THE IMPACT OF ADAPTATION ON THE STABILITY OF AN EMISSIONS TREATY

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ABSTRACT

The availability of adaptation as a private mechanism to defend against damage caused by climate change can affect the outcome of an International Environmental Agreement (IEA) in terms of its stability as well as emissions reduction. In this paper we show, that an increase in the relative cost of adaptation, improves the prospect of a stable treaty in terms of how much emissions reduction it can achieve over the Nash equilibrium. We examine how a change in adaptation cost affects the equilibrium strategies constituting the gains from cooperation and show that the availability of an emissions trading program can help capture a fraction of these potential gains as well as increase global surplus, even if the treaty achieves zero reduction in emissions. The results suggest that policies directed towards making abatement technologies cheaper relative to adaptation will eventually improve the prospects of a stable treaty in terms of aggregate emissions reduction.

1. Introduction

Does the availability of adaptation help or hinder the prospects of a stable climate treaty? The answer to this question is not straightforward because of the inherent complexity adaptation brings in the strategic interactions in an emissions mitigation game and how it impacts the incentive to cooperate in the presence of heterogeneity. Moreover, how differences in adaptation costs & potential heterogeneity in adaptation cost relative to abatement cost among the countries impact the climate change game makes the task more challenging. The existing literature studying emissions abatement, climate treaties & their stability in the *absence* of adaptation is extensive and provides some valuable insights. However, the literature on the *availability* of adaptation and its associated impacts on climate treaties and strategic interactions is still growing and there are few clear answers to these questions.

The main methods involved in the literature considering cooperation on climate mitigation are game theoretic models, integrated assessment models and computable general equilibrium models[1]. We look at the role of adaptation on the incentives to cooperate in an International Environmental Agreement (IEA) & its impact on the performance of a single-parameter allowance-allocation rule (SPAAR) treaty on emissions.

The term "adaptation", within this context, refers to adjustments in ecological, social or economic systems for reducing potential damage from climate change (Parry et al. 2007). We show that an increase in the cost of adaptation improves the prospects of what a treaty can achieve in terms of emissions reductions. We also analyze how the relative cost of adaptation affects the equilibrium cost in the noncooperative & cooperative setting and how heterogeneity in damage affects the scope of control effect.

Our theoretical model focuses on the asymmetric scope of control, that large countries have over small countries, over the global climate destiny and precisely why small countries are concerned

[1] The most important research topics in this extensive literature are: burden-sharing and equity, the stability of the climate coalition and international environmental agreements, technology transfer and climate finance, the linkage of carbon markets, and the response to noncooperation behavior. See (Zhang and Liang 2020) for a bibliometric analysis of the recent progress of the literature.

by the possibility of the largest emitters choosing to over adapt than mitigate emissions. The heterogeneity among the countries in terms of scope of control is the key driver of our results.

In a simple model in which abatement and adaptation are both binary actions, Barrett (2008a) finds that the ability to adapt promotes abatement (in the cooperative setting). Because adaptation lowers the returns to abatement, hence it requires more countries in the treaty to cooperate for abatement to be attractive. He points out that if abatement and adaptation are substitutes in the damage function, the availability of adaptation can reduce the gains to cooperative action on emissions. This reduces the gains to free-ride which in turn reduces the incentive for any one country to remain outside the international treaty. Barret's simulation results show that when climate damages are high and cooperation is most valuable, the size of a stable coalition is marginally larger when adaptation is available compared to when it is not available. Moreover, a stable treaty produces only very small gains over the non-cooperative equilibrium under either scenario.

Bruin, Weikard, and Dellink (2011) combine game theory and Integrated Assessment Modelling to create an applied three-stage cartel formation model assuming sequential decisions about adaptation and abatement with a damage function linear in emissions. They find that it is important to include adaptation allocations in a treaty alongside emissions allocations, because if adaptation is determined outside the treaty, then countries can over-adapt thus the treaty would require higher mitigation level and the size of stable coalitions reduces. Hence, if adaptation choice can also be locked that same way the treaty locks down emissions abatement levels through allowances, this incentive to over-adapt can be reduced. Simulating all possible 4084 coalitions with 12 heterogeneous regions, they find that the best performing stable coalition[2] in their model is able to achieve 48% of the potential cooperation gains. Furthermore, they find that certain countries can influence the stability of the best performing stable coalition by going beyond the credible level of adaptation and over-adapting extremely.

Our paper is closest in spirit to (Benchekroun, Marrouch, and Ray Chaudhuri 2011) and (Lazkano, Marrouch, and Nkuiya 2016). (Benchekroun, Marrouch, and Ray Chaudhuri 2011) - henceforth

[2] The regions under consideration in their simulation models are: USA, Japan, European Union15, Other OECD regions, Eastern European Regions (EET), former Soviet Union, Energy Exporting regions, China (CHN), India (IND), Dynamic Asian economies (DAE), Brazil and rest of the world (ROW). The best performing stable coalition in their model is the coalition between USA, EET, CHN, DAE and IND.

