2. PARETO EFFICIENCY AND ECONOMIC WELFARE

OUTLINE

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2.1 INTRODUCTION

- A central question in this course:
  - under what conditions do markets allocate resources to their most valuable use?
- That in turn raises a more fundamental question:
  - what do we mean by "most valuable use"?

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- Ideally, the most valuable use is that which maximizes "social welfare".

- But how do we measure social welfare?

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- We will soon see that we <u>cannot</u> measure social welfare.

- However, we <u>can</u> say that some resource allocations are better than others, according to the Pareto criterion.

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2.2 PARETO EFFICIENCY

- An allocation of resources is **Pareto efficient** if it is <u>not</u> possible to reallocate those resources in a way that makes at least one person better-off and no person worse-off.

- An allocation is **inefficient** if and only if it is not Pareto efficient.

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- It is helpful to cast these definitions in terms of a closely-related concept.
- A **Pareto improvement** is a reallocation of resources that makes at least one person better-off and no person worse off.
- Thus, we can say that an allocation of resources is Pareto efficient if and only if there are no Pareto improvements available.

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- We can also say that if moving from one allocation (**B**) to an alternative allocation (**A**) is a Pareto-improvement, then allocation **B** is **Pareto-dominated** by allocation **A**.
- Thus, an allocation is Pareto efficient if and only if it is not Pareto-dominated by an alternative allocation.

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- Two allocations **A** and **B** can be **Pareto-ranked** if and only if one allocation Pareto-dominates the other.

- This basis for ranking allocations is typically called the **Pareto criterion**.

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- **Example**: allocations in a simple two-person exchange economy

- Consider Figure 2-1(the Edgeworth box)
  – allocation A is Pareto-efficient
  – allocations A and B can be Pareto-ranked: allocation A Pareto-dominates allocation B (moving from B to A is a Pareto improvement)
  – thus, allocation B is inefficient

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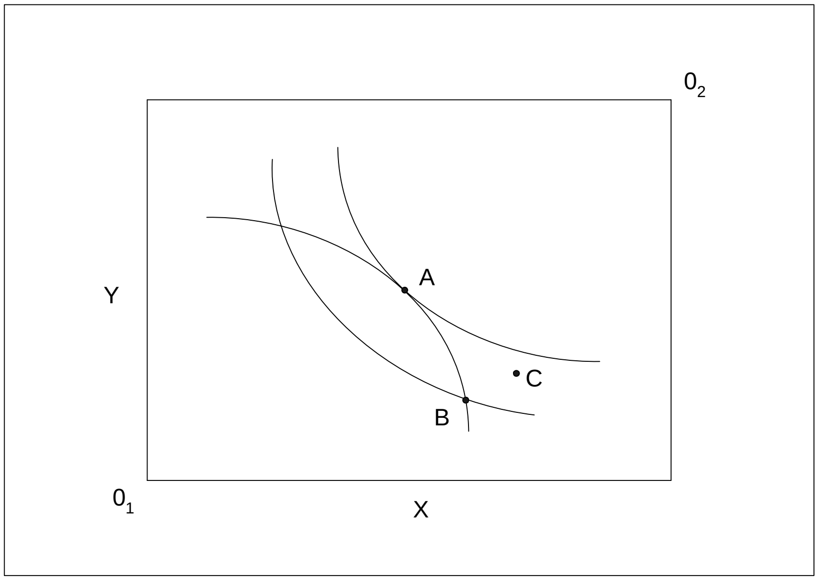

Figure 2-1

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- allocations A and C can be Pareto-ranked: allocation A Pareto-dominates allocation C (moving from C to A is a Pareto improvement)
- thus, allocation C is inefficient
- allocations B and C cannot be Pareto-ranked: B does not Pareto-dominate C; and C does not Pareto-dominate B.

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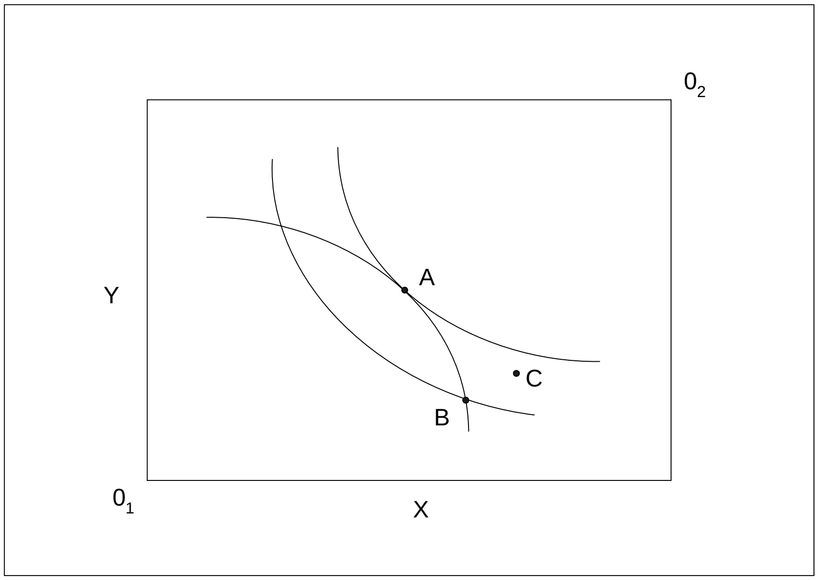

Figure 2-1 (repeat)

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SOME IMPORTANT RELATED CONCEPTS

- The **Pareto frontier** is the set of all Pareto efficient allocations.

- By definition, allocations on the Pareto frontier cannot be Pareto-ranked.

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- In the simple exchange economy, the Pareto frontier is the locus of tangencies between indifference curves; see Figure 2-2.

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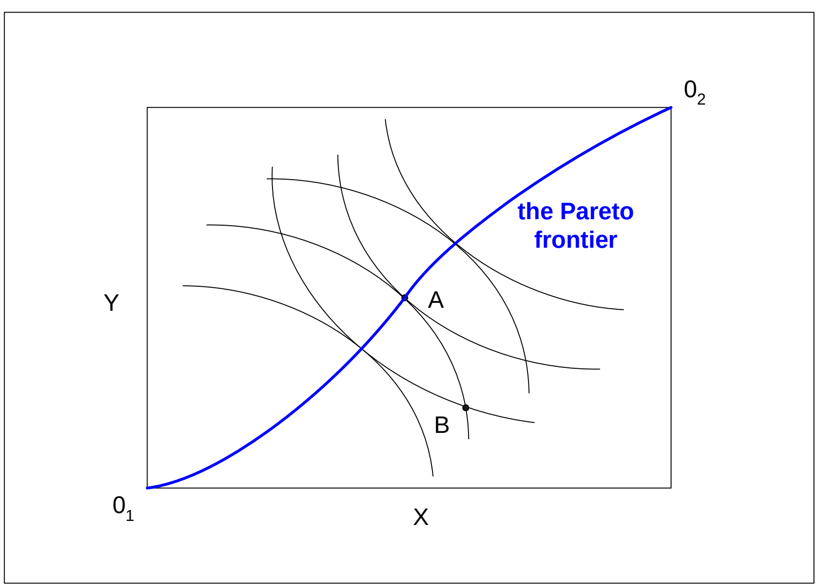

Figure 2-2

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- Why is it the locus of tangencies?

