

CONSUMER BEHAVIOUR EXAMPLES

1. Consider the following utility function:

$$u = x_1 + x_2^{1/2}$$

(a) Show that if $p_1 \leq 2(mp_2)^{1/2}$ then

$$x_1(p, m) = \frac{4mp_2 - p_1^2}{4p_1p_2}$$

$$x_2(p, m) = \frac{p_1^2}{4p_2^2}$$

$$v(p, m) = \frac{4mp_2 + p_1^2}{4p_1p_2}$$

(b) Show that if $p_1 > 2(mp_2)^{1/2}$ then

$$v(p, m) = \left(\frac{m}{p_2} \right)^{1/2}$$

(c) Show that if $p_1 \leq 2(mp_2)^{1/2}$ then

$$e(p, u) = \frac{(4up_2 - p_1)p_1}{4p_2}$$

$$h_1(p, u) = \frac{2up_2 - p_1}{2p_2}$$

$$h_2(p, u) = \frac{p_1^2}{4p_2^2}$$

(d) Show that if $p_1 > 2(mp_2)^{1/2}$ then

$$e(p, u) = p_2u^2$$

2. Consider the following utility function:

$$u = ax_1 - \frac{b}{x_2}$$

Show that if $m \geq (p_1 p_2)^{1/2} (b/a)^{1/2}$ then

$$x_1(p, m) = \frac{m - (p_1 p_2)^{1/2} (b/a)^{1/2}}{p_1}$$

$$x_2(p, m) = \left(\frac{b p_1}{a p_2} \right)^{1/2}$$

$$e(p, u) = \frac{p_1^{1/2} (2b p_2 + u (p_1 p_2)^{1/2} (b/a)^{1/2})}{a p_2^{1/2} (b/a)^{1/2}}$$

$$h_1(p, u) = \frac{(b p_2 + u (p_1 p_2)^{1/2} (b/a)^{1/2})}{a p_1^{1/2} p_2^{1/2} (b/a)^{1/2}}$$

$$h_2(p, u) = \frac{b p_1^{1/2}}{a p_2^{1/2} (b/a)^{1/2}}$$

3. Consider the following indirect utility function:

$$v(p, m) = \frac{m^2}{p_1(p_1 + p_2)}$$

Show that

$$x_1(p, m) = \frac{m(2p_1 + p_2)}{2p_1(p_1 + p_2)}$$

$$x_2(p, m) = \frac{m}{2(p_1 + p_2)}$$

$$e(p, u) = (u p_1^2 + u p_1 p_2)^{1/2}$$

$$h_1(p, u) = \frac{u^{1/2}(2p_1 + p_2)}{2p_1^{1/2}(p_1 + p_2)^{1/2}}$$

$$h_2(p, u) = \frac{u^{1/2}p_1^{1/2}}{2(p_1 + p_2)^{1/2}}$$

4. Consider the following expenditure function:

$$e(p, u) = (p_1 p_2)^{1/2} u$$

Show that

$$h_1(p, u) = \frac{up_2^{1/2}}{2p_1^{1/2}}$$

$$h_2(p, u) = \frac{up_1^{1/2}}{2p_2^{1/2}}$$

$$v(p, m) = \frac{m}{(p_1 p_2)^{1/2}}$$

$$x_1(p, m) = \frac{m}{2p_1}$$

$$x_2(p, m) = \frac{m}{2p_2}$$