ECONOMICS 313

SECOND MIDTERM EXAM

FALL 2022

Answer each of the following 20 questions. Each question is worth one point. This examination accounts for 25% of your assessment on this course. Time allowed: 40 minutes.

(1) This exam is closed-book and this means that you may not consult any material other than the examination paper itself.

(2) You are to work alone on this exam. You may not accept nor solicit assistance of any kind from any other person or entity.

Questions 1 to 12 relate to the following information

Consider am economy with two production sectors (Y and X), two factors of production (K and L), and *N* individuals. The production technology in sector Y is

$$Y = f_Y(K,L) = K^{\frac{1}{2}} L^{\frac{1}{4}}$$

and there is a quasi-fixed managerial-labour input requirement of $F_Y = 4$. The production technology in sector X is

$$X = f_X(K,L) = K^{\frac{1}{2}} L^{\frac{1}{4}}$$

and there is a quasi-fixed managerial-labour input requirement of $F_X = 2$.

Individuals have identical preferences represented by utility function

$$u(x,y,l) = xy^2 l^3$$

Recall that the TRS for a Cobb-Douglas production function of the form

$$f(K,L) = K^a L^b$$

is

$$TRS = \frac{bK}{aL}$$

Recall that the MRS functions for a Cobb-Douglas utility function of the form

$$u(x,y,l) = x^{\alpha} y^{\beta} l^{\delta}$$

are

$$MRS_{xy} = \frac{\alpha y}{\beta x}$$

and

$$MRS_{ly} = \frac{\delta y}{\beta l}$$

Your ultimate goal here is to find the free-entry competitive equilibrium in this economy, where labour is specified as the numeraire. With that in mind, set $w = 1$ from the very beginning; this will simplify the algebra.

Production in Sector X

1. The conditional demand for non-managerial labour by a firm in sector X is

A. $\hat{L}_X(x, w, r) = x^{\frac{4}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$ ****

B. $\hat{L}_X(x, w, r) = x^{\frac{1}{3}}(2r)^{\frac{2}{3}}$

C. $\hat{L}_X(x, w, r) = x^{\frac{2}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$

D. $\hat{L}_X(x, w, r) = x^{\frac{4}{3}}\left(\frac{r}{2}\right)^{\frac{1}{3}}$

2. The conditional demand for capital by a firm in sector X is

A. $\hat{K}_X(x, w, r) = \dfrac{x^{\frac{4}{3}}}{(2r)^{\frac{2}{3}}}$

B. $\hat{K}_X(x, w, r) = \dfrac{x^{\frac{2}{3}}}{(2r)^{\frac{1}{3}}}$

C. $\hat{K}_X(x, w, r) = x^{\frac{4}{3}}\left(\frac{2}{r}\right)^{\frac{1}{3}}$ ****

D. $\hat{K}_X(x, w, r) = x^{\frac{2}{3}}\left(\frac{2}{r}\right)^{\frac{2}{3}}$

The cost function for a firm in sector X can now be found by substituting these conditional factor demands into the expression for cost. That cost function is

$$c_X(x, w, r) = 3x^{\frac{4}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}} + 2$$

3. The marginal cost function for a firm in sector X is

A. $MC_X(x, w, r) = 2x^{\frac{1}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}} + 2$

B. $MC_X(x, w, r) = 4x^{\frac{1}{3}}\left(\frac{r}{2}\right)^{\frac{2}{3}}$ ****

C. $MC_X(x, w, r) = 3x^{\frac{1}{3}}(2r)^{\frac{1}{3}}$

D. $MC_X(x, w, r) = 2x^{\frac{1}{3}}(2r)^{\frac{2}{3}}$

4. The supply function for a firm in sector X is

A. $x(p_X, w, r) = \frac{p_X^3}{16r}$

B. $x(p_X, w, r) = \frac{p_X^2}{8r}$

C. $x(p_X, w, r) = \frac{p_X^3}{8r^2}$

D. $x(p_X, w, r) = \frac{p_X^3}{16r^2}$ **********

The factor demands for a firm in sector X can now be found by substituting the supply function into the conditional factor demands. These factor demands are

$$K_X(p_X, w, r) = \frac{p_X^4}{32r^3}$$

and

$$L_X(p_X, w, r) = \frac{p_X^4}{64r^2}$$

for capital and non-managerial labour respectively.