- From the definition of Pareto efficiency, if we hold the utility of person 2 fixed (to ensure that no reallocation makes them worse off) then Pareto efficiency requires that the utility of person 1 is maximized subject to that constraint on utility for person 2 (or else we could reallocate to make person 1 better off).

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- Thus, we can think of finding a Pareto efficient allocation as solving a constrained optimization problem:
  – maximize the utility of person 1 subject to holding the utility of person 2 fixed at some given level
  – see Figure 2-3

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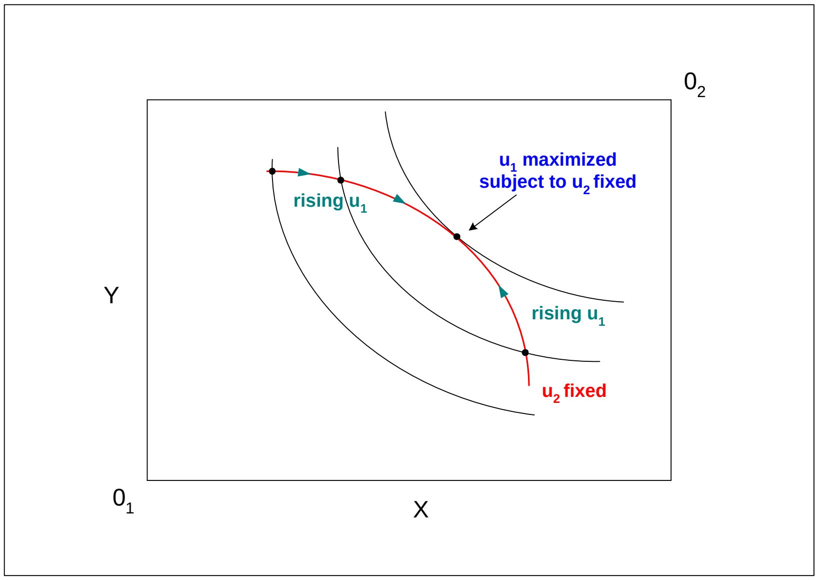

Figure 2-3

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- The solution to that optimization problem is a tangency, where the slopes of the indifference curves – the marginal rates of substitution – are equated.
- For each level of u_2 that we pick, there is a different tangency solution, and the locus of all these tangency solutions is the Pareto frontier; see Figure 2-2.

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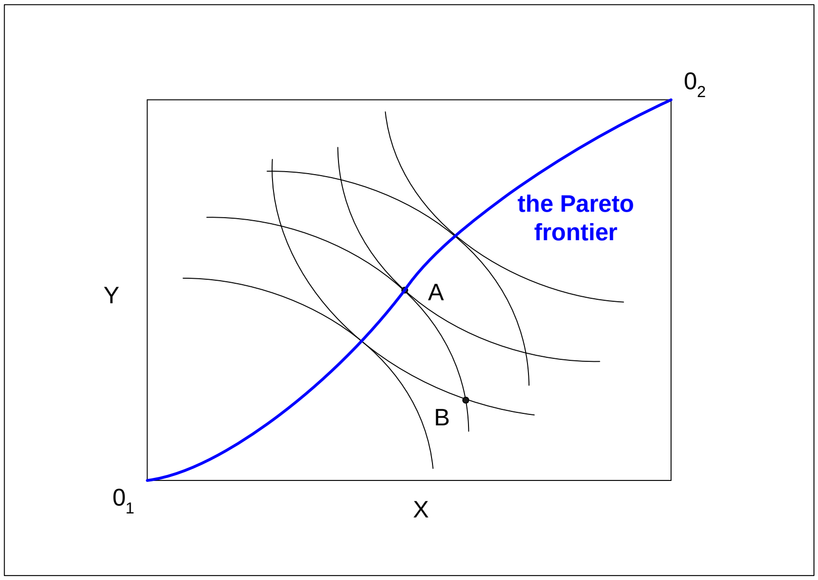

Figure 2-2 (repeat)

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- Relative to some existing allocation **B**, the **region of mutual benefit** is the set of all allocations that Pareto-dominate **B**.

- In the simple exchange economy, this region is shaped like a lens; see Figure 2-4.

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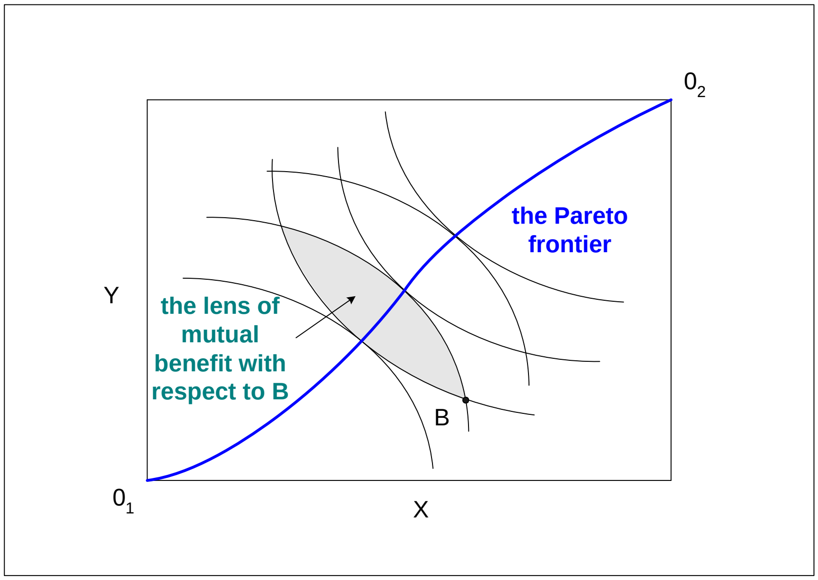

Figure 2-4

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- Relative to some existing allocation **B**, the **core with respect to B** is the set of Pareto-efficient allocations that Pareto-dominate **B**.
- Thus, the core is the intersection of two sets: the Pareto frontier, and the region of mutual benefit.
- See Figure 2-5 for the exchange economy case.

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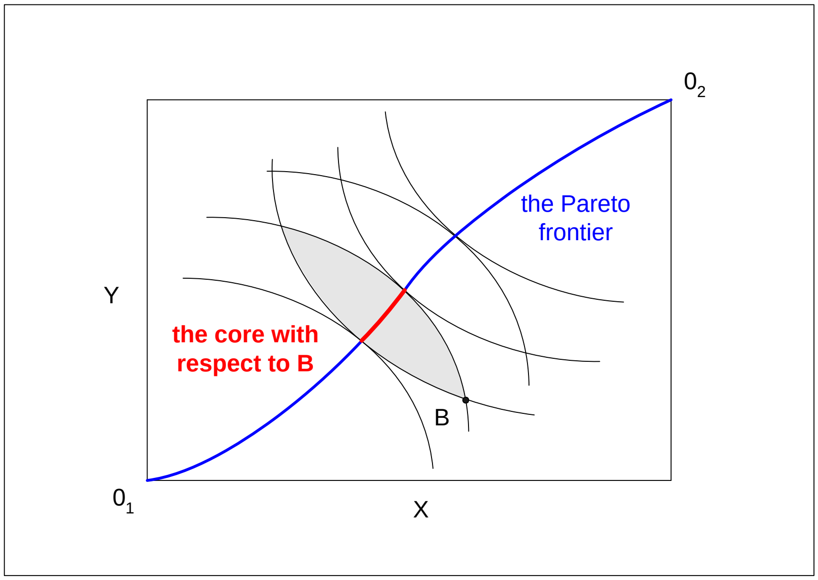

Figure 2-5

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- Note that the core is a <u>subset</u> of the Pareto frontier.
- This has two important implications.
- First, allocations in the core cannot be Pareto-ranked (because they are all Pareto efficient).

