TOPIC 3 – PART 2 REVIEW QUESTIONS

There are two sets of questions here. Each set of questions takes you through the derivation of the competitive equilibrium for a specified economy. Calculating every component needed to solve for the equilibrium takes a lot of time, and it would be unreasonable to expect you to do that in an exam setting. Instead, in an exam setting you will be expected to derive some of the key components while other components will be provided to you. The review questions here take that same approach, thereby giving you a guide as to what to expect in the exam setting. Within each set of questions are additional "suggested exercises" intended to provide extra practice at questions of this type.

Questions 1 to 15 relate to the following information

Consider a economy with two production sectors (Y and X), two factors of production (K and L), and *N* individuals. The production technology in sector Y is

$$Y = f_Y(K,L) = K^{\frac{1}{2}}L^{\frac{1}{4}}$$

and there is a quasi-fixed managerial-labour input requirement $F_Y = 64$. The production technology in sector X is

$$X = f_X(K,L) = K^{\frac{1}{2}}L^{\frac{1}{4}}$$

and there is a quasi-fixed managerial-labour input requirement of $F_X = 4$.

Individuals have identical preferences represented by utility function

$$u(x,y,l) = xy^3l^5$$

Recall that the TRS for a Cobb-Douglas production function of the form

$$f(K,L) = K^aL^b$$

is

$$TRS = \frac{bK}{aL}$$

Recall that the MRS functions for a Cobb-Douglas utility function of the form

$$u(x, y, l) = x^{\alpha} y^{\beta} l^{\delta}$$

are

$$MRS_{xy} = \frac{\alpha y}{\beta x}$$

and

$$MRS_{ly} = \frac{\delta y}{\beta l}$$

Your ultimate goal here is to find the free-entry competitive equilibrium in this economy, where labour is specified as the numeraire. With that in mind, set $w = 1$ from the very beginning; this will simplify the algebra.

Production in Sector Y

1. The conditional demand for non-managerial labour by a firm in sector Y is

A. $\hat{L}_Y(y, w, r) = y^{\frac{4}{3}}(2r)^{\frac{1}{3}}$

B. $\hat{L}_Y(y, w, r) = y^{\frac{4}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$

C. $\hat{L}_Y(y, w, r) = y^{\frac{2}{3}}(2r)^{\frac{1}{3}}$

D. $\hat{L}_Y(y, w, r) = y^{\frac{2}{3}}\left(\frac{r}{2}\right)^{\frac{1}{3}}$

2. The conditional demand for capital by a firm in sector Y is

A. $\hat{K}_Y(y,w,r) = y^{\frac{4}{3}}\left(\frac{2}{r}\right)^{\frac{1}{3}}$

B. $\hat{K}_Y(y,w,r) = \frac{y^{\frac{4}{3}}}{(2r)^{\frac{2}{3}}}$

C. $\hat{K}_Y(y,w,r) = \frac{y^{\frac{2}{3}}}{(2r)^{\frac{1}{3}}}$

D. $\hat{K}_Y(y,w,r) = y^{\frac{2}{3}}\left(\frac{2}{r}\right)^{\frac{2}{3}}$

We can now find the cost function for a firm in sector Y by substituting these conditional factor demands into the expression for cost. That cost function is

$$c_Y(y,w,r) = 3y^{\frac{4}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}} + 64$$

Suggested exercise: Confirm that the reported cost function is correct.

