MICROECONOMIC THEORY
PRACTICE SECOND MIDTERM
ANSWER GUIDE

Answer to Question 1

(a) The certainty-equivalent wealth is the solution to an indifference equation:

$$v(\hat{m}) = \mathrm{E}[v(m)]$$

In this case,

$$v(\hat{m}) = \log(\hat{m})$$

and

$$\mathbf{E[}v(m)] = \pi \log(m_1) + (1-\pi)\log(m_2)$$

Solving for $\hat{m}$ yields

$$\hat{m} = m_1^{\pi} m_2^{1-\pi}$$

For the values in the Question: $m_1 = 10000$, $m_2 = 4096$ and $\pi = 0.25$. Thus, $\hat{m} = 8000$

Expected wealth is

$$\mathbf{E[}m] = \pi m_1 + (1-\pi)m_2$$

For the values in the Question: $\mathbf{E[}m] = 8524$

The risk premium is

$$R \equiv \mathrm{E}[m] - \hat{m}$$

For the values in the Question: $R = 524$

(b) If she purchases full insurance for a total premium *T*, then her expected utility is

$$\mathbf{E}[v(m)_F] = \pi \log(m_1 - T) + (1-\pi)\log(m_1 - T) = \log(m_1 - T)$$

In the absence of insurance, her expected utility is

$$\mathbf{E}[v(m)_0] = \pi \log(m_1) + (1-\pi)\log(m_2)$$

$$= \log(m_1^{\pi} m_2^{1-\pi})$$

The maximum total premium she is willing to pay solves the indifference equation

$$\mathbf{E}[v(m)_F] = \mathbf{E}[v(m)_0]$$

Thus,

$$T_{\max} = m_1 - m_1^{\pi} m_1^{1-\pi}$$

This can be expressed as

$$T_{\max} = \mathbf{E}[L] + R$$

where $\mathbf{E}[L]$ is expected loss:

$$\mathbf{E}[L] = (1-\pi)(m_1 - m_2)$$

This relationship between $T_{\max}$ and *R* holds for any utility function.

Answer to Question 2

(a) Set up the utility maximization problem:

$$\max_{c} \; c_1^{1/2} + \beta c_2^{1/2} \quad st \quad c_1 + pc_2 = w$$

where

$$p = \frac{1}{1+r} \quad \text{and} \quad w = y_1 + py_2 \,.$$

The FOCs with respect to c_1 and c_2 yield the standard tangency condition:

$$c_2 = \frac{\beta^2 c_1}{p^2}$$

Substitution into the wealth constraint then yields the Marshallian demands (and they *are* just special cases of Marshallian demands):

$$c_1(p,w) = \frac{pw}{p+\beta^2} \quad \text{and} \quad c_2(p,w) = \frac{\beta^2 w}{p(p+\beta^2)}$$

The agent is a lender in period 1 iff $c_1(p,w) < y_1$. Making the substitutions for p and w in $c_1(p,w)$ then yields the following necessary and sufficient condition:

$$r > \frac{1}{\beta}\left(\frac{y_2}{y_1}\right)^{1/2} - 1$$

(b) False. This production function is homogenous of degree ½ (and hence exhibits DRS) regardless of the value of *a* and *b*. In particular,

$$f(tx) = a(tx_1)^{1/2} + b(tx_2)^{1/2} = t^{1/2}[ax_1^{1/2} + bx_2^{1/2}] = t^{1/2} f(x)$$

Answer to Question 3

(a) Set up the cost minimization problem (expressed more generally than in the question):

$$\min_x \; w_1x_1 + w_2x_2 \quad \text{s.t.} \quad ax_1^{1/2} + bx_2^{1/2} = y$$

The FOCs with respect to x_1 and x_2 yield the tangency condition:

$$\frac{w_1}{w_2} = \frac{ax_2^{1/2}}{bx_1^{1/2}}$$

The constraint is then used to solve for conditional input demands:

$$x_1(w,y) = y^2\left(\frac{aw_2}{a^2w_2 + b^2w_1}\right)^2$$

$$x_2(w,y) = y^2\left(\frac{bw_1}{a^2w_2 + b^2w_1}\right)^2$$

The cost function is

$$c(y,w) = w_1x_1(y,w) + w_2x_2(y,w) = y^2\frac{w_1w_2}{a^2w_2 + b^2w_1}$$

Verification of Shephard's lemma:

$$\frac{\partial c(w,y)}{\partial w_1} = y^2\left(\frac{(a^2w_2 + b^2w_1)w_2 - w_1w_2b^2}{(a^2w_2 + b^2w_1)^2}\right) = y^2\left(\frac{a^2w_2^2}{(a^2w_2 + b^2w_1)^2}\right) = x_1(w,y)$$

and similarly for $x_2(w,y)$.

(b) Set up the direct profit maximization problem (since this is simpler than the two-stage approach):

$$\max_x \quad p[\log x_1 + x_2^{1/2}] - w_1x_1 - w_2x_2$$

The FOCs yield the input demands:

$$x_1(p,w) = \frac{p}{w_1} \quad \text{and} \quad x_2(p,w) = \left(\frac{p}{2w_2}\right)^2$$

The supply function is

$$y(p,w) = f(x_1(p,w), x_2(p,w)) = \log\left(\frac{p}{w_1}\right) + \left(\frac{p}{2w_2}\right)$$

and the profit function is

$$\pi(p,w) = py(p,w) - w_1x_1(p,w) - w_2x_2(p,w) = p\left(\log\left(\frac{p}{w_1}\right) + \left(\frac{p}{2w_2}\right)\right) - p - \frac{p^2}{4w_2}$$

Verification of Hotelling's lemma (with respect to *p*):

$$\frac{\partial \pi}{\partial p} = p\cdot\frac{1}{p} + \log\left(\frac{p}{w_1}\right) + \frac{p}{w_2} - 1 - \frac{p}{2w_2} = \log\left(\frac{p}{w_1}\right) + \frac{p}{2w_2} = y(p,w)$$