

4. EXTERNALITIES

PART 1: UNILATERAL EXTERNALITIES

OUTLINE

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4.1 INTRODUCTION

Introduction

- An **externality** (or external effect) is an impact associated with an action that is not felt by the agent taking that action.

- Externalities can be positive (an external benefit) or negative (an external cost).

5

Introduction

- A **source agent** undertakes the action that has the associated externality.

- An **external agent** is an agent impacted by the externality.

6

Introduction

- Externalities can be unilateral or reciprocal.

- **Unilateral externalities**:
  - the externality operates in only one direction: from source agent to external agents
  - example: a firm discharges a pollutant that flows downstream, to the detriment of downstream water-users.

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Introduction

- **Reciprocal externalities**:
  - the externality operates in both directions: source agents are also external agents, and external agents are also source agents.
  - examples: GHG emissions; traffic congestion.

8

Introduction

- Part 1 of this topic will focus on unilateral externalities.
- Part 2 will examine reciprocal externalities.

9

Introduction

- We begin our treatment of a unilateral externality with a simple setting in which there is a single source agent and a single external agent.

10

Introduction

- The source agent undertakes some continuously variable activity, the amount of which is denoted y.
- We first characterize the **private optimum** and then examine how and why it diverges from the **social optimum**.

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A Note on Correct Terminology

- The private optimum is a particular allocation of resources; thus, “private optimum” is a noun.
- The defining characteristic of that allocation is that is it privately optimal; thus, “privately optimal” is a compound adjective.

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A Note on Correct Terminology

- Similarly, "social optimum" is a noun; "socially optimal", a compound adjective.

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4.2 THE PRIVATE OPTIMUM

The Private Optimum

- Let $PB(y)$ and $PC(y)$ denote the **private benefit** and **private cost** respectively to the source agent from an activity undertaken in the amount y.

- Consider two examples that we will use throughout this topic.

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The Private Optimum

- <u>Example A</u>: y is the number of hectares that a land-owner protects as wildlife habitat.
- $PB(y)$ measures the personal enjoyment the land-owner gets from wildlife.
- $PC(y)$ measures the foregone revenue from leaving the land undeveloped (for farming or suburban housing).

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The Private Optimum

- <u>Example B</u>: y is the amount of output produced in a factory.
- $PB(y)$ measures revenue from the sale of that output
- $PC(y)$ measures the cost to the factory of the materials and labour used in production.

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The Private Optimum

- We assume that $PB(y)$ is increasing in y at a decreasing rate, and that $PC(y)$ is increasing in y at an increasing rate, as depicted in Figure 4-1.

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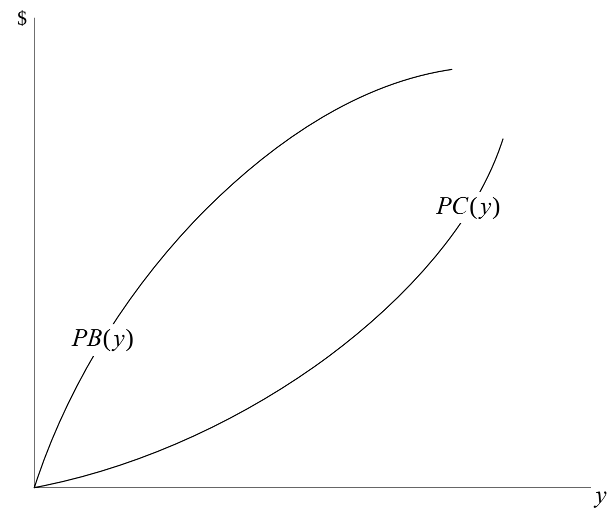

FIGURE 4-1

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The Private Optimum

- **Net private benefit** (or private surplus) is

$$NPB(y) = PB(y) - PC(y)$$

- This is the vertical distance between the two curves in Figure 4-1.

20

The Private Optimum

- The **private optimum** is the value of y at which $NPB(y)$ is maximized; it is denoted $\hat{y}$.
- The private optimum occurs where the rate of change of $PB(y)$ is just equal to rate of change of $PC(y)$; see Figure 4-2.

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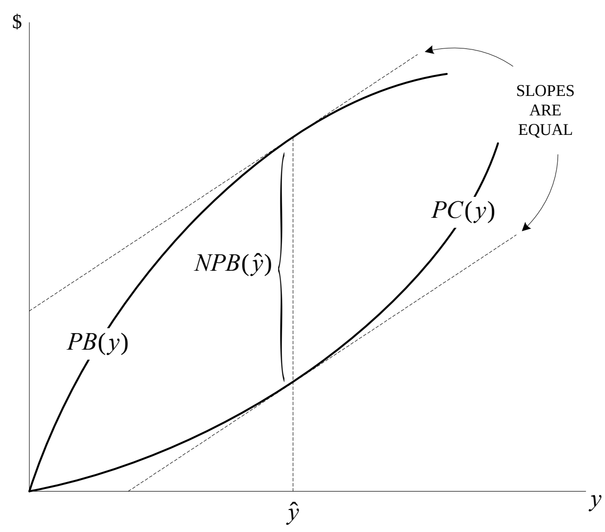

FIGURE 4-2

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The Private Optimum

- Let us now characterize the private optimum directly in terms of the slopes of *PB*(*y*) and *PC*(*y*).

23

The Private Optimum

- Let *MPB*(*y*) denote the **marginal private benefit** *y*, defined as the rate of change (or slope) of *PB*(*y*).

- Let *MPC*(*y*) denote the **marginal private cost** of *y*, defined as the rate of change (or slope) of *PC*(*y*).

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The Private Optimum

- We have assumed that $PB(y)$ is increasing in y at a decreasing rate, and this means that $MPB(y)$ is negatively-sloped.
- We have assumed that $PC(y)$ is increasing in y at an increasing rate, and this means that $MPC(y)$ is positively-sloped.
- See Figure 4-3.

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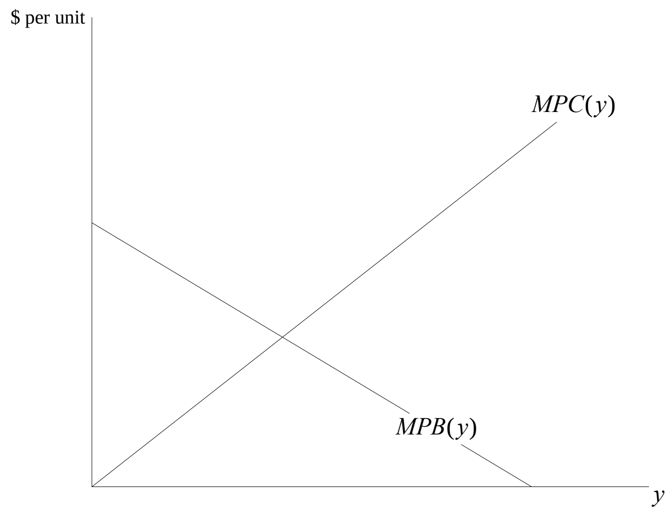

FIGURE 4-3

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The Private Optimum

- The schedules depicted in Figure 4-3 are linear – and we will often work with examples that make this assumption for the sake of simplicity – but our general analysis does not depend on this assumption in any way.

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The Private Optimum

- The **private optimum** is $\hat{y}$, where,

$$MPB(\hat{y}) = MPC(\hat{y})$$

- See Figure 4-4.

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THE PRIVATE OPTIMUM

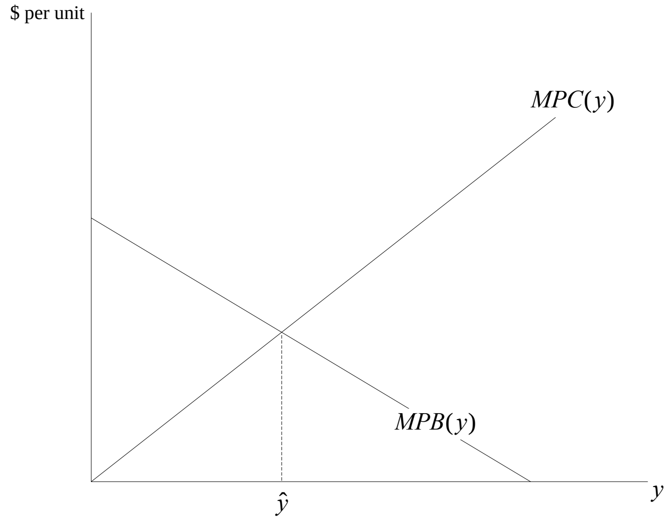

FIGURE 4-4

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The Private Optimum

- The private benefit at the private optimum is the area (or definite integral),

$$PB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy$$

- See Figure 4-5.

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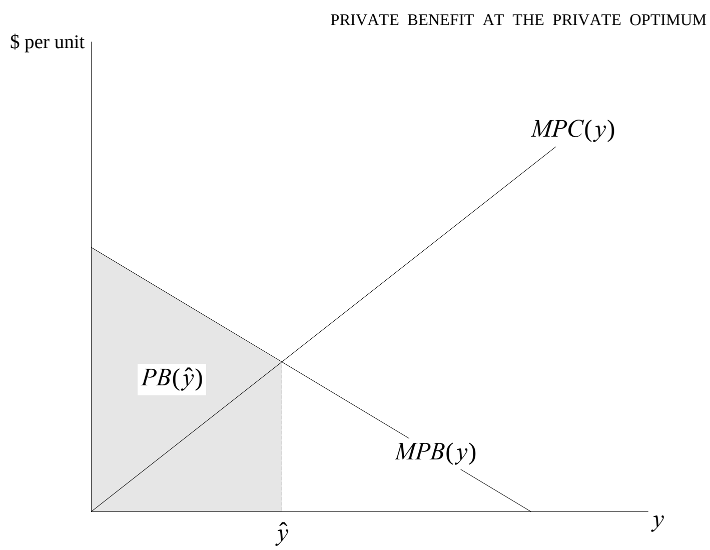

FIGURE 4-5

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The Private Optimum

- The private cost at the private optimum is the definite integral

$$PC(\hat{y}) = \int_0^{\hat{y}} MPC(y)dy$$

- See Figure 4-6.

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PRIVATE COST AT THE PRIVATE OPTIMUM

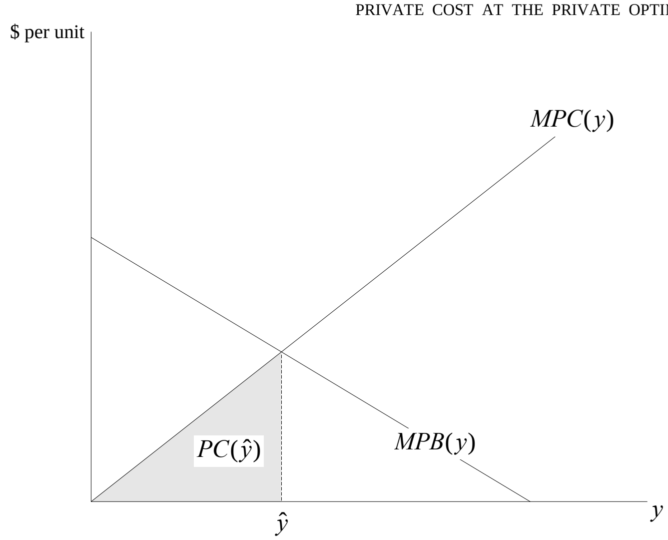

FIGURE 4-6

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The Private Optimum

- The net private benefit at the private optimum is

$$NPB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy - \int_0^{\hat{y}} MPC(y)dy$$

- See Figure 4-7.

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NET PRIVATE BENEFIT AT THE PRIVATE OPTIMUM

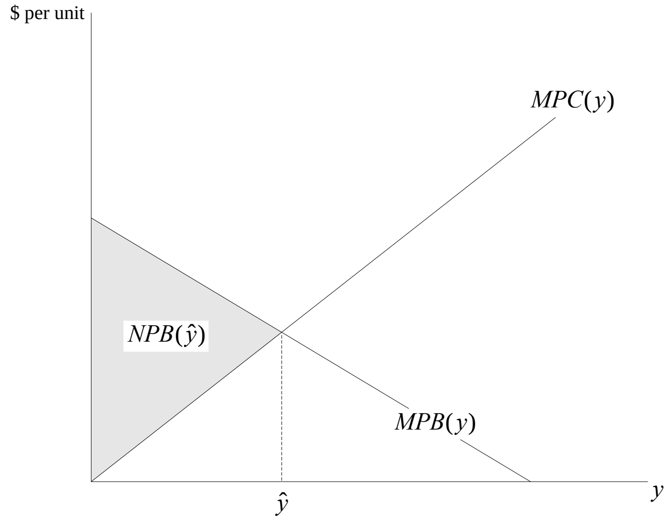

FIGURE 4-7

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4.3 THE PRIVATE OPTIMUM: AN ALTERNATIVE PRESENTATION

The Private Optimum: An Alternative Presentation

- The private optimum is at $\hat{y}$, where

$$MPB(\hat{y}) = MPC(\hat{y})$$

or equivalently, where

$$MPB(\hat{y}) - MPC(\hat{y}) = 0$$

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The Private Optimum: An Alternative Presentation

- Define the **marginal net private benefit**

$$MNPB(y) = MPB(y) - MPC(y)$$

- Then the private optimum is $\hat{y}$, where

$$MNPB(\hat{y}) = 0$$

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The Private Optimum: An Alternative Presentation

- Graphically, MNPB is constructed as the vertical difference between MPB and MPC.

- See Figure 4-8.

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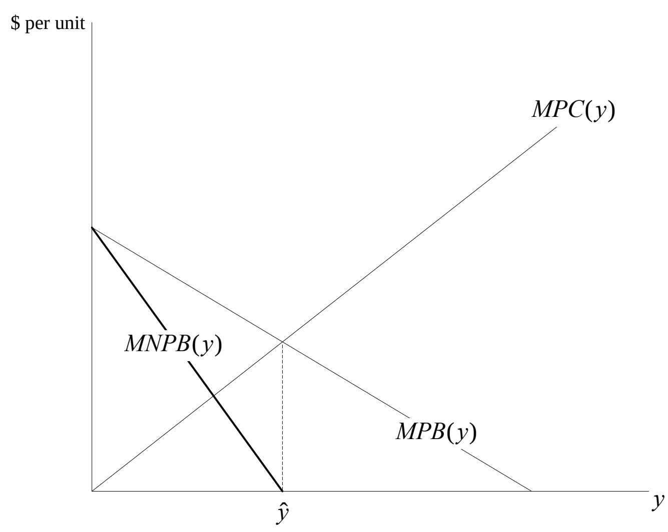

FIGURE 4-8

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The Private Optimum: An Alternative Presentation

- Net private benefit at the private optimum is

$$NPB(\hat{y}) = \int_0^{\hat{y}} MNPB(y)dy$$

- See Figure 4-9.

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NET PRIVATE BENEFIT AT THE PRIVATE OPTIMUM

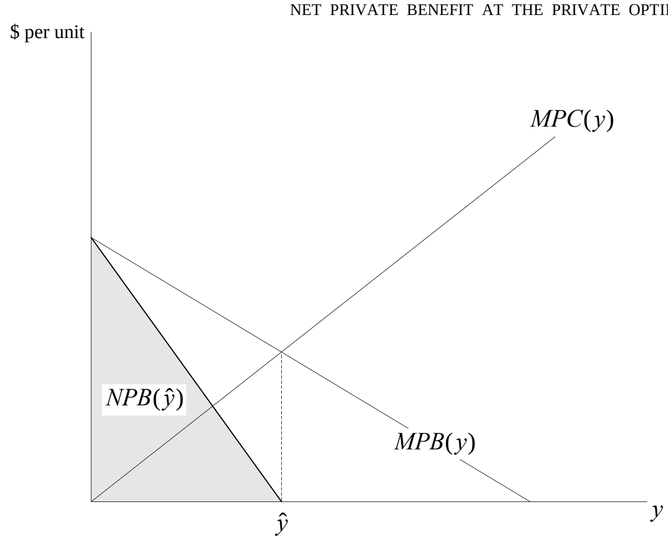

FIGURE 4-9

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The Private Optimum: An Alternative Presentation

- Note that the shaded area in Figure 4-9 is necessarily equal to the shaded area in Figure 4-7.
- This can be verified from the figures themselves using basic geometry if the MPB and MPC schedules are linear.

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NET PRIVATE BENEFIT AT THE PRIVATE OPTIMUM

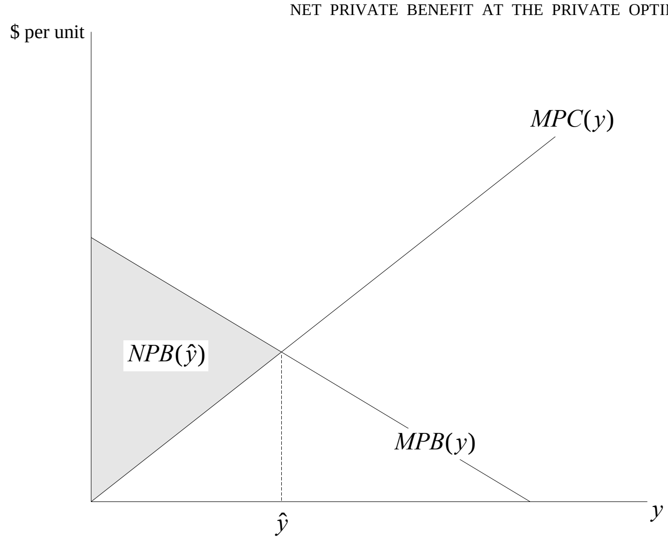

FIGURE 4-7 (repeat)

44

4.4 THE SOCIAL OPTIMUM

The Social Optimum

- If the activity bestows an **external benefit** $G(y)$ then the **social benefit** at y is the sum of the private benefit and the external benefit:

$$SB(y) = PB(y) + G(y)$$

46

The Social Optimum

- We assume that $G(y)$ is increasing at a decreasing rate, so given our assumption on $PB(y)$ from s.18, it follows that $SB(y)$ is also increasing at a decreasing rate.

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The Social Optimum

- If the activity imposes an **external cost** $D(y)$ then the **social cost** at y is the sum of the private cost and the external cost:

$$SC(y) = PC(y) + D(y)$$

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The Social Optimum

- We assume that $D(y)$ is increasing at an increasing rate, so given our assumption on PC(y) from s.18, it follows that $SC(y)$ is also increasing at an increasing rate.

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The Social Optimum

- The **net social benefit** (or **social surplus**) from the activity is the difference between social benefit and social cost:

$$NSB(y) = SB(y) - SC(y)$$

50

The Social Optimum

- The **social optimum** is the value of y at which net social benefit is maximized; it is denoted y^*.

- We can characterize this social optimum in terms of marginal social benefit and marginal social cost.

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The Social Optimum

- Let $MSB(y)$ denote the **marginal social benefit** at y. This is defined as the rate of change of $SB(y)$.

- Let $MSC(y)$ denote the **marginal social cost** at y. This is defined as the rate of change of $SC(y)$.

52

The Social Optimum

- The **social optimum** is y^*, where,

$$MSB(y^*) = MSC(y^*)$$

- See Figure 4-10.

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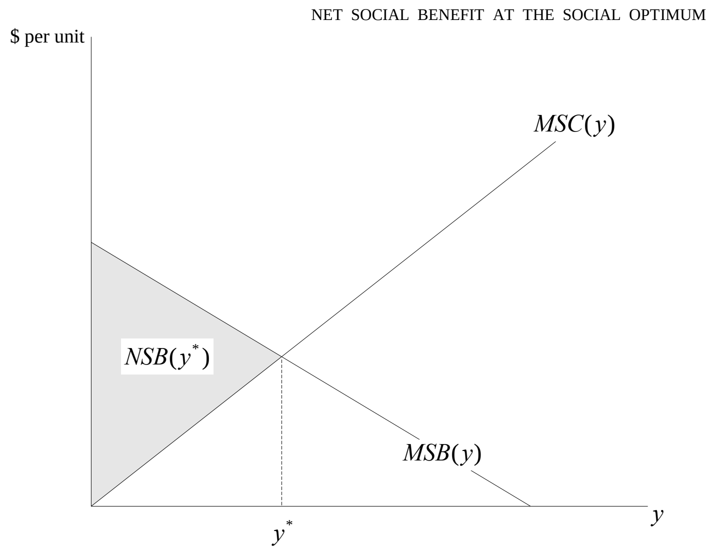

FIGURE 4-10

54

The Social Optimum

- Net social benefit at the social optimum is

$$NSB(y^*) = \int_0^{y^*} MSB(y)dy - \int_0^{y^*} MSC(y)dy$$

- See Figure 4-10.

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NET SOCIAL BENEFIT AT THE SOCIAL OPTIMUM

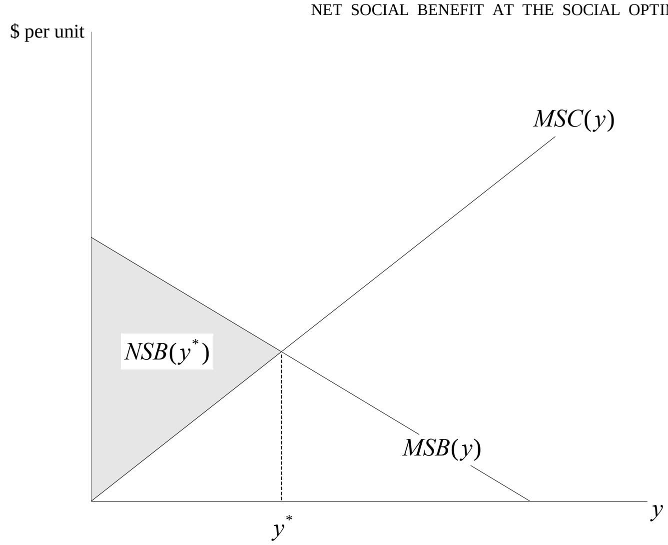

FIGURE 4-10 (repeat)

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The Social Optimum

- If an activity has no external benefit <u>and</u> no external cost (that is, if $G(y) = 0$ at all values of y, and $D(y) = 0$ at all values of y) then the private optimum and the social optimum coincide.

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The Social Optimum

- Conversely, if $G(y) \neq 0$ or $D(y) \neq 0$ at some values of y then the social optimum and the private optimum will typically <u>not</u> coincide.

- Let us consider each case in turn.

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4.5 A POSITIVE EXTERNALITY

A Positive Externality

- Consider a setting where the activity has an external benefit but no external cost: that is, $G(y)>0$ but $D(y)=0$.
- For example, if y is hectares of protected wildlife habitat then $G(y)$ might be the enjoyment that local residents get from wildlife viewing in the area.

60

A Positive Externality

- Since there is no external cost,

$$SC(y) = PC(y)$$

and it follows that

$$MSC(y) = MPC(y)$$

61

A Positive Externality

- Conversely, social benefit is

$$SB(y) = PB(y) + G(y)$$

62

A Positive Externality

- We can decompose the rate of change of $SB(y)$ into two components:

$$MSB(y) = MPB(y) + MEB(y)$$

where $MEB(y)$ is the **marginal external benefit** of the activity at y.

63

A Positive Externality

- Graphically, $MEB(y)$ at any given value of y (say y^0), is the vertical distance between $MSB(y)$ and $MPB(y)$ at $y^{0.}$

- See Figure 4-11.

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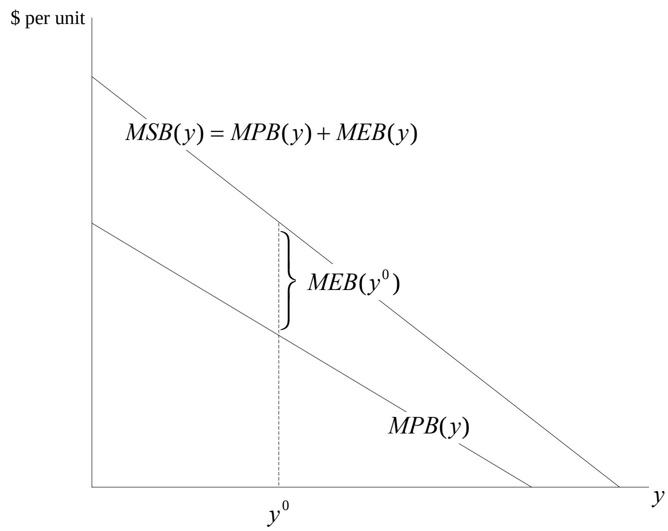

FIGURE 4-11

65

A Positive Externality

- Recall from s.47 our assumption that $G(y)$ is increasing at a decreasing rate.

- This is reflected in Figure 4-11: $MEB(y)$ declines as y rises; the gap between $MSB(y)$ and $MPB(y)$ becomes smaller.

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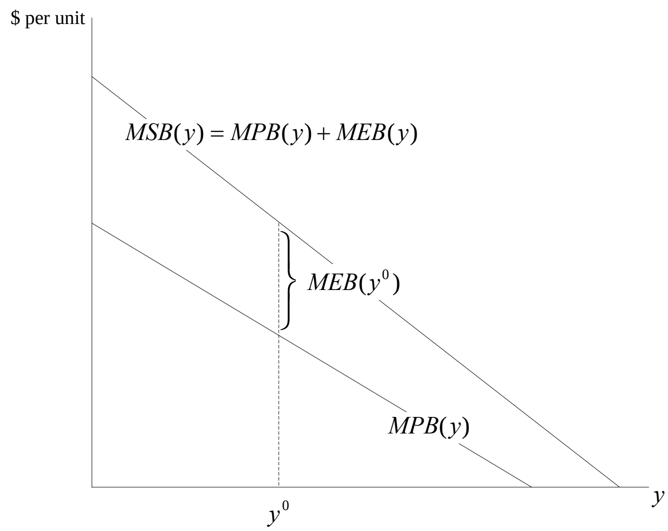

FIGURE 4-11 (repeat)

67

A Positive Externality

- We will later find it useful to depict *MEB*(*y*) as a separate graph.
- It is constructed by graphing the vertical distance between *MSB*(*y*) and *MPB*(*y*) at every value of *y*.
- See Figure 4-12.

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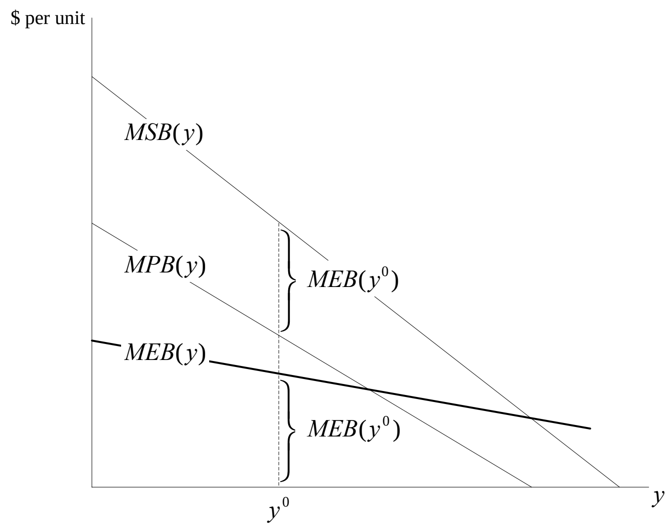

FIGURE 4-12

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A Positive Externality

- Now let us consider the impact of this external benefit on the relationship between the private and social optima.
- See Figure 4-13.