3. The marginal cost function for a firm in sector Y is

A. $MC_Y(y,w,r) = 4y^{\frac{1}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$

B. $MC_Y(y,w,r) = 2y^{\frac{1}{3}}(2r)^{\frac{1}{3}}$

C. $MC_Y(y,w,r) = 2y^{\frac{1}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$

D. $MC_Y(y,w,r) = 2y^{\frac{1}{3}}(2r)^{\frac{2}{3}}$

4. The supply function for a firm in sector Y is

A. $y(p_Y, w, r) = \frac{p_Y^3}{16r}$

B. $y(p_Y, w, r) = \frac{p_Y^2}{8r}$

C. $y(p_Y, w, r) = \frac{p_Y^3}{8r^2}$

D. $y(p_Y, w, r) = \frac{p_Y^3}{16r^2}$

We can now find the factor demands for a firm in sector Y by substituting the supply function into the conditional factor demands. These factor demands are

$$K_Y(p_Y, w, r) = \frac{p_Y^4}{32r^3}$$

and

$$L_Y(p_Y, w, r) = \frac{p_Y^4}{64r^2}$$

for capital and non-managerial labour respectively.

Suggested exercise: Confirm that the reported factor demands are correct.

5. The demand for non-managerial labour is

A. decreasing in *r* because the output effect outweighs the substitution effect.

B. increasing in *r* because the output effect outweighs the substitution effect.

C. decreasing in *r* because the substitution effect outweighs the output effect.

D. increasing in *r* because the substitution effect outweighs the output effect.

We can now find the profit function for a firm in sector Y by substituting the supply function into the expression for profit. That profit function is

$$\pi_Y(p_Y, w, r) = \frac{p_Y^4}{64r^2} - 64$$

Suggested exercise: Confirm that the reported profit function is correct.

Production in Sector X

Following the same procedures we used for calculating the production-related functions for a firm in sector Y, we can find the comparable functions for a firm in sector X. The key functions we need are the following.

Supply function:

$$x(p_X, w, r) = \frac{p_X^3}{16r^2}$$

Factor demands:

$$K_X(p_X, w, r) = \frac{p_X^4}{32r^3} \quad \text{and} \quad L_X(p_X, w, r) = \frac{p_X^4}{64r^2}$$

The profit function:

$$\pi_X(p_X, w, r) = \frac{p_X^4}{64r^2} - 4$$

Suggested exercise: Confirm that the reported functions are correct.

Consumption

6. Demand for *y* by an individual with wealth M_i is

A. $y_i(p_X, p_Y, w, r) = \frac{M_i}{9p_Y}$

B. $y_i(p_X, p_Y, w, r) = \frac{7M_i}{9p_Y}$

C. $y_i(p_X, p_Y, w, r) = \frac{M_i}{3p_Y}$

D. $y_i(p_X, p_Y, w, r) = \frac{7M_i}{3p_Y}$

7. Demand for *x* by an individual with wealth M_i is

A. $x_i(p_X, p_Y, w, r) = \frac{7M_i}{3p_X}$

B. $x_i(p_X, p_Y, w, r) = \frac{M_i}{6p_X}$

C. $x_i(p_X, p_Y, w, r) = \frac{M_i}{3p_X}$

D. $x_i(p_X, p_Y, w, r) = \frac{M_i}{9p_X}$

8. In the absence of profits, aggregate wealth in this economy is $V(w, r) = r\tilde{K} + \tilde{L}$.

A. True.

B. False.

Equilibrium

9. In a free-entry equilibrium,

A. entry of firms into sector X continues until profit in that sector is driven to zero.

B. entry of firms into sector Y continues until profit in that sector is driven to zero.

C. no dividend income accrues to individuals.

D. All of the above.

Henceforth, assume there is free entry.

10. The equilibrium price of *y*, as a function of *r*, is

A. $p_Y(r) = 4r^{\frac{3}{2}}$

B. $p_Y(r) = 3r^{\frac{2}{3}}$

C. $p_Y(r) = 8r^{\frac{1}{2}}$

D. $p_Y(r) = 4r^{\frac{1}{2}}$

11. The equilibrium number of firms in sector Y, as a function of *r*, is

A. $n_Y(r) = \frac{\tilde{L} + r\tilde{K}}{768}$

B. $n_Y(r) = \frac{\tilde{L} + r\tilde{K}}{339}$

C. $n_Y(r) = \frac{2(\tilde{L} + r\tilde{K})}{339}$

D. $n_Y(r) = \frac{3(\tilde{L} + r\tilde{K})}{678}$

Following the same procedures used for calculating $p_Y(r)$ and $n_Y(r)$, we can find comparable functions for sector X. These functions are

$$p_X(r) = 4r^{\frac{1}{2}}$$

and

$$n_X(r) = \frac{\tilde{L} + r\tilde{K}}{144}$$

Suggested exercise: Confirm that the reported functions are correct.