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- Second, a Pareto efficient allocation does not necessarily Pareto-dominate an inefficient one.

- For example, consider allocation P in Figure 2-6. This allocation is Pareto efficient but it does <u>not</u> Pareto-dominate allocation B.

- That is, P is not in the core with respect to B even though P is Pareto efficient.

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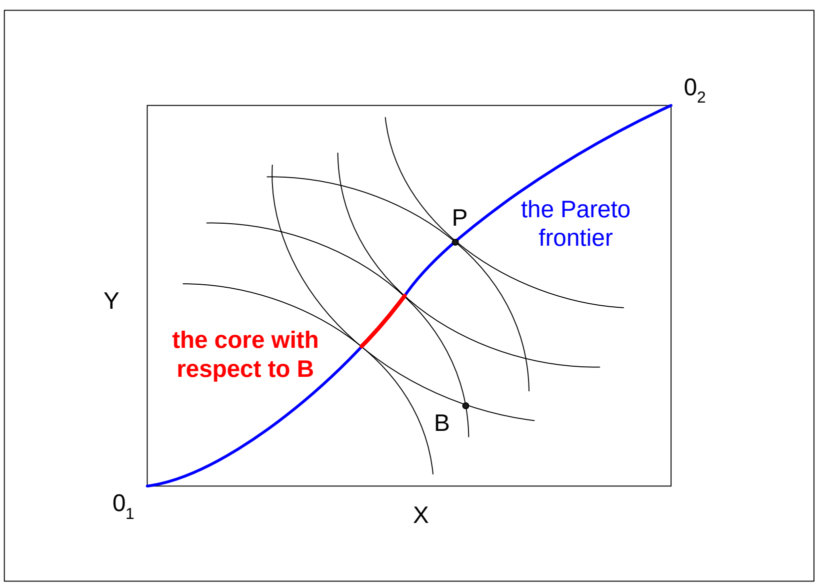

Figure 2-6

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- The Pareto criterion tells us that if we are at an inefficient allocation (call it allocation B) then we should move to an allocation in the core with respect to B.

- The Pareto criterion does not tell us that any Pareto efficient allocation is always better than an inefficient one.

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2.3 "SOCIAL PREFERENCES" AND ARROW'S IMPOSSIBILITY THEOREM

- Can we rank Pareto-efficient allocations to determine which one has the highest social welfare?

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- Ideally we would like to construct “social preferences”, based on individual preferences, and use these social preferences to derive a social ranking or social choice rule.

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- This ideal is <u>not</u> possible.

- Roughly speaking, **Arrow’s Impossibility Theorem** tells us that it is not possible to derive a complete and consistent social choice rule derived exclusively from individual preferences, except dictatorship.

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ARROW'S IMPOSSIBILITY THEOREM

- No social choice rule for ranking alternative allocations can simultaneously satisfy the following five requirements:
  - no dictatorship
  - completeness, reflexivity, transitivity (CRT)
  - unrestricted domain (any set of individual preferences that are CRT is permissible)

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- Pareto efficiency
- independence of irrelevant alternatives: the social ranking over two allocations ***x*** and ***y*** is independent of individual rankings over ***x*** and ***z***, and ***y*** and ***z*** (where ***z*** is the "irrelevant alternative")

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- Interpretation of independence of irrelevant alternatives:
  – the preference ranking for an individual between ***x*** and ***z***, and ***y*** and ***z*** should be irrelevant for the social ranking of ***x*** and ***y***.

  – the only difference between rankings ***x*** > ***y*** > ***z***, and ***x*** > ***z*** > ***y*** is a difference in intensity of preference.

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- However, we cannot observe intensity of preference directly because individual utility cannot be measured cardinally.

- It is not possible to demonstrate, for example, that person **A** derives five units of happiness from a particular allocation, while person **B** derives only four units of happiness from that allocation.

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- This is sometimes called the impossibility of “interpersonal utility comparisons”:
  - we cannot measure utility directly in any objective way that allows a comparison of utilities across different individuals

- This fundamental problem lies at the heart of the impossibility theorem.

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- Arrow’s theorem means that there is no compelling social rule for ranking Pareto efficient allocations.

- It is generally not possible to identify a unique “best” allocation of resources that in any sense “maximizes social welfare”.

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WHAT ABOUT ETHICS AND MORALS?

- A common response by some people to the impossibility theorem is that a “higher” criterion should be used for making social rankings, such as an “ethical” or “moral” criterion that transcends preferences.

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- This is a purely semantic argument: relabeling the preferences of some subset of individuals as “ethics” does not resolve the problem.

- An “ethical” solution is simply one based on the preferences of a subset of individuals (who effectively act as a collective dictatorship).

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- Note too that the possibility of a so-called "benevolent dictator" is eliminated by the impossibility theorem:
  – though possibly well-intentioned, the dictator is also faced with the impossibility of choosing an allocation based on the individual preferences of the subjects to whom she feels benevolent

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DOES VOTING SOLVE THE PROBLEM?

- Voting outcomes do not reflect preferences alone; they jointly reflect preferences <u>and</u> the structure of the voting rules in place (including the voting *agenda*).

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- That is, voting rules place a constraint on how many votes each individual has, and how those votes translate into direct influence over resource allocation.

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A VOTING EXAMPLE

- Example:
  - three available allocations: A, B and C
  - preference ordering for person 1: $A > B > C$
  - preference ordering for person 2: $B > C > A$
  - preference ordering for person 3: $C > A > B$

  - what is the <u>social</u> preference ordering?

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- Consider a candidate social choice rule:
  – a two-step pair-wise majority voting rule
- Step 1: *A* vs. *B*
  – *A* wins by 2 votes to 1 and *B* is eliminated
- Step 2: *A* vs. *C*
  – *C* wins by 2 votes to 1

- Implied social ranking: *C* > *A* > *B*

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- But this outcome is **agenda-dependent.**

- In particular, suppose we reverse the steps.
- Step 1: *A* vs. *C*
  – *C* wins by 2 votes to 1 and *A* is eliminated
- Step 2: *C* vs. *B*
  – *B* wins by 2 votes to 1

- Implied social ranking: *B* > *C* > *A*

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- Thus, the social ranking over *A* and *B* is reversed if we choose a different agenda.

- So why not simply vote over the agenda?

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- Because a third agenda is also possible.
- Step 1: *B* vs. *C*
  - *B* wins by 2 votes to 1 and *C* is eliminated
- Step 2: *B* vs. *A*
  - *A* wins by 2 votes to 1

- Implied social ranking: $A > B > C$

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- Thus, the three different agenda yield three different social rankings (each one corresponding to the preference ordering of one of the three voters).