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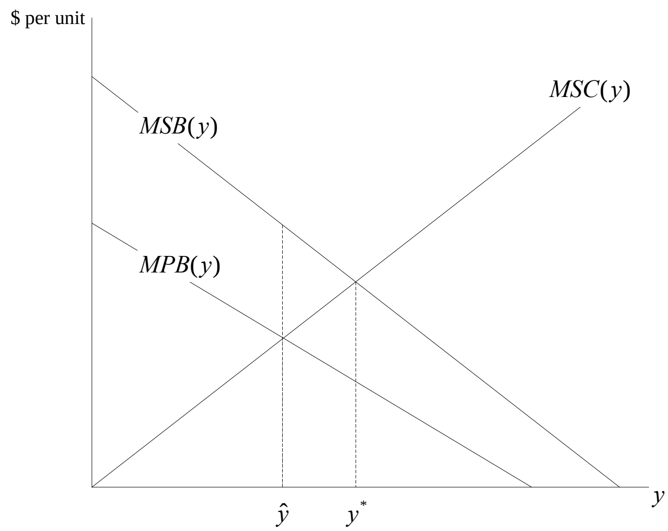

FIGURE 4-13

71

A Positive Externality

- The presence of the external benefit means:

$$\hat{y} < y^*$$

 – *ie*. the privately optimal level of activity is lower than the socially optimal level.

72

A Positive Externality

- Intuition:
  - the source agent does not take into account the benefit she bestows on the external agent when she chooses her action, and so her chosen level of the action is too low from a social perspective.

73

A Positive Externality: A Numerical Example

- Suppose

$$MPB(y) = 50 - 2y$$

$$MPC(y) = 3y$$

$$MEB(y) = 28 - y$$

74

A Positive Externality: A Numerical Example

- First find the <u>private optimum</u>, given by $\hat{y}$ such that

$$50 - 2\hat{y} = 3\hat{y}$$

which solves for

$$\hat{y} = 10$$

75

A Positive Externality: A Numerical Example

- Next find the <u>social optimum</u>.
- *MSB*(y) is the sum of *MPB*(y) and *MEB*(y):

$$MSB(y) = (50 - 2y) + (28 - y) = 78 - 3y$$

- Since there is no external cost here, *MSC*(y) is simply equal to *MPC*(y).

76

A Positive Externality: A Numerical Example

- Thus, the social optimum is y^* such that

$$78 - 3y^* = 3y^*$$

which solves for

$$y^* = 13$$

- See Figure 4-14.

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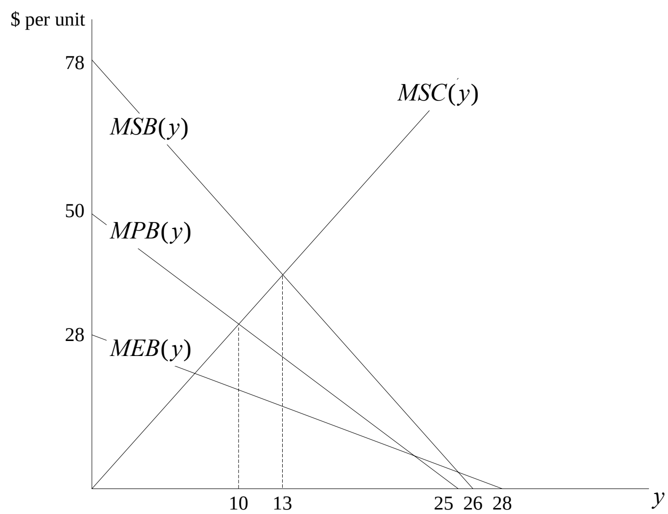

FIGURE 4-14

78

The Impact of a Regulated Increase in *y*

- Now suppose a third party (such as a government regulator) could force the source agent to raise her activity level from $\hat{y}$ to y^*.

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The Impact of a Regulated Increase in *y*

- We will show that this forced increase yields a potential Pareto improvement:
  - the source agent loses but the external agent gains by more than enough to compensate for that loss.

80

The Impact of a Regulated Increase in y

- Consider first the gain to the external agent (the increase in external benefit).
- We derive this by first calculating the external benefit at the social optimum, and then the external benefit at the private optimum, and then we take the difference.

81

The Impact of a Regulated Increase in y

- External benefit at the private optimum is the area under $MEB(y)$ from zero to $\hat{y}$:

$$G(\hat{y}) = \int_0^{\hat{y}} MEB(y)dy$$

82

The Impact of a Regulated Increase in *y*

- Since

$$MEB(y) = MSB(y) - MPB(y)$$

we can find the area under *MEB*(*y*) as the difference between the area under *MSB*(*y*) and the area under *MPB*(*y*).

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The Impact of a Regulated Increase in *y*

- Thus,

$$G(\hat{y}) = \int_0^{\hat{y}} MEB(y)dy$$
$$= \int_0^{\hat{y}} MSB(y)dy - \int_0^{\hat{y}} MPB(y)dy$$

- See Figure 4-15.

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EXTERNAL BENEFIT AT THE PRIVATE OPTIMUM

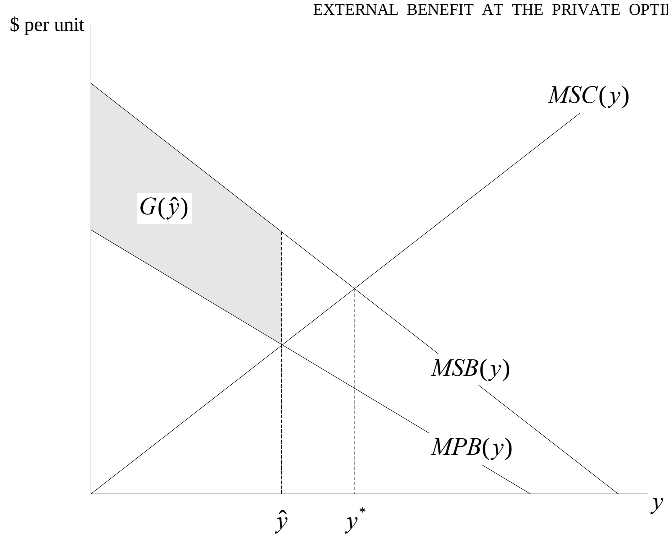

FIGURE 4-15

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The Impact of a Regulated Increase in *y*

- External benefit at the <u>social</u> optimum is

$$G(y^*) = \int_0^{y^*} MEB(y)dy$$

$$= \int_0^{y^*} MSB(y)dy - \int_0^{y^*} MPB(y)dy$$

- See Figure 4-16.

86

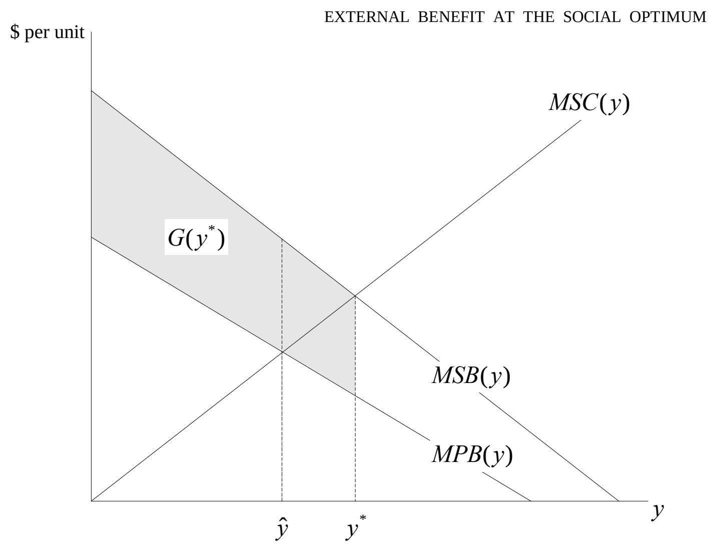

FIGURE 4-16

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The Impact of a Regulated Increase in *y*

- Hence, the <u>increase</u> in external benefit is

$$G(y^*) - G(\hat{y}) = \int_0^{y^*} MEB(y)dy - \int_0^{\hat{y}} MEB(y)dy$$

- This is the **gain to the external agent**.

- See *area*(*abcd*) in Figure 4-17.

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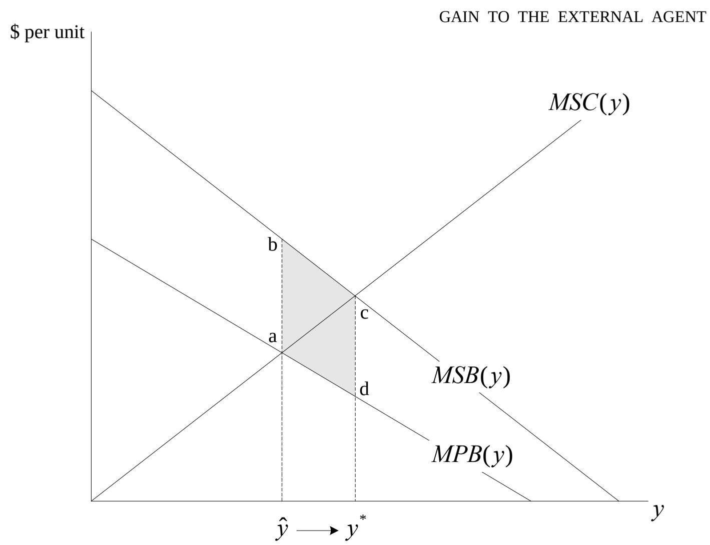

FIGURE 4-17 89

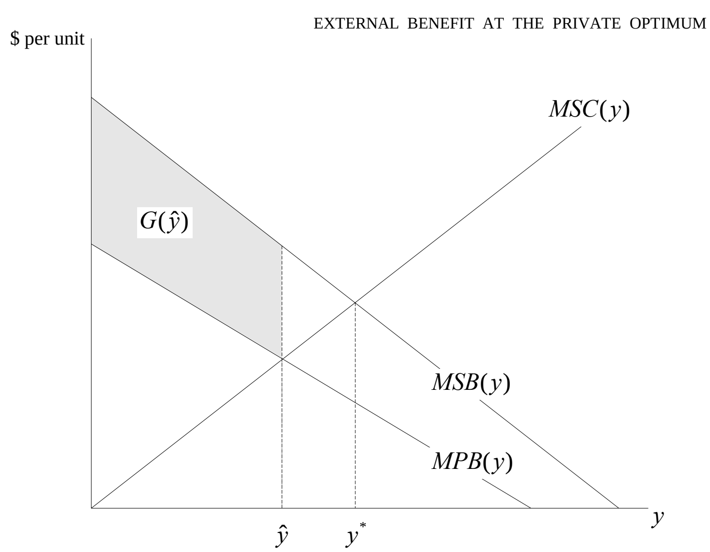

FIGURE 4-15 (repeat) 90

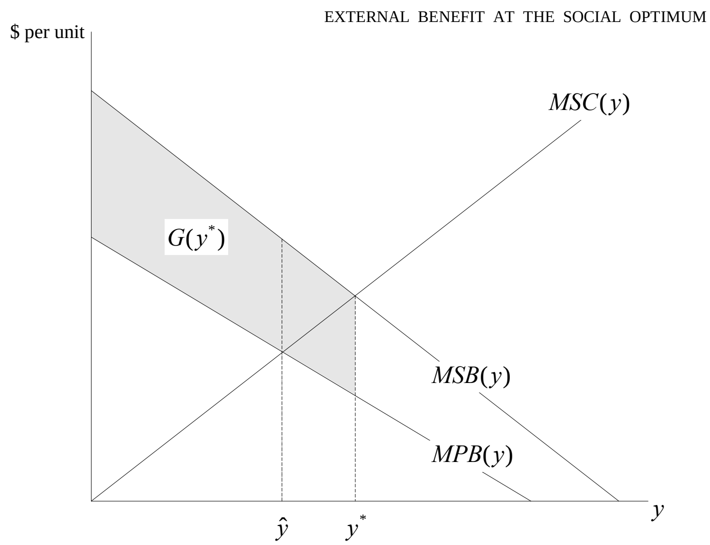

FIGURE 4-16 (repeat) 91

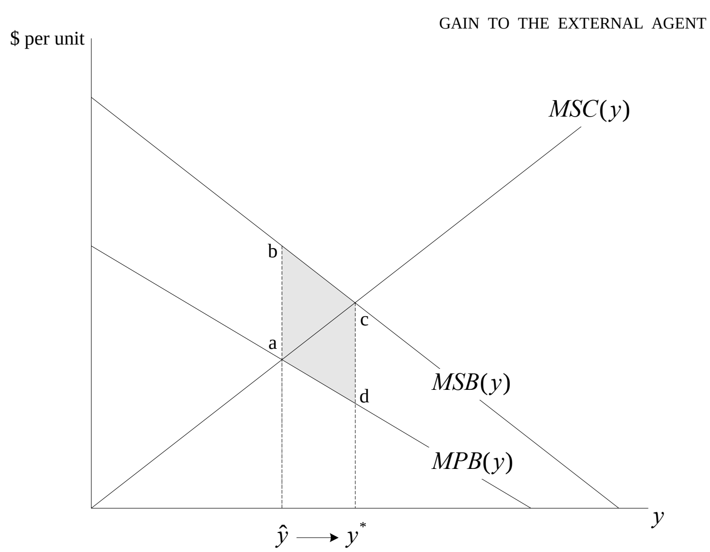

FIGURE 4-17 (repeat) 92

The Impact of a Regulated Increase in y

- Note that this gain to the external agent can also be written as

$$G(y^*) - G(\hat{y}) = \int_{\hat{y}}^{y^*} MEB(y)dy$$

- This definite integral is the area under $MEB(y)$ between $\hat{y}$ and y^*; see Figure 4-18.

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GAIN TO THE EXTERNAL AGENT

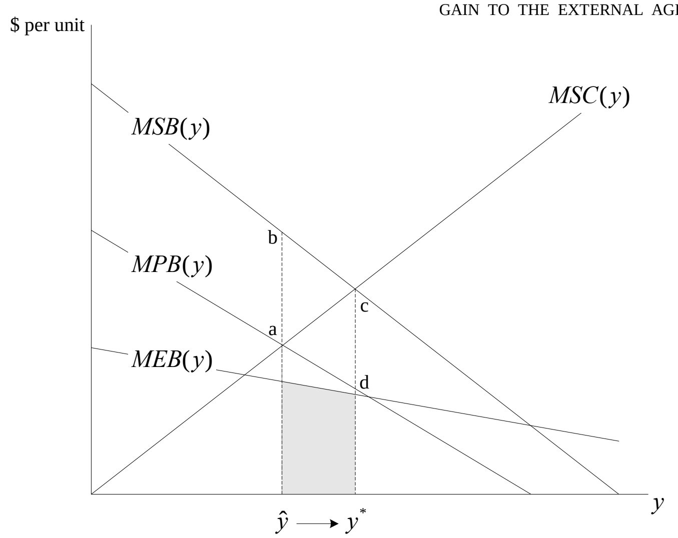

FIGURE 4-18

94

The Impact of a Regulated Increase in *y*

- The shaded areas in Figures 4-17 and 4-18 are necessarily equal; they are alternative graphical representations of the gain to the external agent.
- It is important to understand both representations.

95

The Impact of a Regulated Increase in *y*

- Next consider the <u>reduction</u> in net private benefit for the source agent.

96

The Impact of a Regulated Increase in *y*

- Recall that the private benefit to the source agent at the <u>private</u> optimum is the area under *MPB*(*y*) between zero and $\hat{y}$:

$$PB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy$$

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The Impact of a Regulated Increase in *y*

- In comparison, private benefit to the source agent at the <u>social</u> optimum is

$$PB(y^*) = \int_0^{y^*} MPB(y)dy$$

98

The Impact of a Regulated Increase in *y*

- Thus, the <u>increase</u> in private benefit to the source agent is

$$PB(y^*) - PB(\hat{y}) = \int_{\hat{y}}^{y^*} MPB(y)dy$$

- See Figure 4-19.

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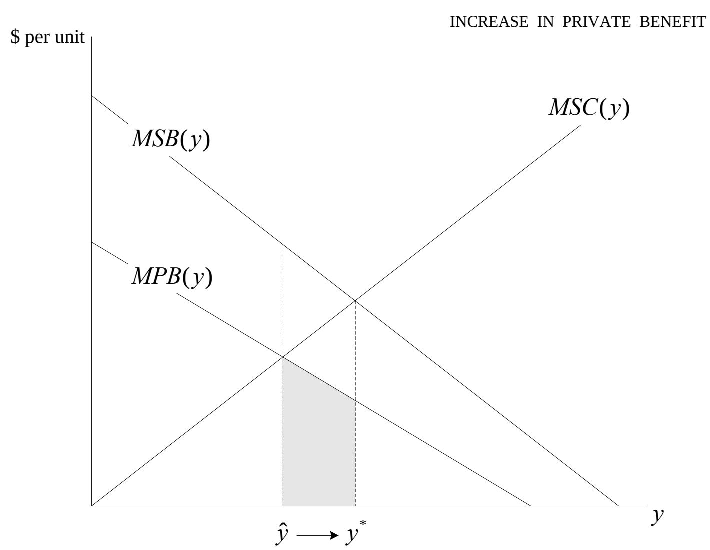

FIGURE 4-19

100

The Impact of a Regulated Increase in *y*

- By the same logic, the increase in private cost to the source agent is

$$PC(y^*) - PC(\hat{y}) = \int_{\hat{y}}^{y^*} MPC(y)dy$$

- See Figure 4-20.

101

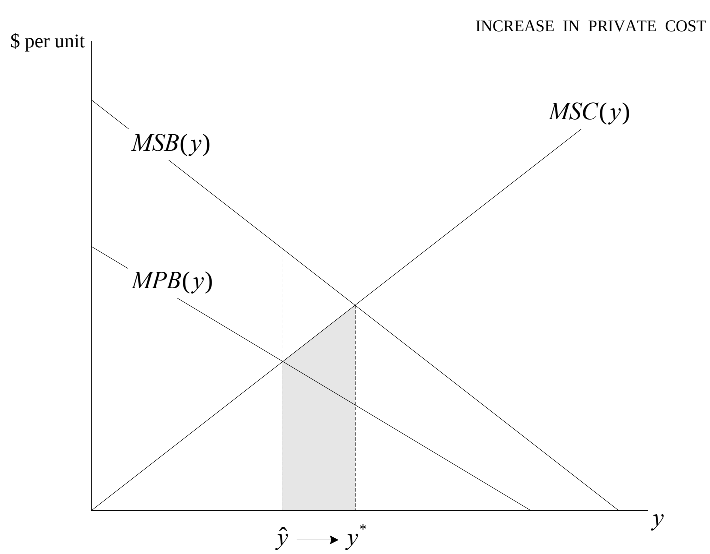

FIGURE 4-20

102

The Impact of a Regulated Increase in *y*

- It is clear from Figures 4-19 and 4-20 that the increase in private cost exceeds the increase in private benefit.
- Thus, the overall change in <u>net</u> private benefit for the source agent is negative.
- See Figure 4-21.

103

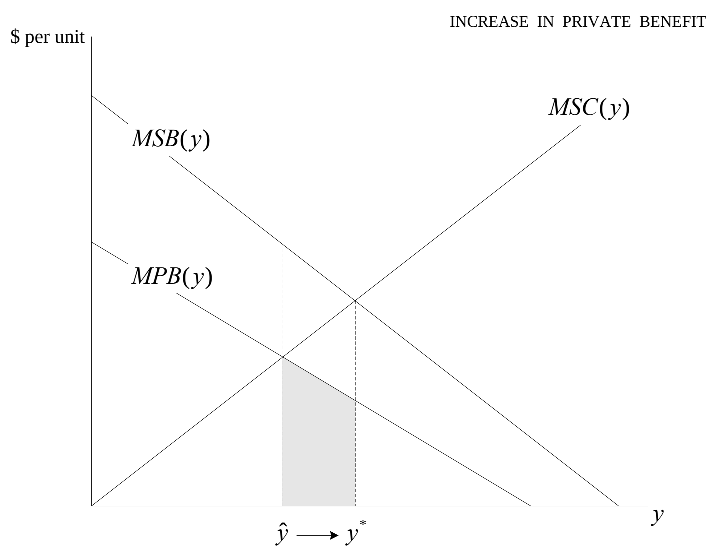

FIGURE 4-19 (repeat)

104

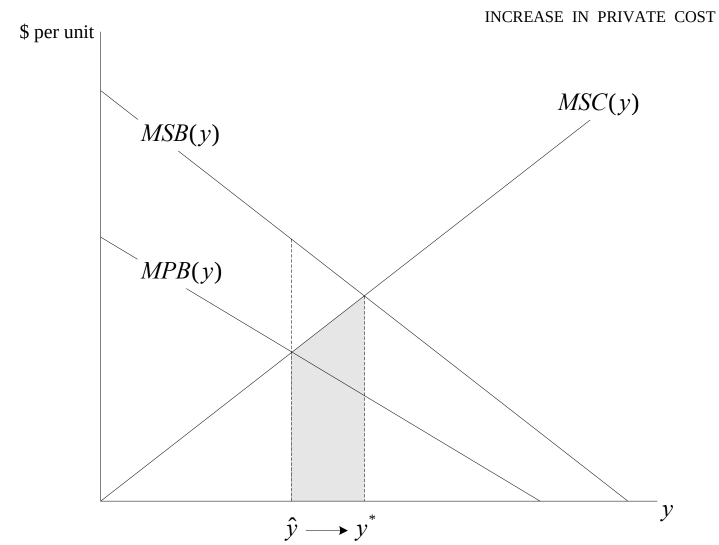

FIGURE 4-20 (repeat) 105

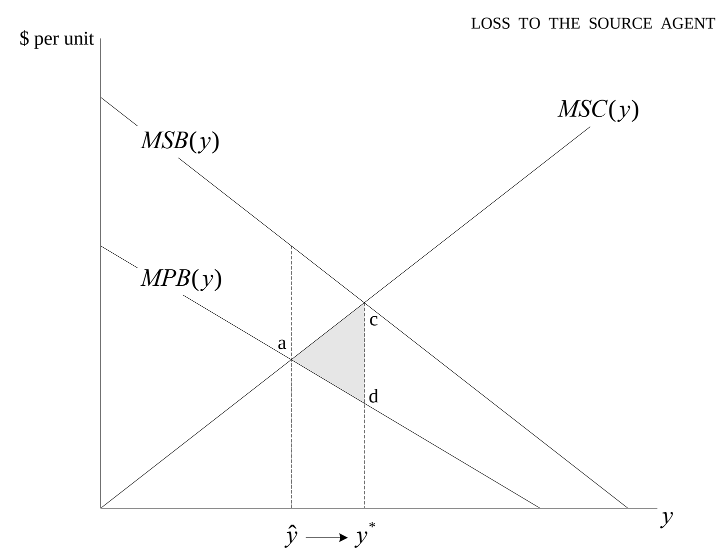

FIGURE 4-21 106

The Impact of a Regulated Increase in *y*

- The source agent is made worse-off because she is forced to move away from her private optimum, and there is no offsetting compensation.

107

The Impact of a Regulated Increase in *y*

- In summary, from Figures 4-17 and 4-21:
  - the gain to the external agent = *area*(*abcd*)
  - the loss to the source agent = *area*(*acd*)

- Thus, the overall **gain in social surplus** = *area*(*abc*)

- See Figure 4-22.

108

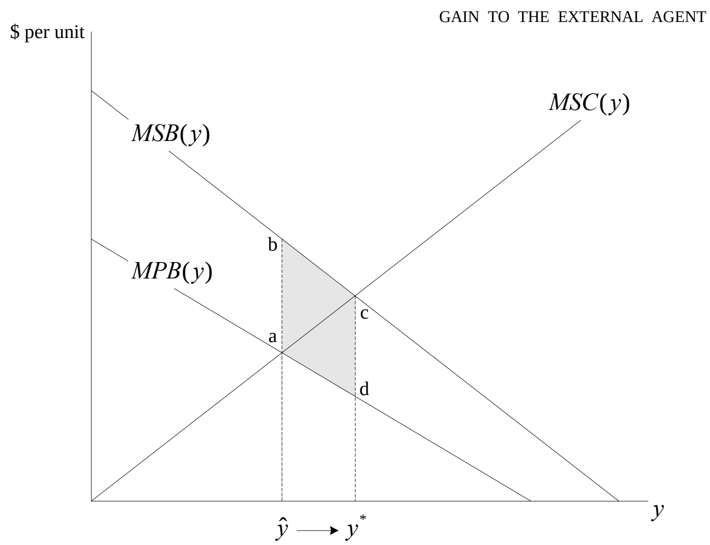

FIGURE 4-17 (repeat) 109

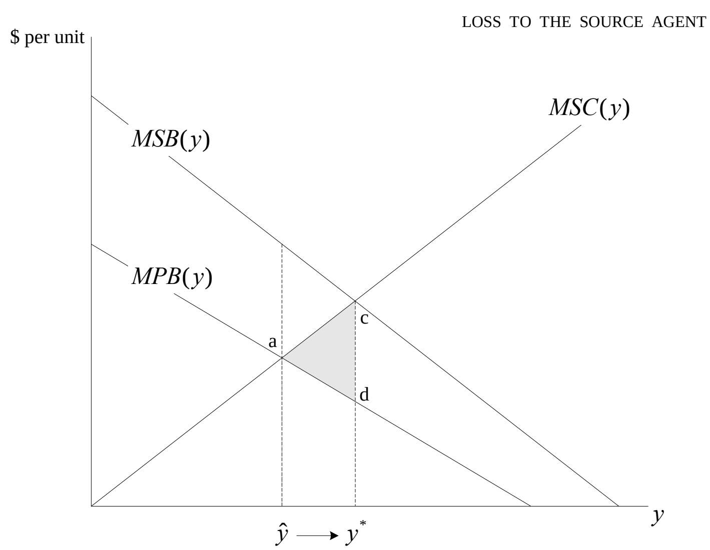

FIGURE 4-21 (repeat) 110

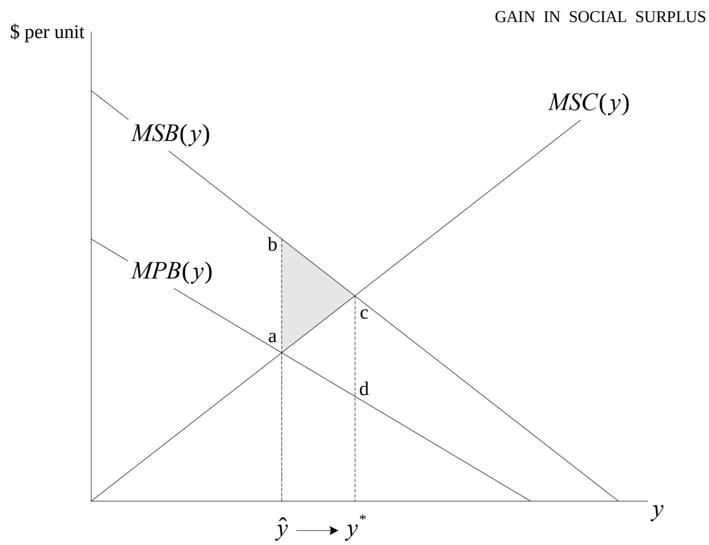

FIGURE 4-22

111

The Impact of a Regulated Increase in *y*

- What can we say about welfare overall?
- Recall the definition of a **Pareto improvement**:
  - a reallocation of resources that makes at least one person better-off and leaves no person worse-off.

