Answer to Question 1

(a) Set up the Lagrangean and derive the first-order conditions for an interior solution:

$$\frac{1}{x_1^2} = \lambda p_1$$

$$1 = \lambda p_2$$

Solution of these equations in combination with the budget constraint yields

$$x_1(p,m) = \left(\frac{p_2}{p_1}\right)^{1/2}$$

$$x_2(p,m) = \frac{m}{p_2} - \left(\frac{p_1}{p_2}\right)^{1/2}$$

These solutions are only valid if $m \geq (p_1^{1/2} p_2^{1/2})$. If this condition does not hold then the solution is at a corner, in which case

$$x_1(p,m) = \frac{m}{p_1}$$

$$x_2(p,m) = 0$$

See **Figure A1-1.** Note that x_1 is an income neutral good (at the interior solution).

(b) Substitute the interior branch of the solution into the utility function to obtain

$$v(p,m) = \frac{m}{p_2} - 2\left(\frac{p_1}{p_2}\right)^{1/2}$$

Set $v(p,m) = u$ and solve for *m*:

$$e(p,u) = up_2 + 2(p_1 p_2)^{1/2}$$

This is homogeneous of degree one in *p*:

$$e(tp,u) = u(tp_2) + 2(tp_1 tp_2)^{1/2} = te(p,u)$$

(c) By Shephard's lemma:

$$h_1(p,u) = \frac{\partial e(p,u)}{\partial p_1} = \left(\frac{p_2}{p_1}\right)^{1/2}$$

$$h_2(p,u) = \frac{\partial e(p,u)}{\partial p_1} = u + \left(\frac{p_1}{p_2}\right)^{1/2}$$

Note that $h_1(p,u)$ is independent of *u* because x_1 is income neutral; the tangency between any indifference curve and any iso-expenditure line (for given prices) occurs at the same value of x_1; see **Figure A1-1.**

The Hicksian demand measures the substitution effect. See **Figure A1-2.**

Answer to Question 2

(a) By Shephard's lemma:

$$h_i = \frac{\partial e}{\partial p_i} \text{ and } h_j = \frac{\partial e}{\partial p_j}. \text{ Then}$$

$$\frac{\partial h_i}{\partial p_j} = \frac{\partial^2 e}{\partial p_i \partial p_j} \text{ and } \frac{\partial h_j}{\partial p_i} = \frac{\partial^2 e}{\partial p_j \partial p_i}$$

but these second cross-partials are equal, by Young's theorem.

(b) False. Express Engel aggregation in elasticity form:

$$\sum_{i=1}^{n} w_i \eta_i = 1$$

where $w_i = \frac{p_i x_i}{m}$ is the "expenditure share" for good *i*. This cannot be satisfied if all goods are luxuries ($\eta_i > 1 \;\; \forall i$), but it can satisfied if $\eta_i > 1$ for some *i* provided $\eta_i < 1$ for some *i*. It is *not* necessary that $\eta_i < 0$ for some i.

(c) Recall that goods *i* and *j* are substitutes if

$$\frac{\partial x_i}{\partial p_j} > 0$$

By Cournot aggregation:

$$p_1 \frac{\partial x_1}{\partial p_1} + p_2 \frac{\partial x_2}{\partial p_1} = -x_1$$

Divide through by x_1 to obtain

$$\varepsilon_{11} + \frac{p_2}{x_1} \frac{\partial x_2}{\partial p_1} = -1$$

Rearranging, we have

$$\frac{\partial x_2}{\partial p_1} = (-1 - \varepsilon_{11}) \frac{x_1}{p_2}$$

Since $\varepsilon_{11} < 0$ (by normality of x_1) and $|\varepsilon_{11}| > 1$, the RHS must be positive.

