

## TOPIC 4 – PART 1 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions.

### A SOLVED EXAMPLE

Consider the following setting with a negative externality.

$$MPB(y) = 1600 - 200y$$

$$MPC(y) = 120y$$

$$MEC(y) = 80y$$

Our goal is characterize the private and social optima, and compare all relevant private and social surplus measures under the two optima.

### Method 1

This method uses with the MPC and MPB functions. (Method 2 following is simpler; it uses the MNPB function).

Find marginal social cost

$$MSC(y) = MPC(y) + MEC(y) = 120y + 80y = 200y$$

See Figure R4-1. In order to illustrate all key values clearly, the figures here are to scale.

The private optimum

Solve the following for  $\hat{y}$  :

$$MPB(\hat{y}) = MPC(\hat{y})$$

$$1600 - 200\hat{y} = 120\hat{y}$$

$$\hat{y} = 5$$

The social optimum

Solve the following for  $y^*$  :

$$MSB(y^*) = MSC(y^*)$$

$$1600 - 200y^* = 200y^*$$

$$y^* = 4$$

External cost at the private optimum

$$D(\hat{y}) = \int_0^5 MEC(y)dy = \int_0^5 MSC(y)dy - \int_0^5 MPC(y)dy$$

which is equal to the shaded area in Figure R4-2 and in Figure R4-3. It can be calculated without calculus – using the geometry of triangles – from Figure R4-2 as follows:

$$D(\hat{y}) = \int_0^5 MEC(y)dy = \frac{5 * 400}{2} = 1000$$

Alternatively, it can be calculated using the geometry of triangles from Figure R4-3 as follows:

$$D(\hat{y}) = \int_0^5 MSC(y)dy - \int_0^5 MPC(y)dy = \frac{5 * 1000}{2} - \frac{5 * 600}{2} = 1000$$

External cost at the social optimum

$$D(y^*) = \int_0^4 MEC(y)dy = \int_0^4 MSC(y)dy - \int_0^4 MPC(y)dy$$

which is equal to the shaded area in Figure R4-4 and in Figure R4-5. It can be calculated using the geometry of triangles from Figure R4-4 as follows:

$$D(y^*) = \int_0^4 MEC(y)dy = \frac{4 * 320}{2} = 640$$

Alternatively, it can be calculated using the geometry of triangles from Figure R4-5 as follows:

$$D(y^*) = \int_0^4 MSC(y)dy - \int_0^4 MPC(y)dy = \frac{4 * 800}{2} - \frac{4 * 480}{2} = 640$$

Gain to external agents from a regulated move from private to social optimum

$$G_{EXT}(\hat{y} \rightarrow y^*) = \int_4^5 MEC(y)dy = D(\hat{y}) - D(y^*) = 100 - 640 = 360$$

See Figures R4-6 and R4-7. (The shaded areas are equal).

Loss to the source agent from a regulated move from private to social optimum

$$L_S(\hat{y} \rightarrow y^*) = \int_4^5 MPB(y)dy - \int_4^5 MPC(y)dy$$

This is the shaded area in Figure R4-8. It can be calculated using the geometry of triangles as follows:

$$L_S(\hat{y} \rightarrow y^*) = \frac{(5-4) * (800-480)}{2} = 160$$

Gain in social surplus from a regulated move from private to social optimum

$$\Delta SS(\hat{y} \rightarrow y^*) = G_{EXT}(\hat{y} \rightarrow y^*) - L_S(\hat{y} \rightarrow y^*) = 360 - 160 = 200$$

See Figure R4-9. To confirm the calculations, we can calculate the shaded area in Figure R4-9 directly:

$$\Delta SS(\hat{y} \rightarrow y^*) = \frac{(5-4) * (1000-600)}{2} = 200$$

**Method 2**Find the marginal net private benefit function

$$MNPB(y) = MPB(y) - MPC(y) = (1600 - 200y) - 120y = 1600 - 320y$$

See Figure R4-10.

The private optimum

Solve the following for  $\hat{y}$ :

$$MNPB(\hat{y}) = 0$$

$$1600 - 320\hat{y} = 0$$

$$\hat{y} = 5$$

The social optimum

Solve the following for  $y^*$  :

$$MNPB(y^*) = MEC(y^*)$$

$$1600 - 320y^* = 80y^*$$

$$y^* = 4$$

External cost at the private optimum

$$D(\hat{y}) = \int_0^5 MEC(y) dy$$

which is equal to the shaded area in Figure R4-11. It can be calculated using the geometry of triangles from Figure R4-11 as follows:

$$D(\hat{y}) = \int_0^5 MEC(y) dy = \frac{5 * 400}{2} = 1000$$

External cost at the social optimum

$$D(y^*) = \int_0^4 MEC(y) dy$$

which is equal to the shaded area in Figure R4-12. It can be calculated using the geometry of triangles from Figure R4-12 as follows:

$$D(y^*) = \int_0^4 MEC(y) dy = \frac{4 * 320}{2} = 640$$

Gain to external agents from a regulated move from private to social optimum

$$G_{EXT}(\hat{y} \rightarrow y^*) = \int_4^5 MEC(y) dy = D(\hat{y}) - D(y^*) = 1000 - 640 = 360$$

See Figure R4-13.

Loss to the source agent from a regulated move from private to social optimum

$$L_S(\hat{y} \rightarrow y^*) = \int_4^5 MNPB(y) dy$$

This is the shaded area in Figure R4-14. It can be calculated using the geometry of triangles as follows:

$$L_S(\hat{y} \rightarrow y^*) = \frac{(5-4) * 320}{2} = 160$$

Gain in social surplus from a regulated move from private to social optimum

$$\Delta SS(\hat{y} \rightarrow y^*) = G_{EXT}(\hat{y} \rightarrow y^*) - L_S(\hat{y} \rightarrow y^*) = 360 - 160 = 200$$

See Figure R4-15. To confirm the calculations, we can calculate the shaded area in Figure R4-15 directly:

$$\Delta SS(\hat{y} \rightarrow y^*) = \frac{(5-4) * 400}{2} = 200$$

We now also easily calculate gains from trade if property rights are assigned.

The equilibrium price

$$p^* = MEC(y^*) = MNPB(y^*) = 320$$

See Figure R4-16.

Gains from trade to source agent if rights assigned to source agent

$$GFT_S(\hat{y} \rightarrow y^*) = \text{receipts} - \text{cost} = p^*(\hat{y} - y^*) - \int_4^5 MNPB(y) dy = 320 - 160 = 160$$

See Figure R4-17.

Gains from trade to external agents if rights assigned to source agent

$$GFT_{EXT}(\hat{y} \rightarrow y^*) = \text{benefit} - \text{payment} = \int_4^5 MEC(y) dy - p^*(\hat{y} - y^*) = 360 - 320 = 40$$

See Figure R4-18.

Check that total GFT are equal to  $\Delta SS(\hat{y} \rightarrow y^*)$ :

$$GFT_S(\hat{y} \rightarrow y^*) + GFT_{EXT}(\hat{y} \rightarrow y^*) = 160 + 140 = 200$$

Gains from trade to source agent if rights assigned to external agents

$$GFT_S(0 \rightarrow y^*) = \text{benefit} - \text{payment}$$

$$= \int_0^4 MNPB(y)dy - p^* y^* = \frac{4*(1600 - 320)}{2} = 2560$$

See Figure R4-19.

Gains from trade to external agents if rights assigned to external agents

$$GFT_{EXT}(0 \rightarrow y^*) = \text{receipts} - \text{cost} = p^* y^* - \int_0^4 MEC(y)dy = \frac{4*320}{2} = 640$$

See Figure R4-20.

Note that total GFT are higher if property rights are assigned to the external agents in this example. This does not mean that property rights should be assigned to the external agents. It does mean that if trade is impossible for some reason then the net social cost of that absence of trade would be larger when external agents hold the rights than when the source agent holds the rights.

**REVIEW QUESTIONS**

Answer Questions 1 – 8 as a set and then grade yourself on those. Once you have resolved any problems with that first set, do Questions 9 – 19 as a set and grade those. Then do Questions 20 – 28 as a set, followed by Questions 29 – 38 as a set.

**Questions 1 – 8 relate to the following data.**

$$MPB(y) = 318 - 23y$$

$$MPC(y) = 30y$$

$$MEB(y) = 330 - y$$

1. Marginal social benefit is

- A.  $MSB(y) = 618 - 11y$
- B.  $MSB(y) = 648 - 24y$
- C.  $MSB(y) = 198 - 33y$
- D.  $MSB(y) = 420 - 22y$

2. The private optimum is

- A. 4
- B. 5
- C. 6
- D. 7

3. The social optimum is

- A. 7
- B. 9
- C. 10
- D. 12

4. External benefit at the private optimum is

- A. 1962
- B. 2024
- C. 2456
- D. 2784

5. External benefit at the social optimum is

- A. 2764
- B. 3888
- C. 4240
- D. 6260

Now suppose that a regulation forces the source agent to increase his level of the activity to the social optimum. Consider Questions 6 – 8 under this scenario.