BMC- provide more optimistic results. They consider symmetric countries in their analysis of the non-cooperative (NCE) and cooperative equilibrium and focus on grand coalition and partial coalition. Their specification of the damage cost includes two adaptation parameters, one related to level of adaptation another for the effectiveness of adaptation efforts. Damage is strictly convex in global emissions so in the absence of adaptation, emissions are strategic substitutes. They find that when adaptation efforts are effective it decreases the marginal damage of countries, thus making the best response functions (BRFs) flatter in terms of emissions. If one country increases their emissions, the other country can increase their adaptation instead of decreasing their emissions, thus making the BRFs flatter. Hence, this reduces global emissions in the NCE which makes cooperation on emissions less costly. So, a higher effectiveness of adaptation lowers incentive to free ride. Also, since the adaptation cost is convex in adaptation level, failing to cooperate on emissions increases each country's cost on adaptation, thus potential gains from cooperation increase as more adaptation is undertaken. However, they do not investigate whether or not this effect on incentives actually leads to a larger stable coalition.

(Lazkano, Marrouch, and Nkuiya 2016)- henceforth LMN- study a generalization of the BMC model where countries can simultaneously choose their emissions and adaptation levels and countries are heterogeneous with respect to adaptation cost: high cost and low cost. They restrict the possibility of over-adaptation by restricting the relevant parameters and consider a general damage function that can be both linear and non-linear in their model. Their results provide the necessary motivation in the sense that it is still worth-while to pursue emissions treaties because presence of adaptation does not hamper its prospects of reducing emissions. Moreover, heterogeneity in adaptation cost is a good thing in terms of the incentives to form an IEA. An exogenous reduction in the adaptation cost gap between countries leads to higher global emissions when damages are linear. A country's adaptation effort increases as adaptation costs get smaller. Higher adaptation efforts in turn lead to higher individual and global emissions in the linear damage case. When damages are quadratic and adaptation costs are heterogeneous, the willingness to join a coalition changes as the cost gap narrows. When cost heterogeneity is large, a cost reduction may strengthen participation incentives, while when cost heterogeneity is smaller, such reduction weakens participation incentives. Their results also imply that when we consider transfers to reduce the heterogeneity in adaptation costs among countries, we should only do so in a certain interval or limit where enough heterogeneity is maintained that can hold the treaty attractive enough to a large set of countries.

Our perspective is quite different from both BMC and LMN. While both of them study the outcomes in the non-cooperative & cooperative setting, we investigate how adaptation affects the performance of a SPAAR treaty. The critical characteristic across which countries differ in our model is through their economic size, which allows us to maintain the asymmetry in the scope of control among the countries. We also analyze how availability of adaptation and changes in its relative cost affects the global cost in the equilibrium and constitute the gains from cooperation.

Moreover, we allow emissions trading among the member countries of the treaty which distinguishes our work further. Most of the papers discussed above do not model the implications of transfers among the members. A system of transfers among treaty members can dramatically improve the prospects for a treaty when countries differ in terms of their net benefits from the collective action (Hoel and Schneider 1997; Barrett 2001). M. Li, Weng, and Duan (2019) show that, linking EU emissions trading system with China's carbon market could reduce the total abatement costs and the welfare of the EU and China could increase by 0.34% and 0.04% respectively compared to the independent EU emissions trading system.

H. Li and Rus (2019) also study a variant of the BMC model in which countries are heterogeneous with respect to benefits from emissions, vulnerability to damage and effectiveness of adaptation efforts. Their analytical results suggest that incentives to free-ride on an IEA can be reduced through diffusion of technological progress on adaptation among the members of a coalition. They do not consider the possibility of transfers within the coalition. Their results appear at first to be contrary to some of the simulation results obtained by LMN but that could simply reflect the fact that the impact of adaptation on the size of a stable coalition cannot be ascertained merely by examining changes in the incentive for a given country to defect from a candidate coalition of given size, the stability of which might be unaffected by that change in incentives. For example, BMC derive analytical results for the impact of adaptation cost on the incentive to defect from the grand coalition but the grand coalition in their model is never stable.

In terms of modelling features, our work is closest to (Farnham and Kennedy 2015). While they study the welfare implications of adaptation on small economies through their analysis of the non-cooperative & cooperative setting, we investigate an entirely different question. Additionally, their analysis of the scope of control effect introduced through economic size is done in a setting with homogeneous damage across countries. We extend the interpretation of the scope of control effect in the presence of heterogeneous damage and the mechanism through which it affects potential gains from trade.

One important aspect in the outcome of these game theoretic models is the timing of adaptation and abatement decisions. Consistent with the bulk of the literature[3], we consider that adaptation decisions are taken simultaneously with abatement decisions. Divergent to these approaches, Masoudi and Zaccour (2017) consider a setting where members of the agreement concur to fully share their knowledge and determine their investments in R&D by maximizing their joint welfare, (thus choosing their level of adaptation cooperatively) while non-members maximize individual payoffs. Finally, countries non-cooperatively choose their emissions. They find that a large coalition is achievable and that the total emissions are increasing in the size of the agreement.