Answer to Question 3

(a) At any prices, expenditure is minimized where $x_1 = x_2$. Thus, the Hicksian demands are simply given by

$$h_1(p,u) = u$$

$$h_2(p,u) = u$$

The expenditure function is

$$e(p,u) = p_1 h_1(p,u) + p_2 h_2(p,u) = u(p_1 + p_2)$$

(b) At any prices, utility is maximized where $x_1 = x_2$. The constraint is then used to solve for Marshallian demands:

$$x_1(p,u) = \frac{m}{p_1 + p_2}$$

$$x_2(p,u) = \frac{m}{p_1 + p_2}$$

The indirect utility function is

$$v(p,m) = \min[x_1(p,m), x_2(p.m)] = \frac{m}{p_1 + p_2}$$

(c) (i) To summarize: $m=10$, $\{p_1^0, p_2^0\}=\{1,1\}$ and $\{p_1^1, p_2^1\}=\{3,1\}$

Compensating variation:

$$CV = m - e(p^1, u^0) = m - e(p^1, v(p^0, m))$$

where we use $v(p,m)$ evaluated at p^0 to find u^0. In particular,

$$v(p^0, m) = \frac{10}{2} = 5$$

Thus,

$$CV = 10 - 5(3+1) = -10$$

Equivalent variation:

$$EV = e(p^0, u^1) - m = e(p^0, v(p^1, m)) - m$$

where we use $v(p,m)$ evaluated at p^1 to find u^1. In particular,

$$v(p^1, m) = \frac{10}{4} = 2.5$$

Thus,

$$EV = 2.5(1+1) - 10 = -5$$

(ii) Change in consumer surplus:

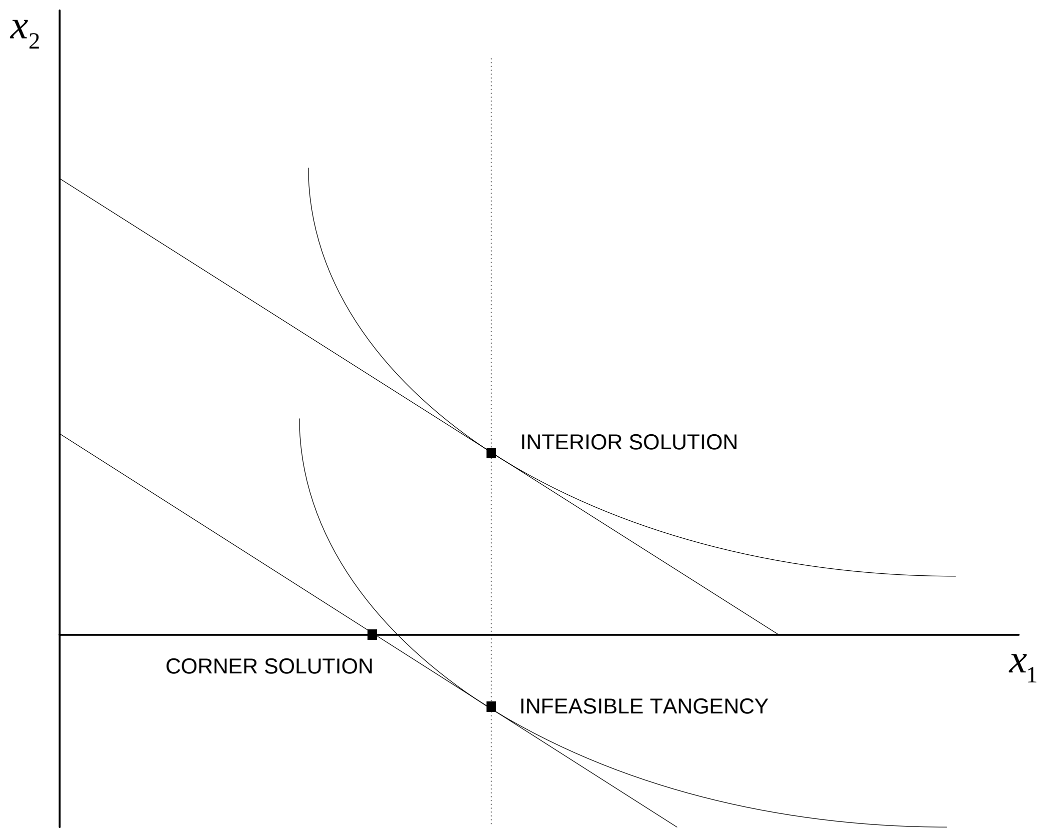
$$\Delta CS = \int_{p_1^1}^{p_1^0} x_1(p,m)dp_1$$

