Answer to Question 1

(a) The certainty-equivalent wealth is the solution to an indifference equation:

$$v(\hat{m}) = \mathrm{E}[v(m)]$$

In this case,

$$v(\hat{m}) = \hat{m}^{\frac{1}{2}}$$

$$\mathbf{E[}v(m)] = \pi(y-L)^{\frac{1}{2}} + (1-\pi)(y)^{\frac{1}{2}}$$

where y is her salary without loss.

The solution for $\hat{m}$ is messy but it is simple to solve with the numerical values in place:

$$\hat{m} = 4078.04$$

Expected wealth is

$$\mathbf{E[}m] = \pi(y-L) + (1-\pi)y$$

For the values in the Question: $\mathbf{E[}m] = 4800$

The risk premium is

$$R \equiv \mathrm{E}[m] - \hat{m}$$

For the values in the Question: $R = 721.96$. (Prices are irrelevant here).

(b) Expected utility with insurance is

$$\mathbf{E[}v(m;q)] = \pi(y - L - rq + q)^{\frac{1}{2}} + (1-\pi)(y - rq)^{\frac{1}{2}}$$

Differentiate with respect to q and solve to yield $q(r)$. The solution is messy but it is simple to solve with the numerical values in place. For $\pi = \frac{1}{4}$ and $r = \frac{1}{2}$,

$$q(r) = 720$$

That is, less than full insurance, because the price is not actuarially fair.

Answer to Question 2

(a) See Figure A2-1. This agent always chooses $c_1 = \beta c_2$ regardless of *r*. She is a lender in period 1 iff $c_1 < y_1$, in which case her income profile must be at a point like *A* in Figure A2-1, *below* the dashed $c_1 = \beta c_2$ threshold. In that region, $y_1 > \beta y_2$. Conversely, she is a borrower in period 1 iff $c_1 > y_1$, in which case her income profile must be at a point like *B* in Figure 1, *above* the dashed $c_1 = \beta c_2$ threshold. In that region, $y_1 < \beta y_2$.

This simple logic yields the answer in this example only because of the Leontief preferences. For less rigid preferences, it is necessary to solve for the consumption functions, and derive a condition under which $c_1 > y_1$. Let us confirm that this approach yields the same answer we have derived above. The utility maximization problem is

$$\max_c \ \min[c_1, \beta c_2] \quad st \quad c_1 + pc_2 = w$$

where $p = \dfrac{1}{1+r}$ and $w = y_1 + py_2$. Setting $c_1 = \beta c_2$ and substituting into the wealth constraint yields the solution for c_1:

$$c_1(w,r) = \frac{\beta w}{\beta + p}$$

Make the substitution for *w* (but leave *p* as is) and rearrange. We then obtain $c_1 < y_1$ iff $y_1 > \beta y_2$. This is the same result we derived above.

(b) False. This is <u>not</u> a homogenous production function. In particular,

$$f(tx) = a\log(tx_1) + b\log(tx_2) = a\log x_1 + b\log x_2 + (a+b)\log t \neq t^k f(x)$$

for any *t* or *k*. This production function does not exhibit any of DRS, CRS or IRS. It has U-shaped AC because $y = 0$ at $x_1 = 1$ and $x_2 = 1$. Thus, there is a quasi-fixed cost of $(w_1 + w_2)$.

Answer to Question 3

(a) At any prices, cost is minimized where $x_1 = x_2$. Thus, the conditional demands are simply given by

$$x_1(w, y) = y$$

$$x_2(w, y) = y$$

The cost function is

$$c(w, y) = w_1 x_1(w, y) + w_2 x_2(w, y) = y(w_1 + w_2)$$

SL: $\dfrac{\partial c(w, y)}{\partial w_i} = x_i(w, y) = y$

(b) Set up the direct profit maximization problem:

$$\max_x \quad p[x_1^{1/2} + x_2^{1/2}] - w_1 x_1 - w_2 x_2$$

The FOCs yield the input demands:

$$x_1(p, w) = \left(\frac{p}{2w_1}\right)^2 \quad \text{and} \quad x_2(p, w) = \left(\frac{p}{2w_2}\right)^2$$