Based on timing, adaptation decisions can be further categorized as further proactive or reactive[4]. Breton and Sbragia (2019) get results similar to (Masoudi and Zaccour 2017). They assume that adaptation requires a prior irreversible investment and use a stylized model where adaptation is treated as a private good and abatement as a public good. They do not consider transfers among the countries. Through simulation results they find that in settings where an IEA includes private adaptive investments settled prior to mitigation decisions and where an IEA decides adaptation decisions only, stable coalitions with a significant number of signatories can be obtained, which is not the case when adaptation and mitigation are decided simultaneously by the IEA. The number of signatories in a stable agreement depends on the relative values of cost parameters. Similar to (Bruin, Weikard, and Dellink 2011), they suggest that including adaptive measures in the scope of environmental agreements may be advisable when these measures require irreversible investments

[3] See (Masoudi and Zaccour 2018; Lazkano, Marrouch, and Nkuiya 2016; Benchekroun, Marrouch, and Ray Chaudhuri 2011; Bayramoglu, Finus, and Jacques 2018)

[4] Proactive adaptation refers to adaptation measures taken before climate change occurs. These measures are often large scale and irreversible. Reactive adaptation takes place in reaction to climate change where costs and benefits are felt simultaneously. For example, building dykes can be considered proactive adaptation while using more water for crop production (with irrigation systems in place) during drought & heat can be considered reactive adaptation. (Bruin, Weikard, and Dellink 2011)

prior to the implementation of mitigation policies. However, this diverts the discussion to another question – to what extent would negotiating countries would actually agree to restrict & share their private adaptation efforts under an IEA?

The absence of clear analytical results on the role of adaptation on treaty formation largely reflects the inherent complexity of standard coalition-formation models. The standard approach assumes that a candidate coalition maximizes the joint welfare of its members given the behavior of non-members. Each candidate coalition is then assessed for its internal and external stability, and unstable coalitions are ruled out. We approach the question from a different perspective. Rather than ask how many countries can we get in a coalition (or the marginal impact on incentives to free-ride), we ask what fraction of the potential gains from cooperation (GFC) and gains from trade (through emissions trading) can be captured by a treaty and how is that affected by the cost of adaptation. We consider the model of heterogeneous countries with available adaptation from (Farnham and Kennedy 2015) and allow them to form a SPAAR treaty with an emissions trading program as examined in (Kennedy 2016). We focus exclusively on the grand coalition and examine its characteristics under this setting.

The rest of our paper is organized as follows. Section 2 describes the theoretical model. Section 3 derives the non-cooperative equilibrium and the first-best solution, and examines the properties of potential gains from full cooperation. Section 4 derives analytical results on treaty composition for a single-parameter allowance allocation rule (SPAAR). Section 5 characterizes the grand coalition SPAAR treaty. Section 6 extends the model to heterogeneous damage. Section 7 provides some concluding remarks. An Appendix contains proofs and derivations not presented in the main text.

2. Model

The basic model in this paper is identical to (Farnham and Kennedy 2015). The key feature of the model is that countries are heterogeneous with respect to their GDP [5] ($y_i > 0$). Emissions from country $i$are denoted e_i. These emissions are a function of its GDP, denoted y_i, and its abatement technology, denoted $x_i \in [0,1)$:

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[5] We take this as fixed, and focus on the costs and benefits of technology-based abatement and adaptation, given that level of output. In practice, countries also have the option of reducing output itself as a way to reduce emissions, but we take a simpler approach so as to focus on the most-easily identified way in which countries differ.

$$e_i = (1 - x_i)y_i \tag{1}$$

Abatement cost is increasing and strictly convex in x_i but linear in output:

$$c_x(y_i, x_i) = x_i^2 y_i \tag{2}$$

Global emissions are denoted $E = \sum_{i=1}^{N} e_i$ where N is the number of countries. In the absence of adaptation, climate change destroys a fraction δE of the output in country i, where δ is the damage parameter[6]. In practice, δ surely varies across countries but we keep the (Farnham and Kennedy 2015) assumption here for two reasons.[7] First, we wish to consider a direct comparison of results; and second, it allows the derivation of sharp analytical results that relate δ to the gains from trade in a cooperative agreement. We discuss the impact of heterogeneity in damage parameters on the interpretation of some of the key quantities in section 6 and report the model solutions in the appendix.