The profit function for a firm in sector X can now be found by substituting the supply function into the expression for profit. That profit function is

$$\pi_X(p_X, w, r) = \frac{p_X^4}{64r^2} - 2$$

Production in Sector Y

Following the same procedures we used for calculating the production-related functions for a firm in sector X, we can find the comparable functions for a firm in sector Y. The key functions we need are the following.

Supply function:

$$y(p_Y, w, r) = \frac{p_Y^3}{16r^2}$$

Factor demands:

$$K_Y(p_Y, w, r) = \frac{p_Y^4}{32r^3} \quad \text{and} \quad L_Y(p_Y, w, r) = \frac{p_Y^4}{64r^2}$$

The profit function:

$$\pi_Y(p_Y, w, r) = \frac{p_Y^4}{64r^2} - 4$$

Henceforth, assume there is free entry into both sectors.

Consumption

5. Demand for *y* by an individual with wealth M_i is

A. $y_i(p_X, p_Y, w, r) = \frac{M_i}{2p_Y}$

B. $y_i(p_X, p_Y, w, r) = \frac{M_i}{p_Y}$

C. $y_i(p_X, p_Y, w, r) = \frac{M_i}{3p_Y}$ ****

D. $y_i(p_X, p_Y, w, r) = \frac{2M_i}{3p_Y}$

6. Demand for *x* by an individual with wealth M_i is

A. $x_i(p_X, p_Y, w, r) = \frac{M_i}{3p_X}$

B. $x_i(p_X, p_Y, w, r) = \frac{M_i}{6p_X}$ *****

C. $x_i(p_X, p_Y, w, r) = \frac{2M_i}{3p_X}$

D. $x_i(p_X, p_Y, w, r) = \frac{M_i}{2p_X}$

Equilibrium

7. The equilibrium price of *x*, as a function of *r*, is

A. $p_X(r) = r^{\frac{3}{2}} 2^{\frac{2}{3}}$

B. $p_X(r) = r^{\frac{2}{3}} 2^{\frac{5}{4}}$

C. $p_X(r) = r^{\frac{1}{2}} 2^{\frac{7}{2}}$

D. $p_X(r) = r^{\frac{1}{2}} 2^{\frac{7}{4}}$ ****

8. The equilibrium number of firms in sector X, as a function of *r*, is

A. $n_X(r) = \dfrac{\tilde{L} + r\tilde{K}}{48}$ *****

B. $n_X(r) = \dfrac{\tilde{L} + r\tilde{K}}{16}$

C. $n_X(r) = \dfrac{\tilde{L} + r\tilde{K}}{32}$

D. $n_X(r) = \dfrac{\tilde{L} + r\tilde{K}}{64}$

Following the same procedures used for calculating $p_X(r)$ and $n_X(r)$, we can find comparable functions for sector Y. These functions are