- This means that voting over the different agenda is equivalent to voting over the outcomes obtained under those agenda, and so we face the same problem all over again.

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WHAT ABOUT ALTRUISM?

- Can we identity a unique social optimum if people have altruistic preferences?

- In general, altruism eliminates some allocations that might otherwise be efficient, but it does not lead to a unique best allocation.

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- Unavoidable bottom line:
  – there is no way to derive a social ranking based on individual preferences alone.

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SOCIAL WELFARE FUNCTIONS

- Despite the impossibility of making inter-personal utility comparisons, many economists still sometimes construct “social welfare functions” that purport to assess aggregate welfare from individual preferences.

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- These “social welfare functions” can sometimes be useful for framing philosophical issues relating to social justice, but it is important to recognize that they can never be made operational for practical purposes.

- See Appendix 2-1 for some examples.

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2.4 POTENTIAL PARETO IMPROVEMENTS AND SOCIAL SURPLUS

- A reallocation creates a **potential Pareto improvement** (PPI) if the winners could in principle make a compensating payment to the losers such that the losers are at least as well off as in the original allocation, and the winners are still better off after making that compensation.

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- For example, in the simple exchange economy depicted in Figure 2-7, a reallocation from B to C is not a Pareto improvement but it is a PPI:
  - person 1 could in principle make a compensating payment (in terms of good X) to person 2 such that person 2 is left no worse off than at allocation B, and person 1 is strictly better off than at allocation B.

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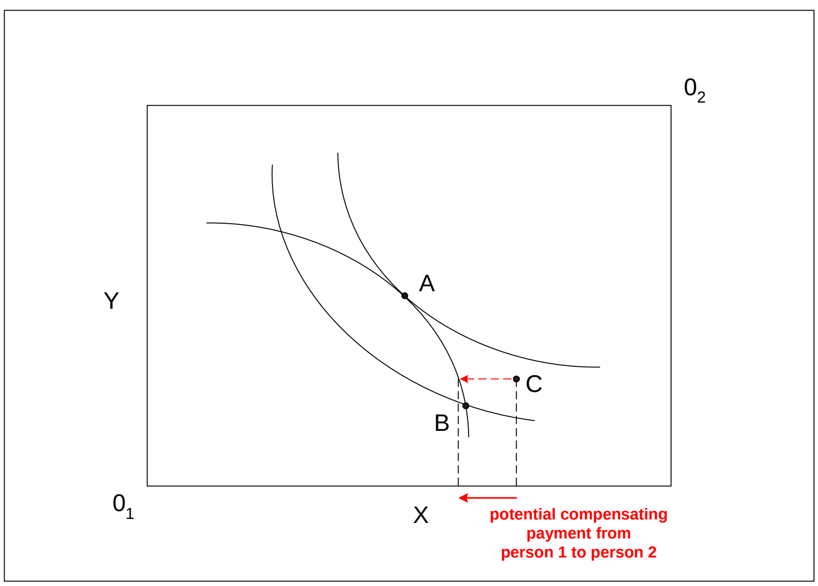

Figure 2-7

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- If the compensating payment is actually made, then we effectively move from allocation B to allocation D in Figure 2-8, and allocation D is a Pareto improvement over allocation B.

- It is in this sense that the move from B to C is a potential Pareto improvement.

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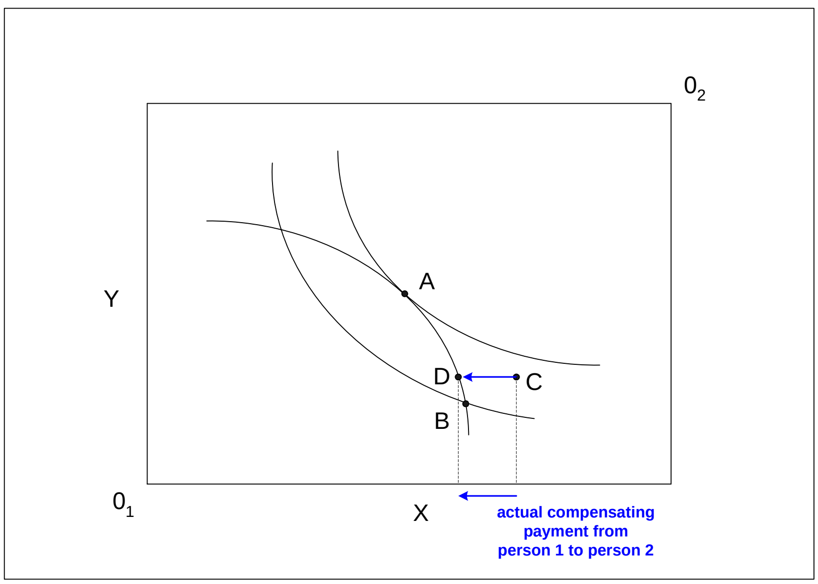

Figure 2-8

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- The difference between the gains to the winners and the losses to the losers, when measured in monetary units, is the **net social benefit** of a reallocation, or the **social surplus** created by the reallocation.

- Thus, if a reallocation creates a PPI then it has a positive net social benefit, or equivalently, it creates social surplus.

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- The potential Pareto improvement criterion is sometimes called the **Kaldor-Hicks criterion**, after the two economists who jointly proposed it.

- It is the central normative criterion in economic welfare analysis.

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- Confusingly, economists often use the term “efficiency” when they actually mean “social surplus maximization”.

- For example, consider the standard monopoly situation depicted in Figure 2-9.

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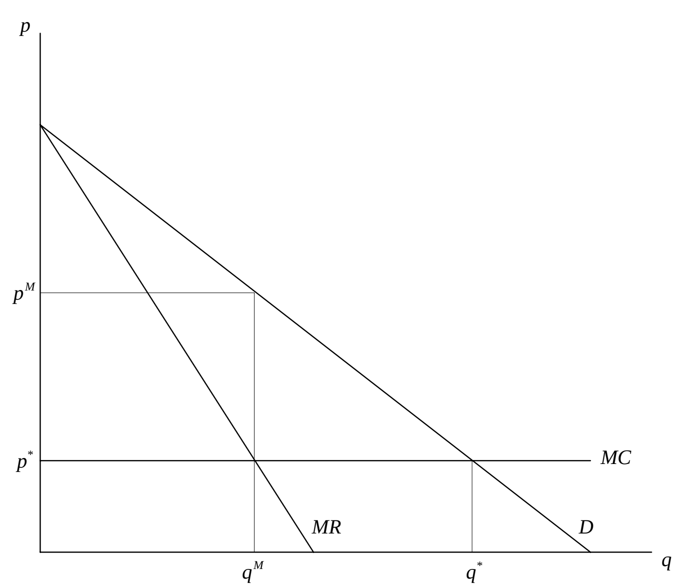

Figure 2-9

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- We often say that the monopoly outcome (denoted q^M in Figure 2-9) is "inefficient" but it is in fact Pareto efficient because it is not possible to make consumers better off without making the monopolist worse off.

- However, the monopoly outcome does not maximize social surplus; social surplus is maximized at q^*.