Adaptation is modeled as defensive action taken by country i to protect some fraction $a_i \in [0,1)$ of its economy from the damaging impact of climate change. Alternatively, adaptation reduces the severity of the impact of climate change.[8] The "undefended" residual fraction of the economy remains subject to damage. Thus, damage in country i is

$$d_i = \delta E(1 - a_i)y_i \tag{3}$$

It is worthwhile to mention that our damage function has the following features: i) damage to country i is increasing in global emissions $\frac{\partial d_i}{\partial E} \geq 0$, ii) marginal damage from global emissions is

[6] In reality, δ will vary across countries, depending on economic composition (especially with respect to dependence on agriculture), and geographic characteristics. We abstract from this source of heterogeneity so as to focus on the impact of economic size.

[7] We solved the model with both heterogeneous and homogeneous damage parameters. Upon investigation of the two cases, we have found that the qualitative implications of our results remain unaffected, hence we have presented the paper with symmetry in damage parameters among the countries, which results in simple and sharp analytical solutions.

[8] Some defensive measures (such as geoengineering) may have significant spillover effects on other countries; see (Barrett 2008b). Here we restrict attention to purely private defensive measures.

decreasing in country i's level of adaptation $\frac{\partial^2 d_i}{\partial E\, \partial \mathrm{a_i}} \leq 0$ and iii) marginal benefit from adaptation is increasing in global emissions $\frac{\partial^2 d_i}{\partial \mathrm{a_i} \partial E} \leq 0$.

The cost of adaptation for country i is strictly convex in its coverage, and proportional to the magnitude of economic activity that must be defended. In particular,

$$c_a(a_i, y_i) = \theta a_i^2 y_i \tag{4}$$

where θ is a parameter reflecting the cost of adaptation relative to the cost of abatement. In reality, θ will likely differ across countries, but abstracting from this potential heterogeneity allows the availability of adaptation to be captured with a single parameter. Both our abatement & adaptation cost functions feature decreasing returns to scale in the level of abatement & adaptation respectively.[9]

Aggregate output is $Y = \sum_i^N y_i$. It will also prove useful to define

$$S \equiv \sum_{i=1}^{N} y_i^2 = N(\mu^2 + \sigma^2) \tag{5}$$

Where μ and σ represent the mean and variance of GDP distribution respectively. Note that $S < Y^2$ for any $N > 1$.

Since our damage function is bi-linear, we impose the following restrictions on key parameters to ensure convexity of the optimization problems.

$$\delta < \frac{2Y}{S} \equiv \delta_{max}\, and \theta > \frac{\delta Y}{2} \equiv \theta_{min} \tag{6}$$

These restrictions ensure that both the non-cooperative equilibrium and the full cooperation outcome are both interior (involving positive quantities of both abatement and adaptation for all countries). We henceforth refer to these conditions as the *interior solution conditions* (ISCs).

3. Potential Gains from Cooperation

[9] The specified cost functions exclude the possibility that the marginal costs of abatement and adaptation are interdependent. In reality they could be related both positively & negatively depending on the context. We abstract from this potential heterogeneity to capture the relative cost with a single parameter.

3.1 The Non-Cooperative Equilibrium (NCE)

The policy problem for country i in a non-cooperative setting is to set a_i and x_i to minimize its total domestic cost (equal to the sum of domestic abatement cost, domestic adaptation cost and domestic damage):

$$\min_{a_i, x_i} x_i^2 y_i + \theta a^2 y_i + \delta[(1 - x_i) y_i + E_{-i}](1 - a_i) y_i \tag{7}$$

where $E_{-i} = \sum_{j \neq i} e_j$ is aggregate emissions from all countries other than country i. The interior solution to this problem yields best-response functions for adaptation and abatement technology choice as follows:

$$a_i(E_{-i}) = \frac{\delta(2y_i - \delta y_i^2 + 2E_{-i})}{4\theta - \delta^2 y_i^2} \tag{8}$$

And

$$x_i(E_{-i}) = \frac{\delta(2\theta - \delta y_i - \delta E_{-i}) y_i}{4\theta - \delta^2 y_i^2} \tag{9}$$

The ISCs in (6) guarantee that these best-response functions solve for an interior and stable equilibrium (see Proposition 2 in Farnham and Kennedy (2015)), characterized by

$$\hat{a}_i(\theta) = \frac{\delta(2Y - \delta S)}{4\theta - \delta^2 S} \tag{10}$$

And

$$\hat{x}_i(\theta) = \varphi(\theta) y_i \tag{11}$$

Where

$$\phi(\theta) \equiv \frac{\delta(2\theta - \delta Y)}{4\theta - \delta^2 S} > 0 \tag{12}$$

Note from (11) that larger economies choose cleaner technologies ($\frac{\delta \hat{x}_i(\theta)}{\delta y_i} > 0$), where $\varphi(\theta)$ captures the strength of this relationship. This reflects that higher GDP countries choose cleaner technologies (scope-of-control effect) while equilibrium adaptation is independent of GDP[10]. While each country has complete control over its adaptation, it can only control a fraction of global emissions that damage its climate. This means, small economies have little control over global emissions, so its best policy is to defend itself via adaptation. In this setting, because of the linearity in the damage function, the availability of adaptation makes emissions strategic complements, while the absence of adaptation would result in dominant strategies in emissions.[11]

It is straightforward to show that $\frac{\delta \widehat{x_i}(\theta)}{\delta\theta} > 0 \; and \frac{\delta \widehat{a_i}(\theta)}{\delta\theta} < 0$; a higher cost of adaptation leads to more abatement and less adaptation in equilibrium policy mix for all countries. It is also worthwhile to mention that equilibrium technology choice for any country depends on the mean and variance of the GDP distribution which is captured by *S*.