6. The gain to the external agents is

- A. 986
- B. 1160
- C. 1926
- D. 2456

7. The loss to the source agent is

- A. 954
- B. 1644
- C. 1756
- D. 2132



8. The gain in social surplus is

- A. 788
- B. 972
- C. 1358
- D. 1674

**Questions 9 – 19 relate to the following data.**

$$MPB(y) = 192 - 60y$$

$$MPC(y) = 4y$$

$$MEC(y) = 192y$$

9. Marginal social cost is

- A.  $MSC(y) = 192 - 64y$
- B.  $MSC(y) = 196y$
- C.  $MSC(y) = 192 + 52y$
- D.  $MSC(y) = 188y$

10. The private optimum is

- A. 4
- B. 5
- C. 3
- D. 8

11. The social optimum is

- A. 0.75
- B. 1.5
- C. 2.5
- D. 1

12. External cost at the private optimum is

- A. 812
- B. 636
- C. 1234
- D. 864

13. External cost at the social optimum is

- A. 728
- B. 54
- C. 142
- D. 256

Now suppose that a regulation forces the source agent to reduce his level of the activity to the social optimum. Consider Questions 14 – 17 under this scenario.

14. The gain to the external agents is

- A. 84
- B. 978
- C. 1092
- D. 810

15. The loss to the source agent is

- A. 384
- B. 454
- C. 162
- D. 26

16. The gain in social surplus is

- A. 594
- B. 648
- C. 794
- D. 58

Now return to the unregulated scenario, and suppose that the source agent is assigned the explicit right to undertake the activity at his private optimum. He may trade that right in whole or in part if he wishes. Suppose a costless contract can be written between the source agent and the external agents that specifies the price for each unit of activity reduced by the source agent. Consider Questions 17 – 19 under this scenario.

17. The equilibrium price at which trade will occur is

- A. 62
- B. 144
- C. 176
- D. 224

18. The gains from trade for the source agent are

- A. 228
- B. 326
- C. 162
- D. 208

19. The gains from trade for the external agents are

- A. 486
- B. 366
- C. 586
- D. 420

**Questions 20 – 27 relate to the following data.**

$$MPB(y) = 72 - 6y$$

$$MPC(y) = 6y$$

$$MEB(y) = 84 - y$$

20. Marginal social benefit is

A.  $MSB(y) = 180 - 30y$

B.  $MSB(y) = 192 - 7y$

C.  $MSB(y) = 156 - 7y$

D.  $MSB(y) = 192 + 5y$

21. The private optimum is

A. 10

B. 6

C. 8

D. 12

22. The social optimum is

A. 14

B. 8

C. 12

D. 14

23. External benefit at the private optimum is

A. 1096

B. 896

C. 524

D. 486

24. External benefit at the social optimum is

- A. 936
- B. 1096
- C. 1160
- D. 1400

Now suppose that a regulation forces the source agent to increase his level of the activity to the social optimum. Consider Questions 25 – 27 under this scenario.

25. The gain to the external agents is

- A. 524
- B. 450
- C. 372
- D. 38

26. The loss to the source agent is

- A. 328
- B. 312
- C. 252
- D. 216

27. The gain in social surplus is

- A. 80
- B. 174
- C. 234
- D. 256

**Questions 28 – 38 relate to the following data.**

$$MPB(y) = 80 - 19y$$

$$MPC(y) = y$$

$$MEC(y) = 60y$$

28. Marginal social cost is

- A.  $MSC(y) = 59y$
- B.  $MSC(y) = 80 - 18y$
- C.  $MSC(y) = 80 - 79y$
- D.  $MSC(y) = 61y$

29. The private optimum is

- A. 4
- B. 5
- C. 3
- D. 8

30. The social optimum is

- A. 3
- B. 2
- C. 2.5
- D. 1

31. External cost at the private optimum is

- A. 526
- B. 480
- C. 328
- D. 412

32. External cost at the social optimum is

- A. 30
- B. 324
- C. 148
- D. 226

Now suppose that a regulation forces the source agent to reduce his level of the activity to the social optimum. Consider Questions 33 – 35 under this scenario.

33. The gain to the external agents is

- A. 300
- B. 264
- C. 450
- D. 102

34. The loss to the source agent is

- A. 90
- B. 112
- C. 146
- D. 84

35. The gain in social surplus is

- A. 360
- B. 18
- C. 188
- D. 152

Now return to the unregulated scenario, and suppose that the source agent is assigned the explicit right to undertake the activity at his private optimum. He may trade that right in whole or in part if he wishes. Suppose a costless contract can be written between the

source agent and the external agents that specifies the price for each unit of activity reduced by the source agent. Consider Questions 36 – 38 under this scenario.

36. The equilibrium price at which trade will occur is

- A. 60
- B. 180
- C. 120
- D. 30

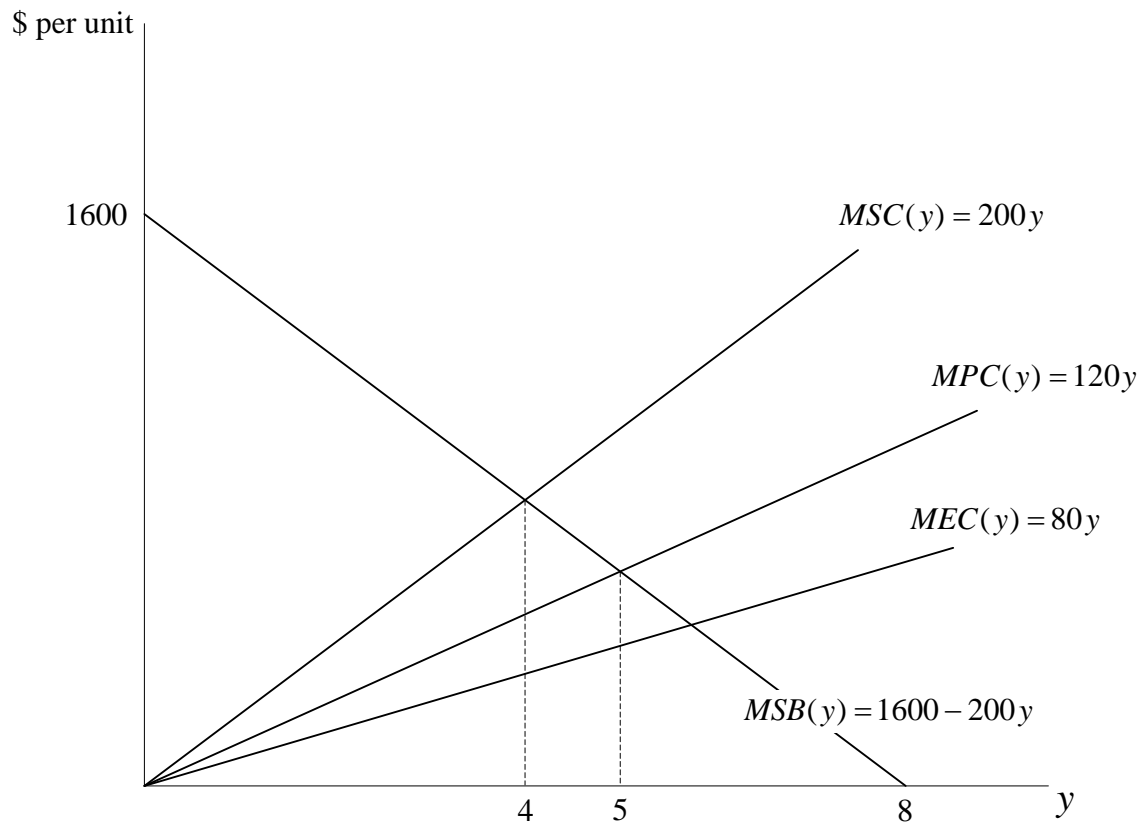
37. The gains from trade for the source agent are

