=-\frac{g(\theta^{\mathrm{LN}})v_0}{qR_{\theta\theta}^{\mathrm{LN}}(\theta^{\mathrm{LN}})}>0$. The inequality follows because $R^{\mathrm{LN}}(\theta)$ is concave at θ^{LN}.

Now, if $\theta_0=0$, then $p_q^{\mathrm{LN}}(q)>0=p_q^{\mathrm{LA}}(q)$ and if $\theta_0>0$ then $\theta^{\mathrm{LN}}(q)\geq\frac{1+\lambda_c}{1+\beta_p}\theta_0$. Putting these statements together, we conclude that

$$p_q^{\mathrm{LN}}(q)=\theta^{\mathrm{LN}}(q)+q\theta_q^{\mathrm{LN}}(q)\geq\theta^{\mathrm{LN}}(q)\geq\frac{1+\lambda_c}{1+\beta_p}\theta_0=p_q^{\mathrm{LA}}(q).\qquad\square$$

Proof of Corollary 1: According to Lemma 4, $R^{\mathrm{LA}}(\theta)$ is decreasing in θ when $\frac{q}{v_0}<1+\lambda_p+\frac{\lambda_c}{1+\lambda_c}$. We need to check that when both θ_0 and θ_1 are PEs for $p=(1+\lambda_c)v_0$, the latter dominates the former. If it does then, by Proposition 2, θ_0 is a PPE for $p=(1+\lambda_c)v_0$. We have $\mathrm{EU}(\theta=0)=\frac{q}{2}-p+v_0-\frac{\lambda_c q}{6}$. Plugging in the price $p=(1+\lambda_c)v_0$ gives $\mathrm{EU}(\theta=0,p=(1+\lambda_c)v_0)=\frac{q}{2}-(1+\lambda_c)v_0+v_0-\frac{\lambda_c q}{6}$, in which case $\mathrm{EU}(\theta=0,p=(1+\lambda_c)v_0)\geq\mathrm{EU}(\theta=1,p=(1+\lambda_c)v_0)=0$ is equivalent to $\frac{6\lambda_c}{3-\lambda_c}\leq\frac{q}{v_0}$. Since $R^{\mathrm{LA}}(\theta)$ is decreasing in θ, there can be no other PE or PPE that yields more revenue than $(1+\lambda_c)v_0$. □

Proof of Lemma 5: We have

$$\frac{d\widetilde{\mathrm{EU}}(q)}{dq} = \mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta').$$

Assumption 2 implies that the $\mathrm{EU}(\theta = 0, p = (1+\lambda_c)v_0) \geq 0$. That is,

$$\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) = v_0 + q\mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0) - (\lambda_c - \beta_c)q\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta') \geq 0,$$

or, equivalently,

$$q\mathbb{E}\theta \geq v_0(\frac{1+\lambda_c}{1+\beta_p} - 1) + \frac{1+\lambda_c}{1+\beta_p}q\theta_0 - (\lambda_c - \beta_c)q\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta'),$$

and, if $\lambda_c \geq \beta_p$, the above inequality implies that

$$\mathbb{E}\theta \geq \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta').$$

We can use our derivations from the proof of Proposition 2 to write

$$\mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta') \geq \mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)(\mathbb{E}\theta - \theta_0)^2 \geq 0.$$

We conclude that $\frac{d\widetilde{\mathrm{EU}}(q)}{dq} \geq (\mathbb{E}\theta - \theta_0)(1 - (\lambda_c - \beta_c)(\mathbb{E}\theta - \theta_0)) - \frac{\lambda_c - \beta_p}{1+\beta_p}\theta_0 \geq 0.$ □

Appendix C: Deviation to No-Consumption; Interior Equilibrium

As mentioned in Section 4.3, it is not possible in general to characterize fully the firms's optimal consumption threshold θ^{LA}. We can, however, make specific statements. Recall that the firm maximizes $R^{\mathrm{LA}}(\theta)$ subject to $\theta \in \mathbf{\Theta}^{\mathrm{PPE}}((v_0 + q\theta)L(\theta))$. We consider the other two relevant cases on Figure 3(a). These cases are also of general interest. In the first case, both θ_0 and θ_1 are PEs at the optimal price which is set such that the consumer is indifferent between the two PEs (and ends up choosing the PE preferred by the firm, θ_0).[15] The firm has to lower the price below p_0^{LA} because θ_1 is a PE that is strictly preferred by the consumer when the price is p_0^{LA}. In Figure 3(a), this corresponds to the section of the curves to the left of the flat segments. An increase in consumption loss aversion reduces the slope of the price schedule.