112

The Impact of a Regulated Increase in y

- The forced move from $\hat{y}$ to y^* is <u>not</u> a Pareto improvement; the source agent is made worse-off.

113

The Impact of a Regulated Increase in y

- In contrast, recall the definition of a **potential Pareto improvement**:
  - a reallocation of resources under which the winners could *in principle* fully compensate the losers and still be better-off

114

The Impact of a Regulated Increase in y

- The forced move from $\hat{y}$ to y^* is a potential Pareto improvement:
  - the winner (the external agent) could in principle fully compensate the loser (the source agent) and still be better-off, by *area*(*abc*)

115

4.6 A NEGATIVE EXTERNALITY

A Negative Externality

- Now consider a setting where the activity has an external cost but no external benefit: $D(y)>0$ but $G(y)=0$.

- For example, if y is output from a factory then $D(y)$ might be the damage associated with the pollution produced as a by-product.

117

A Negative Externality

- Since there is no external benefit,

$$SB(y) = PB(y)$$

and it follows that

$$MSB(y) = MPB(y)$$

118

A Negative Externality

- Conversely, social cost is

$$SC(y) = PC(y) + D(y)$$

119

A Negative Externality

- Accordingly, we can decompose the rate of change of $SC(y)$ into two components:

$$MSC(y) = MPC(y) + MEC(y)$$

where $MEC(y)$ is the **marginal external cost** of y.

120

A Negative Externality

- Graphically, *MEC*(*y*) is the vertical distance between *MSC*(*y*) and *MPC*(*y*).
- See Figure 4-23.

121

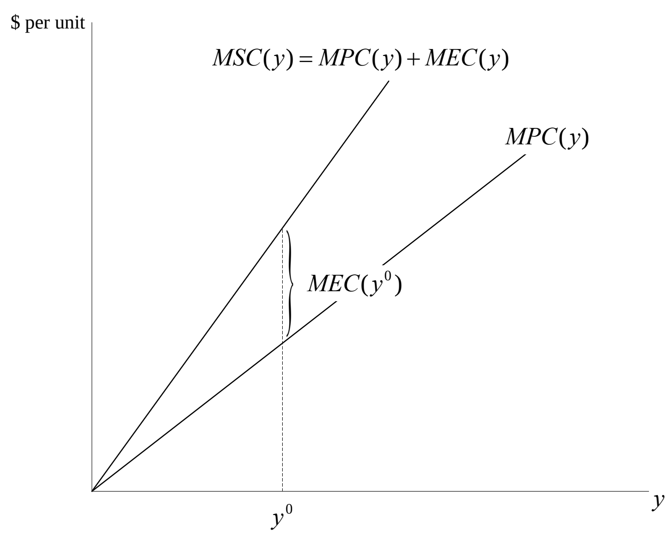

FIGURE 4-23

122

A Negative Externality

- Recall from s.49 our assumption that $D(y)$ is increasing at an increasing rate.

- This is reflected in Figure 4-23: $MEC(y)$ rises as y rises; the gap between $MSC(y)$ and $MPC(y)$ becomes larger.

123

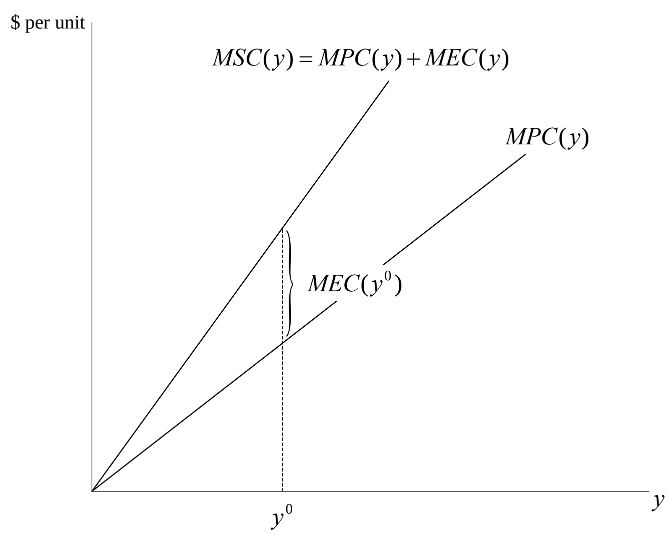

FIGURE 4-23 (repeat)

124

A Negative Externality

- We will later find it useful to depict *MEC*(y) as a separate graph.
- It is constructed by graphing the vertical distance between *MSC*(y) and *MPC*(y) at every value of y.
- See Figure 4-24.

125

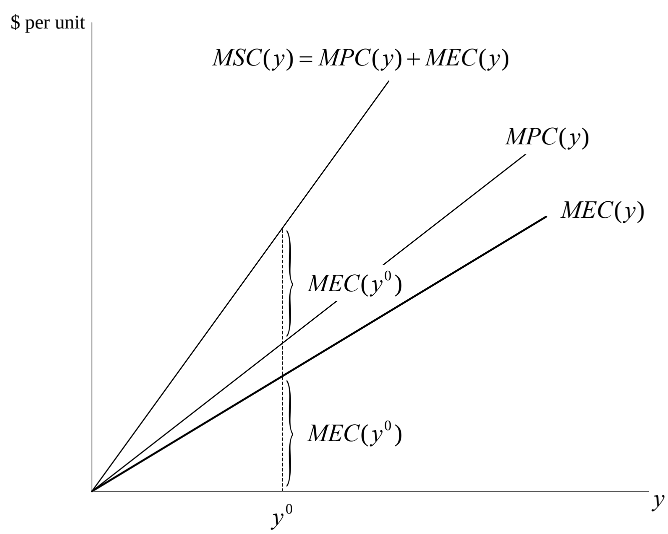

FIGURE 4-24

126

A Negative Externality

- Now let us now consider the impact of this external cost on the relationship between the private and social optima.

- See Figure 4-25.

127

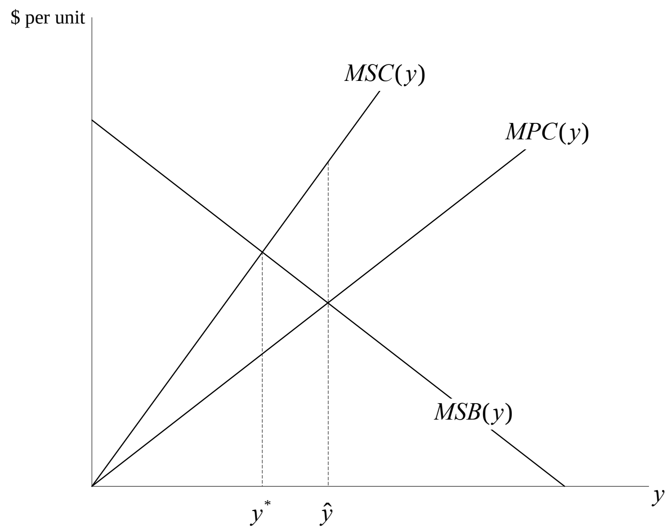

FIGURE 4-25

128

A Negative Externality

- The presence of the external cost means:

$$\hat{y} > y^*$$

  - *ie*. the privately optimal level of activity is greater than the socially optimal level.

129

A Negative Externality

- Intuition:
  - the source agent does not take into account the cost she imposes on the external agent when she chooses her action, and so her chosen level of the action is too high from a social perspective.

130

A Negative Externality:
A Numerical Example

- Suppose

$$MPB(y) = 30 - y$$

$$MPC(y) = \frac{y}{2}$$

$$MEC(y) = y$$

131

A Negative Externality: A Numerical Example

- First derive the private optimum, given by $\hat{y}$ such that

$$30 - \hat{y} = \frac{\hat{y}}{2}$$

which solves for

$$\hat{y} = 20$$

132

A Negative Externality: A Numerical Example

- Next consider the social optimum.
- *MSC*(*y*) is the sum of *MPC*(*y*) and *MEC*(*y*):

$$MSC(y) = \frac{y}{2} + y = \frac{3y}{2}$$

- Since there is no external benefit here, *MSB*(*y*) is simply equal to *MPB*(*y*).

133

A Negative Externality: A Numerical Example

- Thus, the social optimum is y^* such that

$$30 - y^* = \frac{3y^*}{2}$$

which solves for

$$y^* = 12$$

- See Figure 4-26.

134

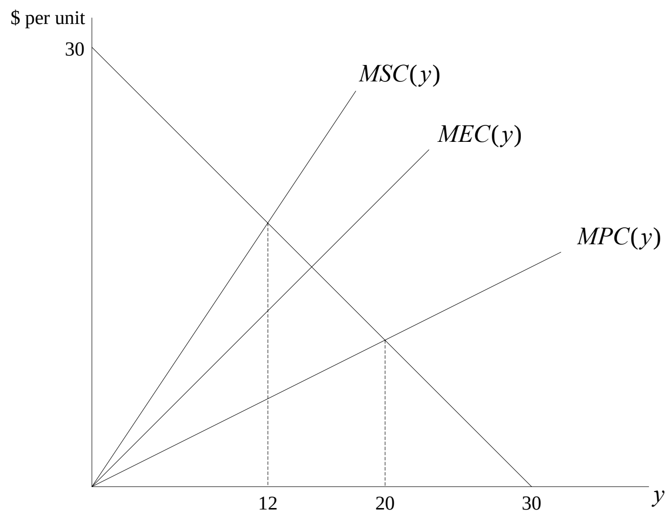

FIGURE 4-26

135

The Impact of a Regulated Reduction in *y*

- Now suppose a third party (such as a government regulator) could <u>force</u> the source agent to reduce her activity level from $\hat{y}$ to y^*.

136

The Impact of a Regulated Reduction in y

- Using the same methodology we used in Section 4.4, we will show that this forced increase yields a potential Pareto improvement:
  - the source agent loses but the external agent gains by more than enough to compensate for that loss.

137

The Impact of a Regulated Reduction in y

- Consider first the gain to the external agent (the reduction in external cost).
- External cost at the private optimum is

$$D(\hat{y}) = \int_0^{\hat{y}} MEC(y)dy$$

138

The Impact of a Regulated Reduction in *y*

- Since

$$MEC(y) = MSC(y) - MPC(y)$$

we can find the area under *MEC*(*y*) as the difference between the area under *MSC*(*y*) and the area under *MPC*(*y*).

139

The Impact of a Regulated Reduction in *y*

- Thus,

$$D(\hat{y}) = \int_0^{\hat{y}} MEC(y)dy$$

$$= \int_0^{\hat{y}} MSC(y)dy - \int_0^{\hat{y}} MPC(y)dy$$

- See Figure 4-27.

140

EXTERNAL COST AT THE PRIVATE OPTIMUM

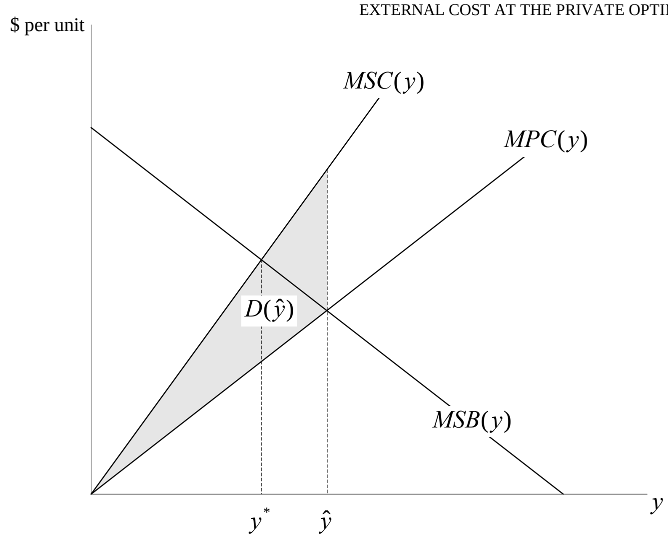

FIGURE 4-27

141

The Impact of a Regulated Reduction in *y*

- External cost at the social optimum is

$$D(y^*) = \int_0^{y^*} MEC(y)dy$$

$$= \int_0^{y^*} MSC(y)dy - \int_0^{y^*} MPC(y)dy$$

- See Figure 4-28.

142

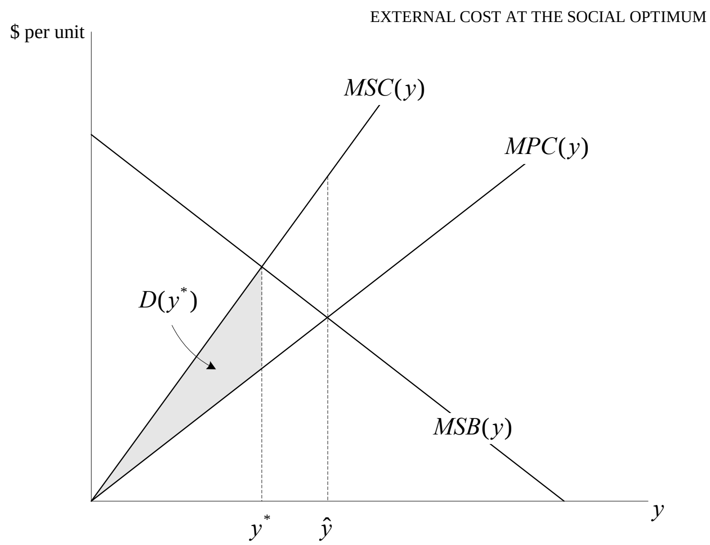

FIGURE 4-28

143

The Impact of a Regulated Reduction in *y*

- Hence, the reduction in external cost is

$$D(\hat{y}) - D(y^*) = \int_0^{\hat{y}} MEC(y)dy - \int_0^{y^*} MEC(y)dy$$

- This is the **gain to the external agent**; it is area *area*(*abcd*) Figure 4-29 (the difference in areas in Figures 4-27 and 4-28).

144

GAIN TO THE EXTERNAL AGENT

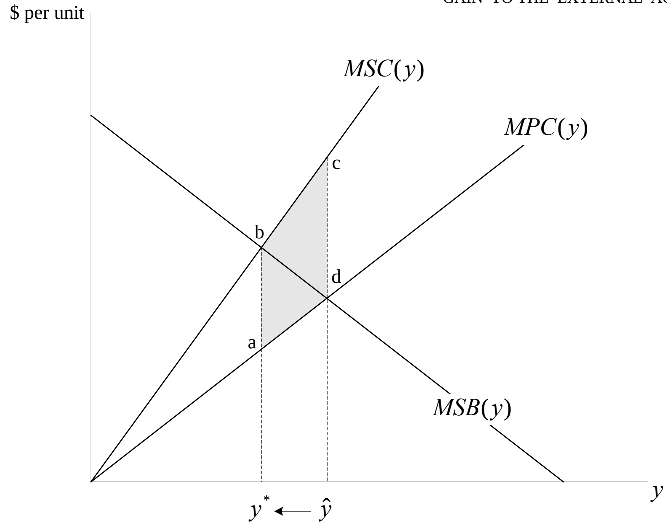

FIGURE 4-29

145

EXTERNAL COST AT THE PRIVATE OPTIMUM

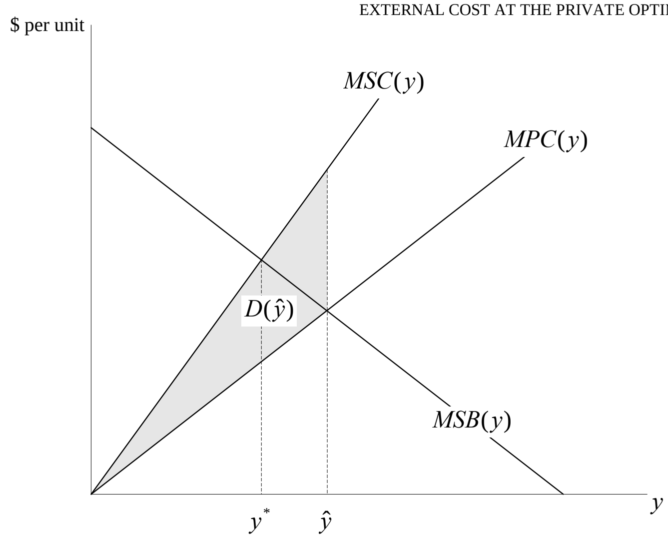

FIGURE 4-27 (repeat)

146

EXTERNAL COST AT THE SOCIAL OPTIMUM

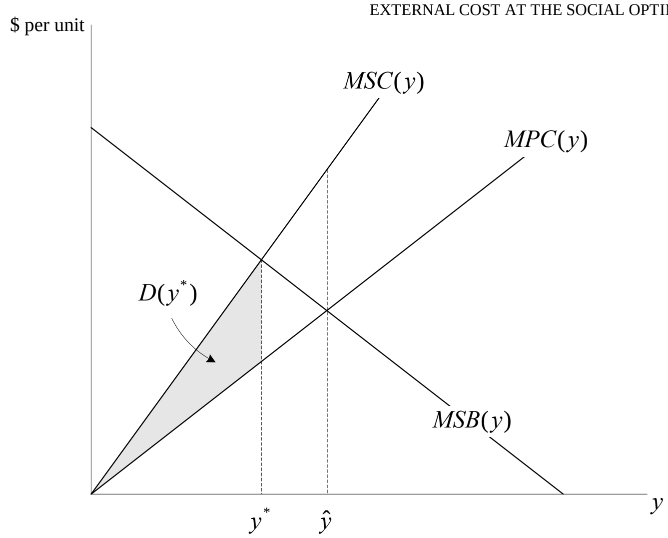

FIGURE 4-28 (repeat) 147

GAIN TO THE EXTERNAL AGENT

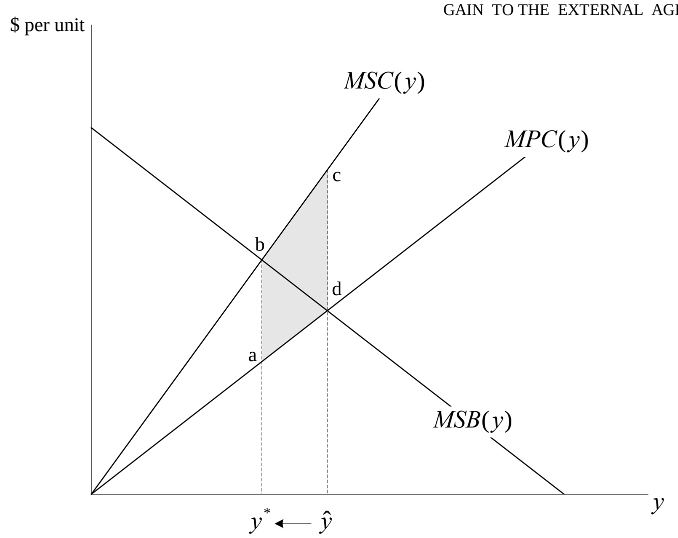

FIGURE 4-29 (repeat) 148

The Impact of a Regulated Reduction in *y*

- Note that this reduction in external cost can also be written as

$$D(\hat{y}) - D(y^*) = \int_{y^*}^{\hat{y}} MEC(y)dy$$

- This definite integral is the area under $MEC(y)$ between y^* and $\hat{y}$; see Figure 4-30.

149

GAIN TO THE EXTERNAL AGENT

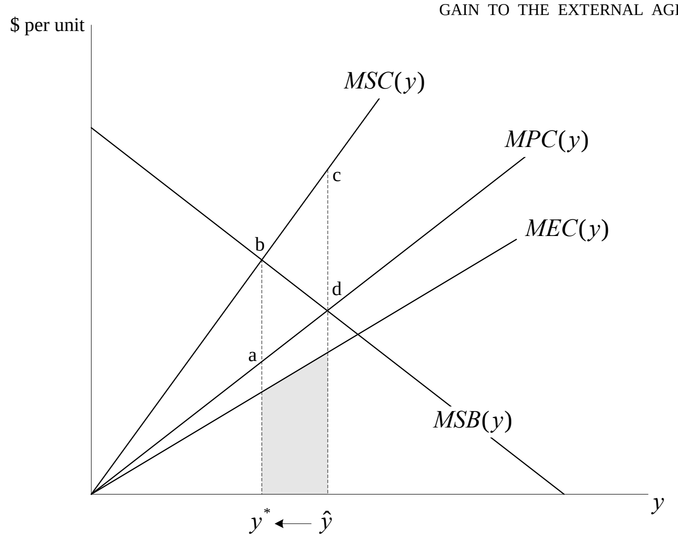

FIGURE 4-30

150

The Impact of a Regulated Reduction in *y*

- Note that the shaded areas in Figures 4-29 and 4-30 are necessarily equal; they are alternative graphical representations of the gain to the external agent.

151

The Impact of a Regulated Reduction in *y*

- Next consider the <u>reduction</u> in net private benefit for the source agent.
- Recall that private benefit to the source agent at the private optimum is

$$PB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy$$

152

The Impact of a Regulated Reduction in *y*

- In comparison, private benefit to the source agent at the social optimum is

$$PB(y^*) = \int_0^{y^*} MPB(y)dy$$

153

The Impact of a Regulated Reduction in *y*

- Thus, the <u>reduction</u> in private benefit to the source agent is

$$PB(\hat{y}) - PB(y^*) = \int_{y^*}^{\hat{y}} MPB(y)dy$$

- See Figure 4-31.

154

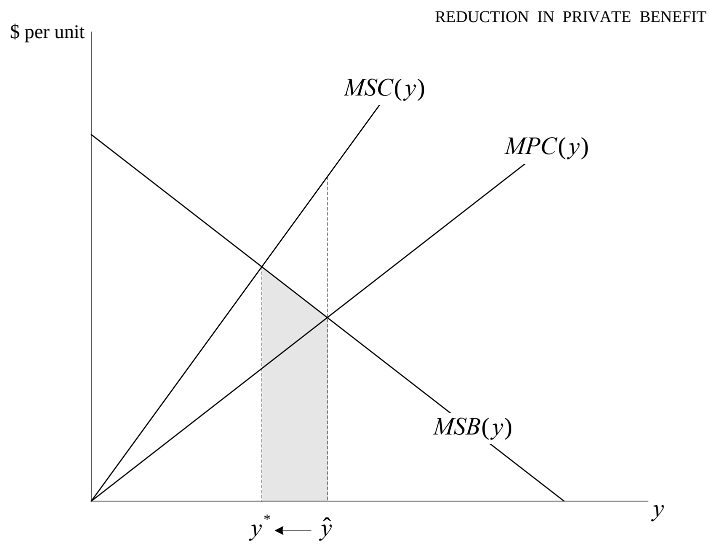

FIGURE 4-31

155

The Impact of a Regulated Reduction in *y*

- By the same logic, the reduction in private cost to the source agent is

$$PC(\hat{y}) - PC(y^*) = \int_{y^*}^{\hat{y}} MPC(y)dy$$

- See Figure 4-32.

156

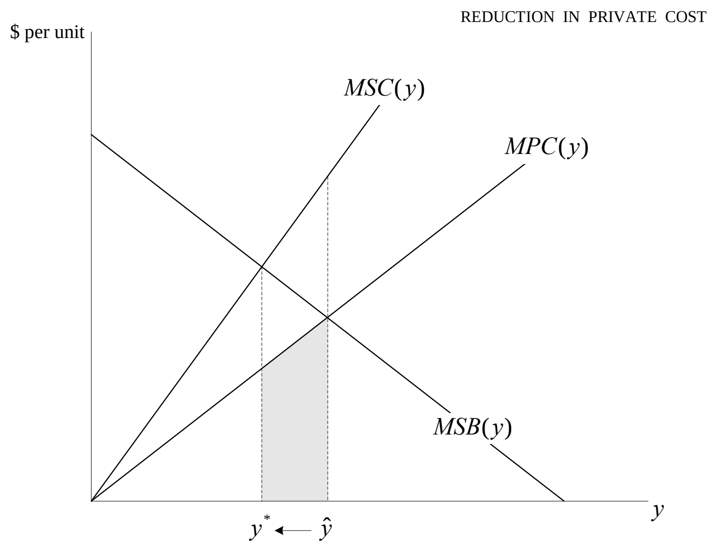

FIGURE 4-32

157

The Impact of a Regulated Reduction in *y*

- It is clear from Figures 4-31 and 4-32 that the reduction in private benefit exceeds the reduction in private cost.
- Thus, the overall change in <u>net</u> private benefit for the source agent is negative.
- See Figure 4-33.

158

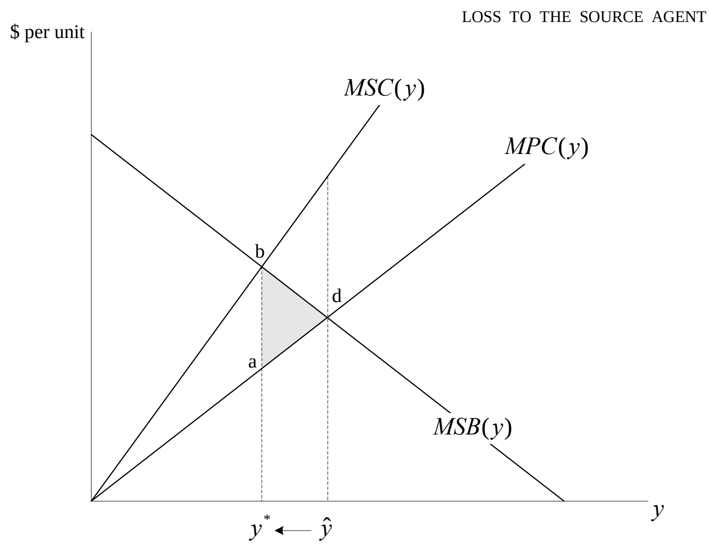

FIGURE 4-33 159

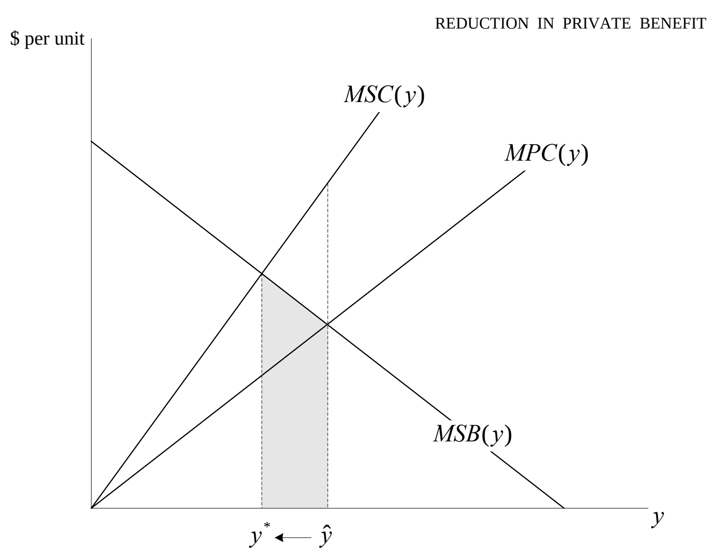

FIGURE 4-31 (repeat) 160

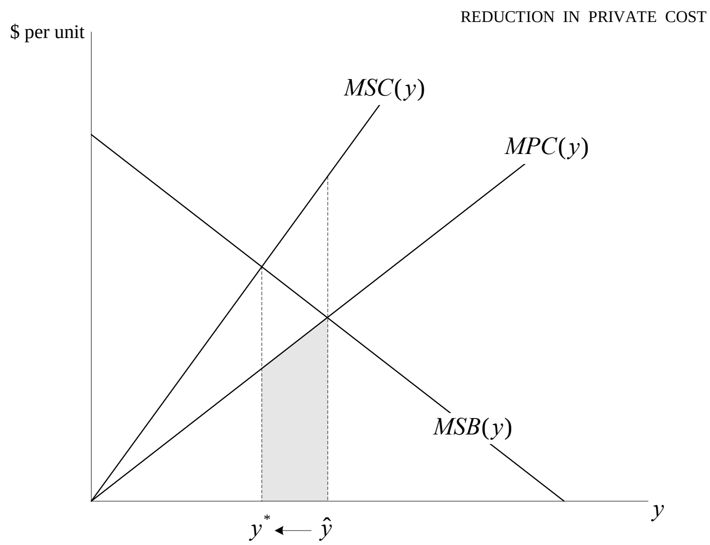

FIGURE 4-32 (repeat) 161

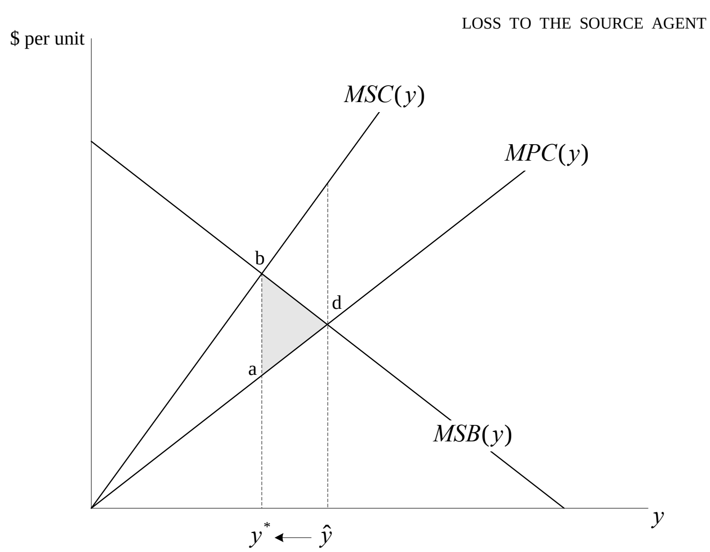

FIGURE 4-33 (repeat) 162

The Impact of a Regulated Reduction in *y*

- The source agent is made worse-off because she is forced to move away from her private optimum, and there is no offsetting compensation.