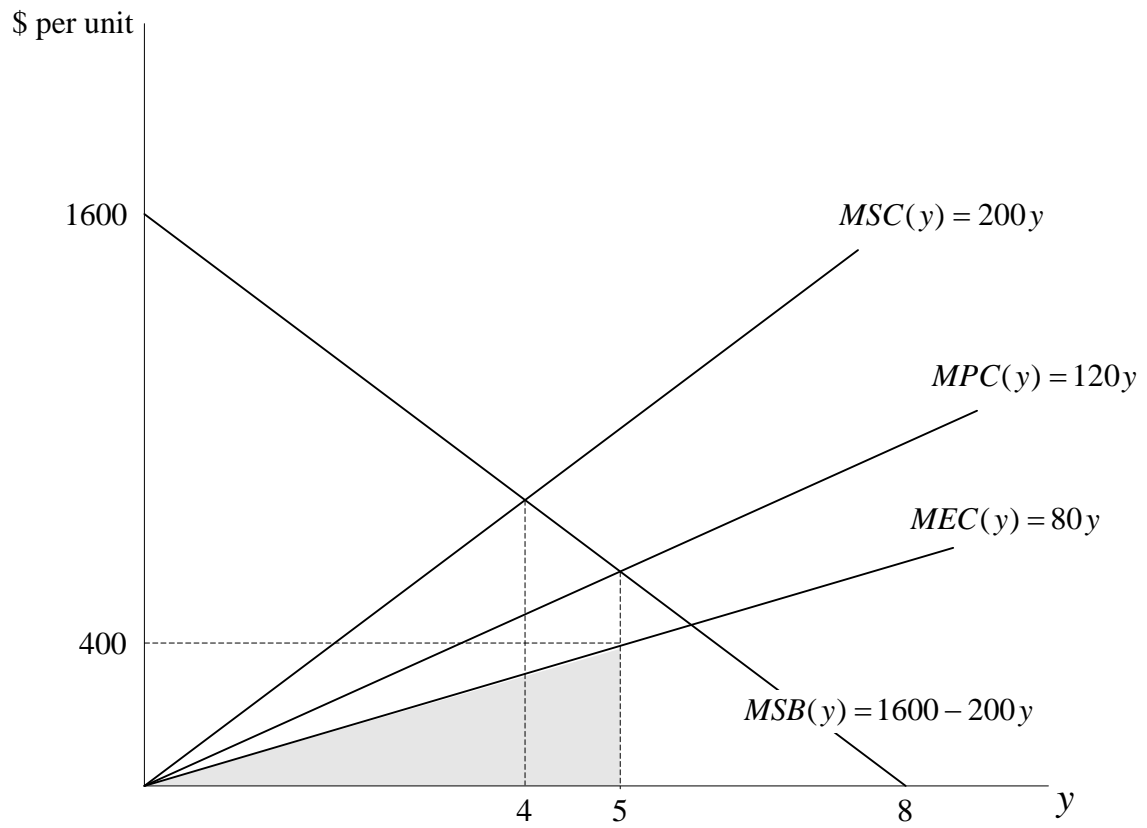
- A. 120
- B. 90
- C. 76
- D. 212

38. The gains from trade for the external agents are

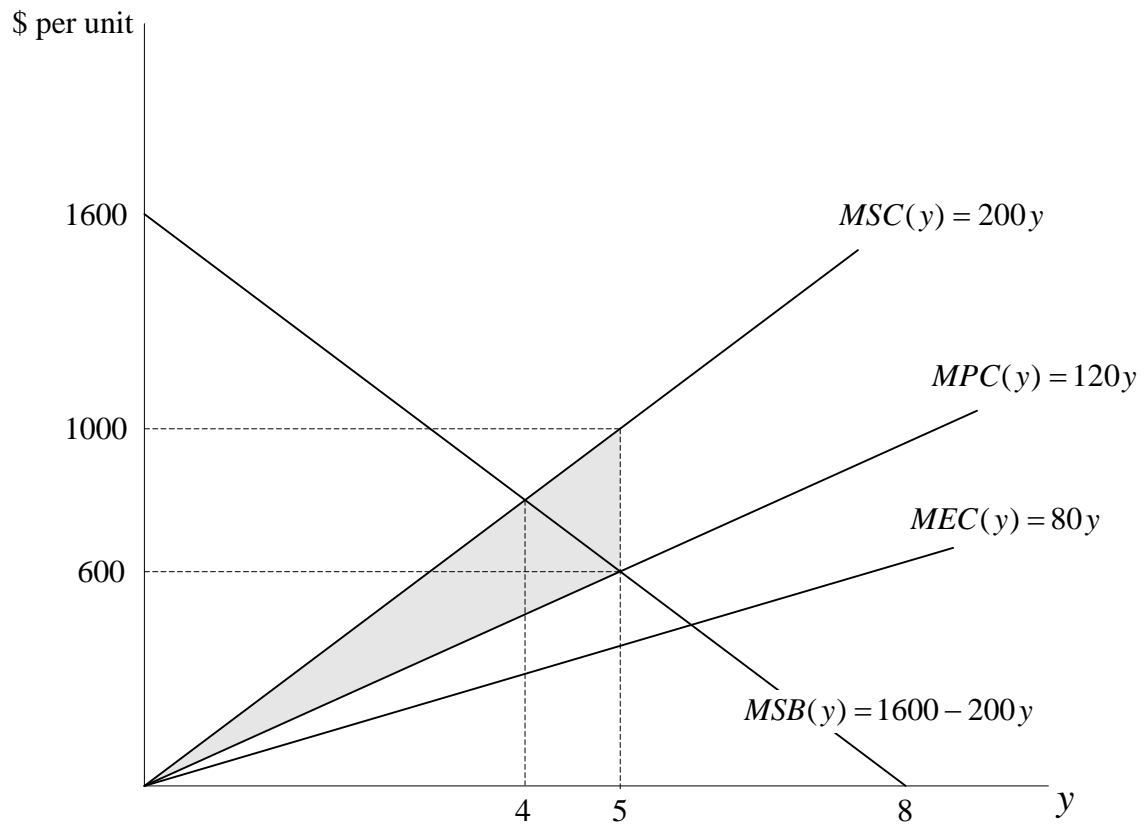
- A. 32
- B. 240
- C. 98
- D. 270