12. Aggregate demand for capital in sector Y, as a function of *r*, is

A. $D_K^Y(r) = \frac{\tilde{L} + r\tilde{K}}{6r}$

B. $D_K^Y(r) = \frac{\tilde{L} + r\tilde{K}}{3r}$

C. $D_K^Y(r) = \frac{2(\tilde{L} + r\tilde{K})}{3r}$

D. $D_K^Y(r) = \frac{3(\tilde{L} + r\tilde{K})}{4r}$

13. Aggregate demand for capital in sector X, as a function of *r*, is

A. $D_K^X(r) = \frac{\tilde{L} + r\tilde{K}}{6r}$

B. $D_K^X(r) = \frac{2(\tilde{L} + r\tilde{K})}{3r}$

C. $D_K^X(r) = \frac{\tilde{L} + r\tilde{K}}{9r}$

D. $D_K^X(r) = \frac{\tilde{L} + r\tilde{K}}{18r}$

14. The equilibrium rental rate is

A. $r^* = \frac{3\tilde{L}}{7\tilde{K}}$

B. $r^* = \frac{2\tilde{L}}{3\tilde{K}}$

C. $r^* = \frac{2\tilde{L}}{7\tilde{K}}$

D. $r^* = \frac{2\tilde{L}}{9\tilde{K}}$

15. The equilibrium GDP for this economy is

A. $GDP = \frac{4\tilde{L}}{9\tilde{K}}$

B. $GDP = \frac{4\tilde{L}}{7}$

C. $GDP = \frac{2\tilde{L}}{9}$

D. $GDP = \frac{2\tilde{L}}{7\tilde{K}}$

Questions 16 to 30 relate to the following information

Consider a economy with two production sectors (Y and X), two factors of production (K and L), and *N* individuals. The production technology in sector Y is

$$Y = f_Y(K,L) = K^{\frac{1}{4}}L^{\frac{1}{2}}$$

and there is a quasi-fixed managerial-labour input requirement $F_Y = 4$. The production technology in sector X is

$$X = f_X(K,L) = K^{\frac{1}{4}}L^{\frac{1}{4}}$$

and there is a quasi-fixed managerial-labour input requirement of $F_X = 2$.