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- In particular, if the monopolist is forced to reduce its price from p^M to p^* it will suffer a loss of profit equal to the shaded area in Figure 2-10 but consumers will gain consumer surplus equal to the shaded area in Figure 2-11.

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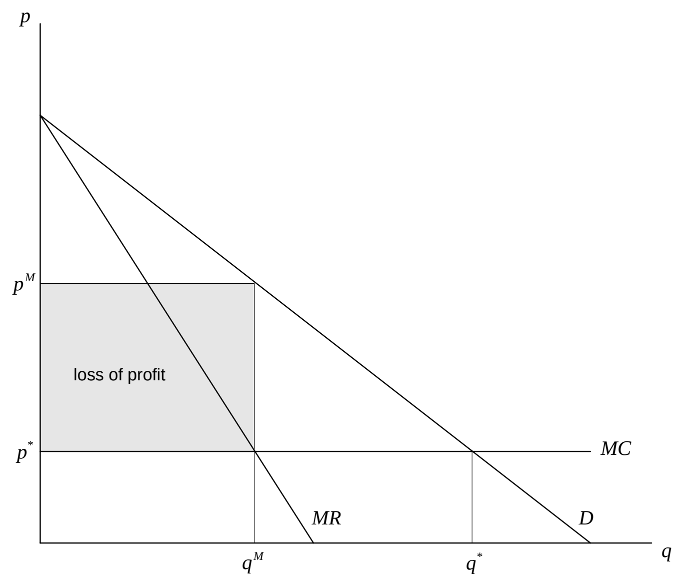

Figure 2-10 65

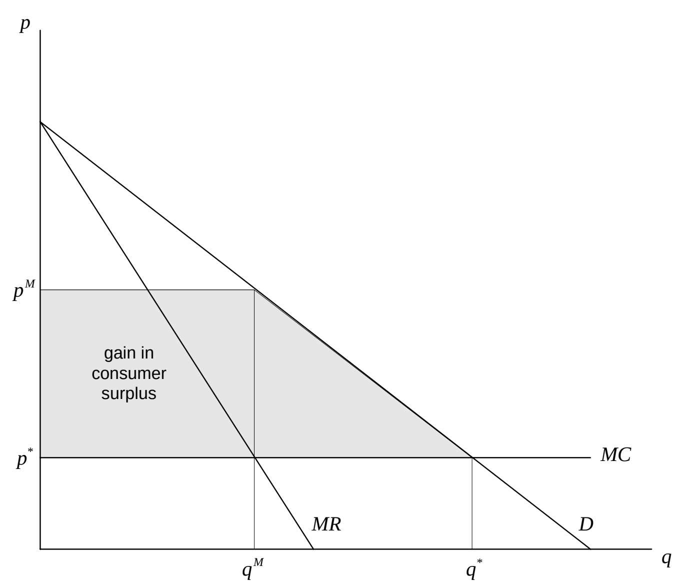

Figure 2-11 66

- Thus, consumers could in principle compensate the monopolist for its loss of profit, and still be better off by the shaded area in Figure 2-12.

- This shaded area is the familiar deadweight loss (DWL) from monopoly; it is the foregone surplus at the monopoly outcome relative to the maximum possible surplus.

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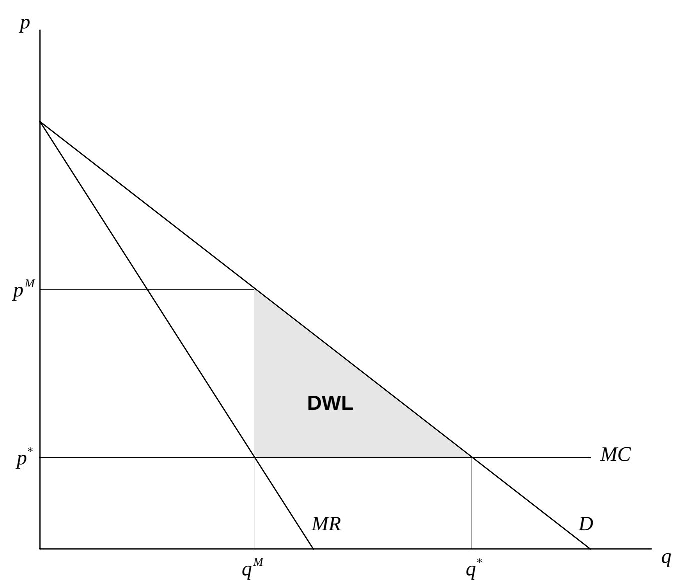

Figure 2-12

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- In general, the term “inefficient” is often mistakenly used to describe an outcome at which social surplus is not maximized even when that outcome is in fact Pareto efficient.
- This confusing terminology is especially common in textbook discussions of externalities, price controls, and other settings where DWLs arise.

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- The confusion in terminology can also lead to misleading statements like “allocation A is <u>more efficient</u> than allocation B”, when what is actually meant is that “allocation A has greater social surplus than allocation B” (and where both allocations could in fact be Pareto efficient).

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- We will try to be more precise in our use of terminology here.

- We will use the term “inefficient” only to describe an allocation that is not Pareto efficient.

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- If we wish to describe an allocation at which social surplus is not maximized, we will say exactly that.

- In particular, we will not describe one allocation as “more efficient” or “less efficient” than another.

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2.5 EXAMPLE: DERIVING A PARETO FRONTIER

Consider a simple two-person exchange economy in which both people have Cobb-Douglas preferences:

(2.1) $$u1 := x1^{a1}\, y1^{b1}$$

(2.2) $$u2 := x2^{a2}\, y2^{b2}$$

The resource constraints in this economy imply that

(2.3) $$x2 := X - x1$$

(2.4) $$y2 := Y - y1$$

Make these substitutions for x_2 and y_2 in (2.2) to yield an expression for u_2 in terms of what person 1 consumes:

(2.5) $$u2 := (X - x1)^{a2}\,(Y - y1)^{b2}$$

We can now find the Pareto frontier as the solution to

(2.6) $$\max_{x_1, y_1} u_1 \text{ subject to } u_2 = U_2$$

where U_2 is some fixed level of utility for person 2.

We can solve this constrained optimization problem using the Lagrange method or we can simply rearrange the constraint and substitute it directly into the objective function. In particular, fix u_2 from (2.5) at some value U_2 and rearrange this constraint to express it in terms of y_1:

(2.7) $$y1 := Y - \frac{U2^{\left(\frac{1}{b2}\right)}}{(X - x1)^{\left(\frac{a2}{b2}\right)}}$$

Now substitute (2.7) into (2.1) to yield

(2.8) $$u1 := x1^{a1}\left(Y - \frac{U2^{\left(\frac{1}{b2}\right)}}{(X - x1)^{\left(\frac{a2}{b2}\right)}}\right)^{b1}$$

We can now choose x_1 to maximize u_1 by differentiating (2.8) with respect to x_1 and setting this derivative equal to zero. The derivate is a bit messy so for the moment we will just write it as

(2.9) $$\frac{du_1}{dx_1} = 0$$

We can now solve (2.9) to derive an explicit solution for x_1 in terms of U_2. To transform this into an expression that we can plot in the Edgeworth box (whose axes measure x_1 and y_1) we can substitute (2.5) for U_2 in (2.9) and then obtain the Pareto frontier as the solution to

(2.10) $$\left.\frac{du_1}{dx_1}\right|_{U_2 = u_2} = 0$$

Note that the substitution for U_2 is made <u>after</u> we differentiate (2.8) with respect to x_1; otherwise we would not be holding u_2 fixed as we change x_1.