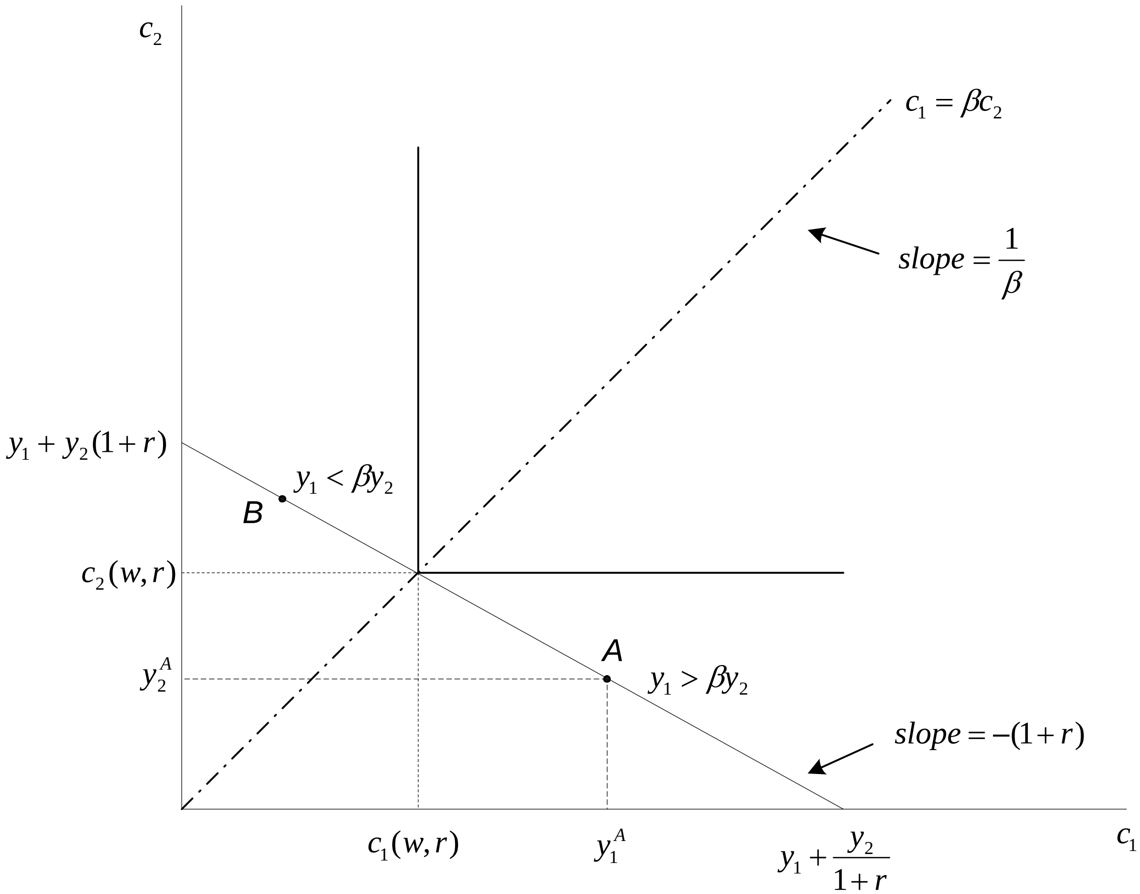
The supply function is

$$y(p, w) = f(x_1(p, w), x_2(p, w)) = \left(\frac{p}{2w_1}\right) + \left(\frac{p}{2w_2}\right) = p\left(\frac{w_1 + w_2}{2w_1 w_2}\right)$$

and the profit function is

$$\pi(p, w) = py(p, w) - w_1 x_1(p, w) - w_2 x_2(p, w) = p^2\left(\frac{w_1 + w_2}{4w_1 w_2}\right)$$

HL: $\dfrac{\partial \pi(p, w)}{\partial p} = y(p, w)$ and $\dfrac{\partial \pi(p, w)}{\partial w_i} = -x_i(p, w)$.



c_2
$c_1 = \beta c_2$
$slope = \frac{1}{\beta}$
$y_1 + y_2(1+r)$
$y_1 < \beta y_2$
B
$c_2(w,r)$
A
y_2^A
$y_1 > \beta y_2$
$slope = -(1+r)$
$c_1(w,r)$
y_1^A
$y_1 + \frac{y_2}{1+r}$
c_1

FIGURE A2-1