163

The Impact of a Regulated Reduction in *y*

- In summary, from Figures 4-29 and 4-33:
  - the gain to the external agent = *area*(*abcd*)
  - the loss to the source agent = *area*(*abd*)
- Thus, the overall **gain in social surplus** = *area*(*bcd*)
- See Figure 4-34.

164

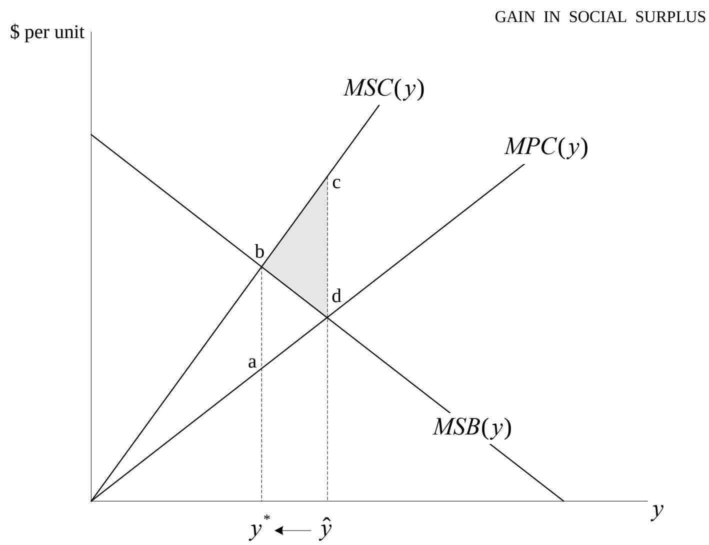

FIGURE 4-34 165

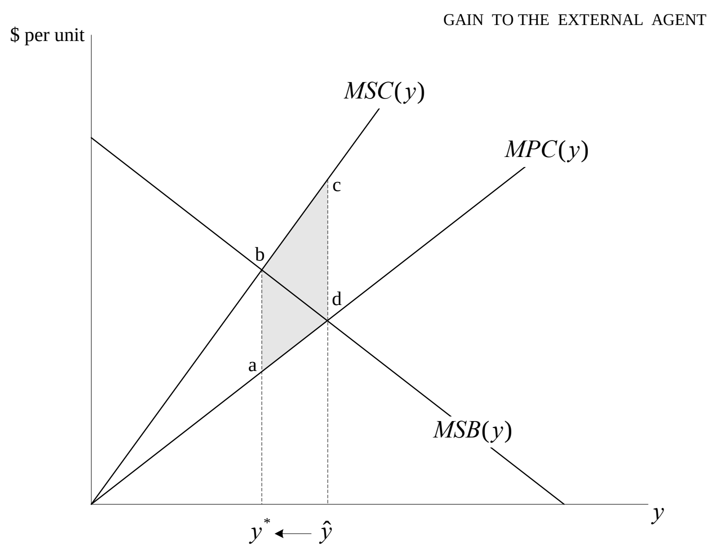

FIGURE 4-29 (repeat) 166

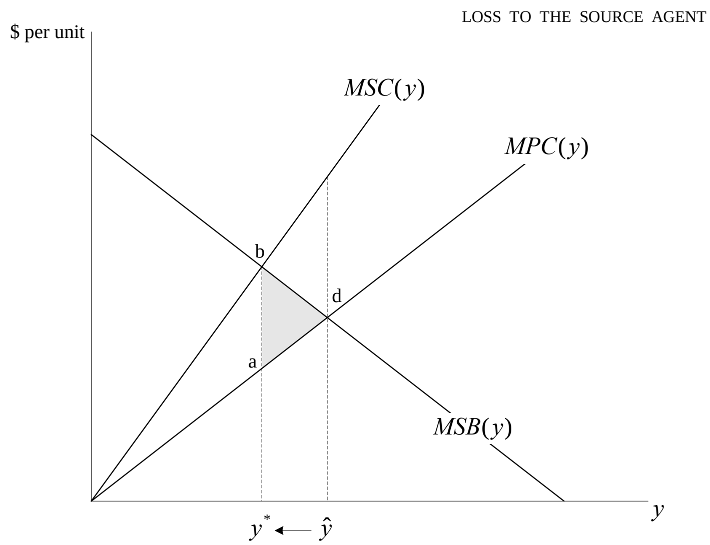

FIGURE 4-33 (repeat) 167

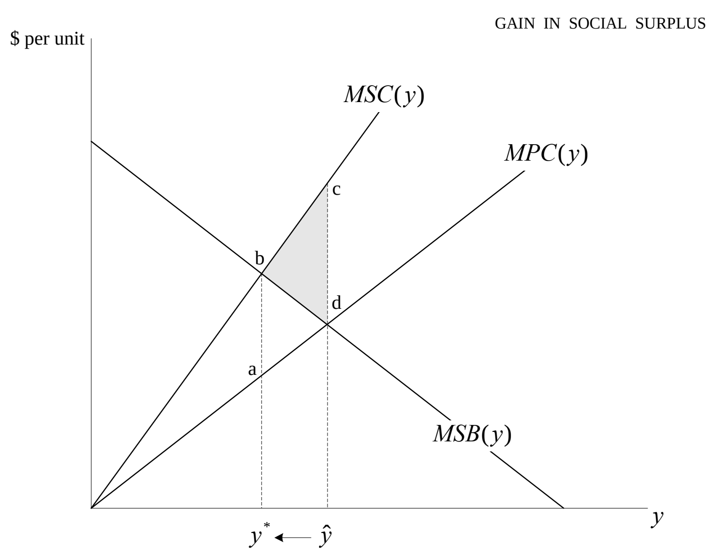

FIGURE 4-34 (repeat) 168

The Impact of a Regulated Reduction in *y*

- What can we say about welfare overall?

169

The Impact of a Regulated Reduction in *y*

- The forced move from $\hat{y}$ to y^* is <u>not</u> a Pareto improvement; the source agent is made worse-off.
- However, it <u>is</u> a potential Pareto improvement:
  - the winner (the external agent) could in principle fully compensate the loser (the source agent) and still be better-off, by *area*(*bcd*)

170

4.7 AN ALTERNATIVE PRESENTATION OF A NEGATIVE EXTERNALITY

A Negative Externality: An Alternative Presentation

- Recall from Section 4.1 that the private optimum can be characterized by

$$MNPB(\hat{y}) = 0$$

where

$$MNPB(y) \equiv MPB(y) - MPC(y)$$

172

A Negative Externality: An Alternative Presentation

- We can use the same approach to characterize the social optimum in the presence of a negative externality.
- In particular, recall that the social optimum is y^* such that

$$MPB(y^*) = MPC(y^*) + MEC(y^*)$$

173

A Negative Externality: An Alternative Presentation

- Subtract $MPC(y^*)$ from both sides to yield

$$MPB(y^*) - MPC(y^*) = MEC(y^*)$$

- The LHS of this equation is $MNPB(y)$ evaluated at the social optimum.

174

A Negative Externality: An Alternative Presentation

- Thus, at the social optimum

$$MNPB(y^*) = MEC(y^*)$$

- In comparison, at the private optimum

$$MNPB(\hat{y}) = 0$$

- See Figure 4-35.

175

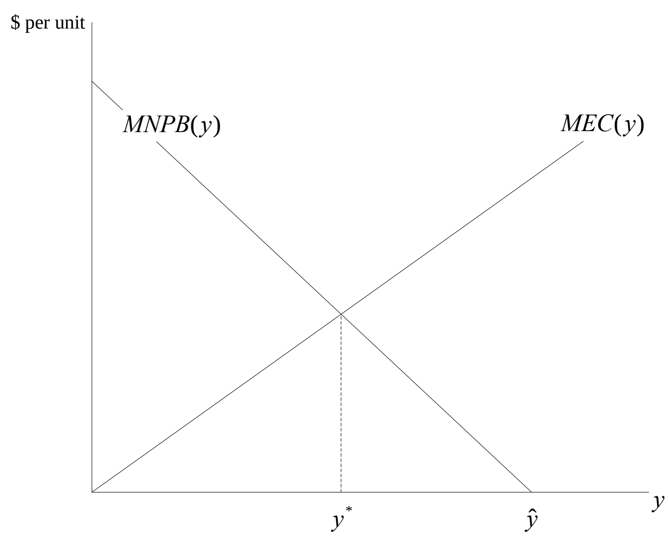

FIGURE 4-35

176

A Negative Externality: An Alternative Presentation

- Now consider a forced move from $\hat{y}$ to y^*.
- The loss to the source agent is

$$\int_{y^*}^{\hat{y}} MNPB(y)dy = area(abd)$$

in Figure 4-36.

177

LOSS TO THE SOURCE AGENT

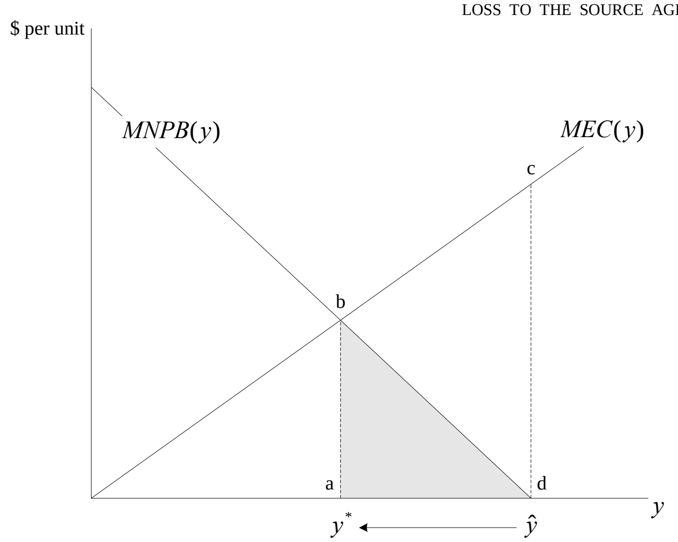

FIGURE 4-36

178

A Negative Externality: An Alternative Presentation

- The gain to the external agent is

$$\int_{y^*}^{\hat{y}} MEC(y)dy = area(abcd)$$

in Figure 4-37.

179

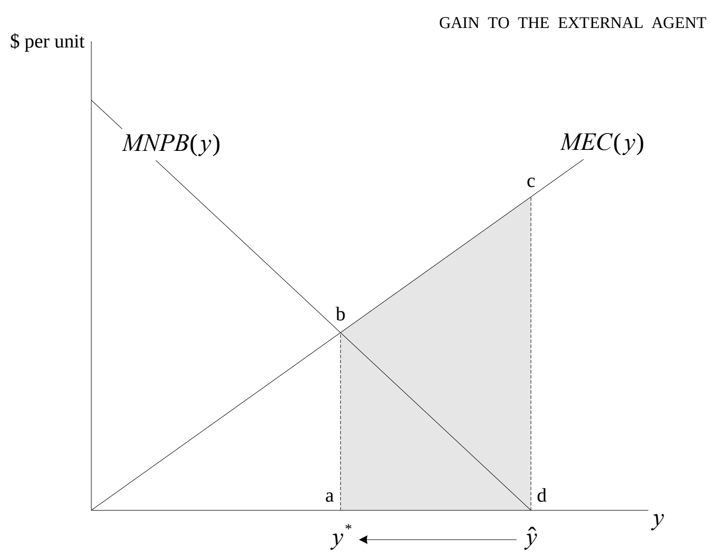

FIGURE 4-37

180

A Negative Externality: An Alternative Presentation

- In summary, from Figures 4-36 and 4-37:
  - – the gain to the external agent = *area*(*abcd*)
  - – the loss to the source agent = *area*(*abd*)
- Thus, the overall **gain in social surplus** = *area*(*bcd*)
- See Figure 4-38.

181

GAIN IN SOCIAL SURPLUS

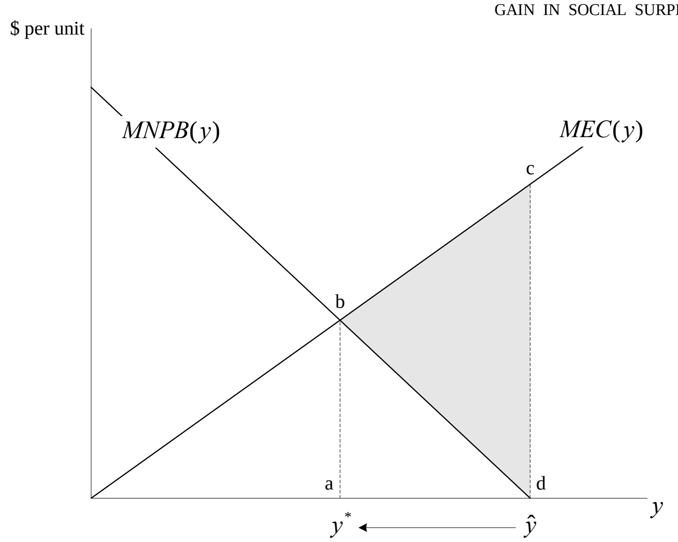

FIGURE 4-38

182

LOSS TO THE SOURCE AGENT

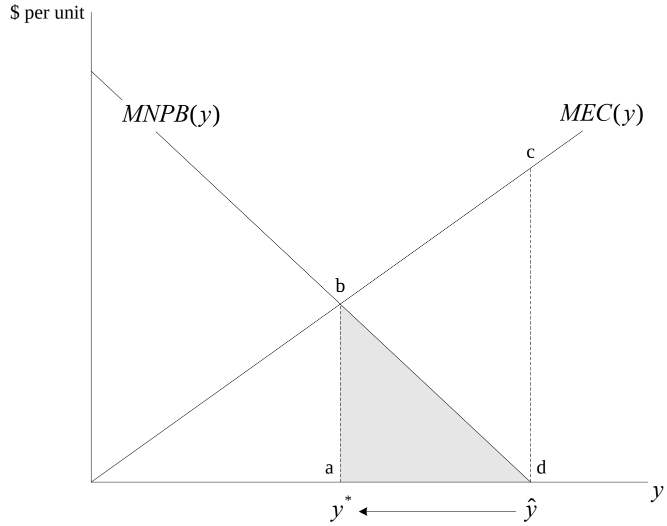

FIGURE 4-36 (repeat) 183

GAIN TO THE EXTERNAL AGENT

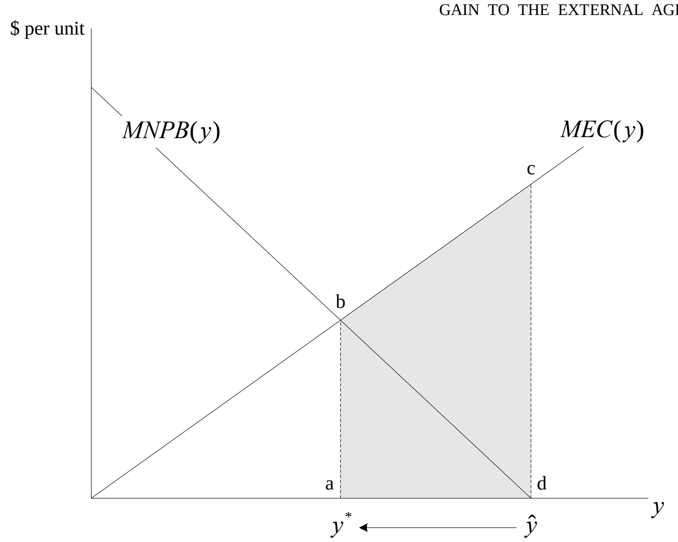

FIGURE 4-37 (repeat) 184

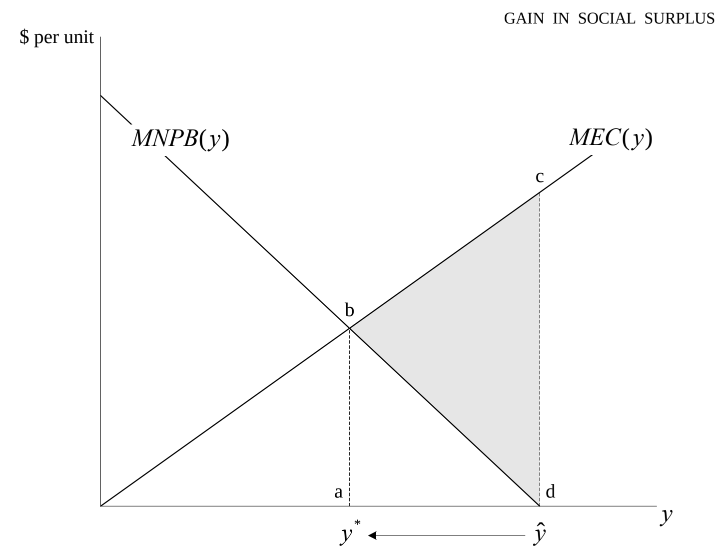

FIGURE 4-38 (repeat)

185

A Negative Externality: An Alternative Presentation

- If all our figures were drawn to the same scale, the shaded area in Figure 4-38 must be equal to the shaded area in Figure 4-34; they are alternative graphical representations of the gain in social surplus when y is reduced from $\hat{y}$ to y^*.

186

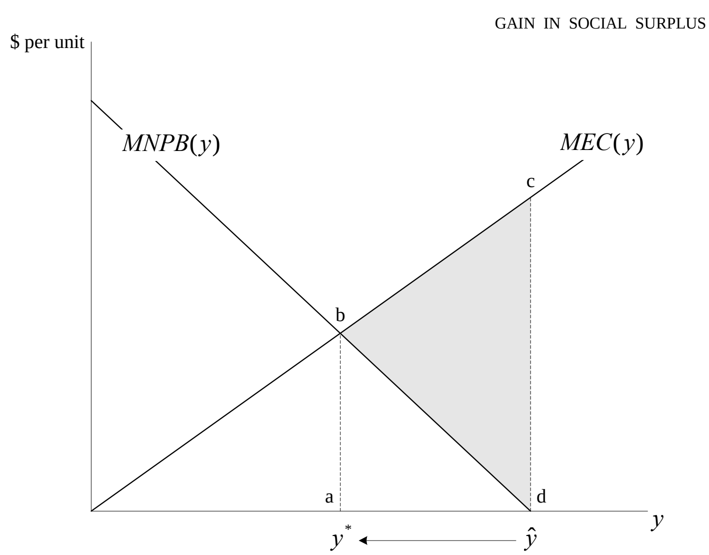

FIGURE 4-38 (repeat) 187

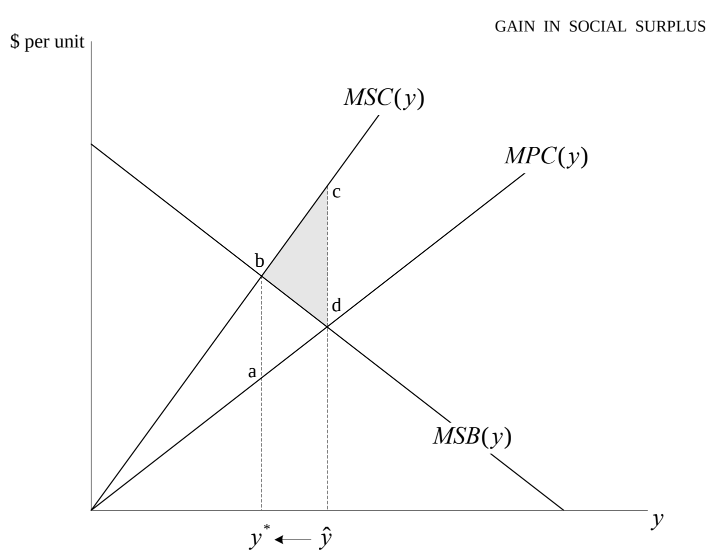

FIGURE 4-34 (repeat) 188

4.8 MULTIPLE EXTERNAL AGENTS

Multiple External Agents

- We have so far restricted attention to a scenario where there is only one external agent.
- Our framework extends easily to a setting with multiple external agents.

190

Multiple External Agents

- Let $D_i(y)$ denote the external cost to external agent i, and suppose there are a total of n external agents affected by the activity.
- In addition, suppose that the activity is a **pure public bad** for the external agents.

191

Multiple External Agents

- This means that the impact of the activity on any one external agent does not depend on how other external agents are affected.
- It does not mean that all external agents suffer the same cost.

192

Multiple External Agents

- For example, oil discharged into a drinking-water source (such as a lake) has an impact on all users of the water.
- The effect of the oil is not eliminated when one user drinks polluted water; the remaining water is still polluted.
- The impact on each user might nonetheless be different, depending on their usage.

193

Multiple External Agents

- Of course, one can envisage settings where the impact on any one external agent <u>does</u> depend on how many agents are affected, as when the physical impact is partly "diluted" when spread across many people.
- In this case, the activity is an **impure** public bad.
- We will not consider such cases here.

194

Multiple External Agents

- If the activity is a pure public bad then the **aggregate external cost** of the activity is

$$D(y) = \sum_{i=1}^{n} D_i(y)$$

195

Multiple External Agents

- The same logic applies to the construction of **aggregate marginal external cost**:

$$MEC(y) = \sum_{i=1}^{n} MEC_i(y)$$

196

Multiple External Agents

- In graphical terms, we take the <u>vertical</u> summation of all the individual MEC schedules to obtain the aggregate MEC schedule.

- See Figure 4-39 for the case of two external agents.

197

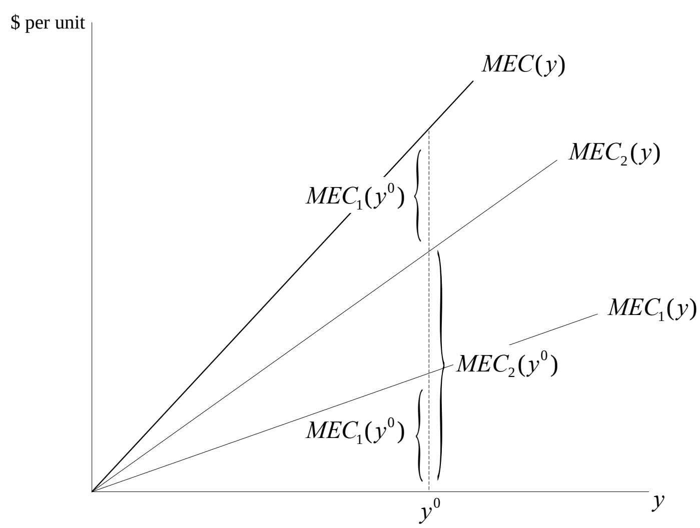

FIGURE 4-39

198

Multiple External Agents

- The aggregate external cost at some level of the activity y^0 is simply the sum of the areas under the two individual MEC schedules.
- See Figures 4-40 through 4-42.

199

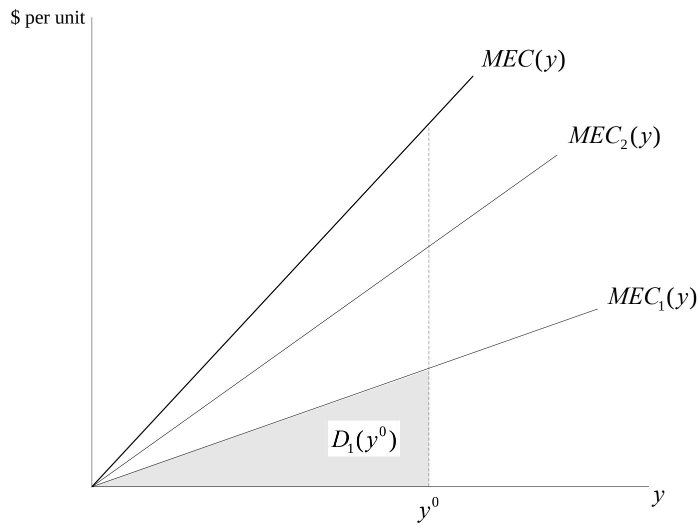

FIGURE 4-40

200

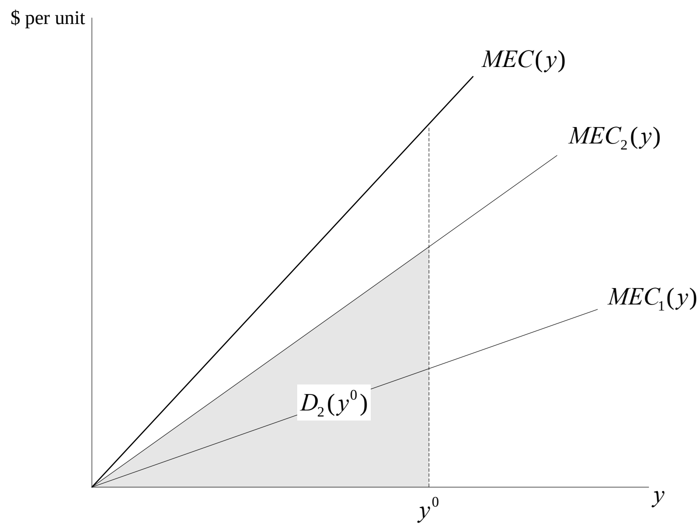

FIGURE 4-41 201

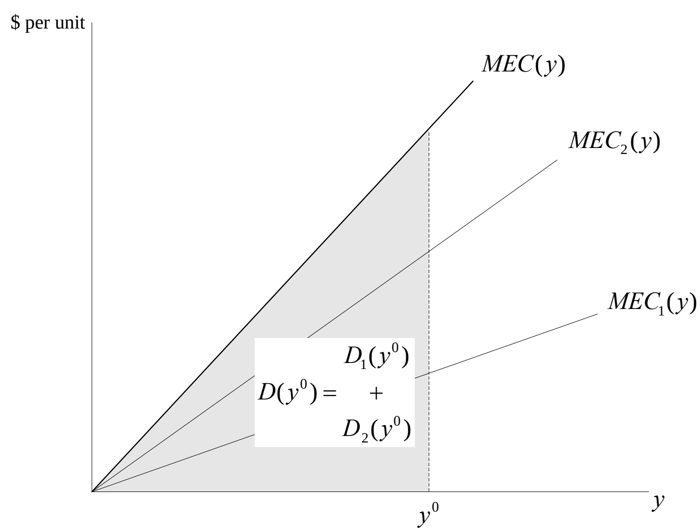

FIGURE 4-42 202

Multiple External Agents

- Consider an example. Suppose

$$MEC_1(y) = 10 + 3y$$

$$MEC_2(y) = 5y$$

- Then

$$MEC(y) = 10 + 8y$$

203

Multiple External Agents

- In a setting with multiple external agents, the social optimum is based on the <u>aggregate</u> marginal external cost:

$$MNPB(y^*) = MEC(y^*)$$

204

Multiple External Agents

- Thus, all of our previous analysis applies equally well to the multiple-agent setting, where MEC is now assumed to mean the aggregate marginal external cost, regardless of how many external agents there are.

