ECONOMICS 531
PROJECT 3

Implementation of an Aggregate Emissions Target via Emissions Trading

Recall the simple model of a price-taking firm from Project 2. Here we examine a setting where there are *n* such firms selling into a market in which output price is determined globally. Let p denote this output price.

Each firm has variable cost of production given by

(1) $$c(y,x) = \omega(1+\zeta x^2)y^2$$

where y is its output and x is its choice of production process, and $\omega > 0$ and $\zeta > 0$ are the same for all firms.

The firms differ with respect to their emissions functions, as determined for example by their technological vintage. In particular, firm *i* has emissions function

(2) $$e_i(y,x) = \psi_i(1-x)y$$

where $\psi_i > 0$ is a fixed technology parameter. Within the set of *n* firms, the distribution of ψ has mean μ and variance σ^2.

Damage is a function of aggregate emissions:

(3) $$D(E) = \frac{\delta E^2}{2}$$

Recall from Project 2 that we derived the marginal abatement cost function for a firm of this type, and showed that it is given by

(4) $$MAC_i(e_i) = \frac{(p\psi_i - 2\omega e_i)\zeta}{(1+\zeta)\psi_i^2}$$

Your task is to code-up this multiple-source model in Maple and examine its properties by completing the three parts described below.

Part 1. Optimal Abatement with Multiple Sources

(a) Using (4), show that the unregulated level of aggregate emissions is

(5) $$\hat{E} = \frac{pn\mu}{2\omega}$$

Now suppose that the regulator wants to reduce aggregate emissions to a level

(6) $$E = \beta\hat{E}$$

where $\beta < 1$, and that it wants to achieve this emissions target at least cost.

(b) Show that if the optimal solution is interior, then the optimal allocation for firm *i* is

(7) $$e_i^*(\beta) = \frac{p\psi_i(\mu^2 + \sigma^2 - \mu(1-\beta)\psi_i)}{2\omega(\mu^2 + \sigma^2)}$$

(c) What restriction must be placed on the range of ψ to ensure that this optimal solution is interior?

(d) Show that marginal aggregate abatement cost at the optimum is

(8) $$MAC(\beta) = \frac{p\mu(1-\beta)\zeta}{(\mu^2 + \sigma^2)(1+\zeta)}$$

(e) Find the optimal value of β, denoted β^*.

(f) Explain the relationship between β^* and σ^2.

Part 2. Implementation via Emissions Trading

Consider an emissions trading program under which the regulator caps aggregate emissions at

(9) $$E = \beta\hat{E}$$

where $\hat{E}$ is the unregulated level of aggregate emissions from (5) above.

Let g_i denote the *gratis* allocation to firm *i*, and suppose

(10) $$g_i = \beta \hat{e}_i$$

where

(11) $$\hat{e}_i = \frac{p\psi_i}{2\omega}$$

is the unregulated level of emissions from firm *i*. Thus, all permits are allocated initially on a *gratis* basis according to a simple "grandfathering" rule.

Let *q* denote the price of permits.

(a) Write down the profit-maximization problem for firm *i* and solve this problem to derive its output and its emissions as functions of the permit price.

(b) Confirm that the unregulated solutions can be recovered by setting $q = 0$. Explain this result.

(c) Show that the aggregate demand for permits is

(12) $$\tilde{E}(q) = \hat{E} - \frac{q(1+\zeta)(\mu^2 + \sigma^2)n}{2\omega\zeta}$$

(d) Find the equilibrium price of permits, denoted $q(\beta)$.

(e) Confirm that $q(\beta)$ is equal to $MAC(\beta)$ from (8) above. Explain this result.

(f) Find a threshold value of ψ (expressed in terms of μ and σ^2) that partitions the set of firms into buyers and sellers. Which firms are buyers?

(g) Bonus question (no points assigned). Show that there must be at least one buyer if $\sigma^2 > 0$. (You have to know your statistics pretty well to be able to show this).

(h) Calculate the difference in profit for a given firm under the regulated and unregulated settings, and find a threshold value of ψ (expressed in terms of μ and σ^2) that partitions the set of firms into those who win from the regulation and those who lose. Which firms are winners?

(i) Find a sufficient condition on β under which there are no winners in the interior solution.

(j) Show that aggregate abatement cost at the trading equilibrium is

(13) $$AC(\beta) = \frac{(1-\beta)^2 \mu^2 p^2 n\zeta}{4\omega(1+\zeta)(\mu^2 + \sigma^2)}$$

Let β^* denote the optimal abatement policy, as determined in Part 1 above.

(k) Show that the equilibrium price under this optimal policy is

(14) $$q(\beta^*) = \frac{np\mu\delta\zeta}{n\delta(1+\zeta)(\mu^2 + \sigma^2) + 2\omega\zeta}$$

(l) Confirm that $q(\beta^*)$ is increasing in δ, and explain why.

Part 3. Cost-Savings from Emissions Trading

Consider an alternative setting where the firms are given the allocation described in (10) above but are not allowed to trade.

(a) Show that aggregate abatement cost in this alternative setting is

(15) $$AC_0(\beta) = \frac{np^2\zeta(1-\beta)^2}{4\omega(1+\zeta)}$$

(b) Now show that relative to the no-trade setting, emissions trading leads to a reduction in aggregate abatement cost equal to

(16) $$R(\beta) = \frac{np^2\zeta(1-\beta)^2\sigma^2}{4\omega(1+\zeta)(\mu^2+\sigma^2)}$$

(c) Explain why $R(\beta)$ is increasing in σ^2, and why $R(\beta)\big|_{\sigma^2=0} = 0$.

Note that the reduction in abatement cost can be expressed instructively as a fraction of the no-trade abatement cost:

(17) $$r \equiv \frac{R(\beta)}{AC_0(\beta)} = \frac{\sigma^2}{\mu^2+\sigma^2}$$

Now let ε denote the mean of *unregulated* emissions, and let ν^2 denote the variance in *unregulated* emissions.

(d) Express *r* from (17) in terms of ε and ν^2.

In a setting where we have no information other than data on unregulated emissions, this last result provides a *very rough* rule for estimating the gains from using emissions pricing to implement an aggregate emissions target, relative to a simplistic proportional-reduction requirement for the regulated entities.