MICROECONOMIC THEORY

PRACTICE FINAL EXAM

ANSWER GUIDE

Question 1

(a) $$\mathrm{L} = x_1^{1/2} + x_2 + \lambda(m - p_1 x_1 - p_2 x_2)$$

First-order conditions:

$$\frac{1}{2} x_1^{-1/2} - \lambda p_1 = 0$$

$$1 - \lambda p_2 = 0$$

Therefore

$$\lambda = \frac{1}{p_2}$$

$$\Rightarrow \quad x_1(p,m) = \frac{p_2^2}{4p_1^2} \quad \text{if} \quad p_1\left(\frac{p_2^2}{4p_1^2}\right) \leq m$$

Then from the budget constraint:

if $m \geq \dfrac{p_2^2}{4p_1}$:

$$x_1(p,m) = \frac{p_2^2}{4p_1^2}$$

$$x_2(p,m) = \frac{m - p_1 x_1}{p_2}$$

$$= \frac{4mp_1 - p_2^2}{4p_1 p_2}$$

if $m < \dfrac{p_2^2}{4p_1}$:

$$x_2(p,m) = 0$$

$$x_1(p,m) = \frac{m}{p_1}$$

That is, a corner solution if $m \le \frac{p_2^2}{4p_1}$.

(b) At $m = 20$ and $\{p_{1,}^0 p_2^0\} = \{1{,}10\}$, $m \le \frac{p_2^2}{4p_1}$. Thus, $x_2(p,m) = 0$. Similarly, at

$\{p_{1,}^1 p_2^1\} = \{1{,}9\}$, $x_2(p,m) = 0$. Thus, $EV = CV = 0$.

Question 2

(a) Calculate expected utility:

$$Eu = \frac{1}{2}(a100^{1/2} + b) + \frac{1}{2}(a64^{1/2} + b) = 9a + b$$

Calculate the certainty-equivalent wealth:

$$a\hat{m}^{1/2} + b = 9a + b$$

$$\Rightarrow \quad \hat{m} = 81$$

Calculate expected wealth:

$$\overline{m} = \frac{1}{2}100 + \frac{1}{2}64 = 82$$

The risk premium is

$$R = \overline{m} - \hat{m} = 82 - 81 = 1$$

The maximum insurance premium she is willing to pay is

$$\hat{T} = E[L] + R = 18 + 1 = 19$$

(b) Write the problem as one where the agent chooses how much to save in period 1:

$$\max_{s} \quad \log(y - s - k_1) + \left(\frac{1}{1+\rho}\right)\log(y + s(1+r) - k_2)$$

Set $\rho = r$ and solve the problem to yield

$$s^* = \frac{k_2 - k_1}{2 + r}$$

This is positive iff $k_2 > k_1$. It is feasible iff $y_1 - s^* - k_1 > 0$ and this requires

$$y > \frac{k_2 + k_1(1+r)}{2 + r}$$

Question 3

(a) Profit maximization yields the factor demand

$$x(p,w) = \left(\frac{p}{2w}\right)^2$$

The supply function is

$$y(p,w) = \frac{p}{2w}$$

The profit function is

$$\pi(p,w) = \frac{p^2}{4w}$$

(b) The easiest solution method is to recognize that we have DRS, so output will be driven down to its minimum feasible level. Thus, aggregate output will be *n*. Set

$$X = a - bp = n$$

and solve for *p*

$$p(n) = \frac{a-n}{b}$$

Substitute for *p* in $y(p,w)$ to yield

$$y(n) = \frac{\left(\frac{a-n}{b}\right)}{2w}$$

Set this equal to one and solve for *n*

$$n = a - 2bw$$

This is positive (and hence aggregate output is positive) iff

$$w < \frac{a}{2b}$$

Question 4

(a) The profit maximization problem for firm *i* is

$$\max_{y_i} \quad [1000-(y_i+\sum_{j\neq i} y_j)]y_i - 400$$

The reaction function (first-order condition) for firm *i* is

$$y_i = \frac{1000-\sum_{j\neq i} y_j}{2}$$

Interpretation: the reaction function describes the profit-maximizing choice of output for firm *i* in response to an expectation of what other firms will produce. It is downward-sloping with respect to output from other firms because higher output from other firms means that price will be lower, and hence, the profitability of output from firm *i* is reduced.

In symmetric Nash equilibrium

$$y_i = y \quad \forall i \quad \text{and} \quad \sum_{j\neq i} y_j = (n-1)y$$

Substitution into the reaction function for firm *i* yields

$$\hat{y} = \frac{1000}{n+1}$$

$$\Rightarrow \qquad \hat{Y} = n\hat{y} = \frac{1000n}{n+1}$$

$$\Rightarrow \qquad \hat{p} = 1000-\hat{Y} = \frac{1000}{n+1}$$

(b) Entry will drive profit to zero:

$$\Pi(n) = \hat{p}\hat{y} - 400$$

Thus, at $\Pi(n) = 0$,

$$\left(\frac{1000}{n+1}\right)\left(\frac{1000}{n+1}\right) = 400$$

$$\Rightarrow \qquad \hat{n} = 49$$

This is a natural monopoly:

$$AC = \frac{400}{y}$$

which is declining for all *y*. Thus, efficiency requires only one firm. However, that firm would need to be regulated or else its exploitation of monopoly power will distort the output choice away from its efficient level.

Question 5

(a) Agent *i* solves the following problem:

$$\max_{x} \ \theta x_i - \delta(x_i + X_{-i})^2$$

The first-order condition is a best-response function:

$$\theta - 2\delta(x_i + X_{-i}) = 0$$

The sole-agent optimum is found by setting $X_{-i} = 0$ in the BRF and solving for x_i:

$$x_i^0 = \frac{\theta}{2\delta}$$

(b) In a symmetric Nash equilibrium, $x_i = \hat{x} \ \ \forall i$ and $X_{-i} = (n-1)\hat{x}$. Making these substitutions and solving yields

$$\hat{x} = \frac{\theta}{2\delta n}$$

If $n = 1$ then $\hat{x} = x^0$; there is no externality if there is only one country.