**Figure R4-1**



**Figure R4-2**



**Figure R4-3**

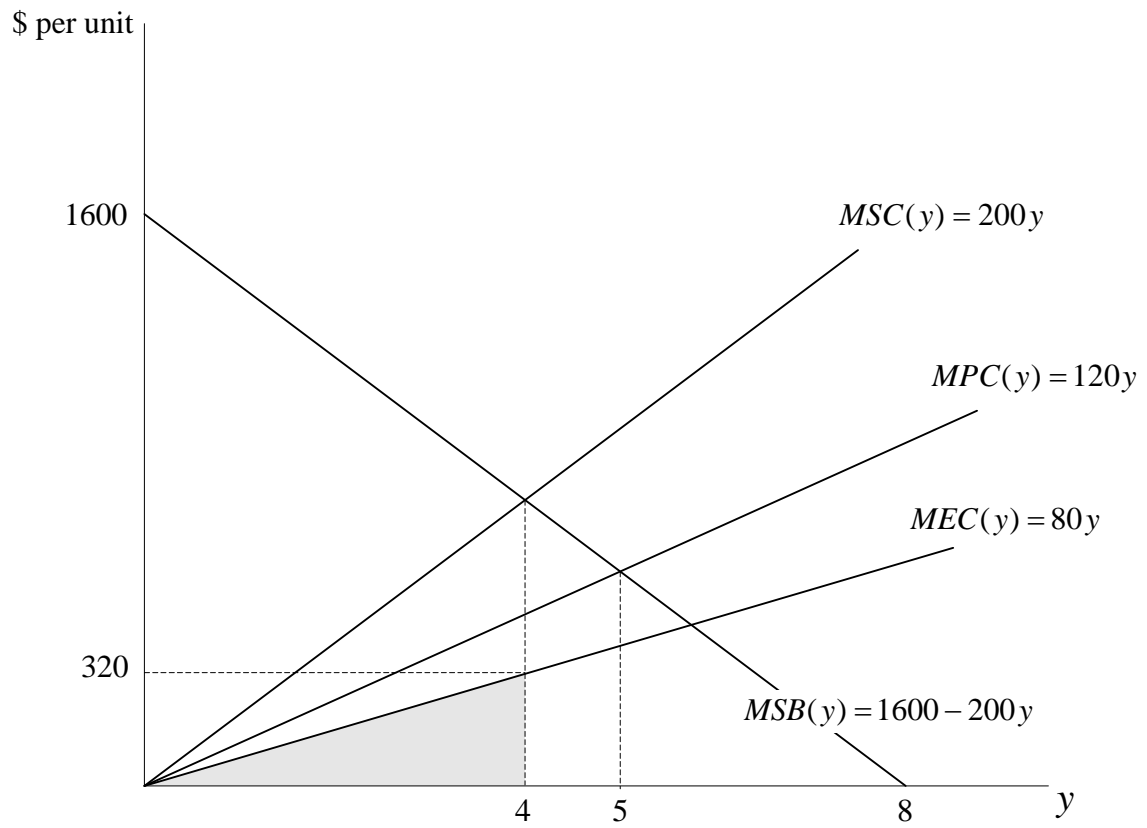


Figure R4-4

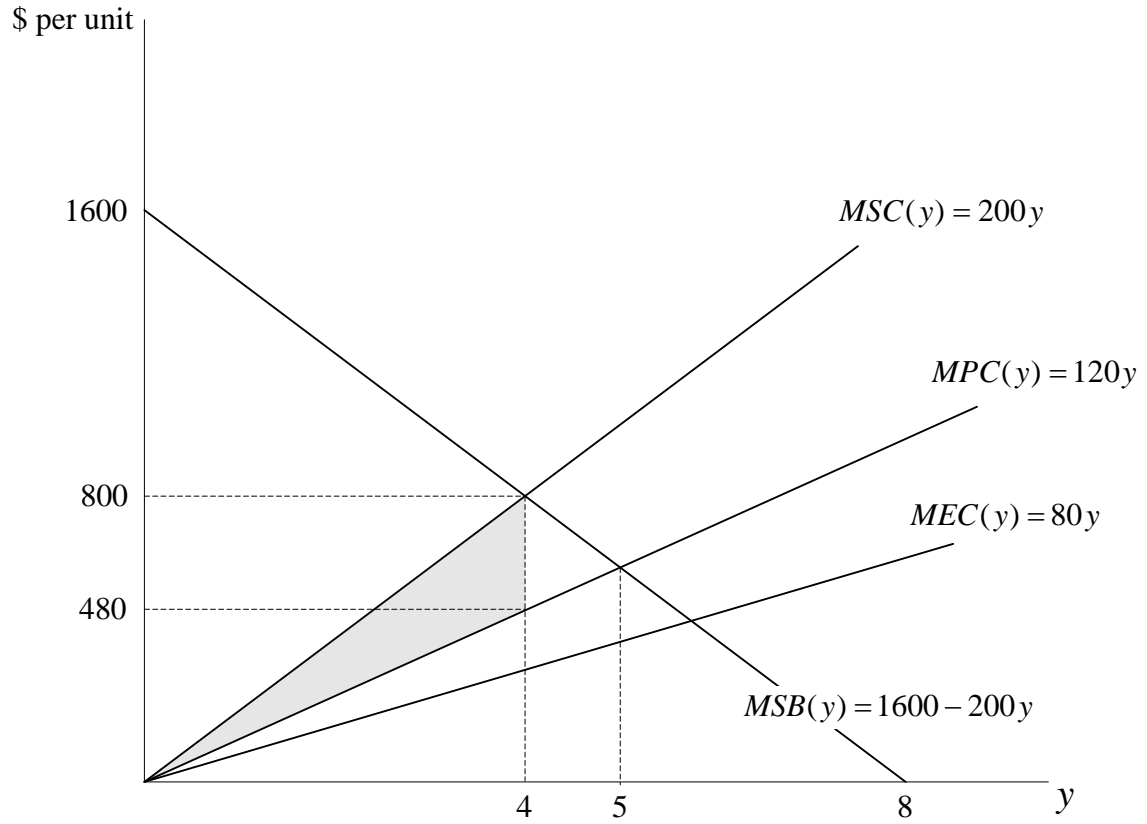


Figure R4-5

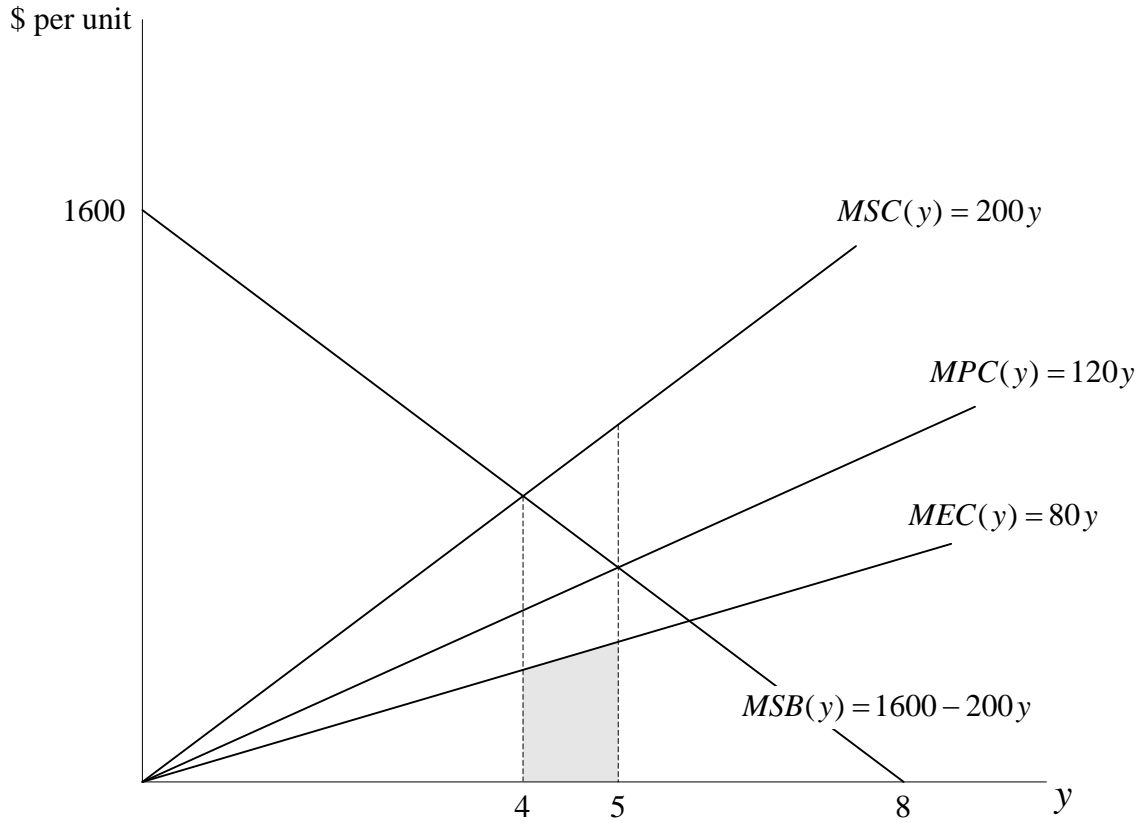


Figure R4-6