205

4.9 WHERE IS THE MARKET FAILURE?

Where is the Market Failure?

- Recall that a shift from the private optimum to the social optimum is a PPI.
- In the case of a negative externality, this requires a reduction in the activity concerned.
- Why is this PPI not realized via a contract among the parties involved?

207

Where is the Market Failure?

- In particular, why don't the source agent and the external agents write a contract under which the source agent reduces her activity to y^* voluntarily, in exchange for a payment from the external agents?

208

Where is the Market Failure?

- There are two reasons in practice:
  - an absence of explicit property rights
  - transaction costs
- Let us consider each in turn.

209

Property Rights

- Trade requires that property rights be defined over the traded good.
- In the case of many externalities, property rights are not well-defined.
- For example, suppose a firm discharges pollution into a lake, and farmers draw water from that lake for irrigation.

210

Property Rights

- Who has property rights over the lake?
- There are two possible extremes:
  - the firm has an unlimited right to use the lake for disposal of its waste
  - farmers have an unlimited right to have access to unpolluted water

211

Property Rights

- In our analysis so far, we have assumed that the firm has an implicit unlimited right to pollute:
  - it is polluting at its private optimum without any requirement to consider the external costs
- The right is implicit in the sense that it reflects existing practice (even though the right may not be written down in law).

212

Property Rights

- Suppose we now make this right explicit, and allow the firm to trade some of those rights to the farmers if it so chooses.

213

Property Rights

- The starting point for negotiations with farmers is $\hat{y}$, the private optimum for the firm.
- Any contract between the farmers and the firm would require that farmers pay the firm to reduce its pollution below $\hat{y}$.
- We will henceforth refer to this reduction in pollution as **abatement**.

214

Property Rights

- What would the contract look like?
- Let us frame the contract negotiations in terms of a **price** per unit of abatement.
- In particular, suppose the contract specifies that the farmers must collectively pay the firm a price *p* for each unit of pollution reduced.

215

Property Rights

- Faced with a price *p*, how many units of abatement will the farmers "buy" from the firm?
- We can describe the decision for the farmers in terms of their marginal costs and marginal benefits.

216

Property Rights

- The marginal cost of purchasing abatement is simply the price that must be paid to the firm for that abatement, p.
- The marginal benefit of abatement is the marginal external cost avoided.

217

Property Rights

- Thus, the farmers will buy abatement from the firm up to the point where

$$MEC(y) = p$$

- See Figure 4-43.

218

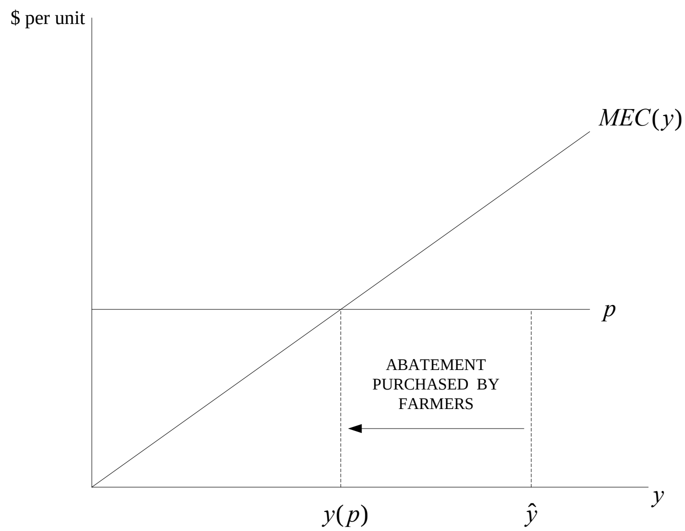

FIGURE 4-43

219

Property Rights

- How many units of abatement is the firm willing to "sell" at price *p*?
- Again, we can frame this decision in terms of marginal costs and marginal benefits.
- The marginal benefit of selling abatement is simply the price received, *p*.

220

Property Rights

- The marginal cost of selling abatement is the marginal net private benefit foregone when the firm cuts pollution.

221

Property Rights

- Thus, the firm will sell abatement to farmers up to the point where

$$MNPB(y) = p$$

- See Figure 4-44.

222

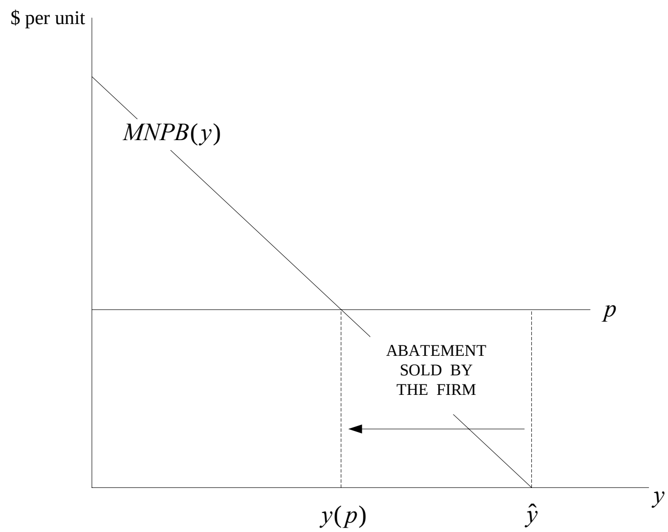

FIGURE 4-44

223

Property Rights

- To reach an agreed price, the amount of abatement that farmers are willing to buy at that price must be equal to the amount of abatement that the firm is willing to sell at that price.
- What is this **equilibrium price**?
- See Figure 4-45.

224

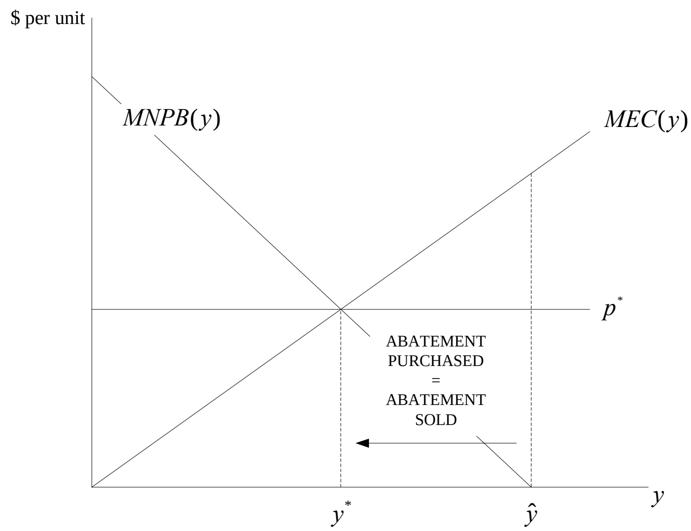

FIGURE 4-45

225

Property Rights

- It is clear from Figure 4-45 that equilibrium is reached at price p^* where

$$MEC(y) = p^* = MNPB(y)$$

226

Property Rights

- The abatement traded in equilibrium is

$$a(p^*) = \hat{y} - y^*$$

- Thus, the contract between the firm and the farmers reduces pollution from $\hat{y}$ to y^*; the contract achieves the social optimum.

227

Property Rights

- How is the associated gain in net social benefit split between the firm and the farmers?

228

Property Rights

- The **gains from trade for the firm**
  = (the revenue from the sale of abatement)
  – (the net private benefit foregone due to that abatement)

$$= p^*(\hat{y} - y^*) - \int_{y^*}^{\hat{y}} MNPB(y)dy$$

- See Figure 4-46.

229

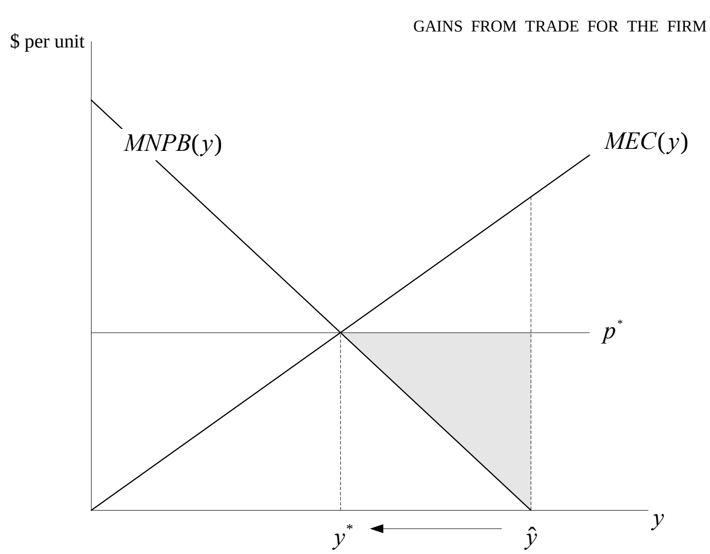

FIGURE 4-46

230

Property Rights

- The **gains from trade for the farmers**
  = (the external cost avoided)
  – (the total payment to the firm)

$$= \int_{y^*}^{\hat{y}} MEC(y)dy - p^*(\hat{y} - y^*)$$

- See Figure 4-47.

231

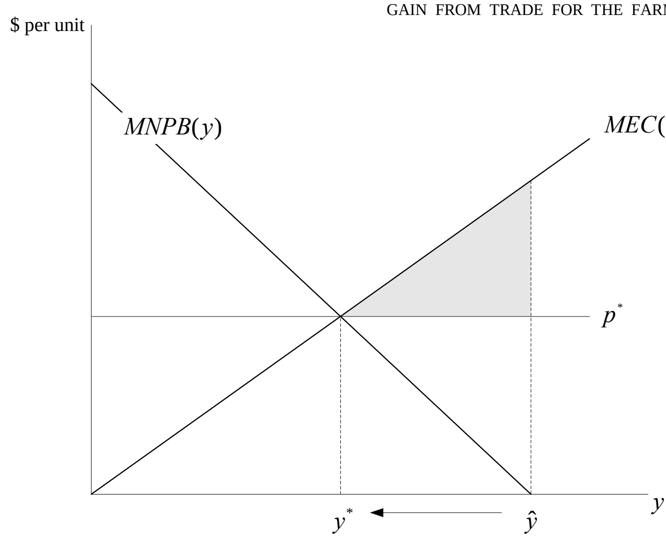

FIGURE 4-47

232

Property Rights

- Comparing Figures 4-46 and 4-47 with Figure 4-38 (repeated next slide) we see that the <u>total</u> gains from trade are exactly equal to the gain in social surplus from a forced reduction in *y*.

233

GAIN IN SOCIAL SURPLUS

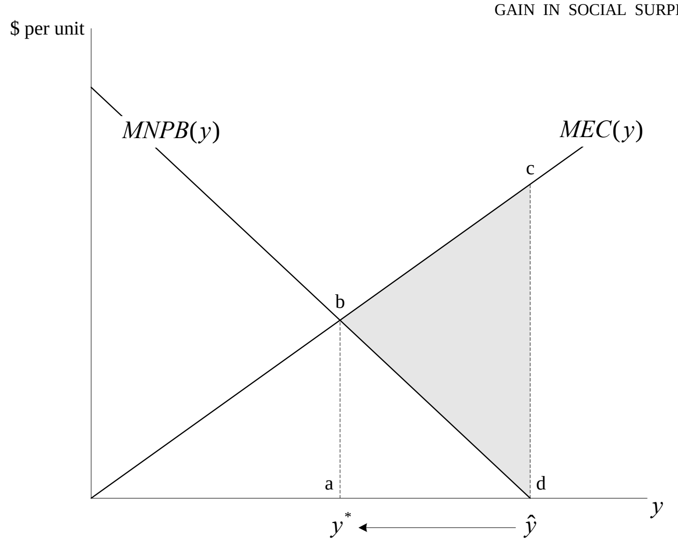

FIGURE 4-38 (repeat)

234

Property Rights

- Thus, the potential Pareto improvement available at the private optimum is fully realized through trade when property rights are made explicit.

235

Property Rights

- This appears to be a straightforward solution to the externality problem; why is policy intervention needed?

236

Property Rights

- There are two potential problems with the "property rights solution":
  - Policy-makers may not be willing to assign explicit pollution rights to polluters even when those rights are currently implicit
  - Transaction costs may create an obstacle to trade
- Let us now explore the second of these.

237

Transaction Costs

- Contracts are costly to construct.
- Significant resources are required to bargain towards an agreement, and then write down that agreement in legally robust terms that cover all possible contingencies.
- These **transaction costs** can be large enough to prevent an otherwise mutually beneficial trade from occurring.

238

Transaction Costs

- Do these transaction costs necessarily justify policy intervention?
- No. If real resources must be used to capture gains from trade through a contract then the true gains from trade are less than they appear; the potential Pareto improvement associated with an externality may be an illusion.

239

Transaction Costs

- However, private trade is not the only mechanism through which resources can be reallocated, and in some settings it may not be the best one.
- In some settings, policy intervention may be a better mechanism in the sense that fewer resources are required to achieve the reallocation than are required via trade.

240

Transaction Costs

- Policy intervention is most likely to have an advantage over trade when there are large numbers of external agents.
- Why?
- Let us explore the answer in the context of the firm versus the farmers.

241

Transaction Costs

- Abatement by the firm is a **public good** from the perspective of the farmers: each farmer benefits even if he does not participate in the bargaining.
- However, a farmer must incur transaction costs in order to participate.
- Thus, each farmer has an incentive to **free-ride** on the bargaining efforts of the others.

242

Transaction Costs

- This free-riding can mean that an abatement agreement is not reached even though it would be to the mutual benefit of all parties involved.

- A better solution might involve direct policy intervention by government.

243

Transaction Costs

- <u>In general</u>, the existence of an externality is not enough on its own to justify policy intervention.
- Any intervention (on efficiency grounds) must be argued on the basis of policy being able to achieve a net social benefit when the market cannot.

244

Transaction Costs

- This is most often true when there are large numbers of external agents, and this is very often true in environmental policy settings.

245

4.10 THE PIGOUVIAN TAX

The Pigouvian Tax

- The logic of placing a tax on an activity that has an associated negative externality was first annunciated by Arthur Pigou, a British economist, in 1924.

247

The Pigouvian Tax

- That logic is compelling:
  - an externality arises from an action because the source agent does not account for the cost imposed on external agents
  - the purpose of the Pigouvian tax is to **internalize** that externality by imposing a tax on the action commensurate with the external cost

248

The Pigouvian Tax

- Let us now explore the details of how a Pigouvian tax works.

249

The Pigouvian Tax

- Recall from Section 4.1 that the private optimum can be characterized by

$$MNPB(\hat{y}) = 0$$

where

$$MNPB(y) \equiv MPB(y) - MPC(y)$$

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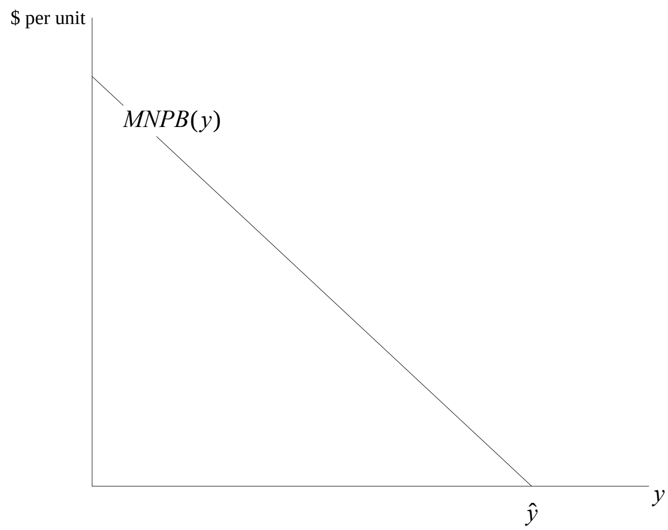

FIGURE 4-48

251

The Pigouvian Tax

- Now suppose the regulatory authority (the "regulator") has the statutory power to levy a tax on this activity at τ dollars per ton.
- Thus, if the source undertakes activity level y then its **total tax payment** is

$$T = \tau y$$

252

The Pigouvian Tax

- The source will respond to this pollution tax by assessing whether it is cheaper to pay the tax on a given unit of this activity or to cut that unit of activity and give up the associated MNPB instead.

253

The Pigouvian Tax

- This assessment on each unit of the activity will lead the source to choose a quantity $y(\tau)$ such that

$$MNPB(y(\tau)) = \tau$$

- See Figure 4-49.

254

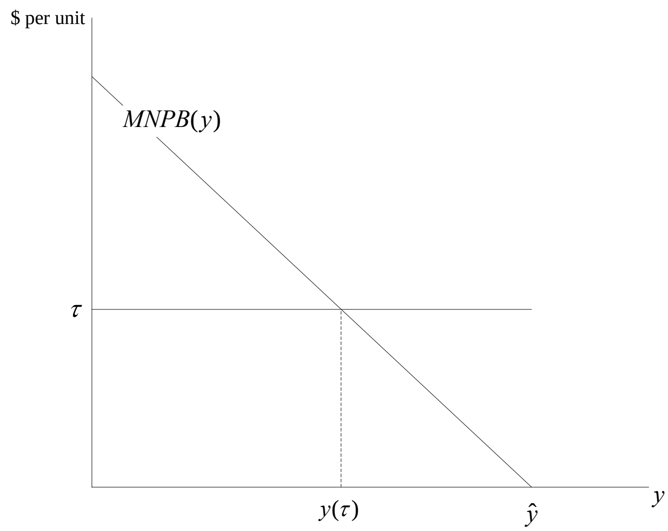

FIGURE 4-49

255

The Pigouvian Tax

- To understand the logic of this response, suppose the source initially chooses a higher level of activity at, $y_0 > y(\tau)$.
- At y_0, the source could reduce its activity to $y(\tau)$ and thereby reduce its total tax payment by more than the associated loss of NPB; see Figures 4-50 through 4-52.

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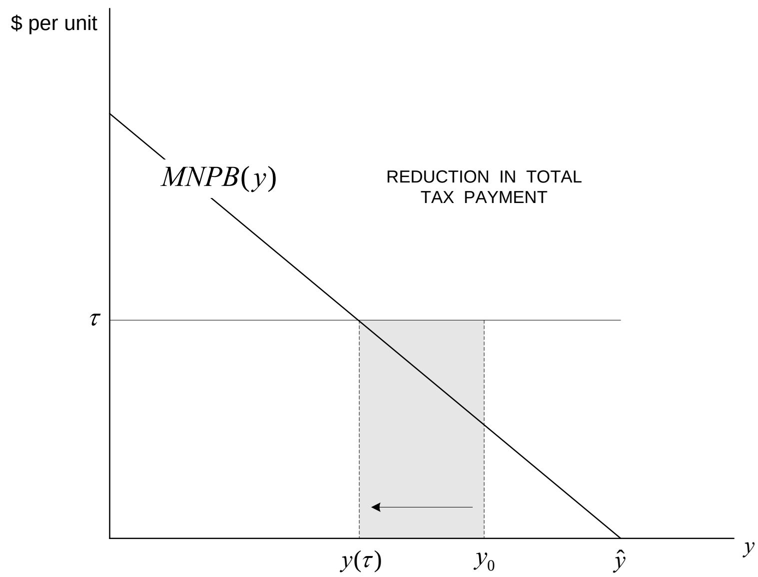

FIGURE 4-50 257

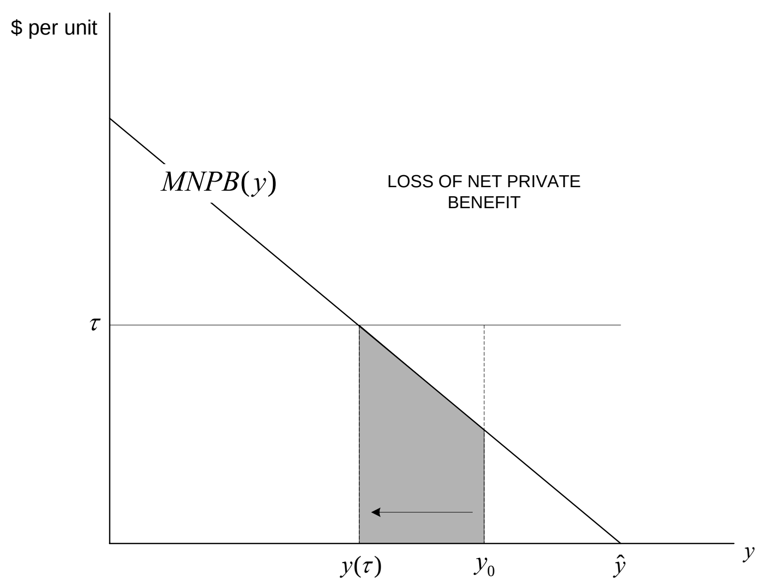

FIGURE 4-51 258

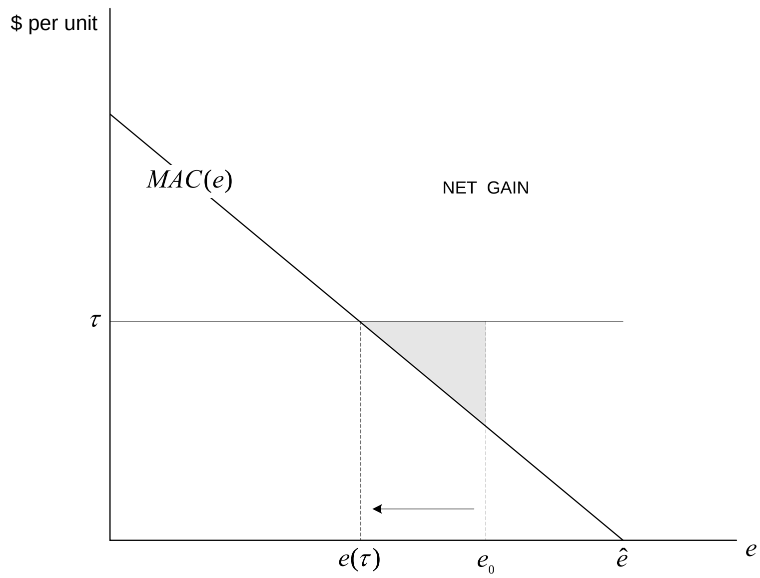

FIGURE 4-52

259

The Pigouvian Tax

- Thus, if the source is initially choosing y_0 it could benefit by reducing emissions to $y(\tau)$.

260

The Pigouvian Tax

- Similarly, if the source initially chooses a lower level of activity, at $y_0<y(\tau)$, then it could increase its activity to $y(\tau)$ and thereby raise its NPB by more than the associated increase in total tax payment; see Figures 4-53 through 4-55.

261

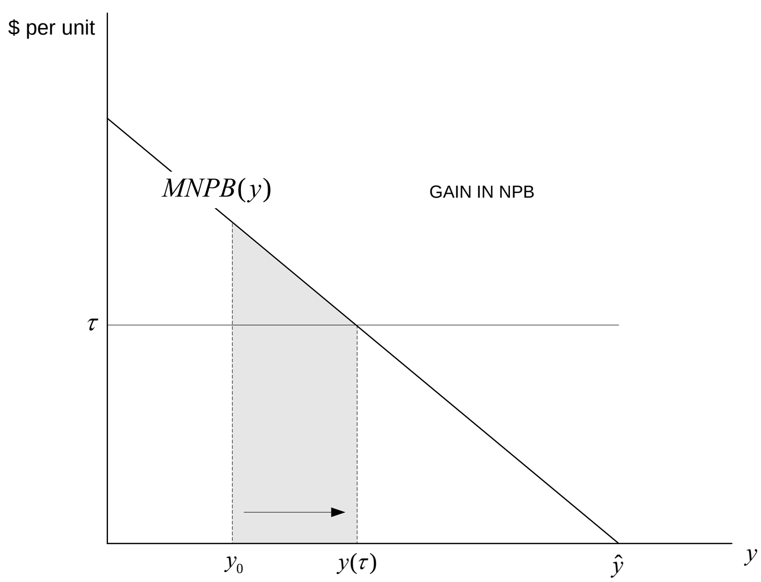

FIGURE 4-53

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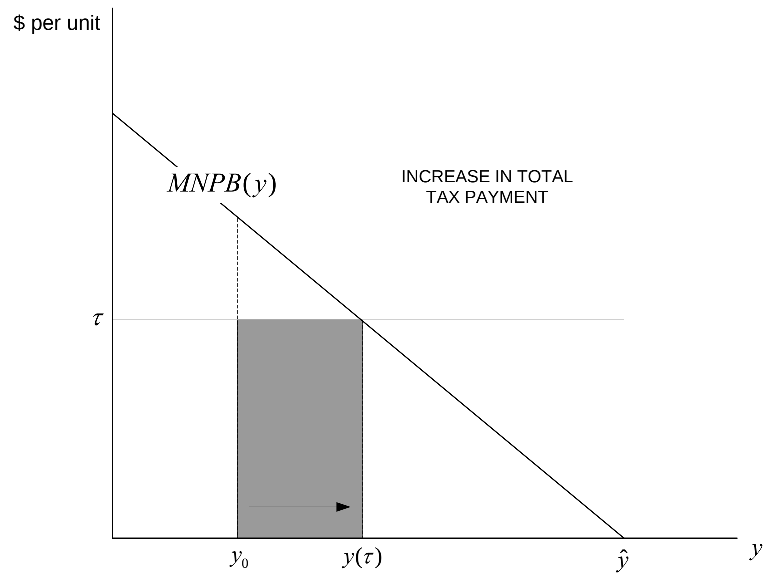

FIGURE 4-54 263

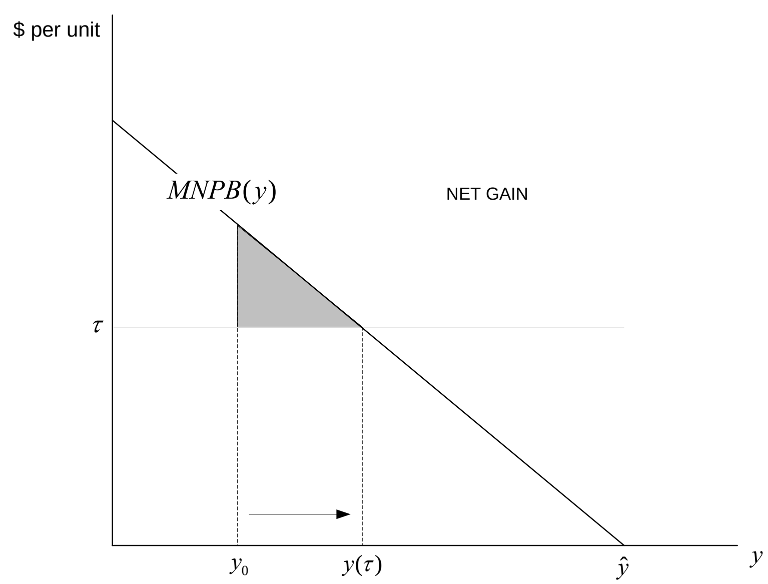

FIGURE 5-55 264

The Pigouvian Tax

- We will refer to $y(\tau)$ as the **corrected private optimum** in the sense that this choice is privately optimal for the source given that its incentives have been corrected by the tax.
- The Pigouvian tax is therefore also known as a corrective tax.

