

MICROECONOMIC THEORY
PRACTICE FIRST MIDTERM EXAM

QUESTION 1

A consumer has the following utility function:

$$u(x) = x_1^{1/3} + x_2^{1/3}$$

(a) Show that the Marshallian demands are given by

$$x_1(p, m) = \frac{m}{p_1 \left(1 + \frac{p_1^{1/2}}{p_2^{1/2}} \right)}$$

$$x_2(p, m) = \frac{m}{p_2 \left(1 + \frac{p_2^{1/2}}{p_1^{1/2}} \right)}$$

To simplify notation from this point forward, write these Marshallian demands as

$$x_1(p, m) = \omega_1(p_1, p_2)m$$

$$x_2(p, m) = \omega_2(p_1, p_2)m$$

(b) Derive the expenditure function.

(c) Let $h_1(p, u)$ denote the Hicksian demand for x_1 . Show that

$$\frac{\partial h_1(p, u)}{\partial p_2} = m \left(\frac{\partial \omega_1}{\partial p_2} + \omega_1 \omega_2 \right)$$

Hint: you do not have to derive the Hicksian demands to show this.

Bonus question: Are the preferences homothetic?

QUESTION 2

A consumer has the following indirect utility function:

$$v(p, m) = \left(\frac{(p_1 + p_2)m}{p_1 p_2} \right)^{1/2}$$

- (a) Find the Marshallian demands. Is x_1 a luxury good? Is the demand for x_1 price-inelastic? Explain your answers.
- (b) Find the expenditure function and the Hicksian demands.
- (c) Are x_1 and x_2 complements or substitutes? Explain your answer.

QUESTION 3

Read all parts of this question before proceeding to answer any part. A consumer has the following utility function:

$$u(x) = \log x_1 + \log x_2$$

- (a) Derive the expenditure function.
- (b) Derive the indirect utility function.
- (c) Suppose income is $m = 10$, the price of x_2 is $p_2 = 1$, and the price of x_1 rises from $p_1^0 = 1$ to $p_1^1 = 2$.
- (i) Calculate the compensating and equivalent variations associated with the price change.
- (ii) Calculate the change in consumer surplus associated with the price change. Explain the relationship between this change in consumer surplus and your answers to part (c)(i).