The associated Nash Equilibrium emissions for country *i* are:

$$\widehat{e_i}(\theta) = y_i - \varphi(\theta)\mathrm{y}_\mathrm{i}^2 \tag{13}$$

Aggregate Emissions in the NCE:

$$\hat{E}(\theta) = \frac{(2Y - \delta S)2\theta}{4\theta - \delta^2 S} \tag{14}$$

Aggregate Global Cost in the NCE:

$$\hat{C}\,(\theta) = \phi(\theta)^2 \, Q \; + \; S\left(\phi(\theta) + \frac{\delta}{2}\right)\left(\frac{Y}{S} - \phi(\theta)\right) Y \tag{15}$$

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[10] While in this setting all countries protect the same fraction of economic activity via adaptation, introduction of heterogeneity in the damage parameter (δ) in this setting would imply that countries would also vary in their adaptation and abatement efforts according to their susceptibility to damage from climate change. The equilibrium solutions with heterogeneous damage parameter are reported in the appendix

[11] (Ebert and Welsch 2011, 2012) consider damage being strictly convex in emission and find that emissions are strategic substitutes in the absence of adaptation while the introduction of adaptation creates the possibility that emissions are strategic complements over some range of the parameter values, which depend on the convexity of the damage function relative to the effectiveness of adaptation.
In terms of technology choices, most of the literature consider adaptation and abatement as substitutes (Tol 2005), however there have been some discussion on the synergies between them. For example, abatement activities like forest & biodiversity conservation reduce vulnerability and thus promote adaptation, while adaptive measures like soil & water conservation contribute to mitigation of climate change (Ravindranath 2007). In reality the empirical relationship between adaptation efforts and abatement measures will vary depending on time, sector and geography (Lazkano, Marrouch, and Nkuiya 2016)

where

$$Q \equiv \sum_{i=1}^{N} y_i^3 = N\,(\widetilde{\mu_3}\,\sigma^3 + \mu^3 + 3\,\mu\sigma^2) \tag{16}$$

The aggregate global cost depends on the skewness ($\widetilde{\mu_3}$) of GDP distribution; this reflects the scope-of-control-effect that the technology choices of the higher GDP countries would drive the cost function more than that of any lower GDP country. This cost is increasing and concave in θ.

3.2 Minimum Cost Solution (MCS)

This is the first-best solution where total global cost (equal to the sum of abatement cost, adaptation cost, and damage for each country, aggregated across countries) is minimized via the choice of abatement technologies and adaptation actions.

$$\min_{a_i, x_i} \sum_{i=1}^{n} x_i^2 y_i + \theta a^2 y_i + \delta\left(\sum_{j=1}^{n} y_j(1 - x_j)\right)(1 - a_i)y_i \tag{17}$$

The interior first-best solutions[12] for abatement and adaptation are as follows:

$$x_i(\theta)^{**} = x(\theta)^{**} = \frac{\delta(2\theta - \delta\, Y)Y}{(4\theta - \delta^2\, Y^2)} \quad \forall\, i \tag{18}$$

and

$$a_i(\theta)^{**} = a(\theta)^{**} = \frac{\delta(2 - \delta\, Y)Y}{(4\theta - \delta^2\, Y^2)} \quad \forall\, i \tag{19}$$

Aggregate Emissions in the MCS:

$$E(\theta)^{**} = \frac{2\,\theta(2-\delta Y)Y}{4\theta-\delta^2 Y^2} \tag{20}$$

Aggregate Global Cost:

$$C(\theta)^{**} = \frac{\delta Y^2(4\theta - (1+\theta)\delta Y)}{4\theta - \delta^2 Y^2} \tag{21}$$

[12] See (Farnham and Kennedy 2015) for complete description including corner and comparative statics. With heterogeneous damage, while abatement technology would be identical across countries, their adaptation efforts would vary as per their vulnerability to climate change.