265

The Pigouvian Tax

- Now that we know how the source will respond to the tax, we can choose the tax rate to ensure that the corrected private optimum implements the policy goal.

266

The Pigouvian Tax

- Recall from section 4.7 that the socially optimal emissions level is y^* such that

$$MNPB(y^*) = MEC(y^*)$$

- And we know that the source chooses $y(\tau)$ such that

$$MNPB(y(\tau)) = \tau$$

267

The Pigouvian Tax

- Thus, $y(\tau) = y^*$ if and only if $\tau = \tau^*$, where

$$\tau^* = MEC(y^*)$$

- That is, the optimal tax rate is set equal to MEC evaluated at the social optimum.

- This is the **Pigouvian rule**. See Figure 4-56.

268

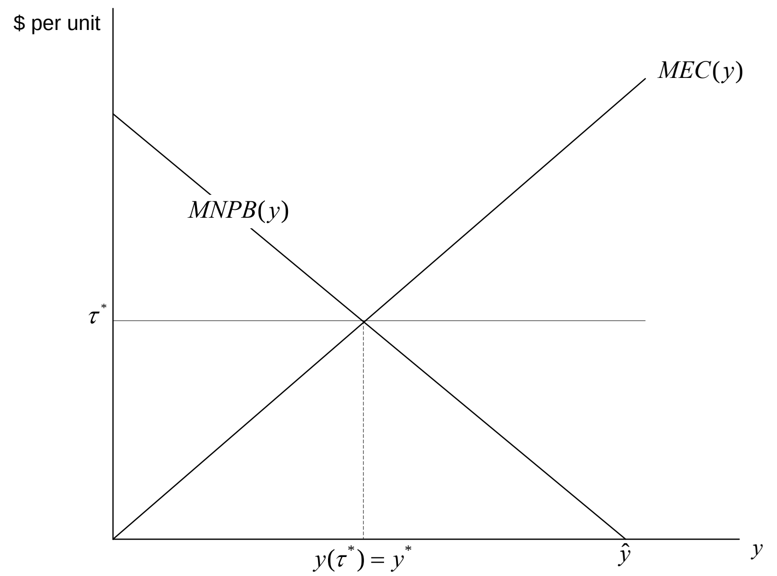

FIGURE 4-56

269

The Pigouvian Tax

- Note that the tax paid on the marginal unit of this activity is just equal to the MEC caused by that unit of activity.
- It is in this sense that the Pigouvian tax internalizes the externality.

270

The Pigouvian Tax

- Who gains and who loses from the implementation of the Pigouvian tax?
- The gain to the external agents is the reduction in external cost. See Figure 4-57.

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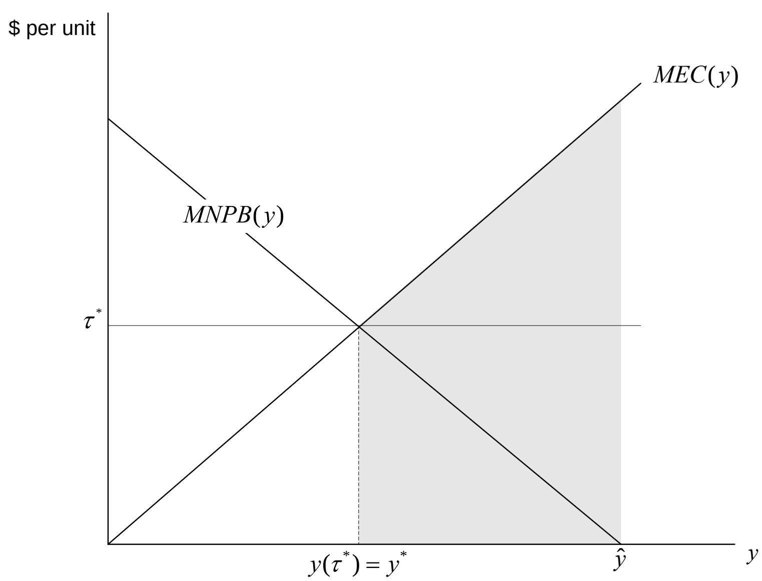

FIGURE 4-57

272

The Pigouvian Tax

- The loss to the source agent comprises two parts:
  - – the loss of NPB (see Figure 4-58); and
  - – the total tax payment (see Figure 4-59)

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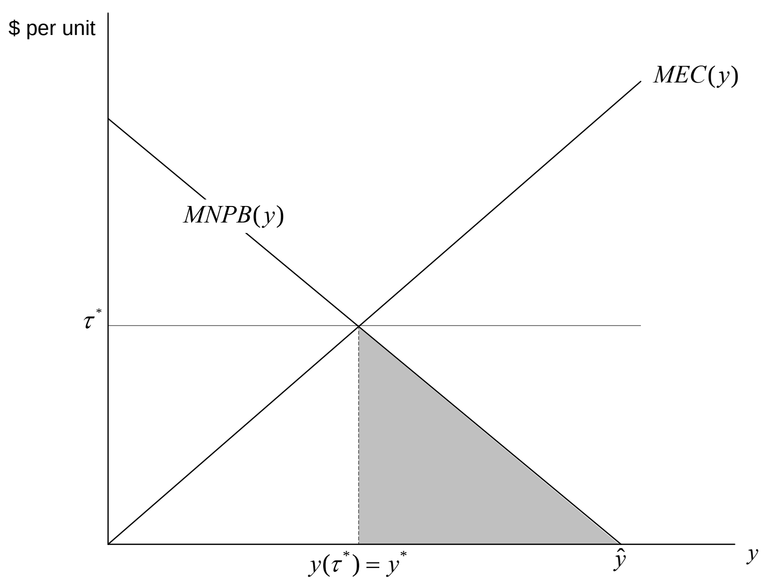

FIGURE 4-58

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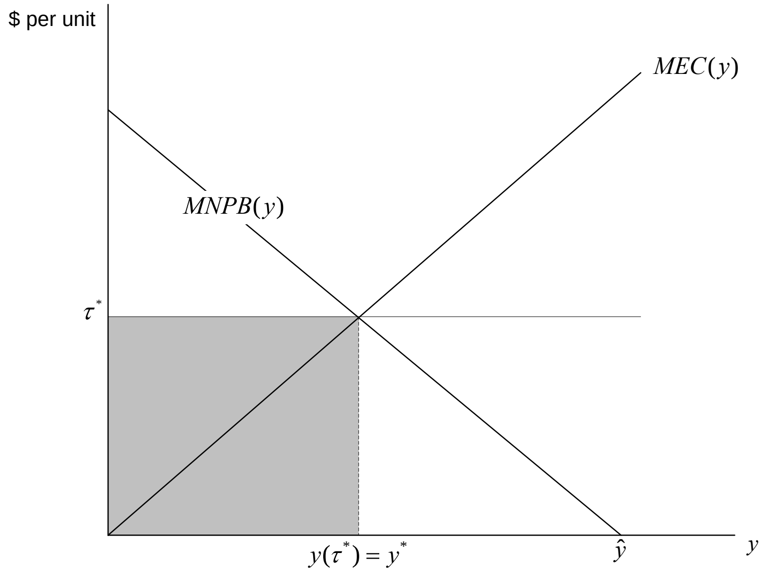

FIGURE 4-59

275

The Pigouvian Tax

- There is now also a third party to consider: the general citizenry.
- This group collects the tax payment paid by the source agent (via the collection agency of the government).

276

The Pigouvian Tax

- Thus, the net social gain (the gain in social surplus) from the implementation of the tax is the reduction in external cost minus the loss of NPB to the source agent. See Figure 4-60.

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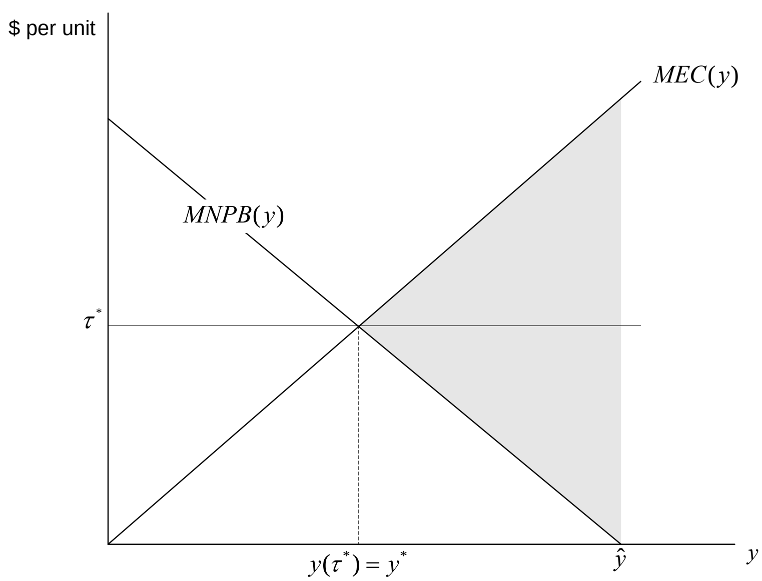

FIGURE 4-60

278

The Pigouvian Tax

- Now let us compare this Pigouvian solution with the trading outcome from section 4-9 where property rights were assigned.
- Recall the equilibrium trading price was

$$MEC(y) = p^{*} = MNPB(y)$$

- Recall Figure 4-45.

279

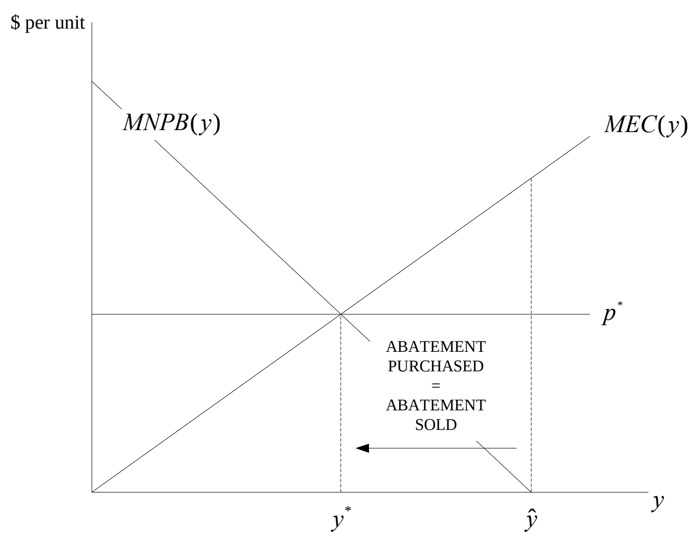

FIGURE 4-45 (repeat)

280

The Pigouvian Tax

- Thus, the Pigouvian tax rate is equal to the equilibrium trading price.

- In that trading solution, the gains from trade are split between the two trading parties. Recall Figures 4-46 and 4-47.

281

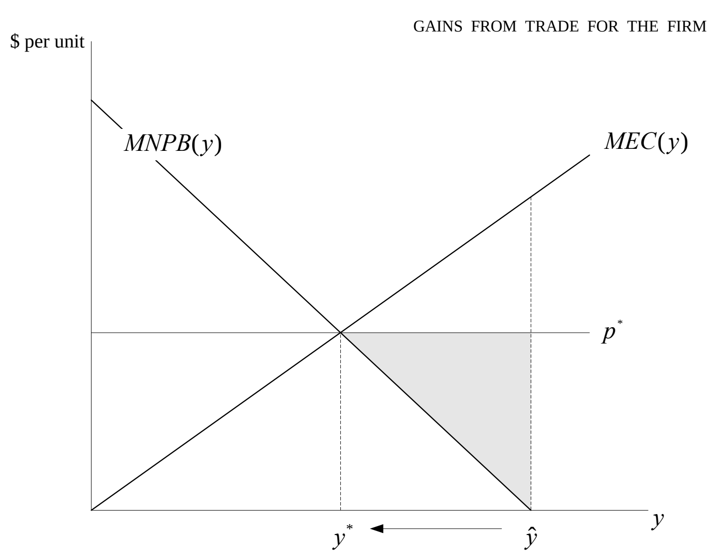

FIGURE 4-46 (repeat)

282

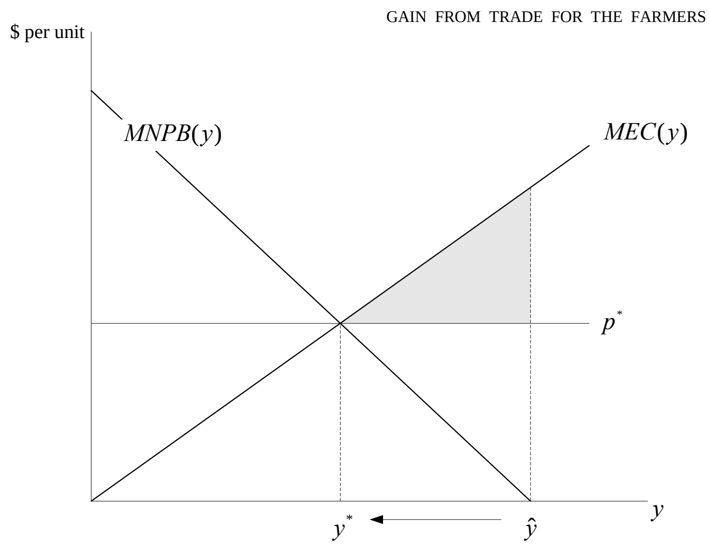

FIGURE 4-47 (repeat)

283

The Pigouvian Tax

- Comparing Figures 4-46 and 4-47 with Figure 4-60, we see that the total gains from trade are exactly equal to the gain in social surplus from the implementation of the Pigouvian tax.

284

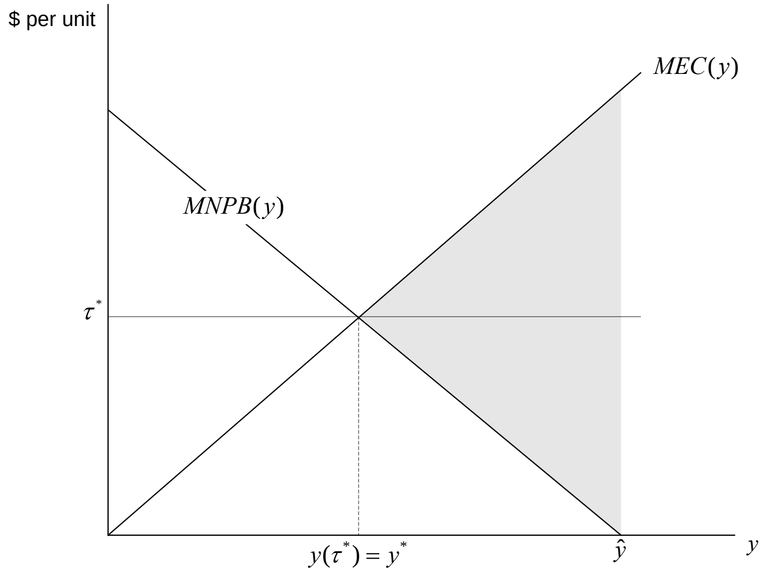

FIGURE 4-60 (repeat) 285

The Pigouvian Tax

- However, the distribution of those total gains is very different under the two settings.
- In particular, the external agents and the general citizenry do much better under the Pigouvian tax, and the source agent does much worse.

END 286

4. EXTERNALITIES – PART 2: RECIPROCAL EXTERNALITIES

Recall that a *reciprocal externality* operates in both directions: source agents are also external agents, and external agents are also source agents. Reciprocal externalities therefore involve an element of strategic interaction between agents that requires us to use some game theory in our analysis.

We begin by briefly reviewing the Nash equilibrium concept. It is the foundation on which game-theoretic analyses of reciprocal externalities are based.

4.11 NASH EQUILIBRIUM

Let s_i be the strategy of player *i*, and let s_{-i} be the vector of strategies of all other players. Let $u_i(s_i, s_{-i})$ be the payoff to player *i*. A *Nash equilibrium* is a vector $\{\hat{s}_i, \hat{s}_{-i}\}$ such that $u_i(\hat{s}_i, \hat{s}_{-i}) \geq u_i(s_i, \hat{s}_{-i}) \quad \forall s_i, \;\; \forall i$.

That is, a NE is an outcome in which each player chooses her strategy to maximize her payoff, given the *equilibrium* strategies of all other players. By definition, no player has an incentive to deviate from the Nash equilibrium.

Note that $u_i(s_i, s_{-i})$ is not a utility function; it is more general than that (though in a game between individuals it would take on that interpretation).

4.12 A TRANSBOUNDARY-POLLUTION GAME

We will develop the key concepts with respect to reciprocal externalities in the context of an example: a transboundary pollution game played between two countries who both generate pollution, and where that pollutions flows across national boundaries.

Consider a setting in which two countries are each engaged in an industrial activity that produces output *y*.

The private (or domestic) benefit to country *i* from this activity is

(4.1) $$PB_i(y_i) = v_i y_i$$

where $v_i > 0$ reflects the value of this activity to the people of country *i*.

The private (or domestic) cost of labour needed to produce y_i is

(4.2) $$L_i(y_i) = \omega_i y_i^2$$

where $\omega_i > 0$ reflects the value of leisure to the people of country *i* (and hence, the opportunity cost of labour).

The production process in country *i* generates pollution $e_i = \theta_i y_i$, and this pollution causes an adverse environmental impact in country *i* and possibly also in country *j*. The parameter θ_i is determined by technology. It could be a choice variable but we will abstract from that here and assume it is fixed.

The cost of the environmental impact in country *i* is

(4.3) $$C_i(e_i, e_j) = \delta_i(e_i + \alpha_{ji} e_j) y_i$$

where δ_i is the "damage parameter" for country *i*, and $\alpha_{ji} \geq 0$ reflects the extent to which pollution from country *j* damages country *i*.

Note from (4.3) that the amount of environmental damage to country *i* from any given level of emissions from country *j* is increasing in y_i. This reflects the fact that this pollutant damages the productivity of the economy, and so its impact is proportional to the size of the economy.

If $\alpha_{ij} = \alpha_{ji} = 1 \ \forall i$ then the pollutant is purely global. That is, damage to either country is a function of total global emissions; the country from which the pollution originates is irrelevant. (Greenhouses gases are of this type).

At the opposite extreme, if $\alpha_{ij} = \alpha_{ji} = 0$ then the pollutant is purely local: it has no transboundary element at all.

Intermediate possibilities include symmetric partial transboundary effects where $\alpha_{ij} = \alpha_{ji} < 1$, and asymmetric transboundary effects where $\alpha_{ij} \neq \alpha_{ji}$. (The latter case applies to many wind-borne and ocean-borne pollutants).

One extreme asymmetric possibility is where $\alpha_{ij} > 0$ and $\alpha_{ji} = 0$. That is, emissions from country *i* damage country *j*, but emissions from country *j* do not damage country *i*. In that case, the externality is unilateral: country *i* is the source agent, and country *j* is the external agent.

That special case highlights an important general point: a unilateral externality problem can always be represented as a limiting case of a reciprocal externality game.

We can incorporate all possibilities with respect to α_{ij} and α_{ji} into our game but the mathematics can get complicated. In these notes we will focus exclusively on the simplest case: where $\alpha_{ij} = \alpha_{ji} = 1$. Recall that this is the global pollutant case.

To keep the algebra manageable we will also assume some other simplifying restrictions. In particular, we will impose symmetry across countries with respect to v_i, ω_i and δ_i. That is, we set $v_1 = v_2 = v$, $\omega_1 = \omega_2 = \omega$, and $\delta_1 = \delta_2 = \delta$. In addition, for simplicity we set $\theta_1 = \theta_2 = 1$. This means that one unit of production generates one unit of pollution: $e_i = y_i$. Thus, we have simplified our game to one between two identical countries, each choosing their level of industrial output.

4.13 PAYOFFS IN THE GAME BETWEEN SYMMETRIC PLAYERS

We can now construct the net private benefit for country *i*:

(4.4) $$NPB_i(y_i, y_j) = PB_i(y_i) - L_i(y_i) - C_i(y_i, y_j)$$

Making the substitutions from (4.2) – (4.3) above, and imposing our simplifying restrictions, yields

(4.5) $$NPB_i(y_i, y_j) = v y_i - \omega y_i^2 - \delta(y_i + y_j) y_i$$

This net private benefit for country *i* is the **payoff function** for country *i* in the game between the two countries. Crucially, the payoff for country *i* depends on the actions of country *j*, and this introduces the strategic interaction between the two countries.

We will henceforth write the net private benefit for country *i* as

(4.6) $$u_i(y_i, y_j) \equiv NPB_i(y_i, y_j) = v y_i - \omega y_i^2 - \delta(y_i + y_j) y_i$$

This is the payoff to country *i*.

It will later prove helpful to isolate a component of this payoff. The external cost that country *j* imposes on country *i* is

(4.7) $$D_{ji}(y_j) = \delta y_i y_j$$

Note that this external cost is a more complicated object than the external cost we described in the context of a unilateral externality. Here the cost imposed on country *i* by country *j* depends on the action taken by country *i* itself (via its output choice). Thus, the external agents here are not just passive agents; they respond to the cost imposed on them by source agents.

4.14 ISOPAYOFF CONTOURS

To provide a better sense of how the interaction between the two countries determines the payoff to each one, **Figure 4-61** depicts **isopayoff contours** for country 1 in (y_1, y_2) space.

An isopayoff contour is somewhat like an indifference curve. It is a locus of points along which the payoff is constant (in the same sense that an indifference curve is a locus of points along which utility is constant).

The equation for an isopayoff contour for country 1 can be found simply by setting $u_1(y_1, y_2) = u$ and solving for y_2 as a function of y_1:

(4.8) $$y_2(y_1, u) = \frac{v y_1 - (\delta + \omega) y_1^2 - u}{\delta y_1}$$

This is the function plotted in **Figure 4-61**, where the different contours in the figure correspond to different values of *u*. Lower contours correspond to higher payoffs because smaller values of y_2 make country 1 better off (because a smaller value of y_2 means there is less pollution coming from country 2 to damage country 1).

An expression analogous to (4.8) can be found to describe an isopayoff contour for country 2.

4.15 BEST-RESPONSE FUNCTIONS

The two countries choose their outputs at the same time. This makes the game a "simultaneous move game". Thus, neither country sees what the other country does before it must make its own choice. Each country must therefore form an expectation of what the other country will do, and then make its own choice.

The choice problem for country *i* is to choose its output to maximize its own payoff, conditional on its expectation of output from country *j*.

This optimal choice for country *i* is characterized by

(4.9) $$\frac{\partial u_i(y_i, y_j)}{\partial y_i} = 0$$

where y_j is taken as given.

This optimality condition solves for a *best-response function* (BRF) for country *i*. The BRF function identifies the optimal choice for country *i* in response to the choice it anticipates will be made by country *j*. (It is sometimes called a "reaction function").

It is important to stress that the terminology here is not meant to suggest that country *i* responds to country *j* in a sequential-move sense; recall that this is a simultaneous move game. Instead, country *i* responds to its own expectation of what country *j* will choose. With *common knowledge of rationality*, country *i* can correctly anticipate that choice by country *j*.

Using (4.6) and (4.9) we can find the best-response functions for country 1 and for country 2. These are

(4.10) $$y_1(y_2) = \frac{\nu - \delta y_2}{2(\delta + \omega)}$$

and

(4.11) $$y_2(y_1) = \frac{\nu - \delta y_1}{2(\delta + \omega)}$$

respectively.

The best-response function for country 1 is illustrated in **Figure 4-61**. Note that it passes through the turning points of the isopayoff contours. Why?

Graphically, the best-response function for country 1 represents a solution to the problem of finding the lowest contour (the highest payoff) conditional on facing a given level of

y_2, represented graphically by a horizontal constraint (such as the dashed line in **Figure 4-61**).

The best-response functions for both countries are illustrated together in **Figure 4-62**.

The points labeled y_1^0 and y_2^0 in **Figure 4-62** correspond to the *sole-agent optima* for countries 1 and 2 respectively. That is, y_i^0 is the level of output country *i* would choose if it were the only country in this global economy and thus unaffected by the actions of the other country.

It is straightforward to find y_1^0 and y_2^0. Set $y_2 = 0$ in (4.10) to yield

(4.12) $$y_1^0 = \frac{v}{2(\delta + \omega)}$$

and set $y_1 = 0$ in (4.11) to yield

(4.13) $$y_2^0 = \frac{v}{2(\delta + \omega)}$$

Note that these solutions are equal only because in our example the countries are identical and damage is caused by a global pollutant.

4.16 THE NON-COOPERATIVE EQUILIBRIUM

The Nash equilibrium in this context is called the *non-cooperative equilibrium* (*NCE*). (This term distinguishes the equilibrium from a "treaty equilibrium" in which countries agree to form a treaty to reduce emissions. We will discuss this briefly in section 4.20).

Graphically, the NCE is the intersection of the best response functions, as depicted in **Figure 4-63**. Algebraically, it is the simultaneous solution of (4.10) and (4.11), which yields

(4.14) $$\tilde{y}_1 = \frac{v(2\omega + \delta)}{4(\delta + \omega)^2 - \delta^2}$$

and

(4.15) $$\tilde{y}_2 = \frac{v(2\omega + \delta)}{4(\delta + \omega)^2 - \delta^2}$$

Note again that these solutions are equal in this symmetric global pollutant example. Thus, the NCE lies on the 45^0 line in **Figure 4-63** (along which $y_2 = y_1$).

4.17 EFFICIENCY

There are a *continuum* of efficient allocations in this setting corresponding to different distributions of aggregate payoffs across the two countries.

This set of Pareto efficient allocations – the Pareto frontier in this context – can be derived in a now familiar way: we maximize the payoff to one country subject to maintaining a given payoff to the other country.