Individuals have identical preferences represented by utility function

$$u(x,y,l) = xy^2l^3$$

Recall that the TRS for a Cobb-Douglas production function of the form

$$f(K,L) = K^aL^b$$

is

$$TRS = \frac{bK}{aL}$$

Recall that the MRS expressions for a Cobb-Douglas utility function of the form

$$u(x,y,l) = x^\alpha y^\beta l^\delta$$

are

$$MRS_{xy} = \frac{\alpha y}{\beta x}$$

and

$$MRS_{ly} = \frac{\delta y}{\beta l}$$

Production in Sector Y

16. The conditional demand for non-managerial labour by a firm in sector Y is

A. $\hat{L}_Y(y,w,r) = y^{\frac{4}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$

B. $\hat{L}_Y(y,w,r) = y^{\frac{4}{3}}(2r)^{\frac{1}{3}}$

C. $\hat{L}_Y(y,w,r) = y^{\frac{2}{3}}(2r)^{\frac{1}{3}}$

D. $\hat{L}_Y(y,w,r) = y^{\frac{2}{3}}\left(\frac{r}{2}\right)^{\frac{1}{3}}$

17. The conditional demand for capital by a firm in sector Y is

A. $\hat{K}_Y(y,w,r) = \frac{y^{\frac{4}{3}}}{(2r)^{\frac{2}{3}}}$

B. $\hat{K}_Y(y,w,r) = y^{\frac{4}{3}}\left(\frac{2}{r}\right)^{\frac{1}{3}}$

C. $\hat{K}_Y(y,w,r) = \frac{y^{\frac{2}{3}}}{(2r)^{\frac{1}{3}}}$

D. $\hat{K}_Y(y,w,r) = y^{\frac{2}{3}}\left(\frac{2}{r}\right)^{\frac{2}{3}}$

We can now find the cost function for a firm in sector Y by substituting these conditional factor demands into the expression for cost. That cost function is

$$c_Y(y,w,r) = \frac{3y^{\frac{4}{3}}(2r)^{\frac{1}{3}}}{2} + 4$$

Suggested exercise: Confirm that the reported cost function is correct.

18. The marginal cost function for a firm in sector Y is

A. $MC_Y(y,w,r) = 4y^{\frac{1}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$

B. $MC_Y(y,w,r) = 2y^{\frac{1}{3}}(2r)^{\frac{1}{3}}$

C. $MC_Y(y,w,r) = 2y^{\frac{1}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$

D. $MC_Y(y,w,r) = 2y^{\frac{1}{3}}(2r)^{\frac{2}{3}}$

19. The supply function for a firm in sector Y is

A. $y(p_Y,w,r) = \frac{p_Y^3}{16r^2}$

B. $y(p_Y,w,r) = \frac{p_Y^2}{8r}$

C. $y(p_Y,w,r) = \frac{p_Y^3}{8r^2}$

D. $y(p_Y,w,r) = \frac{p_Y^3}{16r}$

We can now find the factor demands for a firm in sector Y by substituting the supply function into the conditional factor demands. These factor demands are

$$K_Y(p_Y,w,r) = \frac{p_Y^4}{64r^2}$$

and

$$L_Y(p_Y,w,r) = \frac{p_Y^4}{32r}$$

for capital and non-managerial labour respectively.

Suggested exercise: Confirm that the reported factor demands are correct.

We can now find the profit function for a firm in sector Y by substituting the supply function into the expression for profit. That profit function is

$$\pi_Y(p_Y, w, r) = \frac{p_Y^4}{64r} - 4$$

Suggested exercise: Confirm that the reported profit function is correct.

Production in Sector X

Following the same procedures we used for calculating the production-related functions for a firm in sector Y, we can find the comparable functions for a firm in sector X. The key functions we need are the following.

Supply function:

$$x(p_X, w, r) = \frac{p_X}{4r^{\frac{1}{2}}}$$

Factor demands:

$$K_X(p_X, w, r) = \frac{p_X^2}{16r^{\frac{3}{2}}} \quad \text{and} \quad L_X(p_X, w, r) = \frac{p_X^2}{16r^{\frac{1}{2}}}$$

The profit function:

$$\pi_X(p_X, w, r) = \frac{p_X^2}{8r^{\frac{1}{2}}} - 2$$

Suggested exercise: Confirm that the reported functions are correct.

Consumption

20. Demand for *y* by an individual with wealth M_i is

A. $y_i(p_X, p_Y, w, r) = \frac{2M_i}{9p_Y}$

B. $y_i(p_X, p_Y, w, r) = \frac{M_i}{7p_Y}$

C. $y_i(p_X, p_Y, w, r) = \frac{M_i}{3p_Y}$

D. $y_i(p_X, p_Y, w, r) = \frac{2M_i}{3p_Y}$

21. Demand for *x* by an individual with wealth M_i is

A. $x_i(p_X, p_Y, w, r) = \frac{M_i}{6p_X}$

B. $x_i(p_X, p_Y, w, r) = \frac{M_i}{9p_X}$

C. $x_i(p_X, p_Y, w, r) = \frac{M_i}{7p_X}$

D. $x_i(p_X, p_Y, w, r) = \frac{3M_i}{7p_X}$

22. In the absence of profits, aggregate wealth in this economy is $V(w, r) = r\tilde{K} + \tilde{L}$.

A. True.

B. False.

Equilibrium

23. In a free-entry equilibrium,

A. entry of firms into sector X continues until profit in that sector is driven to zero.

B. entry of firms into sector Y continues until profit in that sector is driven to zero.

C. no dividend income accrues to individuals.

D. All of the above.

Henceforth, assume there is free entry.

