MICROECONOMIC THEORY

PRACTICE MIDTERM

ANSWER GUIDE

Answer to Question 1

(a) Set up the Lagrangean

$$L = x_1^{1/3} + x_2^{1/3} + \lambda(m - p_1 x_1 - p_2 x_2)$$

and derive the FOCs:

$$\frac{1}{3x_i^{2/3}} = \lambda p_i$$

Take the ratio for $i = 1,2$ to obtain the tangency condition:

$$\left(\frac{x_2}{x_1}\right)^{2/3} = \frac{p_1}{p_2}$$

which can be rearranged to obtain

$$x_2 = \left(\frac{p_1}{p_2}\right)^{3/2} x_1$$

Substitute this expression into the budget constraint and solve for x_1:

$$x_1(p, m) = \frac{m}{p_1\left(1 + \frac{p_1^{1/2}}{p_2^{1/2}}\right)}$$

The expression for $x_2(p, m)$ follows by symmetry of the utility function.

To simplify notation, write the Marshallian demands as

$$x_1(p, m) = \omega_1(p_1, p_2)m$$

$$x_2(p, m) = \omega_2(p_1, p_2)m$$

Note that these are linear in income because the preferences are homothetic.

(b) Substitute the Marshallian demands into the utility function to derive the indirect utility function:

$$v(p, m) = (\omega_1^{1/3} + \omega_2^{1/3})m^{1/3}$$

Then set $v(p,m)=u$ and $m=e(p,u)$, and solve for $e(p,u)$ to obtain

$$e(p,u)=\left(\frac{u}{\omega_1^{1/3}+\omega_2^{1/3}}\right)^3$$

(c) There are three alternative ways to approach this. The most elegant approach is to use the cross-price Slutsky equation:

$$\frac{\partial x_i}{\partial p_j}=\frac{\partial h_i}{\partial p_j}-x_j\frac{\partial x_i}{\partial m}$$

which can be arranged (with $i=1$ and $j=2$) as

$$\frac{\partial h_1}{\partial p_2}=\frac{\partial x_1}{\partial p_2}+x_2\frac{\partial x_1}{\partial m}=\frac{\partial \omega_1}{\partial p_2}m+\omega_2 m\omega_1$$

Alternatively, begin with the duality property of the consumer problem (from which the Slutsky equation is derived):

$$h_i(p,u)=x_i(p,e(p,u))$$

and differentiate with respect to p_j. The least elegant approach is to use brute force: invoke Shephard's lemma and differentiate $e(p,u)$ with respect to p_i and then with respect to p_j.

Answer to Question 2

Roy's identity:

$$x_i(p,m)=\frac{-\partial v/\partial p_i}{\partial v/\partial m}$$

$$\Rightarrow \qquad x_i(p,m)=\frac{mp_j}{p_i(p_i+p_j)}$$

$$\therefore \qquad x_1(p,m)=\frac{mp_2}{p_1(p_1+p_2)} \quad \text{and} \quad x_2(p,m)=\frac{mp_1}{p_2(p_2+p_1)}$$

A luxury good is one for which $\eta_i>1$, where

$$\eta_i = \left(\frac{\partial x_i}{\partial m}\right)\left(\frac{m}{x_i}\right)$$

In this case, $\eta_1 = 1$. Thus, x_1 is not a luxury.

The demand for x_1 is price-inelastic if $|\varepsilon_{11}| < 1$, where

$$\varepsilon_{11} = \left(\frac{\partial x_1}{\partial p_1}\right)\left(\frac{p_1}{x_1}\right)$$

In this case,

$$|\varepsilon_{11}| = \frac{2p_1 + p_2}{p_1 + p_2} > 1 \text{ at positive prices.}$$

(b) Invert the indirect utility function:

$$e(p,u) = \frac{p_1 p_2 u^2}{(p_1 + p_2)}$$

Then by Shephard's lemma:

$$x_i(p,u) = \frac{\partial e(p,u)}{\partial p_i}$$

$$\therefore \qquad x_1(p,u) = u^2\left(\frac{p_2}{p_1 + p_2}\right)^2$$

and

$$x_2(p,u) = u^2\left(\frac{p_1}{p_1 + p_2}\right)^2$$

(c) They are substitutes:

$$\frac{\partial x_i(p,m)}{\partial p_j} = \frac{m}{(p_i + p_j)^2} > 0$$

Answer to Question 3

(a) This is simpler to solve if $u(x)$ is first transformed to

$$u(x) = x_1 x_2$$

Set up the expenditure minimization problem:

$$\min_x \; p_1 x_1 + p_2 x_2 \quad \text{s.t.} \quad x_1 x_2 = u$$

The FOCs yield the tangency condition:

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

The constraint is then used to solve for Hicksian demands:

$$h_1(p,u) = \left(\frac{up_2}{p_1}\right)^{1/2}$$

$$h_2(p,u) = \left(\frac{up_1}{p_2}\right)^{1/2}$$

The expenditure function is

$$e(p,u) = p_1 h_1(p,u) + p_2 h_2(p,u) = 2(up_1 p_2)^{1/2}$$

(b) Since we transformed the utility function to find the expenditure function in part (a), we must also transform the utility function to solve the utility maximization problem, otherwise the indirect utility function we derive will not be consistent with the "units" in which utility is measured in the expenditure function. This will in turn complicate the interpretation of the units in which EV and CV are measured.

Set up the utility maximization problem:

$$\max_x \; x_1 x_2 \quad \text{s.t} \quad p_1 x_1 + p_2 x_2 = m$$

The FOCs yield the tangency condition:

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

The constraint is then used to solve for Marshallian demands:

$$x_1(p,u) = \frac{m}{2p_1}$$

$$x_2(p,u) = \frac{m}{2p_2}$$

The indirect utility function is

$$v(p,m) = x_1(p,m) \cdot x_2(p.m) = \frac{m^2}{4p_1p_2}$$

Note that this is convex in *p*. (The Hessian matrix is positive definite)

(c) (i) To summarize: $m = 10$, $\{p_1^0, p_2^0\} = \{1,1\}$ and $\{p_1^1, p_2^1\} = \{2,1\}$

Compensating variation:

$$CV = m - e(p^1, u^0) = m - e(p^1, v(p^0, m))$$

where we use $v(p,m)$ evaluated at p^0 to find u^0. In particular,

$$v(p^0, m) = \frac{10^2}{4} = 25$$

Thus,

$$CV = 10 - 2(25 \cdot 2 \cdot 1)^{1/2} = -4.14$$

Equivalent variation:

$$EV = e(p^0, u^1) - m = e(p^0, v(p^1, m)) - m$$

where we use $v(p,m)$ evaluated at p^1 to find u^1. In particular,

$$v(p^1, m) = \frac{10^2}{8} = 12.5$$

Thus,

$$EV = 2(12.5 \cdot 1 \cdot 1)^{1/2} - 10 = -2.93$$

(ii) Change in consumer surplus:

$$\Delta CS = \int_{p_1^1}^{p_1^0} x_1(p,m)dp_1$$

In this case,

$$\Delta CS = \int_{p_1^1}^{p_1^0} \left(\frac{m}{2p_1}\right)dp_1 = \frac{m}{2}\left[\log(p_1)\right]_2^1 = 5[\log(1) - \log(2)] = -5\log(2) = -3.47$$

Thus, we have $|CV| > |\Delta CS| > |EV|$, as expected for a price rise for a normal good.