It makes no difference whether we maximize $u_1(y_1, y_2)$ and hold $u_2(y_1, y_2)$ constant, or *vice versa*. Here we will maximize $u_2(y_1, y_2)$. Thus, our planning problem is

(4.16) $$\max_{y_1, y_2} \; vy_2 - \omega y_2^2 - \delta(y_2 + y_1)y_2$$

$$\text{subject to } \; vy_1 - \omega y_1^2 - \delta(y_1 + y_2)y_1 = u$$

The closed-form solution to this problem is quite complicated, and reporting here is not instructive. It does however have a simple graphical representation, as illustrated in **Figure 4-64**.

The Pareto frontier – labeled *PF* in the figure – is the locus of tangencies of the isopayoff contours for the two countries.

The logic of that solution is the same as that underlying the derivation of the Pareto frontier in the exchange economy from Topic 2. In particular, if we hold $u_1(y_1, y_2)$ fixed – corresponding to a particular isopayoff contour for country 1 – and then maximize $u_2(y_1, y_2)$, then the solution is a point of tangency between an isopayoff contour for country 2 and the isopayoff contour for country 1 corresponding to the fixed value of $u_1(y_1, y_2)$. As we vary the value at which $u_1(y_1, y_2)$ is fixed, we trace out a continuum of such tangency points. That continuum is the Pareto frontier.

Note from **Figure 4-64** that the Pareto frontier is anchored at the sole-agent choices, y_1^0 and y_2^0.

Why? Setting $y_2 = 0$ as part of an efficient solution effectively makes country 1 a sole agent, and we know that its own payoff in that case is maximized at $y_1 = y_1^0$. Similarly, y_2^0 is the efficient value for country 2 when $y_1 = 0$.

Figure 4-65 overlays the Pareto frontier on the best-response functions and the corresponding NCE. The key message from this figure is that the NCE is inefficient; it lies above the Pareto frontier.

Why? Each country ignores the cost that its production imposes on the other country precisely because that cost is external. This external cost is nonetheless part of the true global social cost of the activity, and efficiency requires that it be taken into account.

It is important to recognize that all points on the Pareto frontier are Pareto efficient (by definition) but not all points on the Pareto frontier *Pareto dominate* the NCE.

This point is highlighted in **Figure 4-66**, which overlays on **Figure 4-65** the isopayoff contours passing through the NCE. These contours correspond to the payoffs at the NCE.

Points on the Pareto frontier that <u>do</u> Pareto-dominate the NCE are represented in **Figure 4-66** by the heavily drawn segment of the frontier; this is the *core* with respect to the NCE. This core is the segment of the frontier that lies within the shaded lens-shaped region bounded by the two-isopayoff contours; this region is the *region of mutual benefit* because it constitutes the sets of points that Pareto-dominate the NCE.

These concepts are the same as those we have seen before in the context of the simple exchange economy in Topic 2. In that context the payoff were utilities, and since we cannot compare utility across different individuals, we could not say that some points on the Pareto frontier are better than others.

In contrast, in the current setting the payoffs are in terms of dollars (the net value of production), and so we can compare payoffs across countries. This means that we can rank points on the Pareto frontier in terms of social surplus.

4.18 MAXIMUM SOCIAL SURPLUS

The social surplus (or net social benefit) is the sum of the two payoffs, and we can show that some Pareto-efficient allocations have higher social surplus than others.

To see this, consider a planning problem that chooses y_1 and y_2 to maximize social surplus:

$$\max_{y_1, y_2} \; (vy_1 - \omega y_1^2 - \delta(y_1 + y_2)y_1) + (vy_2 - \omega y_2^2 - \delta(y_2 + y_1)y_2) \tag{4.17}$$

Let $\{y_1^*, y_2^*\}$ denote the solution to this problem. We will henceforth call this solution the *social optimum*, reflecting the terminology we used in the case of unilateral externalities (but it is important to remember that it is a just one Pareto-efficient point among many).

The planning problem in (4.17) can be simplified in a way that makes it very easy to solve. In particular, because we have focused on the case where countries are identical and the pollutant is global, the implied symmetry means that $y_1^* = y_2^*$; that is, the social optimum will be symmetric.

We can impose that symmetry on the problem in (4.17) by setting $y_1 = y_2 = y$ to obtain a simplified problem:

(4.18) $$\max_y \; 2(vy - \omega y^2 - 2\delta y^2)$$

We can now solve this problem easily by setting the derivative with respect to *y* equal to zero, and then solving for *y* to yield

(4.19) $$y_1^* = y_2^* = y^* = \frac{v}{2\omega + 4\delta}$$

Graphically, this social optimum is the point on the Pareto frontier where it crosses the 45^0 line, labeled *MSS* in **Figure 4-67**).

The Social Optimum vs. the NCE

Comparing the social optimum with the NCE in **Figure 4-67** reveals two key properties of the NCE in the symmetric global pollutant case.

First, both countries produce too much in the NCE. At the NCE, neither country takes into account the negative impact its own output has on the other country. In contrast, that negative externality is fully internalized at the social optimum, by definition.

Second, the social optimum lies in the core with respect to the NCE. That is, the social optimum is Pareto efficient, and it Pareto dominates the NCE. See **Figure 4-68**.

In a more general setting with asymmetry between the two countries, neither of these properties will necessarily hold. In particular, the social optimum could potentially involve higher output for one of the countries than at the NCE (though total output at the NCE will always be too high). In addition, the social optimum may not lie in the core.

To demonstrate these possibilities would require us to relax our symmetry assumptions, which introduces more complicated mathematics, so we will not do it here.

However, the unilateral externality from Part 1 provides some useful intuition. That setting is an extreme case of asymmetry, where one country is damaged by the action of the other country but not *vice versa*. We have already seen in our graphical analysis of the unilateral externality that the social optimum does not Pareto dominate the private optimum in that setting. That is, the social optimum is not in the core. We could introduce less extreme asymmetry into the reciprocal externality problem and observe a similar result.

4.19 THE PIGOUVIAN SOLUTION: A GLOBAL EMISSIONS TAX

Imagine for a moment that there exists a global government that can impose a tax on emissions in both countries. In our simple model we have assumed that there is a one-to-one relationship between emissions and output, so a tax on emissions is equivalent to a tax on output.

What is the Pigouvian tax rate in this setting? That is, what tax on output would implement the corrected NCE as the social optimum?

To investigate this question, first recall from (4.6) the payoff function to country *i* in the non-cooperative game, repeated here as

(4.20) $$u_i(y_i, y_j) = vy_i - \omega y_i^2 - \delta(y_i + y_j)y_i$$

If this country now faces a tax at rate *t* on its output, its revised tax-inclusive payoff function is

(4.21) $$u_i(y_i, y_j, t) = vy_i - \omega y_i^2 - \delta(y_i + y_j)y_i - ty_i$$

This can be written instructively as

(4.22) $$u_i(y_i, y_j, t) = (\nu - t)y_i - \omega y_i^2 - \delta(y_i + y_j)y_i$$

That is, if we replace ν in the original payoff function with $(\nu - t)$ then we obtain the revised tax-inclusive payoff function. This means we can obtain the tax-corrected NCE output values simply by replacing ν with $(\nu - t)$ in our original NCE values from (4.14) and (4.15).

These tax-corrected NCE outputs are

(4.23) $$\tilde{y}_1(t) = \frac{(\nu - t)(2\omega + \delta)}{4(\delta + \omega)^2 - \delta^2}$$

and

(4.24) $$\tilde{y}_2(t) = \frac{(\nu - t)(2\omega + \delta)}{4(\delta + \omega)^2 - \delta^2}$$

We can now choose the tax rate to ensure that these corrected equilibrium values implement the social optimum. That is, we set $\tilde{y}_1(t) = y_1^*$,

(4.25) $$\frac{(\nu - t)(2\omega + \delta)}{4(\delta + \omega)^2 - \delta^2} = \frac{\nu}{2\omega + 4\delta}$$

and then solve for *t* to yield the Pigouvian tax rate:

(4.26) $$t^* = \frac{\delta\nu}{2\omega + 4\delta}$$

In graphical terms, the Pigouvian tax shifts the best-response functions so that their new intersection coincides with the social optimum; see **Figure 4-69**.

In Section 4.10 we saw that in the unilateral externality setting, the Pigouvian tax is set equal to marginal external cost evaluated at the social optimum. Does that same rule apply here?

Recall from Section 4.13 that the external cost imposed on country *i* by country *j* is

(4.27) $$D_{ji}(y_j) = \delta y_i y_j$$

Thus, the marginal external cost of y_j is

(4.28) $$MEC_{ji}(y_j) \equiv \frac{\partial D_{ji}(y_j)}{\partial y_j} = \delta y_i$$

Note that this is actually independent of y_j. That is, if we plot $MEC_{ji}(y_j)$ against y_j we obtain a graph like **Figure 4-70**. This is not especially important but it is worth highlighting to ensure that there is no confusion about the meaning of $MEC_{ji}(y_j)$; it is the marginal external cost of output from country *j*.

(In our graphical analysis of the unilateral externality we assumed that MEC was upward-sloping but we could have also considered a case where it is flat, as in **Figure 4-70**, and nothing important would change).

If we evaluate $MEC_{ji}(y_j)$ at the social optimum, where $y_i = y^*$, we obtain

(4.29) $$MEC_{ji}(y_j)\Big|_{y^*} = \delta\left(\frac{v}{2\omega + 4\delta}\right)$$

Comparing (4.29) and (4.26) tells us that the Pigouvian tax rate is equal to marginal external cost evaluated at the social optimum. That is, the same Pigouvian rule applies in the reciprocal externality setting as applies in the unilateral setting.

Note from (4.26) that the Pigouvian tax rate is increasing in δ, as illustrated in **Figure 4-71**. This reflects the fact that the marginal external cost imposed by one country on the other is proportional to the size of the damage parameter.

But why is t^* increasing at a decreasing rate, as depicted in **Figure 4-71**? This reflects the fact that the emissions from a given country also damage that country itself, and that impact is also increasing in δ. This means that each country curtails its own output somewhat as δ rises, acting out of self-interest alone. Consequently, the tax rate needed to correct the externality does not need to rise at a linear rate as δ rises.

Welfare Gains

Are both countries necessarily better off at the tax-corrected equilibrium than at the NCE? This depends on whether or not the tax revenue is refunded.

To investigate this question, let us first calculate the payoff to each country at the uncorrected NCE. To make this calculation we simply substitute $\tilde{y}_1$ and $\tilde{y}_2$ from (4.14) and (4.15) respectively into the payoff function from (4.20). After some simplification, this yields

(4.30) $$\tilde{u}_i = \frac{v^2(\omega+\delta)}{(2\omega+3\delta)^2} \quad \forall i$$

To calculate the payoff to each country at the tax-corrected NCE (before any tax refunds), we substitute y_1^* and y_2^* from (4.19) and (4.15) into (4.22) and then set $t = t^*$ in that expression to yield

(4.31) $$\tilde{u}_i(t^*)_0 = \frac{v^2(\omega+\delta)}{4(\omega+2\delta)^2} \quad \forall i$$

where the “0” subscript indicates “ no refunds”.

It is straightforward to show that $\tilde{u}_i(t^*) < \tilde{u}_i$. That is, the payoff to each country is lower under the tax than at the NCE, if tax revenue is not refunded.

Now suppose the tax revenue is refunded. How much revenue is available for refunding?

We can calculate the tax revenue collected from each country as

(4.32) $$r_i^* = t^* y_i^* = \frac{\delta v^2}{4(\omega+2\delta)^2}$$

If this revenue is refunded to each country then the payoff to each country under the tax becomes

(4.33) $$\tilde{u}_i(t^*)_R = \frac{v^2}{4(\omega + 2\delta)} \quad \forall i$$

This payoff is unambiguously higher than the payoff at the NCE.

There is one last noteworthy point about the Pigouvian solution. Recall that the tax is designed to implement the social optimum as a corrected equilibrium. Suppose instead the global government could impose that social optimum directly, by dictating that both countries choose y^* from (4.19).

Under that scenario the associated payoff to each country is calculated by substituting $y_1 = y^*$ and $y_2 = y^*$ directly into (4.20). This yields

(4.34) $$u_i^* = \frac{v^2}{4(\omega + 2\delta)} \quad \forall i$$

This is equal to the payoff under the tax policy with refunds from (4.33). That is, the tax policy with revenue refunds gives us exactly the same outcome as imposing the social optimum directly.

4.20 SELF-ENFORCING COOPERATION

In practice, there is no global government that can impose a tax on emissions or dictate output levels for sovereign countries. Any agreement to reduce emissions below the NCE must be self-enforcing; that is, both countries must prefer to be part of a cooperative treaty than to remain outside that treaty and act non-cooperatively.

The study of treaties (using a game-theoretic framework called "coalition theory") is beyond the scope of this course but we can get a sense of how hard it can be to achieve a cooperative treaty in practice.

Suppose both countries tentatively agree to reduce emissions from the NCE level to the social optimum. We know that both countries would be better off doing so than to stay at the NCE.

However, the relevant question is: if one country commits to reduce output to y^*, what is the best action for the other country? Agree to do the same, or do something different?

To answer that question, suppose country 1 commits to y^*. Then the best response for country 2 is dictated by its best-response function, from (4.11) above, repeated here as

(4.35) $$y_2(y_1) = \frac{\nu - \delta y_1}{2(\delta + \omega)}$$

Setting $y_1 = y^*$ in (4.35) yields the best-response by country 2:

(4.36) $$y_2(y^*) = \frac{\nu(2\omega + 3\delta)}{4(\omega + 2\delta)(\omega + \delta)}$$

Not only is this best-response higher than y^*, it is higher even than $\tilde{y}_2$. That is, in response to a commitment by country 1 to reduce output to y^*, country 2 finds it privately optimal to *raise* its output above its NCE level. See **Figure 4-72**.

These countries effectively face a "prisoners' dilemma". Both countries would be better off in a binding cooperative agreement to reduce emissions to the social optimum, but neither country finds it in their private interests to join a treaty that aims to achieve that cooperative outcome.

In practice, the situation is not as grim as this simple analysis suggests in the context of global emissions. In a setting with heterogeneous countries (unlike the identical-country case here), there is greater scope for building a self-enforcing treaty that can reduce global emissions. However, some of the key results from coalition theory tells us that achieving the social optimum is almost never possible.

4.21 A NUMERICAL EXAMPLE

Consider a transboundary pollution game between two identical countries where the payoff to country 1 is

(4.37) $$u_1(y_1, y_2) = 100y_1 - y_1^2 - 2(y_1 + y_2)y_1$$

and the payoff to country 2 is

(4.38) $$u_2(y_1, y_2) = 100y_2 - y_2^2 - 2(y_1 + y_2)y_2$$

Thus, in this example, $v = 100$, $\omega = 1$ and $\delta = 2$.

To find the best-response function for country 1, we choose y_1 to maximize $u_1(y_1, y_2)$, taking y_2 as given. Thus, we set the derivative of $u_1(y_1, y_2)$ with respect to y_1 equal to zero:

(4.39) $$\frac{\partial u_1}{\partial y_1} = 100 - 2y_1 - 4y_1 - 2y_2 = 0$$

Solving for y_1 yields

(4.40) $$y_1(y_2) = \frac{100 - 2y_2}{6}$$

We derive the best response function for country 2 in exactly the same way, to yield

(4.41) $$y_2(y_1) = \frac{100 - 2y_1}{6}$$

We can now find the NCE by setting $y_2 = y_2(y_1)$ in $y_1(y_2)$ to yield

(4.42) $$y_1(y_2) = \frac{100 - 2\left(\frac{100 - 2y_1}{6}\right)}{6}$$

We can then solve this for y_1 to obtain the NCE output for country 1:

(4.43) $$\tilde{y}_1 = \frac{25}{2}$$

Making this substitution for y_1 in $y_2(y_1)$ then yields the NCE output for country 2:

(4.44) $$\tilde{y}_2 = \frac{25}{2}$$

These are equal because the countries are identical and the pollutant is global.

We derive the social optimum (where social surplus is maximized) as the solution to

(4.45) $$\max_{y_1, y_2} \; u_1(y_1, y_2) + u_2(y_1, y_2)$$

Since countries are identical and the pollutant is global, the implied symmetry means that the social optimum will also be symmetric. We impose that symmetry on the problem in (4.45) by setting $y_1 = y_2 = y$ to obtain a simplified problem:

(4.46) $$\max_{y} \; 2(100y - y^2 - 4y^2)$$

We can now solve this problem by setting the derivative with respect to *y* equal to zero, and solving for *y* to yield

(4.47) $$y_1^* = y_2^* = y^* = 10$$

In comparison, recall that output in the NCE is 12.5. Thus, output in the NCE is 125% of output at the social optimum.

What is the Pigouvian tax rate in this setting? There are two ways we can find it: (i) solve for the NCE when the countries face a tax, and then choose the tax rate to ensure that the corrected NCE implements the social optimum; or (ii) calculate marginal external cost at the social optimum and invoke the Pigouvian rule.

Method 1

When we impose a tax on output from country 1, its tax-inclusive payoff becomes

(4.48) $$u_1(y_1, y_2, t) = 100y_1 - y_1^2 - 2(y_1 + y_2)y_1 - ty_1$$

and this can be rewritten as

(4.49) $$u_1(y_1, y_2, t) = (100 - t)y_1 - y_1^2 - 2(y_1 + y_2)y_1$$

Similarly, the tax-inclusive payoff for country for country 2 is

(4.50) $$u_2(y_1, y_2, t) = (100 - t)y_2 - y_2^2 - 2(y_1 + y_2)y_2$$

We can find the tax-corrected NCE by substituting $(100 - t)$ for 100 in the best-response functions from (4.40) and (4.41) to obtain

(4.51) $$y_1(y_2) = \frac{100 - t - 2y_2}{6}$$

We derive the best response function for country 2 in exactly the same way, to yield

(4.52) $$y_2(y_1) = \frac{100 - t - 2y_1}{6}$$

We then find the tax-corrected NCE as the simultaneous solution to (4.51) and (4.52). This yields

(4.53) $$\tilde{y}_1(t) = \frac{4(100 - t)}{32}$$

and

(4.54) $$\tilde{y}_2(t) = \frac{4(100 - t)}{32}$$

Setting these equal to y^* yields $t^* = 20$

Method 2

Expand (4.37) to obtain

(4.55) $$u_1(y_1, y_2) = 100y_1 - y_1^2 - 2(y_1)y_1 + 2(y_2)y_1$$

The last term is the external cost imposed on country 1 by country 2:

(4.56) $$D_{21}(y_2) = 2y_2y_1$$

Thus, the marginal external cost of output from country 2 is

(4.57) $$MEC_{21}(y_2) = 2y_1$$

Evaluate this at $y_1 = y^*$ to obtain $t^* = 20$. Thus, the two methods give us the same solution for the Pigouvian tax rate.