This global cost function is increasing and strictly concave in θ. Countries under full-cooperation internalize the pollution externality, which leads to lower aggregate emissions in the equilibrium, even when adaptation is available (Lazkano, Marrouch, and Nkuiya 2016). The cooperative equilibrium also ensures equalization of the marginal abatement costs among the countries in order to minimize global cost. Since, different countries face different development stages, their costs of emission reduction are also different. Implementation of an emission reduction policy separately by each country leads to countries with smaller emission reduction potentials facing higher marginal abatement costs than countries with larger emission reduction potentials while achieving the same emission reduction targets. (Mehling, Metcalf, and Stavins 2018)

It will be useful to express the gap between aggregate emissions under full cooperation $E(\theta)^{**}$ and aggregate emissions under the non-cooperative equilibrium $\widehat{E(\theta)}$ in terms of the fraction by which $\widehat{E(\theta)}$ would need to fall to achieve $E(\theta)^{**}$:

$$\beta(\theta)^{*} \equiv 1 - \frac{E(\theta)^{**}}{\widehat{E(\theta)}} \tag{22}$$

In other words, the fraction $\beta(\theta)^{*}$ represents the potential gains from full cooperation in terms of aggregate emissions reduction.

Proposition 1: The gap between aggregate emissions in non-cooperative equilibrium and full-cooperation is increasing in cost of adaptation relative to cost of abatement, i.e.,

$$\beta(\theta)^{*} \textit{ is increasing in } \theta$$

Proof: See the appendix

As adaptation becomes cheaper in the future relative to abatement, the gap between Aggregate Emissions in the NCE and MCS is going to get narrower. In Barret's way of interpretation: as adaptation becomes more expensive relative to abatement, gains from cooperation increases & the prospects of a stable coalition increases.

If θ approaches zero, then adaptation becomes universally free. Then both the co-operative and non-cooperative equilibrium level of abatement would approach zero. That means, when theta is equal to zero, then aggregate emissions in cooperative & non-cooperative equilibrium must be equal (*i.e., $\beta(\theta)^{*}$ becomes 0*). Now, if adaptation starts to become costlier (*i.e., θ grows larger*), then there is a shift in equilibrium abatement for both the cooperative and non-cooperative setting, such that abatement becomes non-zero. Optimal abatement in the cooperative setting, $x(\theta)^{**}$ grows more quickly because it fully incorporates the externality, but optimal abatement in the non-cooperative setting, $\widehat{x(\theta)}$ does not grow as quickly because of the nature of the free riding inherent in the non-cooperative setting. Accordingly, $\widehat{E(\theta)}$ grows faster than $E(\theta)^{**}$ and hence the gap must grow as theta rises above zero. **Figure 1** depicts this growing gap between $\hat{E}(\theta)$ and $E(\theta)^{**}$ as theta grows, which can be thought of as the potential gains from cooperation in terms of aggregate emissions reduction.

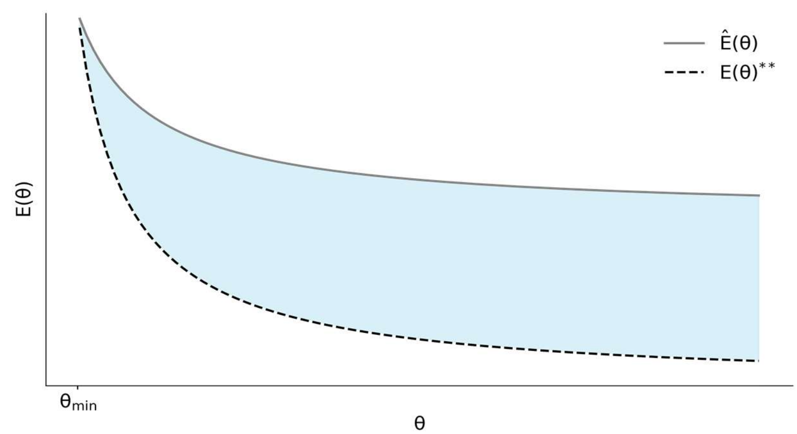

Figure 1: Gap between Aggregate Emissions in NCE & MCS

3.3 Gains from Full Cooperation

The gains from full cooperation are

$$GFFC(\theta) = \widehat{C(\theta)} - C(\theta)^{**} \tag{23}$$

It will be useful to decompose these gains into two parts: (net) gains from equiproportionate abatement (GFA) and gains from equalization (GFE). The GFA arises from the reduction in aggregate emissions from $\widehat{E(\theta)}$ to $E(\theta)^{**}$, with the associated changes in abatement cost, adaptation cost and damage. These would be the only gains in a setting with homogeneous countries. In a setting with heterogeneous countries, aggregate emissions are too high and marginal abatement costs (MACs) are not equated across countries. Thus, there are additional gains to acquire, in the form of gains from equalization of MACs (GFE). We decompose the GFFC by conducting a thought experiment in which the move from the NCE to the MCS is achieved in two steps. In the first step, an equiproportionate increase in abatement is done such that all countries are required to reduce emissions to a level

$$\widetilde{e_i}(\theta) = [1 - \beta(\theta)^*]\, \widehat{e_i}(\theta) \tag{24}$$

This abatement requirement ensures that aggregate emissions are brought down to their MCS level: $\sum_{i=0}^{n} \widetilde{e_i}(\theta) = E(\theta)^{**}$

The second step involves reallocating emissions among countries, keeping aggregate emissions fixed at $E(\theta)^{**}$, to ensure that marginal abatement costs are equated across countries and that aggregate abatement cost is thereby minimized. We refer to the associated reduction in aggregate abatement cost as gains from equalization (GFE). Let us begin with the gains from equiproportionate abatement.