24. The equilibrium price of *y*, as a function of *r*, is

A. $p_Y(r) = 8r^{\frac{1}{2}}$

B. $p_Y(r) = 4r^{\frac{1}{2}}$

C. $p_Y(r) = 4r^{\frac{1}{4}}$

D. $p_Y(r) = 2r^{\frac{1}{4}}$

25. The equilibrium number of firms in sector Y, as a function of *r*, is

A. $n_Y(r) = \frac{\tilde{L} + r\tilde{K}}{678}$

B. $n_Y(r) = \frac{\tilde{L} + r\tilde{K}}{48}$

C. $n_Y(r) = \frac{\tilde{L} + r\tilde{K}}{339}$

D. $n_Y(r) = \frac{7(\tilde{L} + r\tilde{K})}{48}$

Following the same procedures used for calculating $p_Y(r)$ and $n_Y(r)$, we can find comparable functions for sector X. These functions are

$$p_X(r) = 4r^{\frac{1}{4}}$$

and

$$n_X(r) = \frac{\tilde{L} + r\tilde{K}}{24}$$

Suggested exercise: Confirm that the reported functions are correct.

26. Aggregate demand for capital in sector Y, as a function of *r*, is

A. $D_K^Y(r) = \frac{\tilde{L} + r\tilde{K}}{6r}$

B. $D_K^Y(r) = \frac{\tilde{L} + r\tilde{K}}{12r}$

C. $D_K^Y(r) = \frac{\tilde{L} + r\tilde{K}}{9r}$

D. $D_K^Y(r) = \frac{3(\tilde{L} + r\tilde{K})}{7r}$

27. Aggregate demand for capital in sector X, as a function of *r*, is

A. $D_K^X(r) = \frac{\tilde{L} + r\tilde{K}}{24r}$

B. $D_K^X(r) = \frac{\tilde{L} + r\tilde{K}}{18r}$

C. $D_K^X(r) = \frac{\tilde{L} + r\tilde{K}}{9r}$

D. $D_K^X(r) = \frac{\tilde{L} + r\tilde{K}}{48r}$

28. The equilibrium rental rate is

A. $r^* = \frac{2\tilde{L}}{7\tilde{K}}$

B. $r^* = \frac{\tilde{L}}{7\tilde{K}}$

C. $r^* = \frac{2\tilde{L}}{9\tilde{K}}$

D. $r^* = \frac{4\tilde{L}}{7\tilde{K}}$

29. The equilibrium GDP for this economy is

A. $GDP = \frac{4\tilde{L}}{7\tilde{K}}$

B. $GDP = \frac{4\tilde{L}}{9}$

C. $GDP = \frac{4\tilde{L}}{7}$

D. $GDP = \frac{7\tilde{K}}{9}$

30. An exogenous increase in the supply of capital causes no change in the real GDP of this economy.

A. True.

B. False.

ANSWER KEY

1. B
2. A
3. A
4. D
5. A
6. C
7. D
8. A
9. D
10. C
11. A
12. A
13. D
14. C
15. B

This question does take some time. You need to make the substitution for r^* in the equilibrium expressions for p_Y, p_X, n_Y and n_X, and then go on to find equilibrium values for aggregate output of the two goods, whose combined value is the GDP for this economy.

16. B
17. A
18. B
19. D
20. C
21. A
22. A
23. D
24. C

25. B
26. B
27. A
28. B
29. C
30. B