Even though the differentiation yields some messy terms, the solution to (2.10) is actually quite simple. In particular, solving (2.10) for y_1 as a function of x_1 yields an explicit solution for the Pareto frontier that we can plot in the Edgeworth box:

(2.11) $$y1PF := -\frac{Y\,x1\,b1\,a2}{(a1\,b2 - b1\,a2)\,x1 - a1\,b2\,X}$$

For example, suppose we choose the following parameter values: $a_1 = 2$, $b_1 = 3$, $a_2 = 4$, $b_2 = 1$, $X = 20$ and $Y = 10$. Then the plot of the frontier looks like Figure 2-13.

The shape of the frontier depends critically on the relative values of the preference parameters. In particular, if

(2.12) $$\frac{a_1}{b_1} < \frac{a_2}{b_2}$$

then the frontier is strictly concave like the one in Figure 2-13. Conversely, if the inequality in (2.12) is reversed then the frontier is strictly convex. In the special case where

(2.13) $$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

(which implies that the two persons have identical preferences) the frontier is linear.

An Alternative Approach: Solving Directly for a Tangency

We know from the graphical treatment in Section 2.2 that the Pareto frontier is the locus of tangencies of the indifferences curves. Let us now confirm that the solution we derived in (2.11) gives us precisely that.

To begin, let us find the marginal rate of substitution (*MRS*) associated with Cobb-Douglas preferences, which recall from (2.1) are represented by

(2.14) $$u1 := x1^{a1}\, y1^{b1}$$

for person 1. First, rearrange (2.14) to make y_1 the subject, expressed as function of a given value of u_1:

(2.15) $$y1 := \left(\frac{U1}{x1^{a1}}\right)^{\left(\frac{1}{b1}\right)}$$

This is the equation for the indifference curve plotted in (x_1, y_1) space, corresponding to a particular (fixed) value of utility U_1. Now differentiate (2.15) with respect to x_1 to find the slope of this indifference curve, whose negative is the *MRS*:

(2.16) $$MRS^1 = -\frac{dy_1}{dx_1} = \frac{a_1 U_1^{\frac{1}{b_1}} x_1^{-\left(\frac{a_1}{b_1}+1\right)}}{b_1}$$

We know that U_1 is fixed (because we are on an indifference curve) but we also know that U_1 is necessarily equal to u_1, as given by (2.14) . So make the substitution for U_1 from (2.14) in (2.16) and collect terms in x_1. Doing so reduces (2.16) to a very simple expression:

(2.17) $$MRS^1 = \frac{a_1 y_1}{b_1 x_1}$$

A similar expression can be found for MRS^2:

(2.18) $$MRS^2 = \frac{a_2 y_2}{b_2 x_2}$$

We can now find the tangency by setting $MRS^1 = MRS^2$ and imposing the resource constraints:

(2.19) $$\frac{a_1 y_1}{b_1 x_1} = \frac{a_2 (Y - y_1)}{b_2 (X - x_1)}$$

Solving this equation for y_1 yields the Pareto frontier:

(2.20) $$y1PF := -\frac{Y\,x1\,b1\,a2}{(a1\,b2 - b1\,a2)\,x1 - a1\,b2\,X}$$

This is exactly the same expression we derived by solving the constrained optimization problem; see expression (2.11) above.

A Numerical Example

Consider a two-person exchange economy with two goods in fixed amounts $X = 100$ and $Y = 60$. Person 1 has preferences represented by

(2.21) $$u_1 = x_1^{\frac{1}{2}} y_1^3$$

and person 2 has preferences represented by

(2.22) $$u_2 = x_2^2 y_2$$

Suppose the current allocation in this economy is one with an even split of the available goods: { $x_1 = x_2 = 50$, $y_1 = y_2 = 30$ }. Call this allocation E.

Is this allocation Pareto efficient? That is, does allocation E lie on the Pareto frontier? To answer this question, let us first derive the Pareto frontier.

Based on our earlier derivations in (2.17) and (2.18), the MRS for person 1 is

(2.23) $$MRS^1 = \frac{\frac{1}{2} y_1}{3x_1}$$

and the MRS for person 2 is

(2.24) $$MRS^2 = \frac{2y_2}{x_2}$$

Setting $MRS^1 = MRS^2$ and imposing the resource constraints yields

(2.25) $$\frac{\frac{1}{2} y_1}{3x_1} = \frac{2(60 - y_1)}{(100 - x_1)}$$

Solving for y_1 yields the Pareto frontier:

(2.26) $$y_1^{PF} = \frac{720x_1}{11x_1 + 100}$$

We can now ask whether the candidate allocation lies on this frontier. Setting $x_1 = 50$ in (2.26) yields $y_1^{PF} = 55.4$. Thus, an allocation in which $x_1 = 50$ and $y_1 = 30$ is not Pareto efficient. See Figure 2-14.

Does this mean that if person 1 has $x_1 = 50$ then she should have $y_1 = 55.4$ as well?

Not at all. The Pareto criterion tells us that if we are at an inefficient allocation (an allocation off the frontier) then we should move to an allocation in the core with respect to that inefficient allocation (and not just to any old Pareto efficient allocation).

So what is the core with respect to allocation E? To derive the core we need to find where the Pareto frontier intersects the indifference curves that pass through the current allocation. This requires a bit of algebra and will not do it here. However, it can be easily shown that the lower bound of the core is at { $x_1 = 12.4$, $y_1 = 37.8$ } and that the upper bound is at { $x_1 = 23.5$, $y_1 = 47.2$ }. See Figure 2-15.

Moreover, we do not need to identify the core in order to ascertain whether or not an allocation lies in the region of mutual benefit. In particular, we simply need to compare the utility level for each person at allocation E with their utility levels at the candidate allocation.

For example, at allocation E, utility for person 1 is

(2.27) $$u_1^E = (50)^{\frac{1}{2}}(30)^3 = 190918.8$$

and utility for person 2 is

(2.28) $$u_2^E = (50)^2(30) = 75000$$

In comparison, utilities at some allocation A = { $x_1 = 20$, $y_1 = 45$, $x_2 = 80$, $y_2 = 15$ } are

(2.29) $$u_1^A = (20)^{\frac{1}{2}}(45)^3 = 407523.4$$

and

(2.30) $$u_2^E = (80)^2(15) = 96000$$

for persons 1 and 2 respectively. Thus, both persons are better at allocation A than at allocation E, so A must lie in the region of mutual benefit with respect to E.

We also know that A lies in the core with respect to E because A lies on the Pareto frontier: Setting $x_1 = 20$ in (2.26) yields $y_1^{PF} = 45$. Thus, an allocation in which $x_1 = 20$ and $y_1 = 45$ is Pareto efficient. See Figure 2-16.