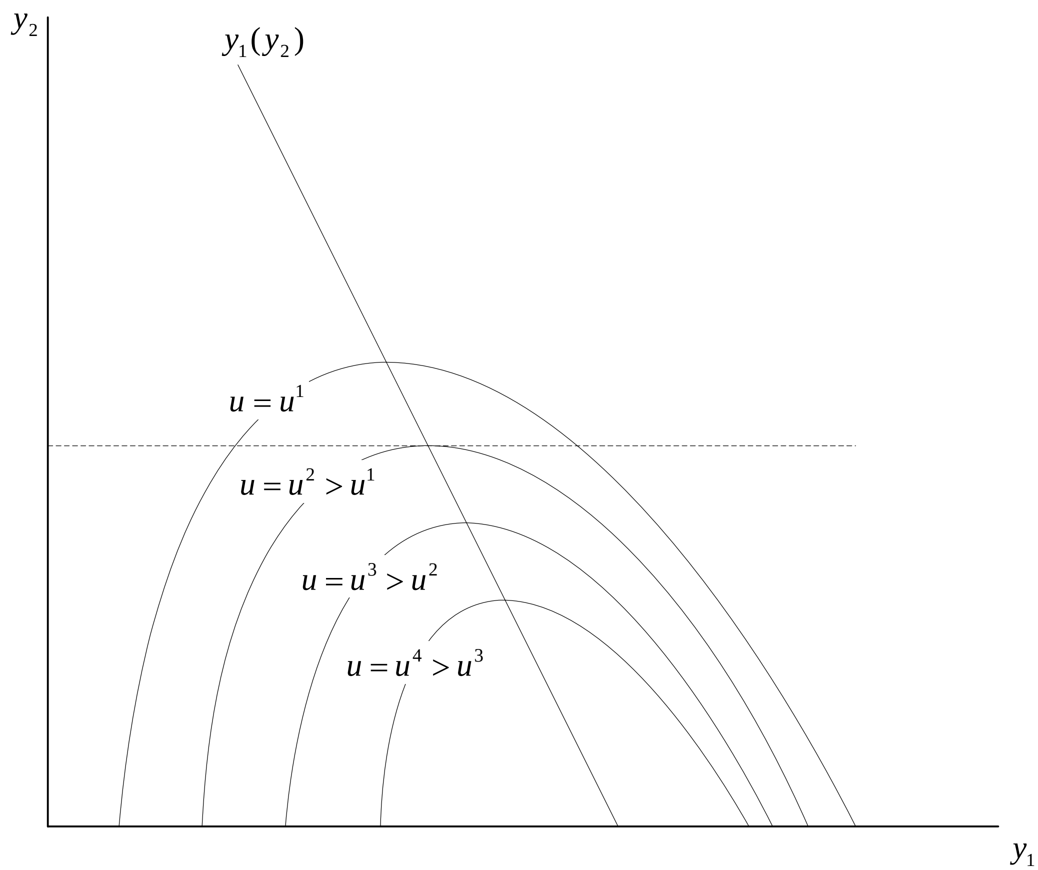

Figure 4-61

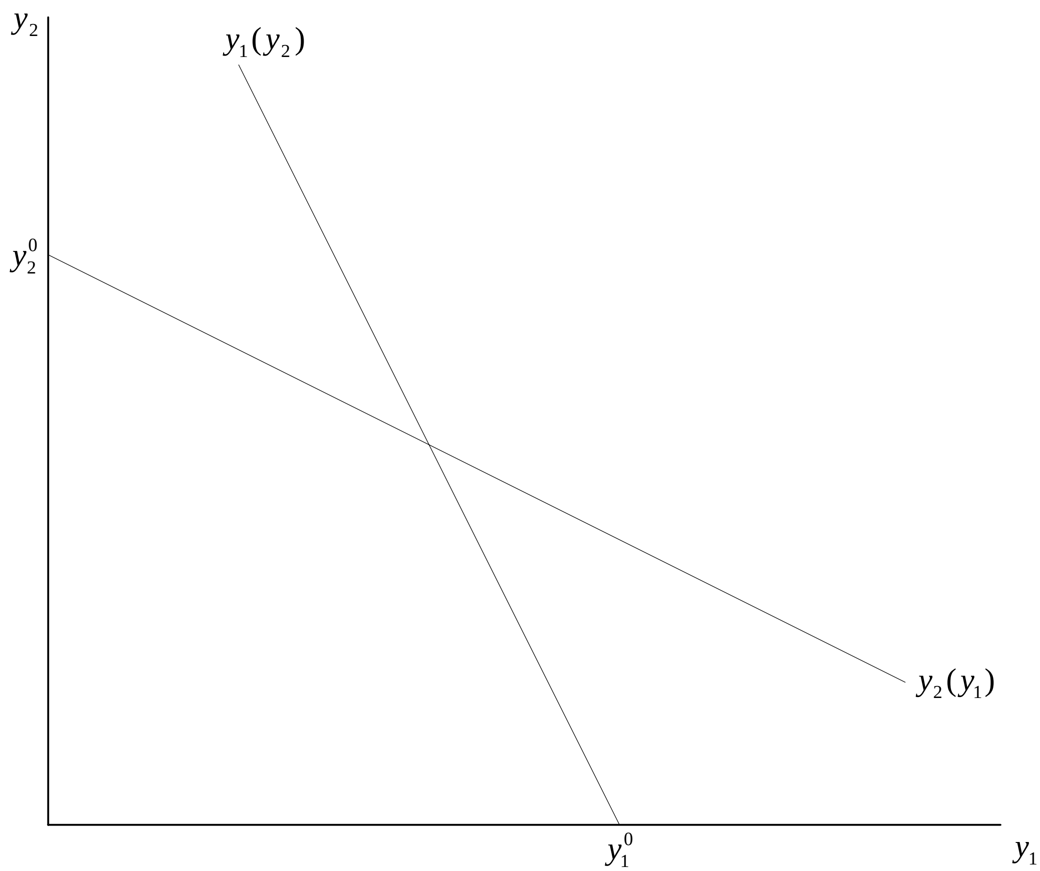

Figure 4-62

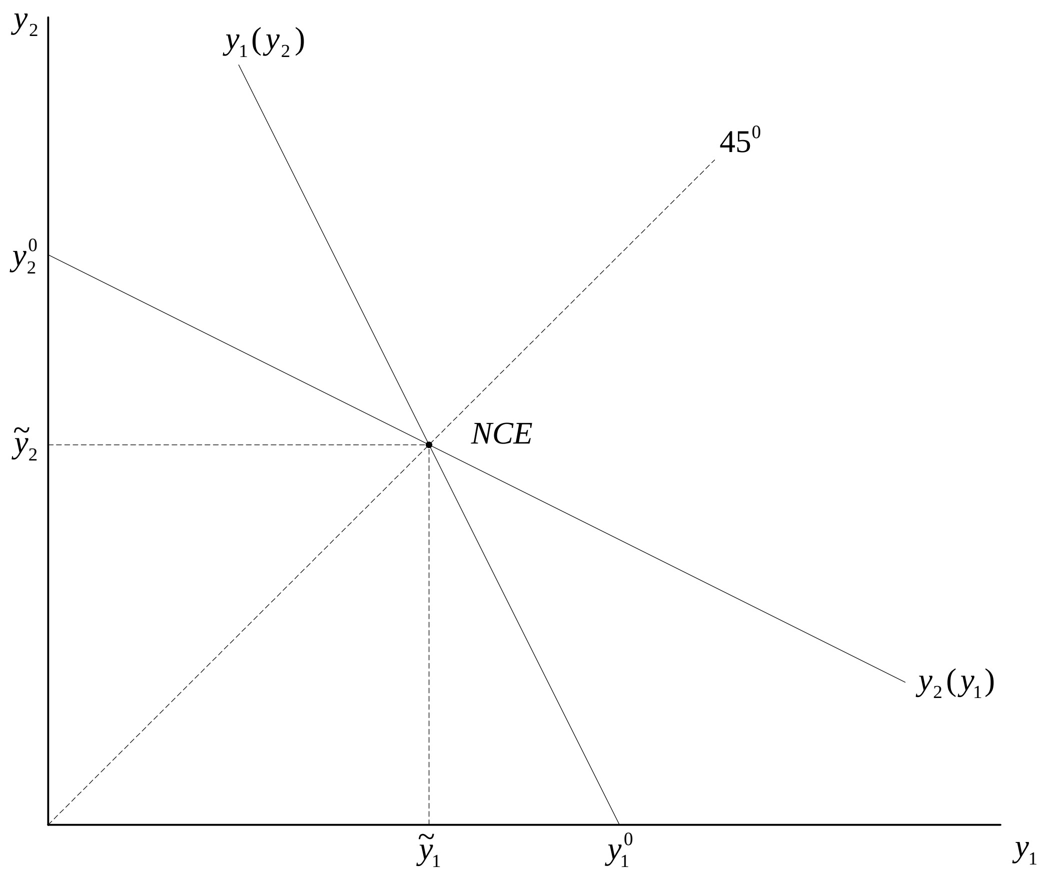

Figure 4-63

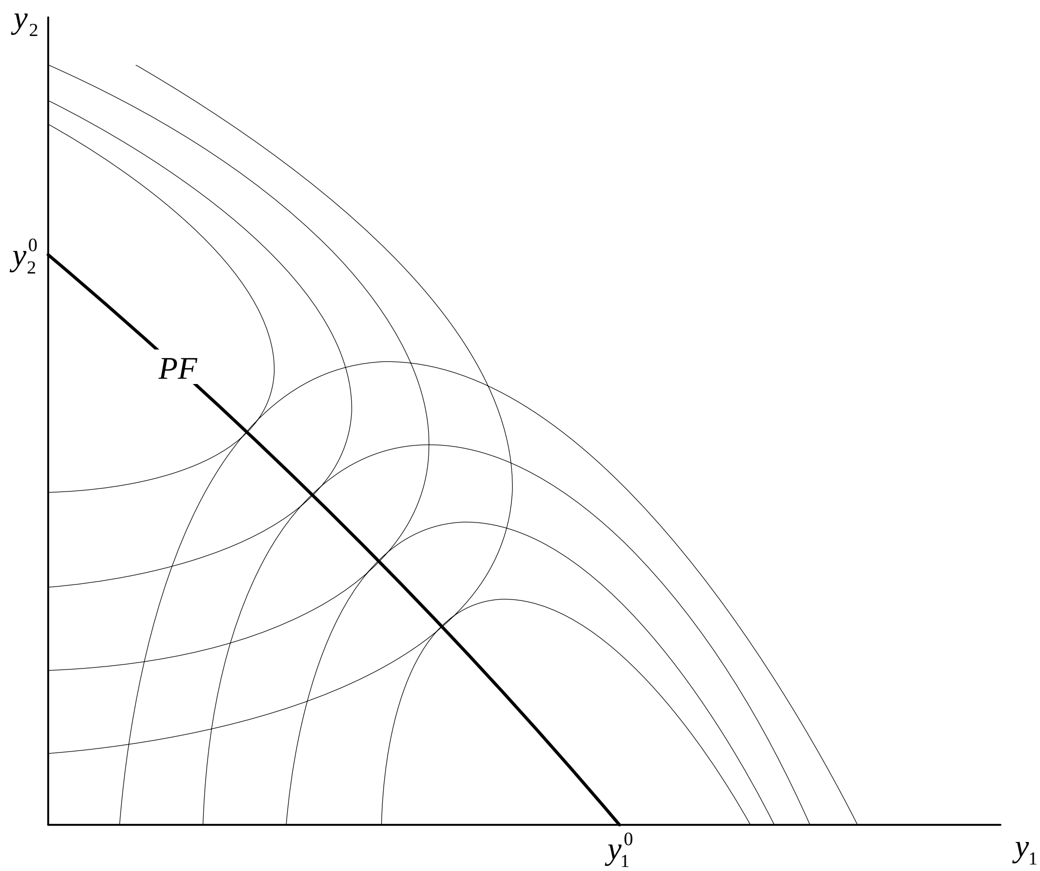

Figure 4-64

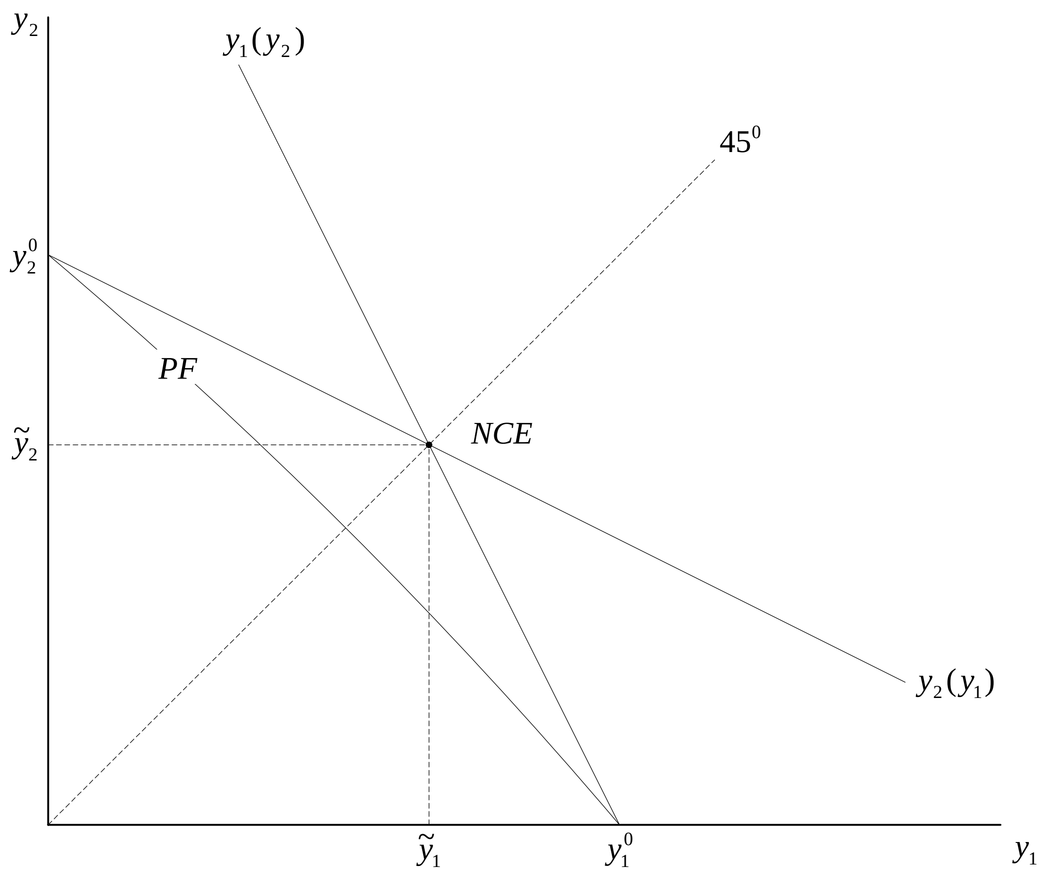

Figure 4-65

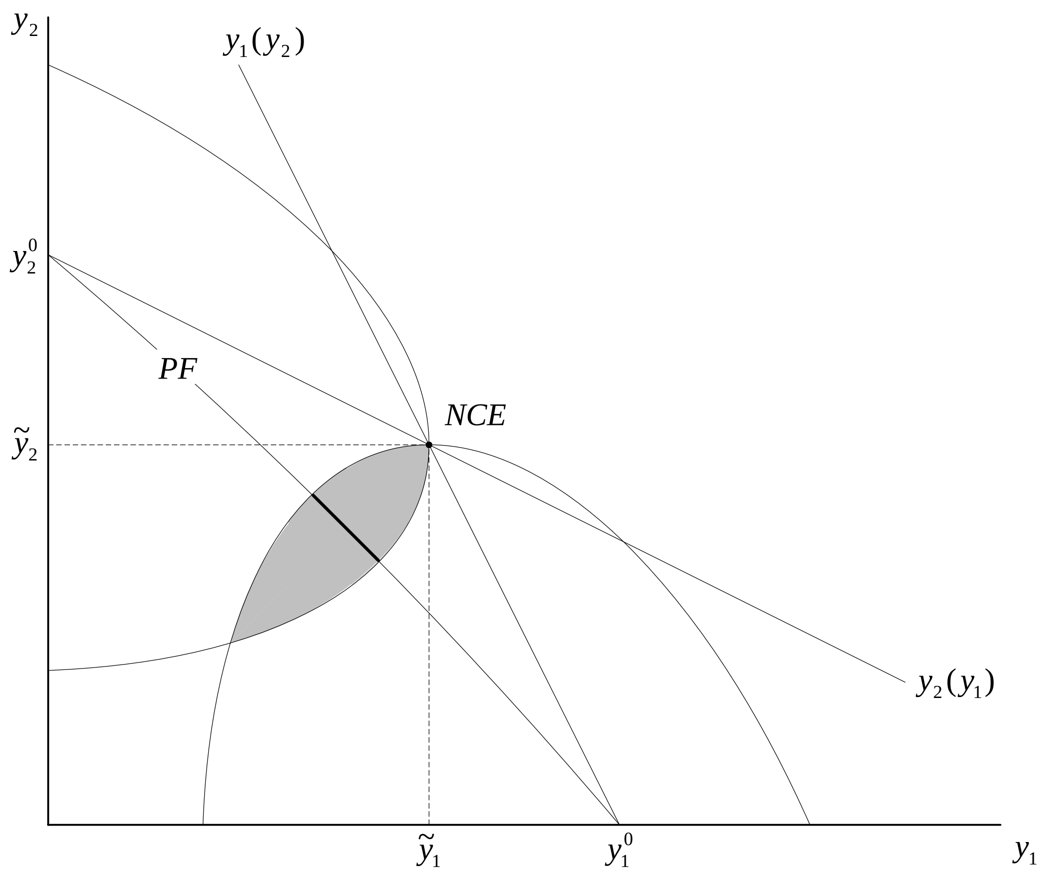

Figure 4-66

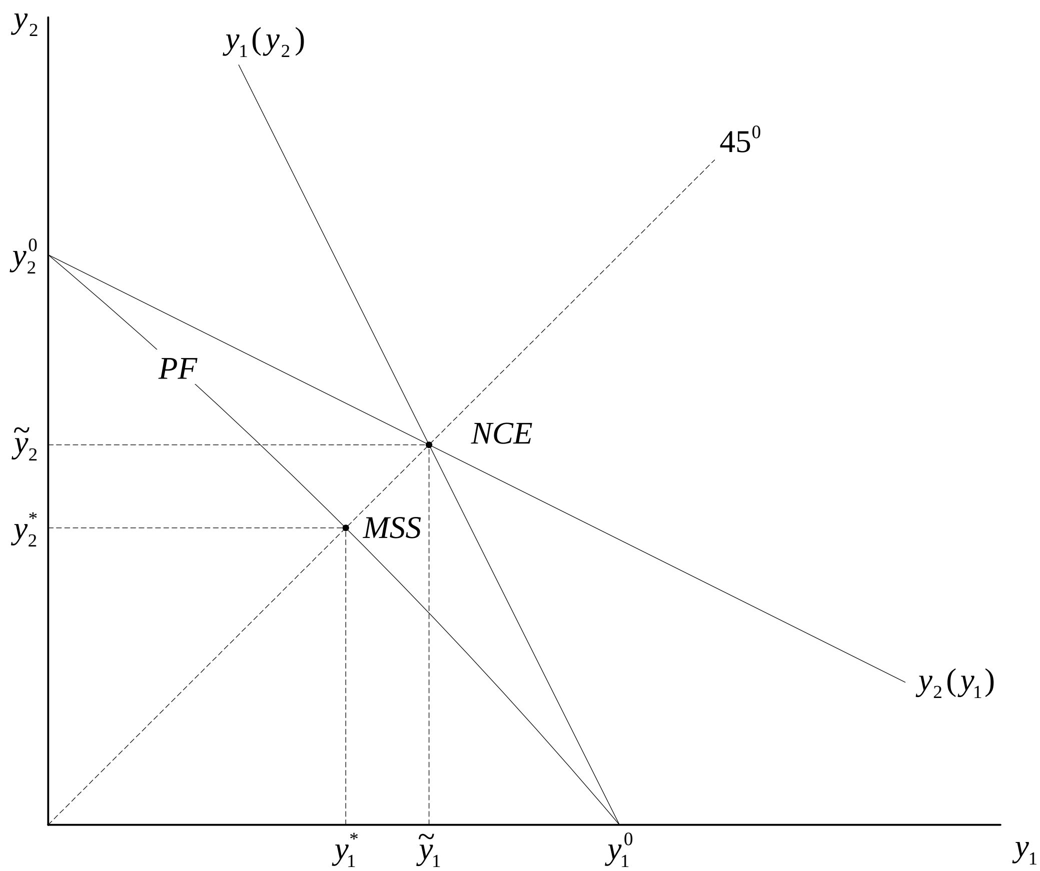

Figure 4-67

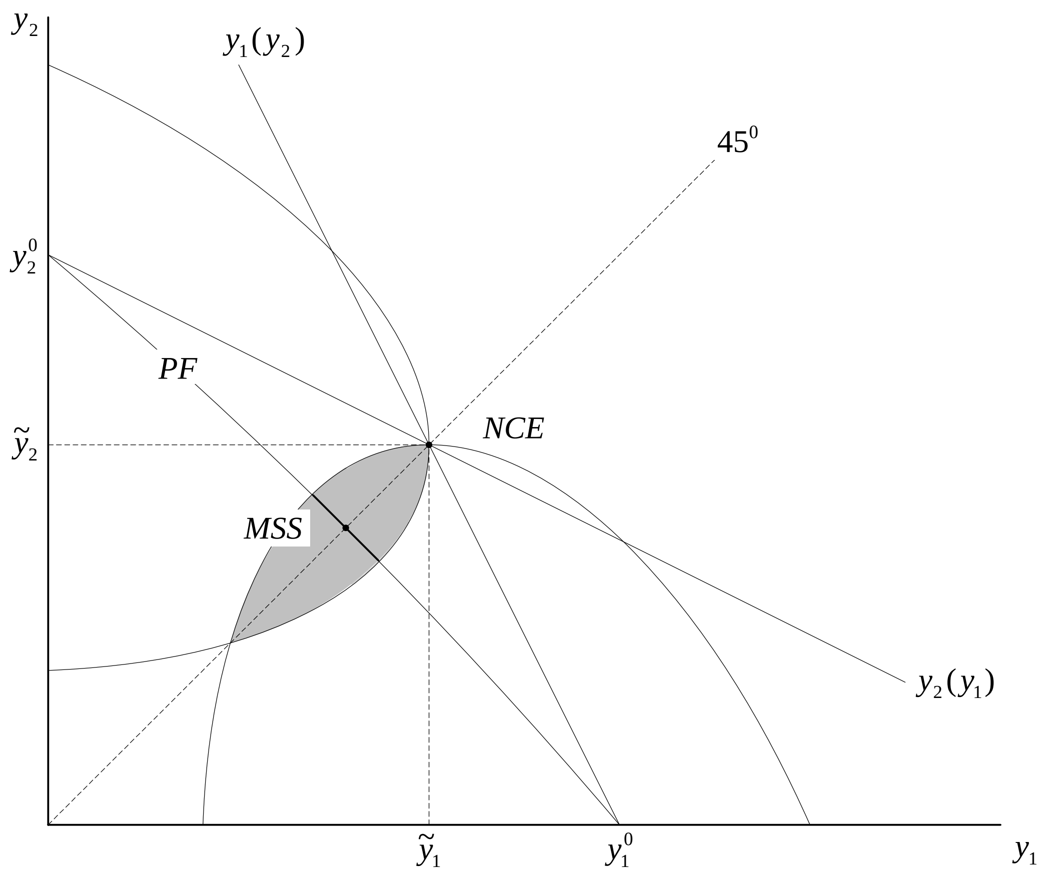

Figure 4-68

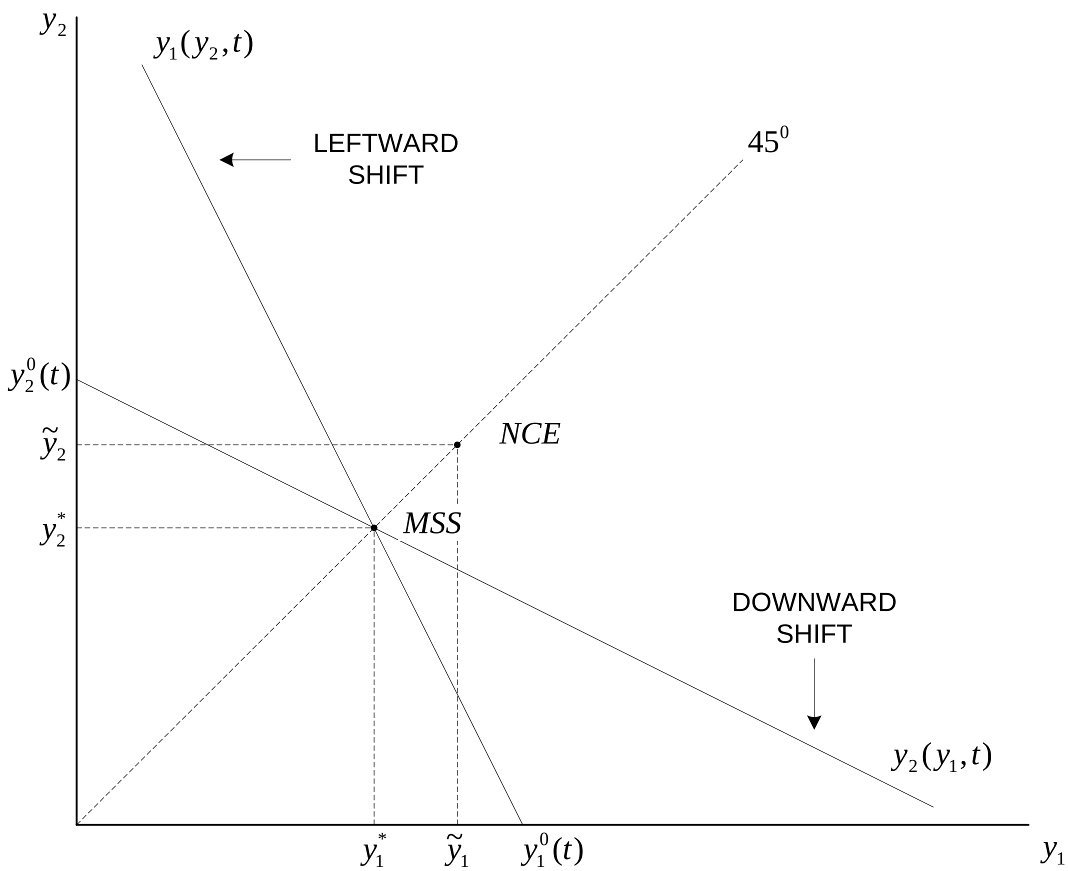
y_2
$y_1(y_2, t)$
LEFTWARD
SHIFT
45^0
$y_2^0(t)$
$\tilde{y}_2$
NCE
y_2^*
MSS
DOWNWARD
SHIFT
$y_2(y_1, t)$
y_1^*
$\tilde{y}_1$
$y_1^0(t)$
y_1

Figure 4-69

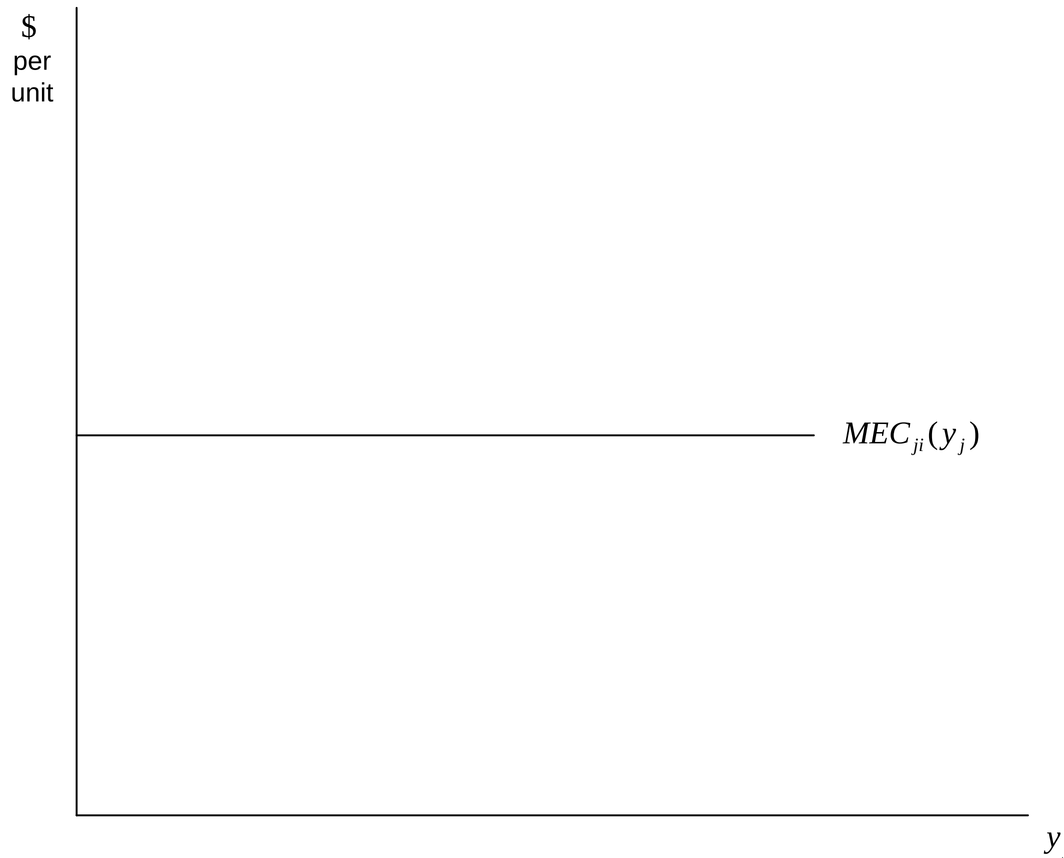

Figure 4-70

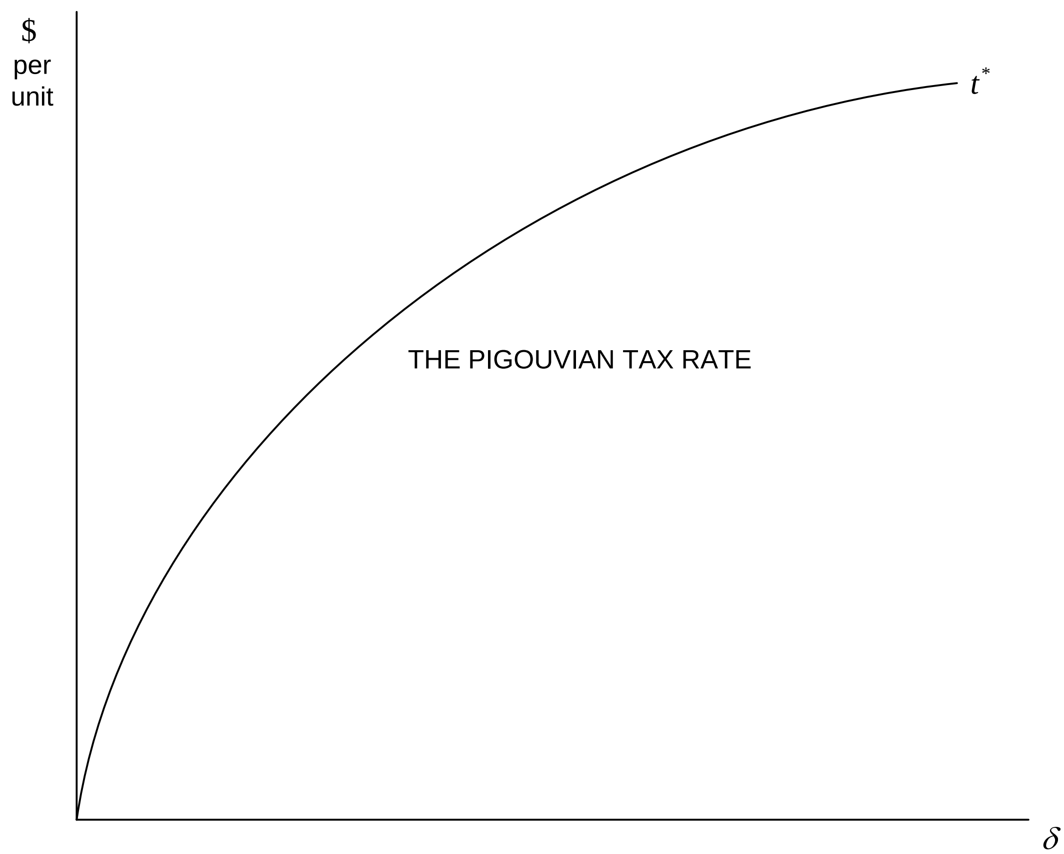

Figure 4-71

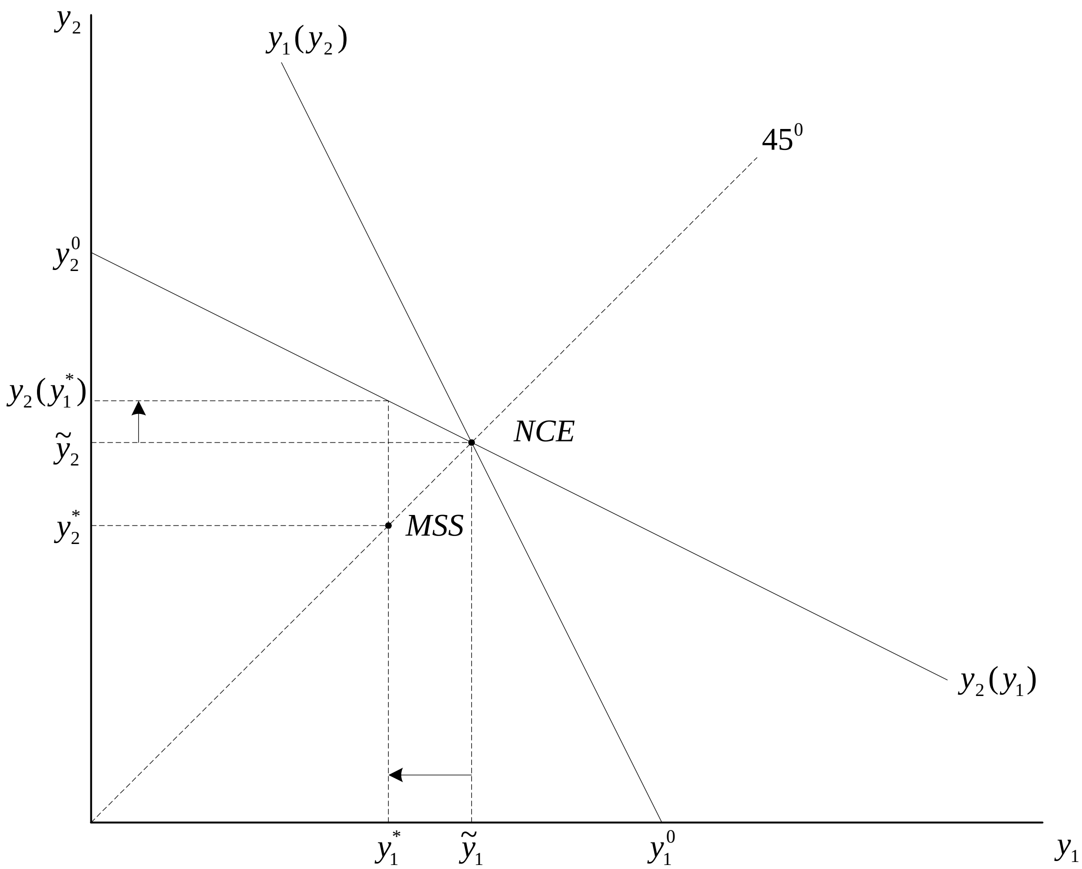

Figure 4-72