$$p_Y(r) = 4r^{\frac{1}{2}}$$

and

$$n_Y(r) = \frac{\tilde{L} + r\tilde{K}}{48}$$

9. Aggregate demand for capital in sector X, as a function of *r*, is

A. $D_K^X(r) = \dfrac{\tilde{L} + r\tilde{K}}{6r}$

B. $D_K^X(r) = \dfrac{2(\tilde{L} + r\tilde{K})}{3r}$

C. $D_K^X(r) = \dfrac{\tilde{L} + r\tilde{K}}{12r}$ *****

D. $D_K^X(r) = \dfrac{\tilde{L} + r\tilde{K}}{18r}$

10. Aggregate demand for capital in sector Y, as a function of *r*, is

A. $D_K^Y(r) = \dfrac{\tilde{L} + r\tilde{K}}{6r}$ ******

B. $D_K^Y(r) = \dfrac{\tilde{L} + r\tilde{K}}{48r}$

C. $D_K^Y(r) = \dfrac{2(\tilde{L} + r\tilde{K})}{3r}$

D. $D_K^Y(r) = \dfrac{\tilde{L} + r\tilde{K}}{32r}$

11. The equilibrium rental rate is

A. $r^* = \dfrac{2\tilde{L}}{3\tilde{K}}$

B. $r^* = \dfrac{\tilde{L}}{3\tilde{K}}$ ****

C. $r^* = \dfrac{\tilde{L}}{2\tilde{K}}$

D. $r^* = \dfrac{\tilde{L}}{6\tilde{K}}$

12. The equilibrium GDP for this economy is

A. $GDP = \dfrac{\tilde{L}}{3\tilde{K}}$

B. $GDP = \dfrac{3\tilde{L}}{2}$

C. $GDP = \dfrac{2\tilde{L}}{3}$ ****

D. $GDP = \dfrac{\tilde{L}}{2\tilde{K}}$

13. Which of the following is the best description of productive efficiency?

A. An allocation is productively efficient if it is not possible, by re-allocating available factors, to produce more of one good without producing less of another. ****

B. An allocation is productively efficient it is not possible to change the number of firms within a sector in a way that allows more aggregate output to be produced in that sector using the same aggregate input values.

C. An allocation is productively efficient if it does not lie outside the production possibility frontier.

D. An allocation is productively efficient if the aggregate quantity of all goods is maximized.

14. Which of the following is the best description of allocative efficiency?

A. An allocation is allocatively efficient if it is not possible, by re-allocating available factors, to produce more of one good without producing less of another.

B. An allocation is allocatively efficient if it lies on the production possibility frontier.

C. An allocation is allocatively efficient if the aggregate quantities of goods produced, and the allocation of those goods across individuals, is such that no person can be made better off without making someone else worse off. ****

D. An allocation is allocatively efficient if the marginal rates of substitution are equated across all persons at that allocation.

15. Recall that allocative efficiency requires

$$MRS_{xy}^{1} = MRS_{xy}^{2} = ... = MRS_{xy}^{N} = MRT_{xy}$$

That is, marginal rates of substitution between *x* and *y* are equated across consumers, and these are equal to the marginal rate of transformation. Which property (or properties) of the competitive equilibrium are critical for this result?

A. Firms maximize profits.

B. Consumers are price-takers in the output markets.

C. Firms are price-takers in their output markets.

D. All of the above. ******

Questions 16 – 20 relate to the following information on a setting where an activity y generates a positive externality:

$$MPB(y) = 200 - 17y$$

$$MPC(y) = 8y$$

$$MEB(y) = 424 - y$$

Assume that there is no negative externality from this activity, so $MSC(y) \equiv MPC(y)$.

16. Marginal social benefit is

A. $MSB(y) = 200 - 25y$

B. $MSB(y) = 424 - 9y$

C. $MSB(y) = 624 - 18y$ ****

D. $MSB(y) = 224 + 16y$

17. The private optimum is

A. 1

B. 5

C. 8 ****

D. 12

18. The social optimum is

A. 12

B. 18

C. 24 *****

D. 30

19. External benefit at the private optimum is

A. 0

B. 2024

C. 2456

D. 3360 *****

20. Now suppose that a regulation forces the source agent to increase his level of the activity to the social optimum. The gain to the external agents is

A. 2764

B. 6528 **********

C. 7240

D. 4260

ANSWER KEY

1. A
2. C
3. B
4. D
5. C
6. B
7. D
8. A
9. C
10. A
11. B
12. C
13. A
14. C
15. D
16. C
17. C
18. C
19. D
20. B