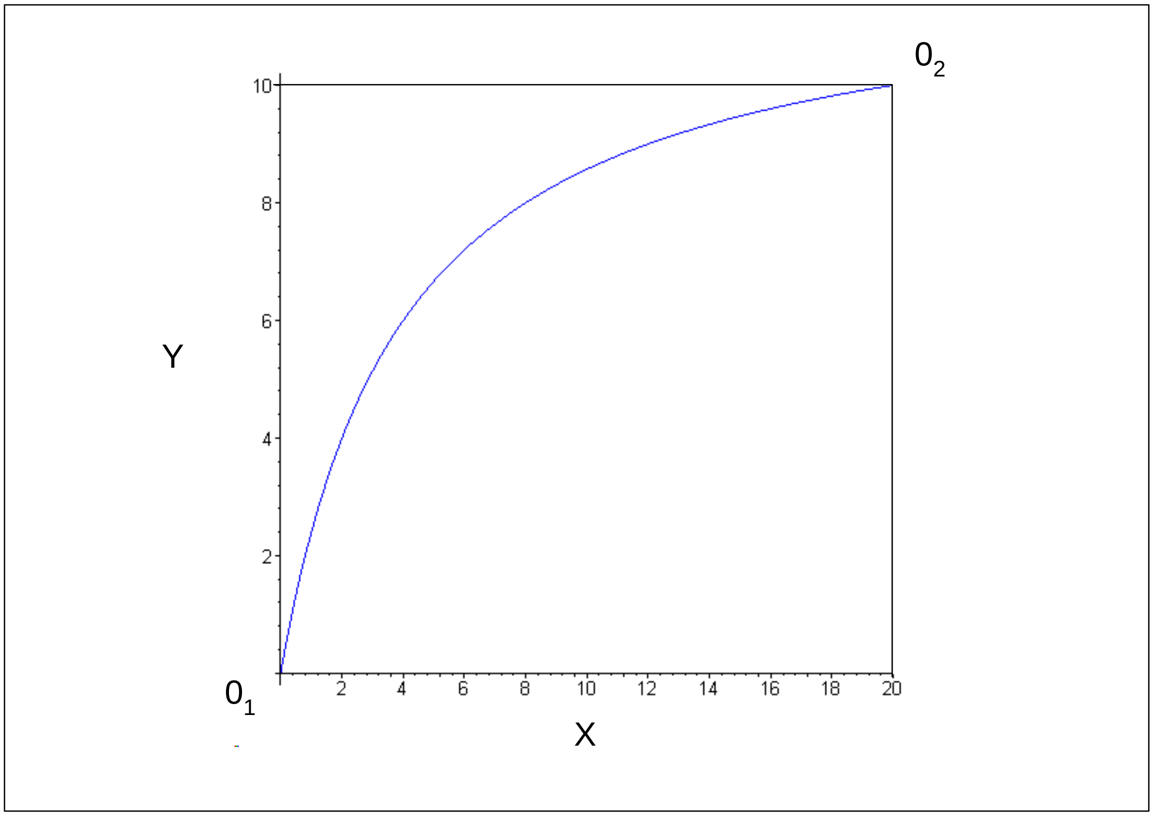

Figure 2-13

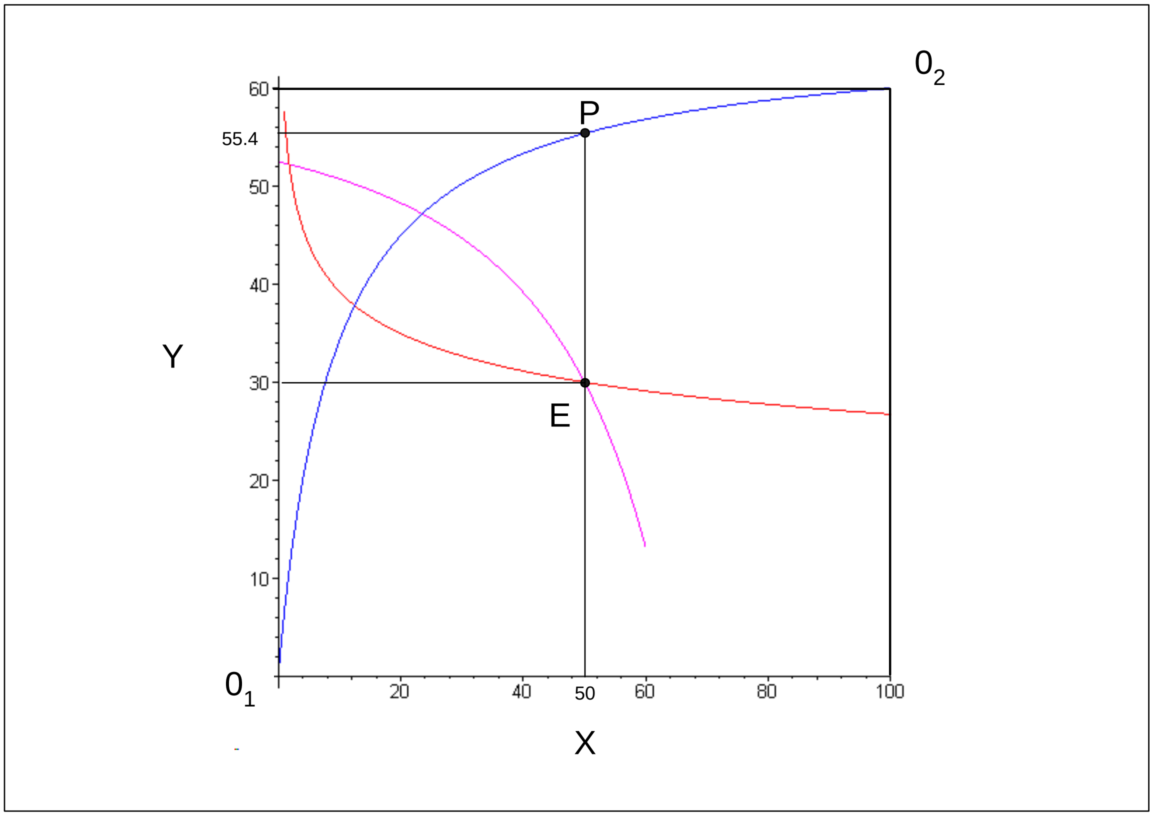

Figure 2-14

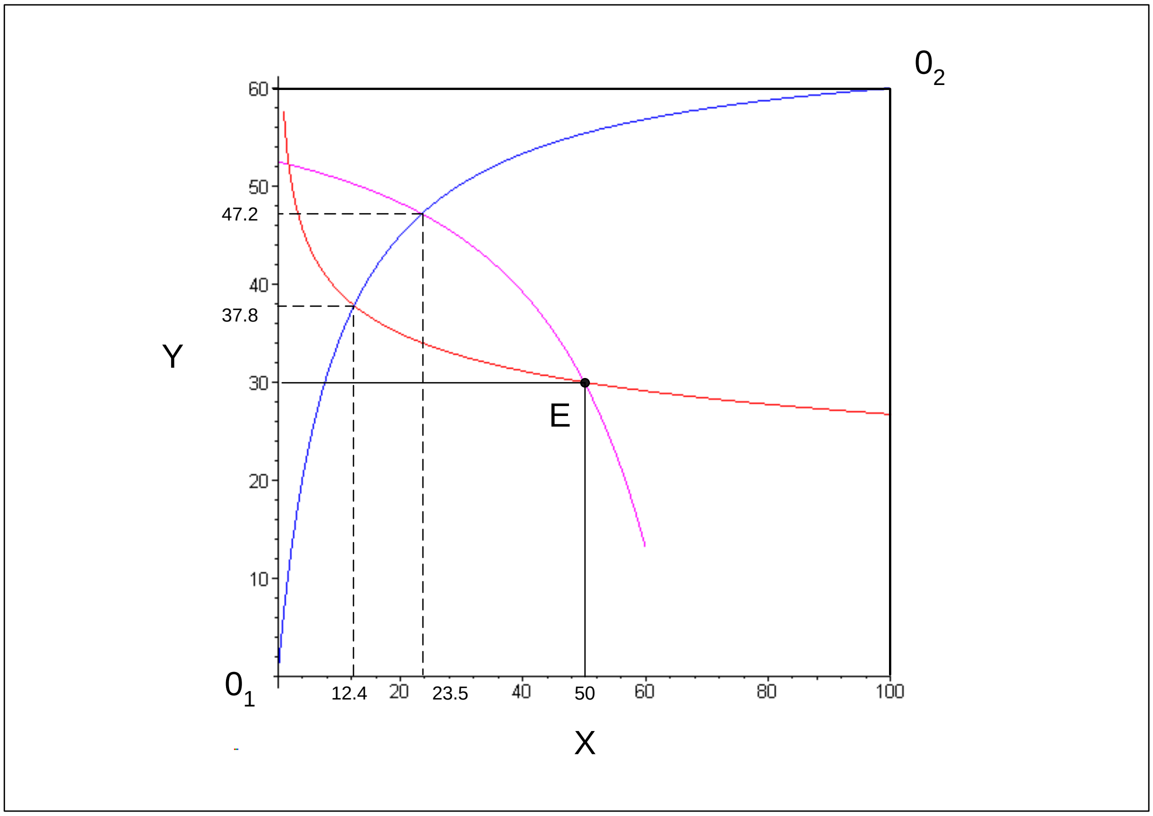

Figure 2-15

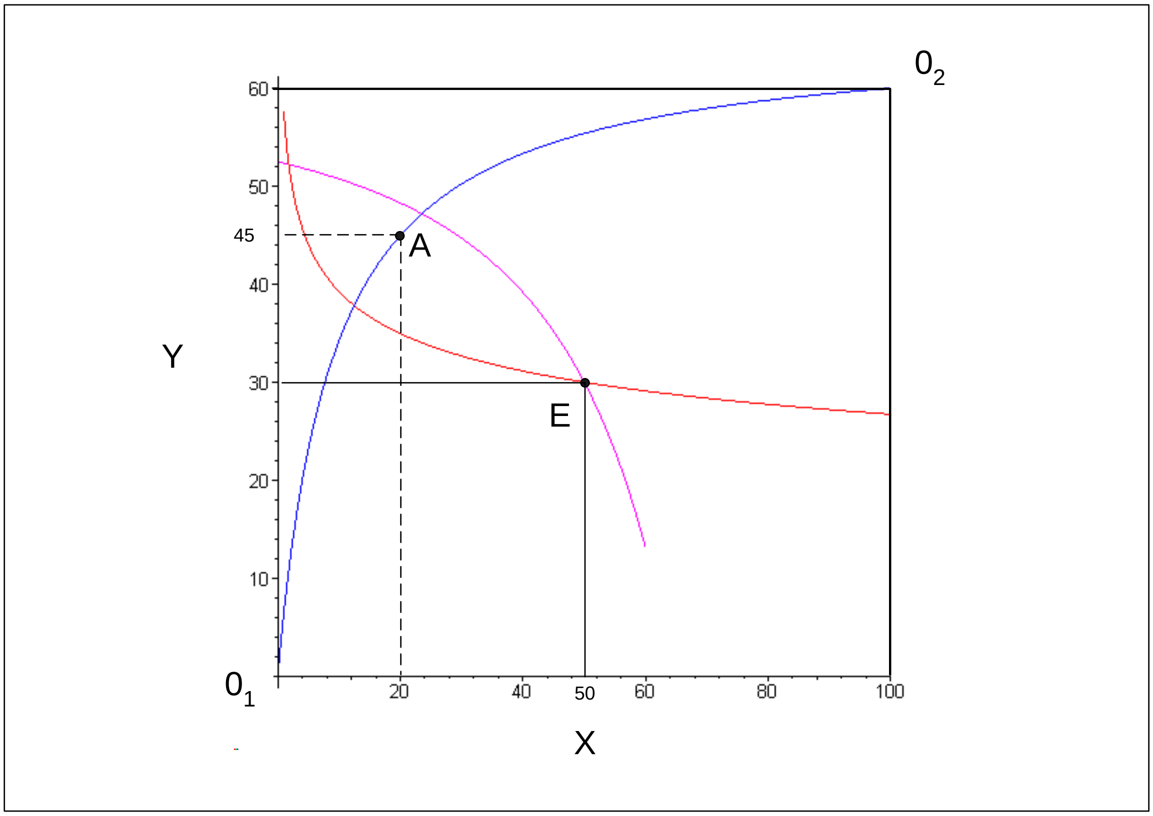

Figure 2-16

APPENDIX A2-1
SOCIAL WELFARE FUNCTIONS

The general form of a social welfare function is

(A2.1) $$W = W(u_1, u_2, \ldots, u_m)$$

where $u_1, u_2, \ldots, u_m$ are the utilities of the *m* individuals in the economy. It must be stressed that such a function is an entirely artificial construct. It is not possible to measure *W* for any given specification of the function *W(.)* because its arguments, the utility of individuals, are not measurable in a cardinal way.

Consider three specific social welfare functions.

1. THE UTILITARIAN (OR BENTHAMITE) WELFARE FUNCTION

This is often associated with Jeremy Bentham, a nineteenth century philosopher.

(A2.2) $$W = \sum_{i=1}^{m} u_i$$

This welfare function reflects the **utilitarian ethic**: everyone's utility should count equally regardless of their level of utility.

2. THE RAWLSIAN WELFARE FUNCTION

This was proposed by John Rawls, in *A Theory of Justice*, (1971)

(A2.3) $$W = \min(u_1, u_2, \ldots, u_m)$$

This reflects the **Rawlsian** ethic: the welfare of society is equal to that of its least well-off member. It can be derived as the allocation rule preferred by infinitely risk averse agents choosing between different rules from behind a "veil of ignorance".

In some sense the Rawlsian ethic is at the opposite end of the concern-for-distribution spectrum to the utilitarian ethic. Somewhere in the middle is the weighted utilitarian function.

3. WEIGHTED UTILITARIAN WELFARE FUNCTION

(A2.4) $$W = \sum_{i=1}^{m} \alpha_i u_i$$

where the weight α_i reflects the "importance" of individual *i* to overall social welfare. The usual interpretation is that changes in the utility of poor people carry more weight in determining a change in social welfare than do changes in the utility of wealthy people.

It is worth reiterating that none of these welfare functions can be made operational for practical purposes because there is no way to measure *W* for any of the three functions.