3.4 (Net) Gains from Equiproportionate Abatement (GFA)

If country i is required to reduce emissions to $\widetilde{e_i}(\theta)$ then it would need to implement abatement technology

$$\widetilde{x_i}(\theta) = 1 - \frac{\widetilde{e_i}(\theta)}{y_i} \tag{25}$$

This requires an increase in abatement cost over the NCE level equal to

$$\widetilde{\Delta} c_{xi}(\theta) = (\widetilde{x_i}(\theta)^2 - \widehat{x_i}(\theta)^2)\, y_i > 0 \tag{26}$$

Aggregating across countries yields the required increase in global abatement cost

$$\widetilde{\Delta} C_x(\theta) = \sum_{i=1}^{n} \widetilde{\Delta}\, c_{xi}(\theta) > 0 \tag{27}$$

This abatement cost rise is increasing in θ because the gap between $E(\theta)^{**}$ and $\widehat{E(\theta)}$ is increasing in θ, as explained in section 3.2 above. The relationship between $\widetilde{\Delta} C_x(\theta)$ and θ is depicted in **Figure 2(b)**.

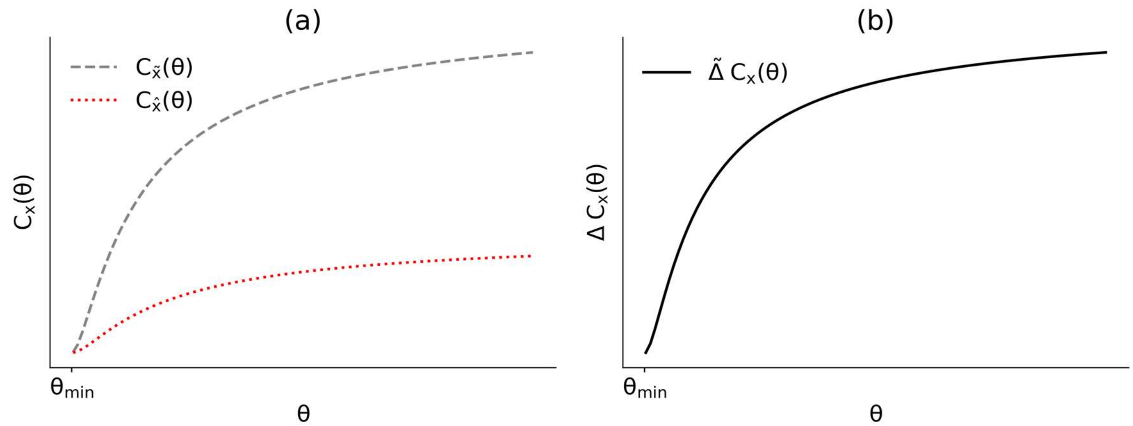

Figure 2: Change in Global Abatement Cost

The least-cost adaptation level required for country *i* in response to $E(\theta)^{**}$ is $a(\theta)^{**}$from (19). Note that this is also the adaptation level that country *i* would willingly choose in response to $E(\theta)^{**}$. Thus, implementing $a(\theta)^{**}$ does not require a planning directive to country *i* beyond the directive to reduce emissions to $\widetilde{e_i}(\theta)$ i.e., independent of how $\widehat{E(\theta)}$ is reduced.

The move from $\widehat{a_i}(\theta)$ to $a(\theta)^{**}$ brings with it a change in adaptation cost for country *i* equal to:

$$\Delta^* c_{ai}(\theta) = \theta\left(a(\theta)^{**^2} - \widehat{a_i}(\theta)^2\right) y_i < 0 \tag{28}$$

Aggregating across countries yields the associated change in global adaptation cost:

$$\Delta^* C_a(\theta) = \sum_{i=1}^{n} \Delta^* c_{ai}(\theta) < 0 \tag{29}$$

The relationship between $\Delta^* C_a(\theta)$ and θ is depicted in **Figure 3(b)**. Note that the relationship is not monotonic. The bigger the difference between $\widehat{E(\theta)}$ and $E(\theta)^{**}$, the larger will be the reduction in adaptation cost due to equiproportionate reduction of emissions. Since this gap is increasing in θ, the decline in adaptation cost also rises as θ rises. Then there is the opposing force which is for any given level of adaptation, the cost of adaptation rises as θ rises. This results in the nonmonotonic relationship between $\Delta^* C_a(\theta)$ and θ.