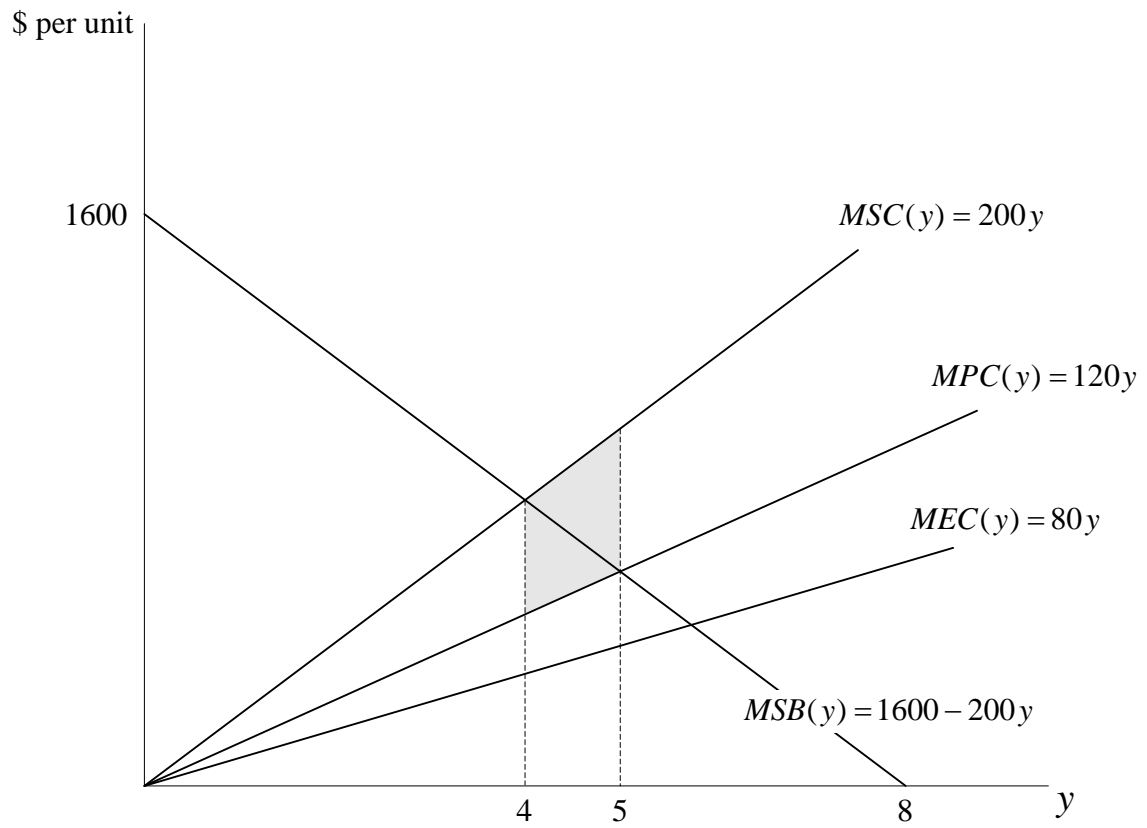


Figure R4-7

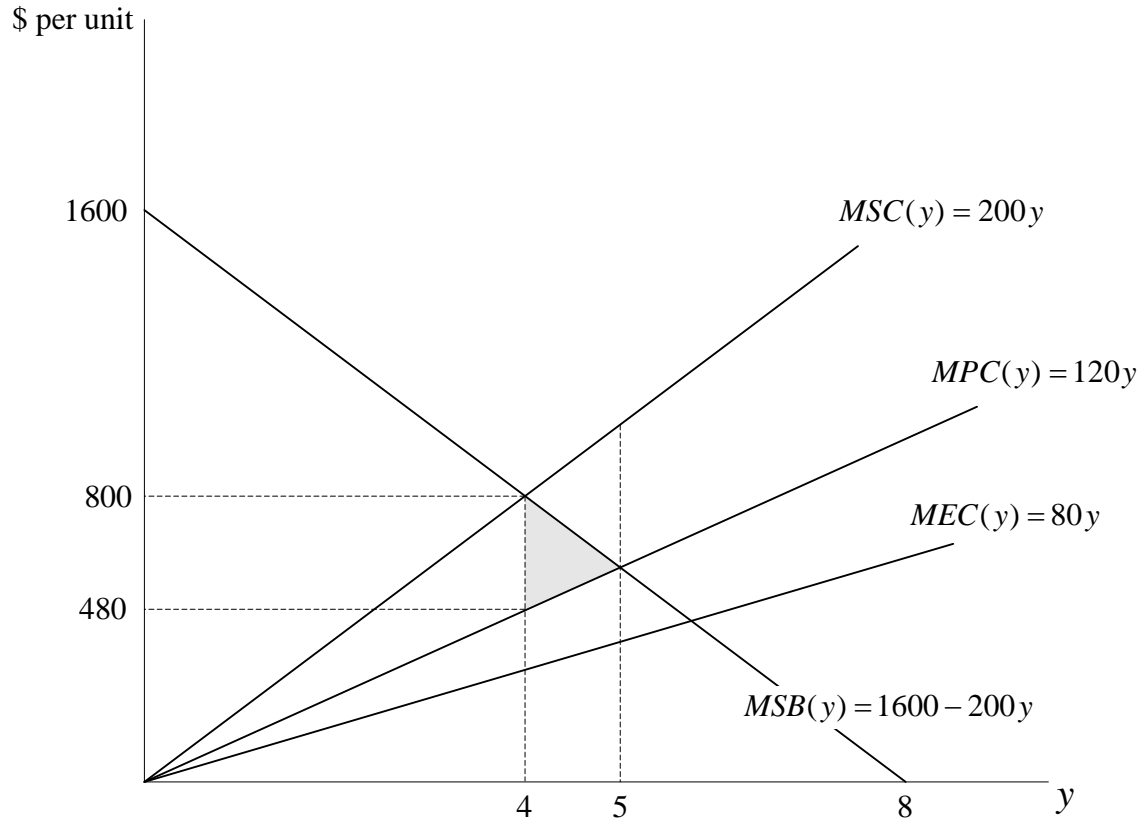


Figure R4-8



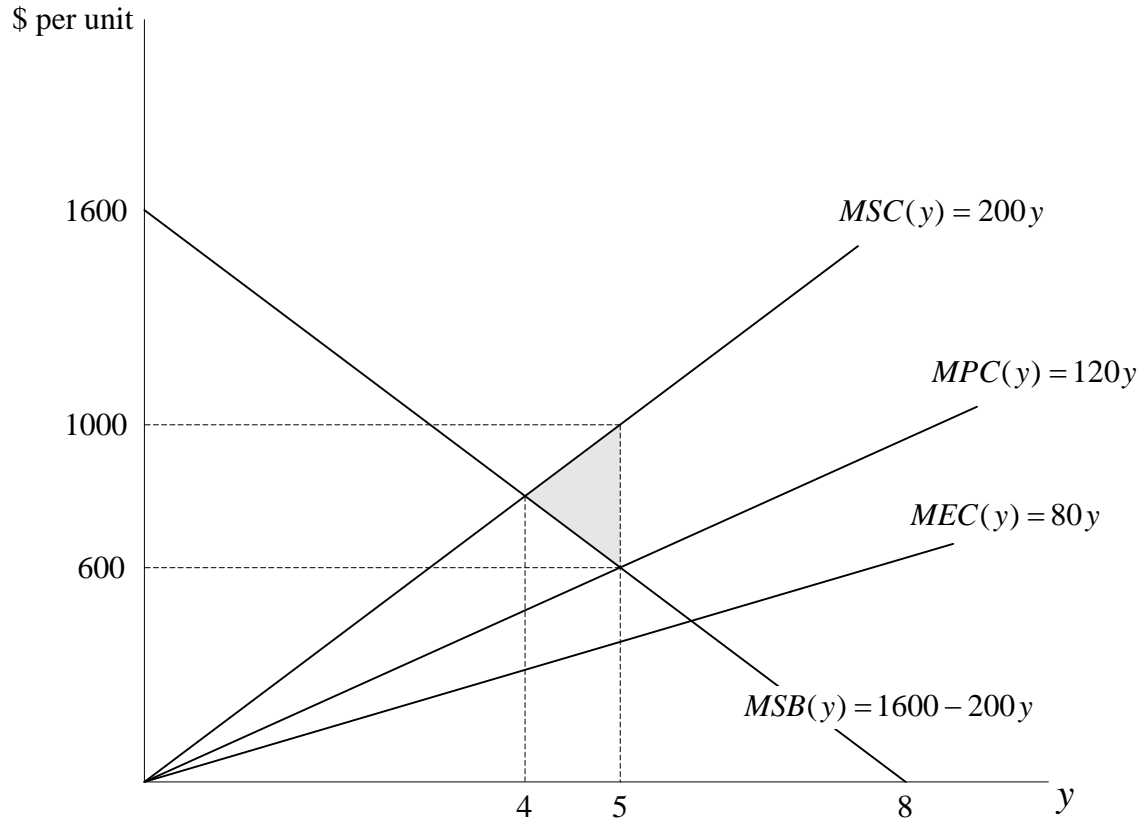
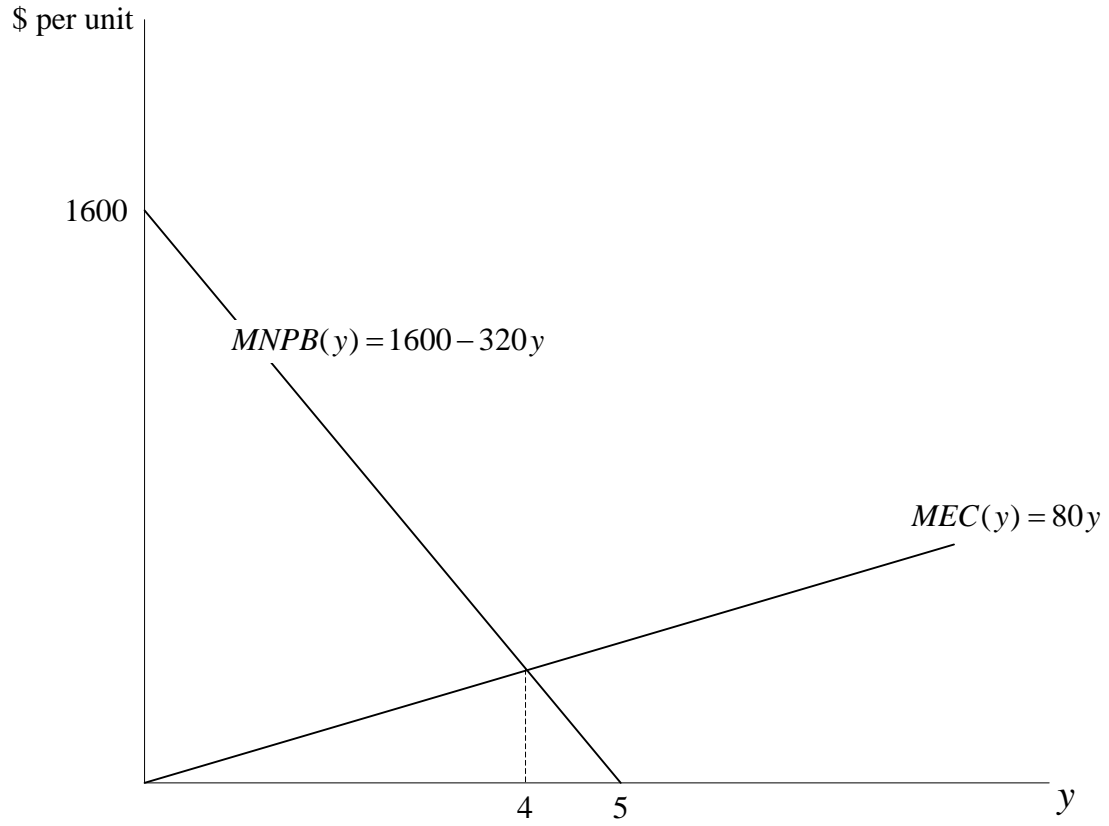
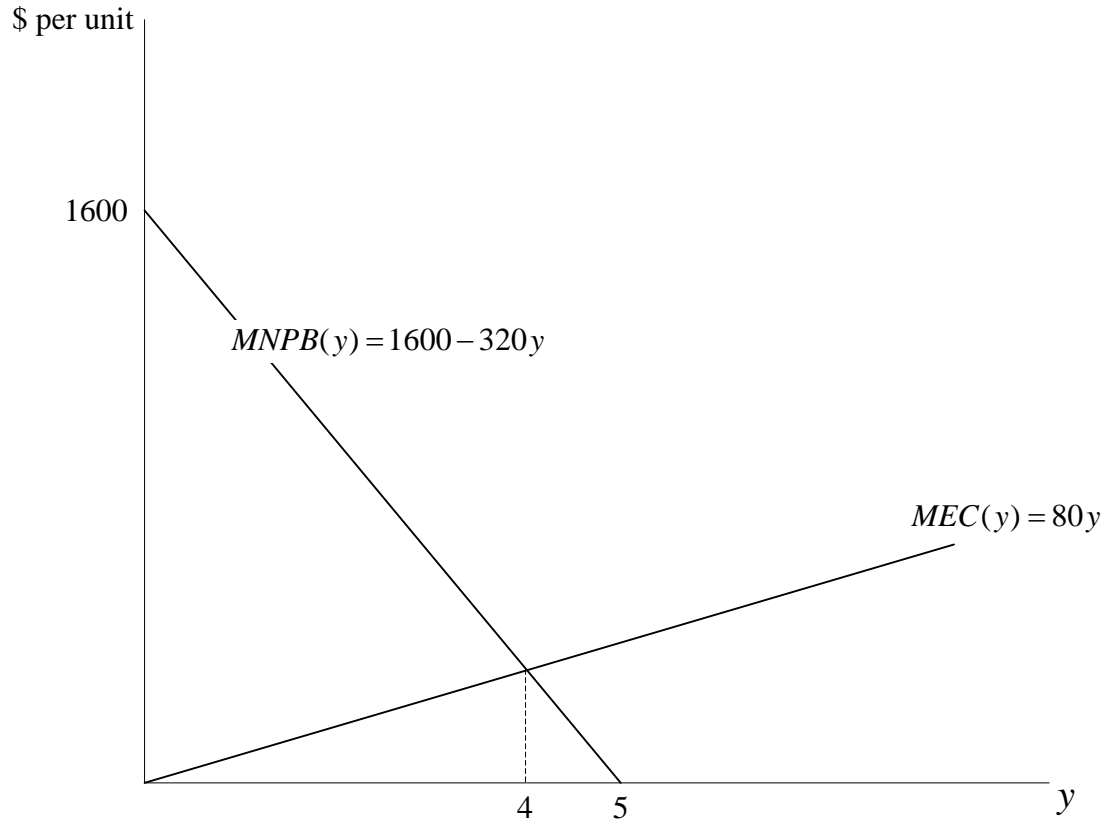


