MICROECONOMIC THEORY

PRACTICE FINAL EXAM

Answer each of the following five questions. All questions are of equal value. This examination accounts for 50% of your overall grade on this course. Time allowed: 3hrs.

QUESTION 1

A consumer has the following utility function:

$$u(x) = x_1^{1/2} + x_2$$

(a) Derive the Marshallian demand functions, and explain why there are two branches to the solution (with the aid of an appropriate diagram).

(b) Suppose $m = 20$ and prices change from $\{p_{1,}^0 p_2^0\} = \{1,10\}$ to $\{p_{1,}^1 p_2^1\} = \{1,9\}$. Find the associated compensating and equivalent variations.

QUESTION 2

The two parts of this question are unrelated.

(a) An individual has indirect utility function

$$v(m) = am^{1/2} + b$$

where the functional dependence on prices has been suppressed. She faces the following prospect over wealth, *m*:

$$\{100,64; ½, ½\}$$

Calculate the risk premium associated with this prospect. What is the maximum premium this agent would be willing to pay for full insurance against the potential wealth loss?

(b) An agent has the following intertemporal utility function

$$u(c) = \log(c_1 - k_1) + \left(\frac{1}{1+\rho}\right)\log(c_2 - k_2)$$

where ρ is her rate of time preference, and k_t is the minimum level of consumption she needs to survive in period *t*. This agent has income y in each period, and can borrow and lend at interest rate r.

Suppose $\rho = r$. Find necessary conditions (in terms of k_1 relative to k_2 and y) under which she will be a saver in period 1. (Hint: Think carefully about when saving is feasible).

QUESTION 3

Consider a firm that uses just one input, and has the following production function:

$$f(x) = x^{1/2}$$

The price of the input is *w*.

(a) Suppose this firm is a price-taker in its product market. Find its supply function and its profit function.

(b) Suppose the output market is perfectly competitive, and entry is free. Suppose also that each firm can produce no less than one unit. Aggregate demand is

$$X = a - bp$$

Find the equilibrium price and the equilibrium aggregate output, and derive a critical value of *w* above which this market cannot support a competitive equilibrium with positive output.

QUESTION 4

The inverse demand curve in an industry is

$$p = 1000 - X$$

There are *n* identical firms each with no variable costs. However, each firm must incur a quasi-fixed cost $F = 400$ in order to produce at all. Firms choose quantity (denoted *y*) and move simultaneously.

(a) Derive the best-response function for a representative firm, and provide an interpretation of this best-response function. Derive the symmetric Nash equilibrium output for a representative firm, and the equilibrium price.

(b) If entry is free, what is the equilibrium number of firms? Is this the efficient number of firms? Explain your answer.

QUESTION 5

Suppose there are *n* agents in an economy, and each agent engages in some activity *x* which produces environmental damage *D*, such that

$$D = \delta X^2 \quad \text{where} \quad X = \sum_{j=1}^{n} x_j$$

The agents derive utility from their own activity but are adversely affected by the environmental damage. In particular, utility for agent *i* is

$$u_i = \theta x_i - D$$

(a) Derive the best-response function (BRF) for country *i*, and find the sole-agent optimum for this agent. (You do <u>not</u> need to find a closed-form solution for the BRF).

(b) Find the symmetric Nash equilibrium level of activity for each agent, denoted $\hat{x}$. Explain the relationship between $\hat{x}$ and the sole-agent optimum when $n = 1$.