Online Scarcity Signals: A Bayesian Persuasion Approach

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Abstract

An Online retailer sends a personalized scarcity signal to persuade a consumer to buy a good. The retailer uses an informative signal only when the consumer's prior is not too pessimistic. The optimal signal partitions the state space into two non-overlapping subsets. The bad realization is more likely as the prior is more pessimistic.

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1 Introduction

Scarcity and pressure tactic, such as limited-quantity offers (remaining stock left, flash sales), cues that prices will increase (Online Travel Agencies), and information about other consumers' searches, are widely used by Online retailers to increase sales (Aggarwal et al., 2011).[1] According to marketers and psychologists, scarcity creates a sense of urgency, it increases desirability and gives a perceived benefit of acting quickly (Brock, 1968; Mullainathan and Shafir, 2013). But these messages also contain valuable information (Courty and Ozel, 2017).

Consistent with this view, we present a Bayesian model of personalized signals (Gentzkow and Kamenica, 2011). A literal interpretation of this approach is that the seller treats each consumer differently. Another interpretation is that there is a representative consumer which is not unreasonable for some uses of Online scarcity signals. Consumers could be equally uninformed despite having different demands. We show that there is a general sense in which the seller can act as if she was dealing with a representative consumer.

The seller increases immediate sales by strategically manipulating the consumer's belief about the benefit of waiting. This paper focuses on personalized signals because this benchmark case is relevant and tractable. In our setup, the consumer is rational and the sender has no private information ex-ante. Consistent with many Online practices, the seller *commits* to a 'signalling policy'.

We show that the optimal signal is binary and maximizes the probability that the '*buy*' realization be sent. Under the '*buy*' posterior, the consumer is just indifferent between buying and waiting. We derive empirically relevant predictions on the link between the consumer's prior and posterior beliefs. To our knowledge this is the first Bayesian theory of scarcity signals.

This paper is an application of the general framework of Gentzkow and Kamenica (2011). One contribution is to demonstrate that their setup applies to scarcity signals. This paper also makes a theoretical contribution. Gentzkow and Kamenica (2011) offer a method to solve for the optimal signal in only specific cases that do not apply in our problem (it applies in the benchmark case with two states of the world). We derive the optimal signal in the continuous case.

[1] For example, Expedia highlights some airfares with the mention that only a few seats are left at the posted price. Some hotel booking websites post the number of consumers who have recently booked featured rooms.

2 Model

An Online merchant sells a good to a single consumer. There are two periods. The consumer receives surplus V^0 from buying the first period. The following period, the consumer receives a state-dependent surplus $v \in [0, V]$. Random variable v is distributed with CDF $F()$. The state captures some fundamental uncertainty that influences the consumer's surplus. It could be uncertainty about price (airline ticket), availability (inventory status on Amazon), or popularity (large demand). The consumer and the merchant share the same prior about the second period surplus. The merchant earns f_0 if she the consumer buys early and $f_1 \leq f_0$ if late. The merchant can find out information about the state. This is reasonable for Online retail because the merchant has exclusive access to information about prices, availability and demand. We investigate whether the merchant should reveal information following the model of Bayesian persuasion proposed by Gentzkow and Kamenica (2011). The merchant *commits ex-ante* to an information disclosure policy that systematically reveals information following a preset rule. We restrict to discrete and finite signals. This is without loss of generality as will be clear soon. Denote τ_n the probability that signal realization $n = 1..N$ is sent. A signal could say, for example, that prices are likely to increase, that inventory is low, or that other consumers are buying the item searched. Bayes plausibility (BP) requires that $\tau_n \in [0, 1]$, $\sum_n \tau_n = 1$, and that the posteriors $F^n()$ be well-defined CDFs, that is, $F^n(v) \in [0, 1]$ and $\frac{d}{dv}F^n(v) \geq 0$, such that

$$F(v) = \sum_n \tau_n F^n(v). \tag{1}$$

We follow Gentzkow and Kamenica (2011) in assuming that the retailer can use any Bayesian signal $(\tau_n, F^n())_{n=1..N}$ that satisfies BP and we characterize the optimal signal. Gentzkow and Kamenica (2011) demonstrate that the optimal signal can be easily derived when sender's expected utility is a convex function of the belief. This condition does not hold in our analysis and one must find a different approach to derive the optimal signal.[2]

Before turning to the analysis, we argue that there is a general sense in which the analysis applies to heterogenous consumers. Assume consumers have different valuations and denote consumer i's surplus in the first period by V_i^0. Denote the second period state by ω. Consumer i's valuation in state ω is $v_i(\omega) = \omega V_i^0$. That is, the state has a multiplicative effect on consumers. Then, consumer i purchases early if $V_i^0 \geq \int_0^V V_i^0 \omega dF(\omega)$. Since this condition is the same for all consumers, the analysis in the personalized signal case applies.

[2]Following Gentzkow and Kamenica (2011)'s notations, the sender's expected utility for belief F is $\hat{v}(F) = f_0 I\left(V^0 > \int_0^V v dF(v)\right) + f_1 I\left(V^0 \leq \int_0^V v dF(v)\right)$ which is not a convex or concave function. Moreover, Proposition 4 which characterizes the properties of the optimal signal does not apply here.

3 Analysis

The consumer buys early when $V^0 \geq \int_0^V v dF(v)$. This holds when the consumer has a pessimist prior $F()$. The retailer can only do worse by revealing information. An informative signal is never used. This straightforward result is a direct application of Proposition 2 in Gentzkow and Kamenica (2011).