PROPOSITION 5. *Assume that at the firm's optimal price, $p^{LA}(q)$, the consumer is weakly indifferent between PE θ_0 and θ_1, that is, $EU(\theta_0, p^{LA}(q)) = 0$. We have: $p^{\mathrm{LA}}_{q,\lambda_c}(q) < 0$.*

Proof of Proposition 5: The constraint that the consumer weakly prefers θ_0 over θ_1 is binding

$$\int_{\theta}^{\theta_1} (v_0 + q\theta' - p^{\mathrm{LA}}(q,\lambda_c))\, dG(\theta') - (\lambda_c - \beta_c)\int_{\theta}^{\theta_1} (G(\theta') - \bar{G}(\theta'))(v_0 + q\theta')\, dG(\theta') - (\lambda_p - \beta_p)p^{\mathrm{LA}}(q,\lambda_c)G(\theta)\bar{G}(\theta) = 0$$

We have $p^{\mathrm{LA}}_{q,\lambda_c}(q) < 0$ and $p^{\mathrm{LA}}_{q,\lambda_p}(q) = 0$. □

In Figure 3(a), the curves get flatter as λ_c increases to the left of the first kink. This response is characteristic of low-quality products and occurs also when the consumption loss aversion coefficient is large enough so that $EU(\theta_0, p_0^{LA})$ from equation (14) is negative. Monetary loss aversion has no effect on the slope of the price schedule, $p^{\mathrm{LA}}_{q,\lambda_p}(q) = 0$.

Next we consider the case of interior consumption thresholds. In Figure 3(a), these correspond to the section of the curves to the right of the flat segments. Interior PPEs are also always chosen for small enough values of loss aversion. Consumption loss aversion and monetary loss aversion increase consumption.

[15] What is happening here is that the set of θ that satisfy the PPE constraint (θ such that $\theta \in \mathbf{\Theta}^{\mathrm{PPE}}((v_0 + q\theta)L(\theta))$) is non-convex: It includes θ_0 and θ_1 but not the thresholds slightly above θ_0.

LEMMA 8. $\frac{\partial\theta^{\text{LA}}}{\partial\lambda_c} < 0$ *and* $\frac{\partial\theta^{\text{LA}}}{\partial\lambda_p} < 0$.

Proof of Lemma 8: Assume that the first-order approach holds. Then the derivative of the firm's revenue, equation (15), with respect to θ is

$$R_\theta^{\text{LA}} = \frac{d}{d\theta}[(v_0 + q\theta)\bar{G}(\theta)]L + (v_0 + q\theta)\bar{G}(\theta)L_\theta.$$

An interior solution is characterized by $R_\theta^{\text{LA}} = 0$ or

$$\frac{\frac{d}{d\theta}[(v_0 + q\theta)\bar{G}(\theta)]}{(v_0 + q\theta)\bar{G}(\theta)} = -\frac{L_\theta}{L}.$$

An increase in either λ_c or λ_p does not change the expression's left-hand side. However, $-\frac{d}{d\lambda_p}\frac{L_\theta}{L} > 0$ (similarly $-\frac{d}{d\lambda_c}\frac{L_\theta}{L} > 0$); that is, an increase in loss aversion increases the RHS. Since $\frac{d}{d\theta}[(v_0 + q\theta)\bar{G}(\theta)]$ is decreasing in θ, it must be that θ decreases. □

Although the consumption threshold decreases with consumption and price loss aversion, the effect on the slope of the price schedule is not possible to sign. For interior equilibria, we have

$$p_q^{\text{LA}}(q) = \theta^{\text{LA}}(q)L(\theta^{\text{LA}}(q)) + \big(qL(\theta^{\text{LA}}(q)) + (v_0 + q\theta^{\text{LA}}(q))L_\theta(\theta^{\text{LA}}(q))\big)\theta_q^{\text{LA}}(q).$$

An increase in either λ_c or λ_p reduces $\theta^{\text{LA}}(q)$, but its effect on the other terms cannot be signed.