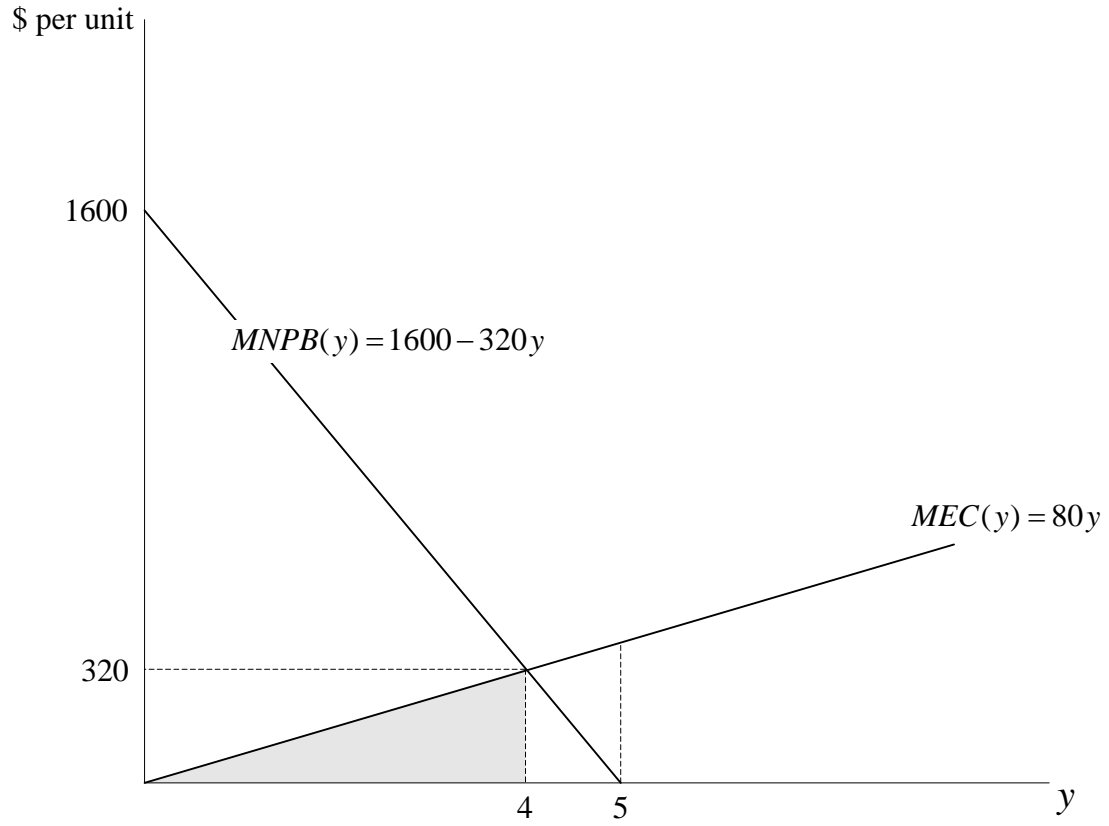
Figure R4-9



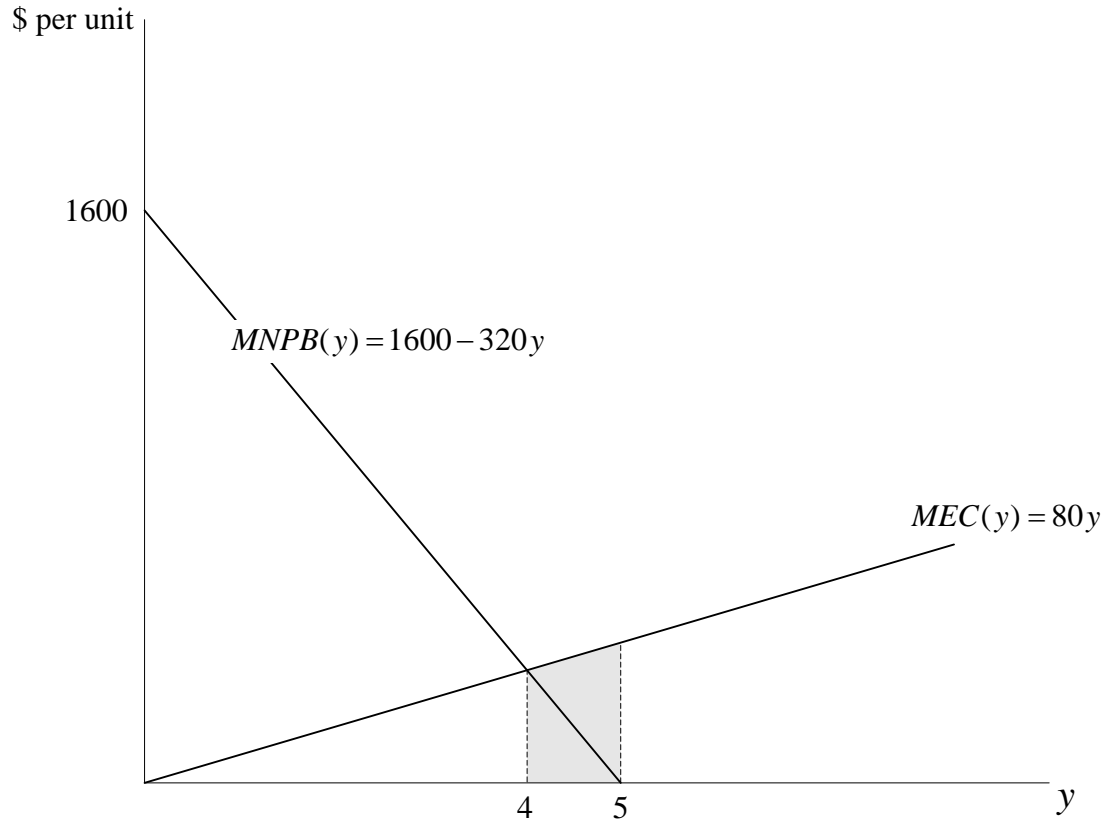
**Figure R4-10**



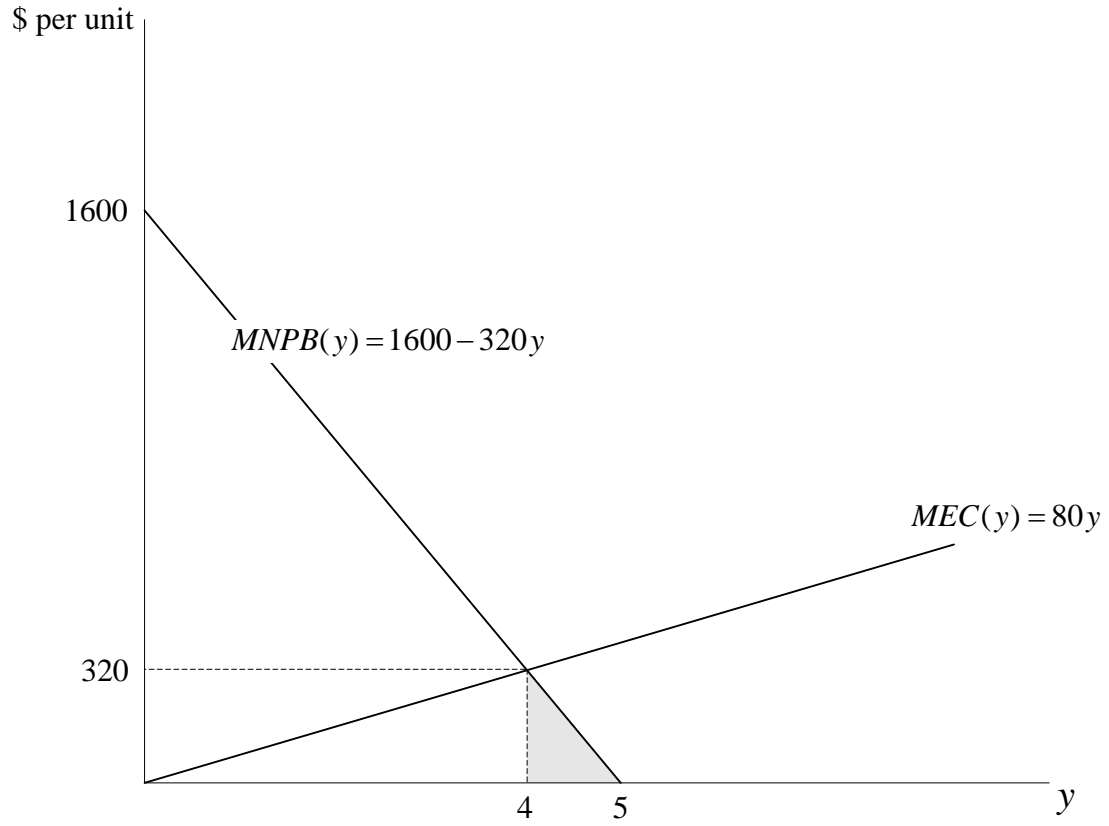
**Figure R4-11**



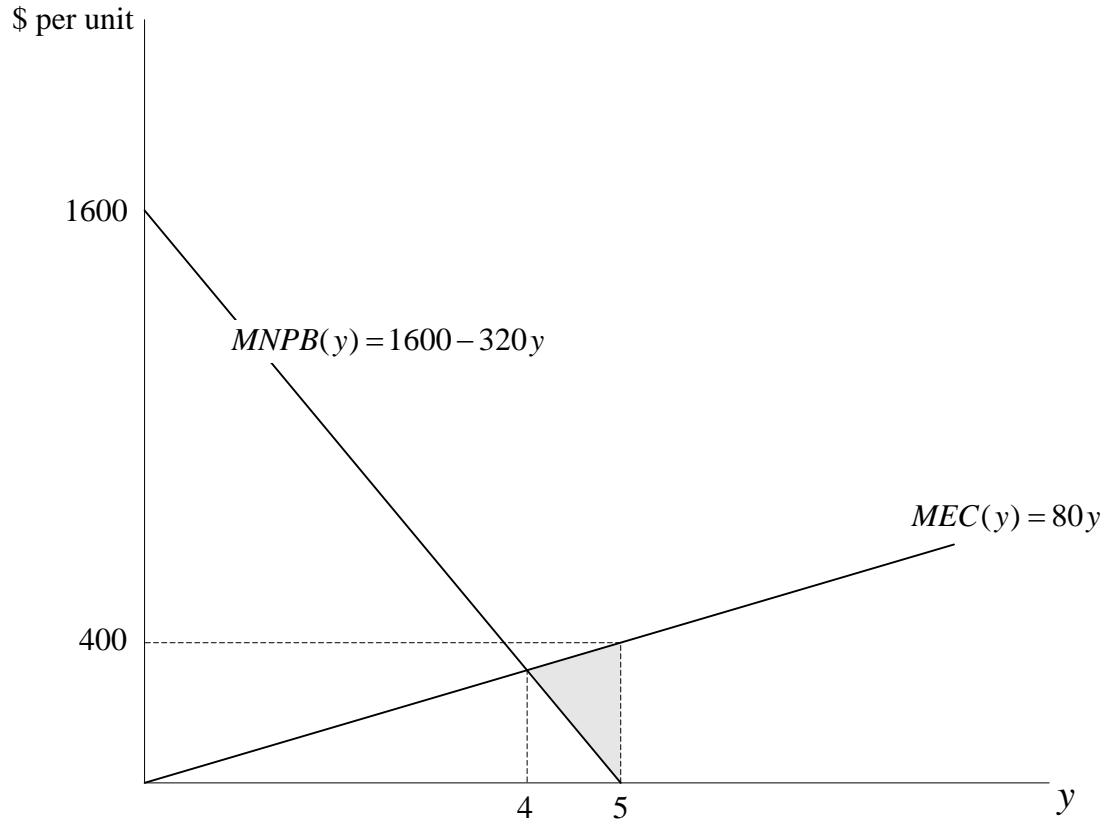
**Figure R4-12**



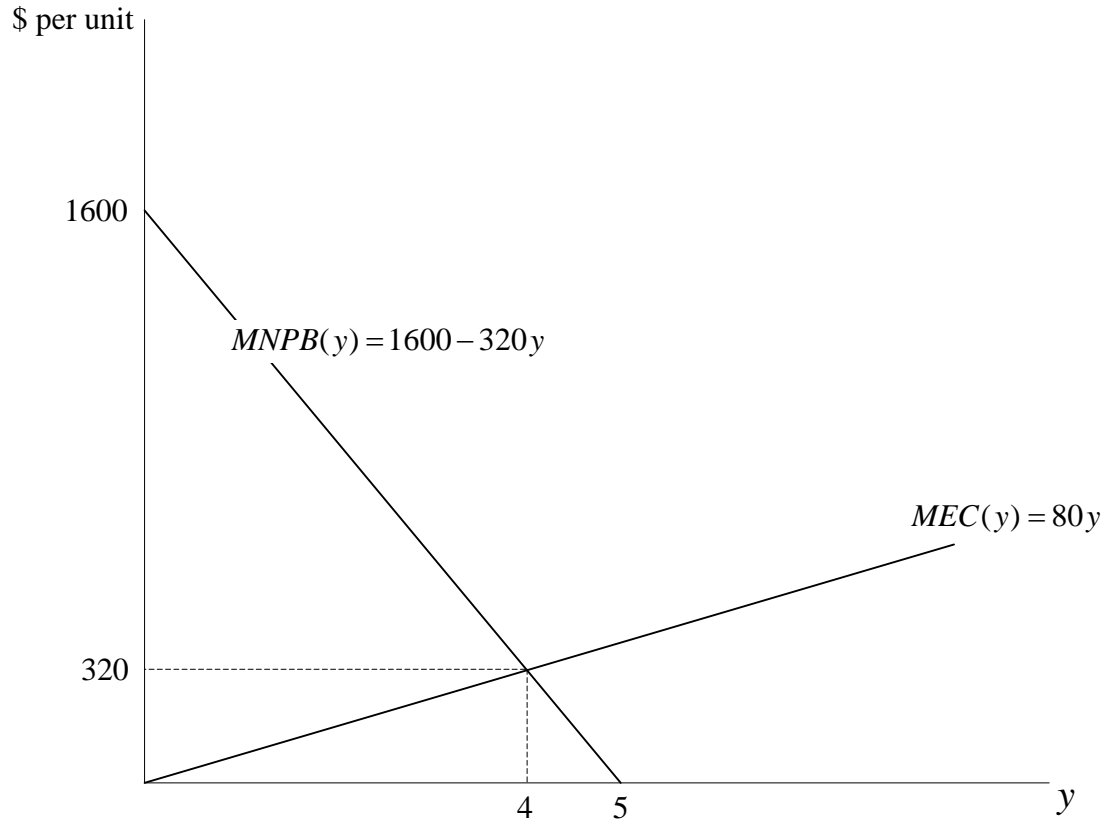