When $V^0 < \int_0^V v dF(v)$, the retailer can restrict attention to binary signals ($N = 2$). To see why, consider a signal with $N > 2$. The consumer buys early for any realization n such that $V^0 \geq \int_0^V v dF^n(v)$. Denote the set of all such realizations X. The retailer can implement the same profits with a binary signal that is such that one of the posteriors is the weighted average of the posteriors F^n for $n \in X$. The other posterior is determined by BP.

Denote the two signal realizations 'g'ood and 'b'ad. The consumer buys when the signal is b as long as $V^0 \geq \int_0^V v dF^b(v)$. We call this inequality the incentive compatibility (IC) constraint. Under g, the consumer waits and subsequently buys with probability $1 - F^g(0)$. The retailer chooses (τ, F^b, F^g) to maximizes $\tau f_0 + (1-\tau)(1 - F^g(0)) f_1$ subject to (IC) and (BP). Denote $I_X()$ the indicator function.

Proposition 1. *The optimal signal is:* $\tau = F(v^*)$, $F^g(v) = \frac{F(v)}{1-\tau} I_{[v^*,\infty]}(v)$ *and* $F^b(v) = \frac{F(v)}{\tau} I_{[0,v^*)}(v)$, *where* v^* *is any solution to* $V^0 = \int_0^{v^*} \left(1 - \frac{F(v)}{F(v^*)}\right) dv$.

Proof of Proposition 1: Let $K(x) = \int_0^x \left(1 - \frac{F(y)}{F(x)}\right) dy$. The function $K(x)$ is weakly increasing and such that $K(0) = 0$ and $K(V) = \int_0^V v dF(v) > V^0$. Thus, a solution v^* to $K(v) = V^0$ exists.

We first show that $F^b(v) \geq F(v) \geq F^g(v)$. Assume not. Using integration by part, (IC) is rewritten $V^0 \geq \int_0^V \left(1 - F^b(v)\right) dv$. For any v such that $F^b(v) < F(v) < F^g(v)$, set $F^b(v) = F(v) = F^g(v)$. Under this new signal, IC and BP hold and profits weakly increase.

Next, we show that IC binds. Assume not. For any v such that $F^b(v) > F(v) > F^g(v)$, consider the new signal $\tilde{\tau} = \tau + \epsilon$, $\tilde{F}^b(v) = F^b(v) - \frac{F^b(v) - F^g(v)}{\tau}\epsilon$ and $\tilde{F}^g(v) = F^g(v) + \frac{\epsilon^2(F^b(v) - F^g(v))}{\tau(1-\tau-\epsilon)}$. BP holds since $\tilde{F}(v) = \tilde{\tau}\tilde{F}^b(v) + (1-\tilde{\tau})\tilde{F}^g(v)$. For ϵ small, IC holds and profits increase. A contradiction.

Finally, we have $F^b(v)(1 - F^g(v)) = 0$. If not, there exist a v such that $0 < F^g(v) \leq F^b(v) < 1$. Consider the alternative signal such that $F^g(v') = 0$ and $F^b(v') = \frac{F(v')}{\tau}$ for any $v' \leq v$. BP holds and IC is strict for this new signal. A contradiction.

The only signal that satisfies all three properties above is the one stated in the Proposition. QED

The signal reveals the partition of states $\{[0, v^*), (v^*, V]\}$. IC binding says that the consumer is just indifferent between buying and waiting when she receives the bad signal, and this in turn implies that the consumer does not benefit from the signal; only the retailer does. This indifference property is implied by Proposition 5 in Gentzkow and Kamenica (2011).

Finally, consider a comparative statics where the prior $F()$ changes. The likelihood to send the bad signal does not change if the prior changes only for surpluses $v > v^*$. The bad posterior does not change either. This is not the case if the prior changes for surpluses $v < v^*$. Assume, for example, that surpluses below v^* are less likely; $F(v)$ decreases for all $v < v^*$. The sender responds by sending the bad signal more often (τ increases). The opposite holds if $F(v)$ increases for $v < v^*$. More generally, we have $\tau_2 > \tau_1$ for any priors (F_1, F_2) such that $\frac{F_2(v')}{F_2(v)} > \frac{F_1(v')}{F_1(v)}$ for any $v > v'$.[3] The bad signal is more likely as the prior puts more weight on low state realizations in a proportional sense.

4 Conclusions

We argue that Online scarcity signals are consistent with a model of Bayesian persuasion. The retailer, who prefers a sale today over an indecisive consumer, nudges the consumer toward buying now by revealing negative information. This information is credible only if the retailer also commits to sometimes reveal positive information that reinforces the consumer's decision to delay. The theory has empirical implications on how the prior influences the optimal signal. The analysis focuses on personalized signals. An interesting question for future research is the case with multiple consumers and multiple states.[4]

References

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[3] Under the stated condition, $K_1(x) - K_2(x) = -\int_0^x \left(\frac{F_1(y)}{F_1(x)} - \frac{F_2(y)}{F_2(x)}\right) dy > 0$ and $K_1(x) > K_2(x)$. Thus $v_1^* < v_2^*$ and $\tau_1 < \tau_2$.

[4] With multiple consumers, the two-state case remains manageable. The seller's profits is written as a function of the consumers' posterior. If the profit function is convex at the prior, the seller uses a binary signal to move to the profit function's convex envelope.

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