In this case,

$$\Delta CS = \int_{p_1^1}^{p_1^0} \left(\frac{m}{p_1 + p_2^0}\right) dp_1 = m\left[\log(p_1 + p_2^0)\right]_3^1 = 10[\log(1+1) - \log(3+1)]$$

$$= 10\log(1/2) = -6.93$$

Thus, we have $|CV| > |\Delta CS| > |EV|$, as expected for a price rise for a normal good.



x_2
INTERIOR SOLUTION
CORNER SOLUTION
x_1
INFEASIBLE TANGENCY

FIGURE A1-1

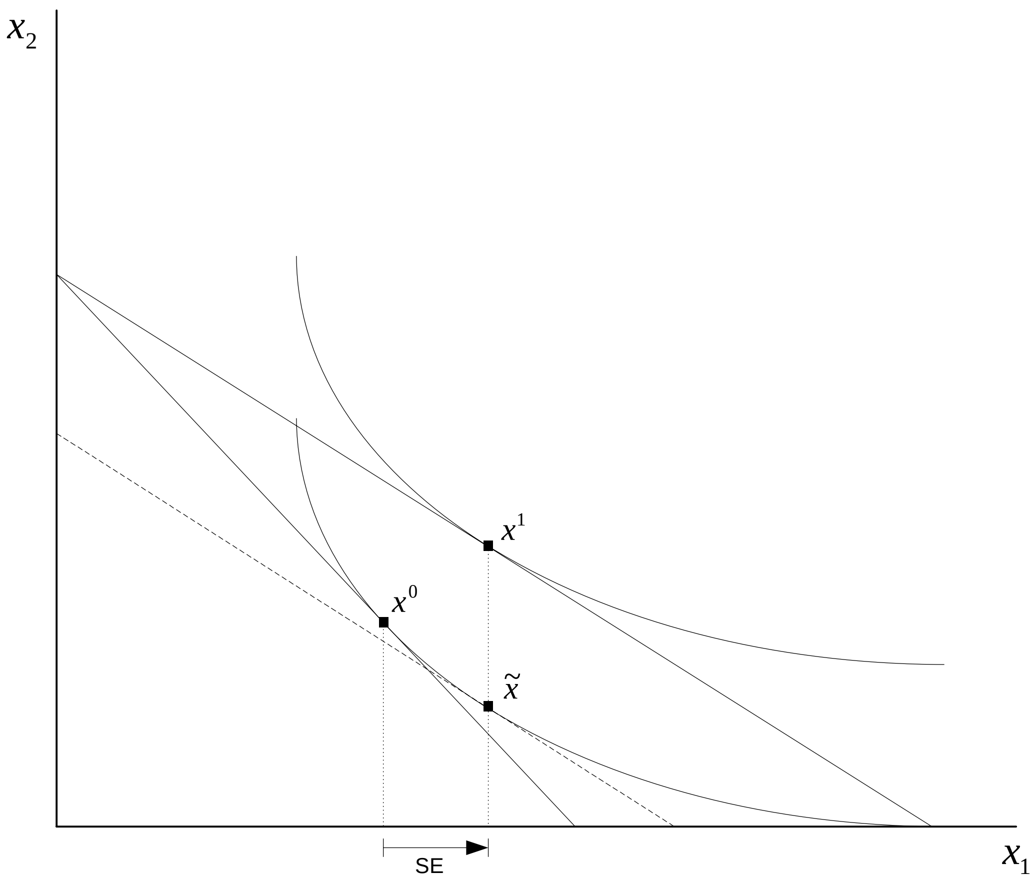


FIGURE A1-2