**Figure R4-13**



**Figure R4-14**

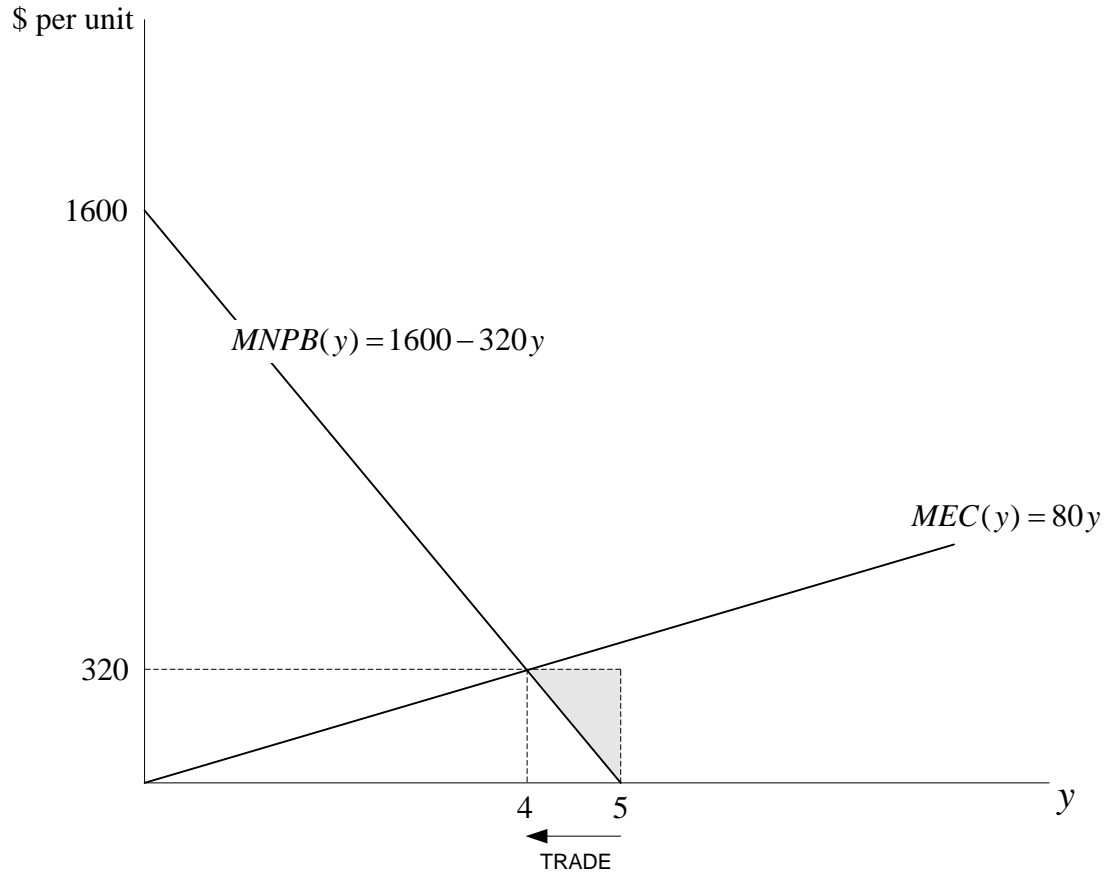


**Figure R4-15**

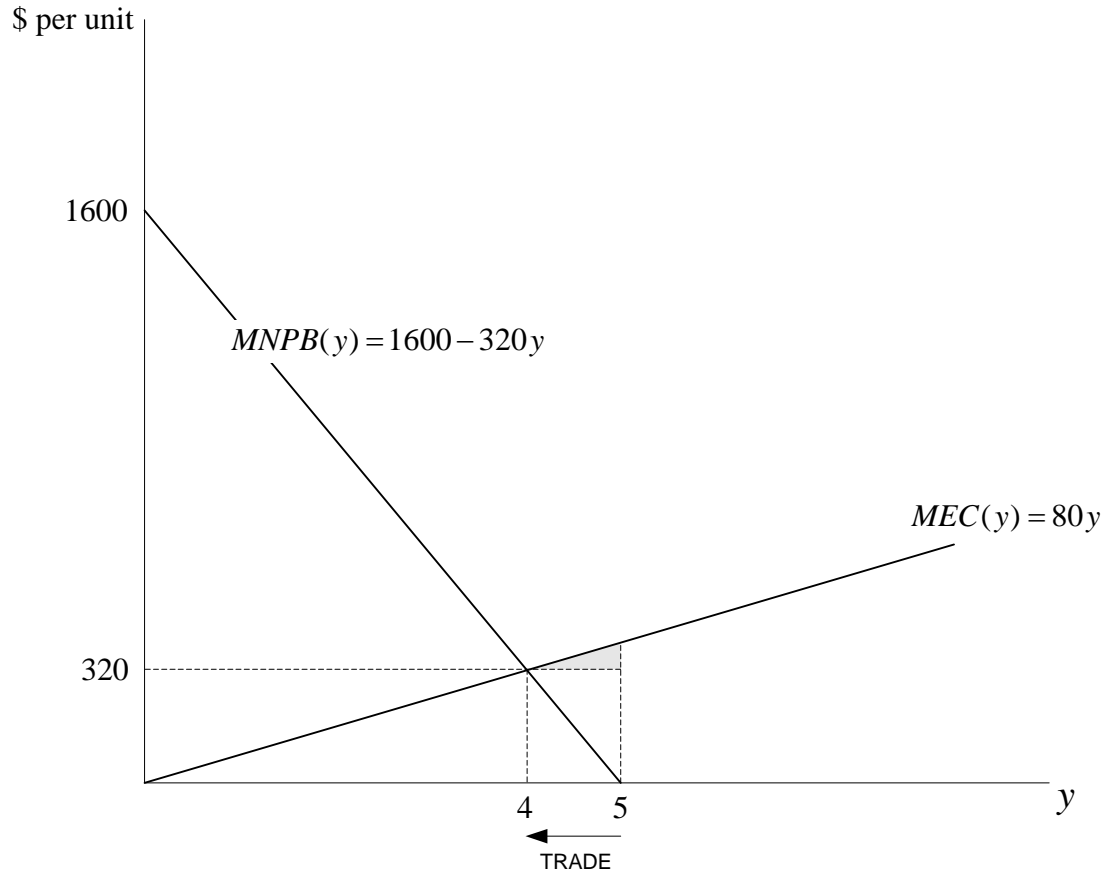


**Figure R4-16**

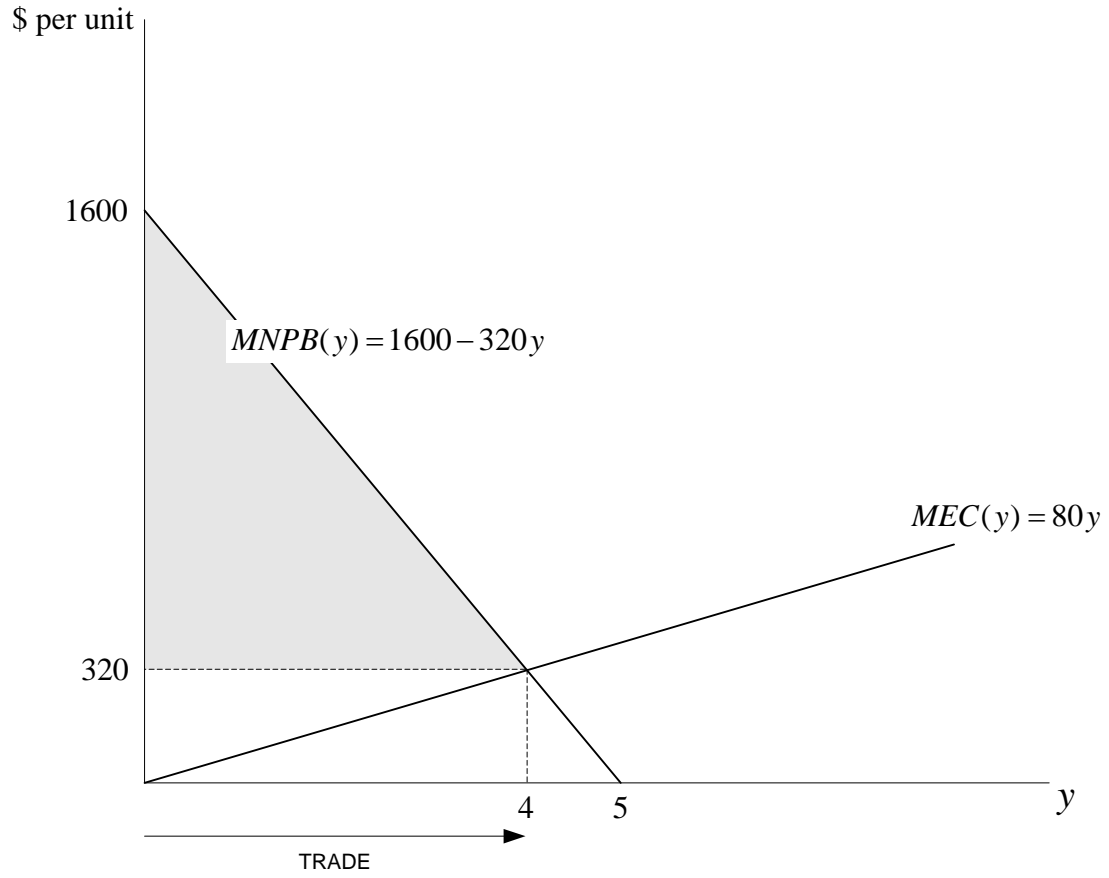




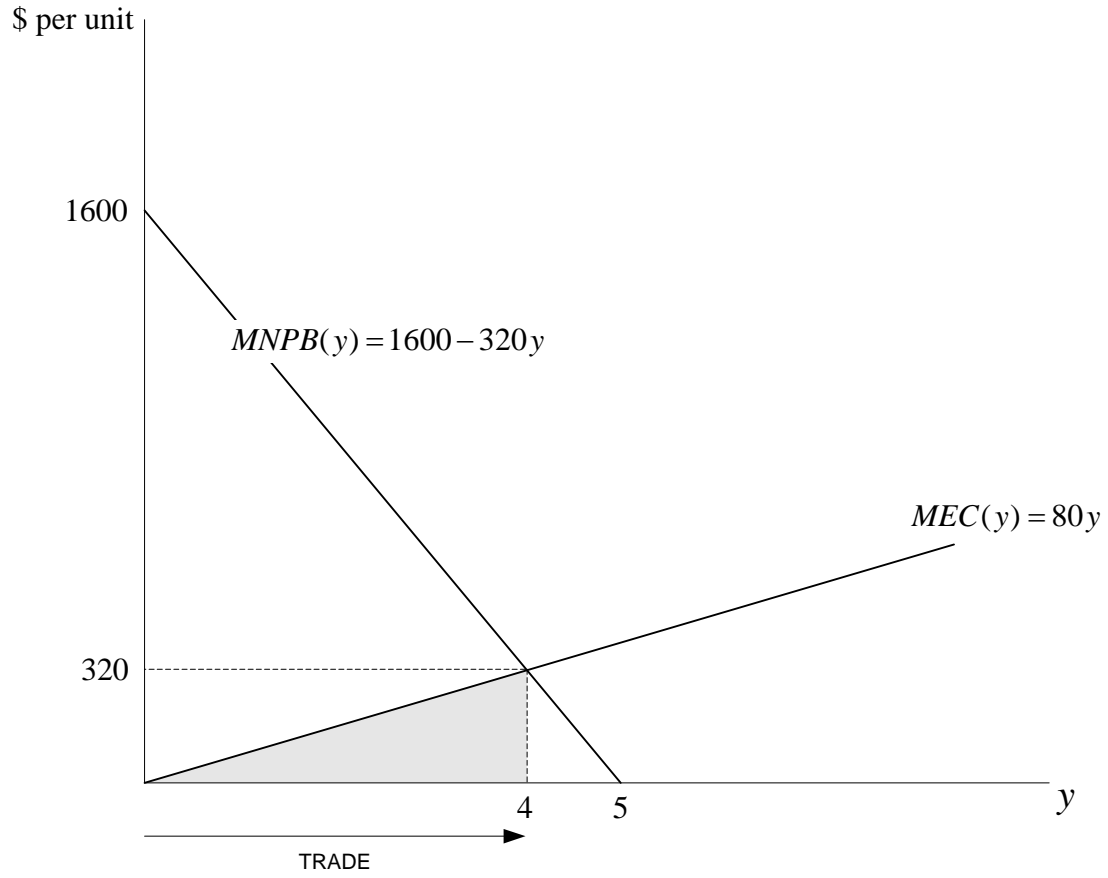
**Figure R4-17**



**Figure R4-18**



**Figure R4-19**



**Figure R4-20**

**ANSWER KEY**

1. B		20. C	
	11. A		30. D
2. C		21. B	
	12. D		31. B
3. D		22. C	
	13. B		32. A
4. A		23. D	
	14. D		33. C
5. B		24. A	
	15. C		34. A
6. C		25. B	
	16. B		35. A
7. A		26. D	
	17. B		36. A
8. B		27. C	
	18. C		37. B
9. B		28. D	
	19. A		38. D
10. C		29. A	