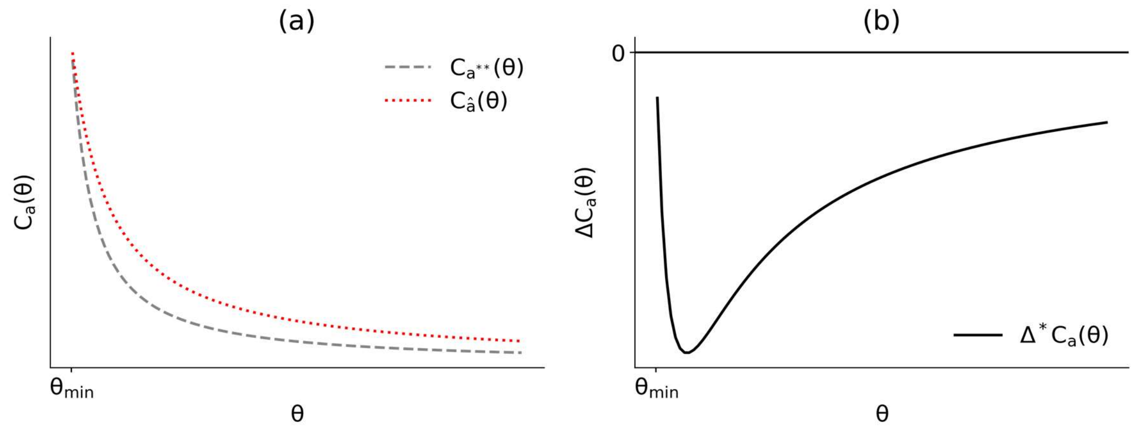

Figure 3: Change in Global Adaptation Cost

The reduction in aggregate emissions from $\widehat{E(\theta)}$ to $E(\theta)^{**}$, coupled with the reduction in adaptation from $\widehat{a_\iota}(\theta)$ to $a(\theta)^{**}$, yields a change in damage equal to:

$$\Delta^* d_i(\theta) = \delta \left[E(\theta)^{**}(1 - a(\theta)^{**}) - \widehat{E(\theta)}\big(1 - \widehat{a_\iota}(\theta)\big)\right] y_i \tag{30}$$

Aggregating across countries yields the associated change in global damage:

$$\Delta^* D(\theta) = \sum_{i=1}^{n} \Delta^* d_i(\theta) \tag{31}$$

This change in global damage is not necessarily negative. In particular, it is straightforward to show that damage actually rises ($\Delta^* D(\theta) > 0$) for $\theta < \bar{\theta}$, where

$$\bar{\theta} = \frac{\delta Y}{2} + \frac{\delta[\,(2 - \delta Y)(2Y - \delta S)Y]^{\frac{1}{2}}}{4} > \theta_{min} \tag{32}$$

The relationship between $\Delta^* D(\theta)$ and θ is depicted in **Figure 4(b)**. It might seem surprising that the MCS level of damage could be higher than the NCE level. However, we know that aggregate emissions in the NCE are higher than the first best solution, as a consequence of which, adaptation is higher in the NCE as well. The move from $\widehat{E(\theta)}$ to $E(\theta)^{**}$ brings about a substantial reduction in aggregate emissions. Since, adaptation is way too high in the NCE, it is possible to observe higher damage in the first-best solution over some range of θ, because of the rapid decline in adaptation.

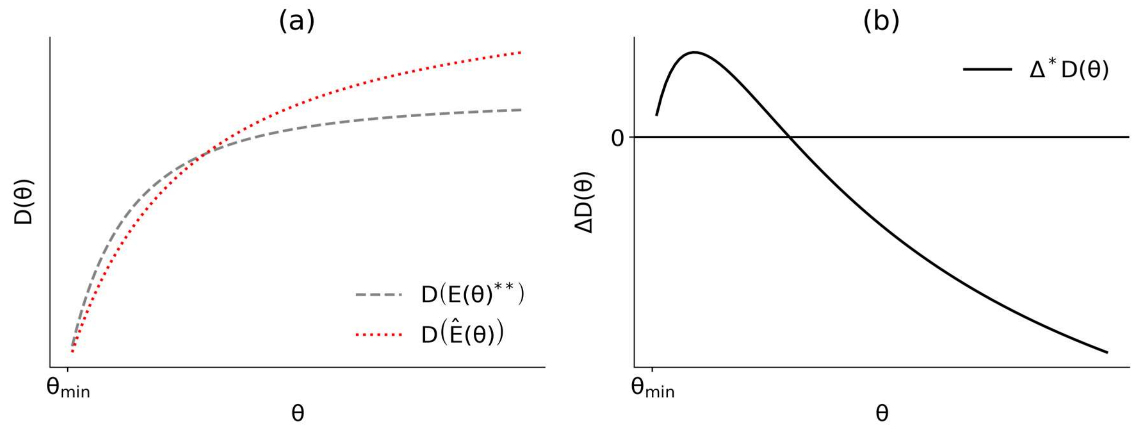

Figure 4: Change in Global Damage

The overall reduction in global cost from the equiproportionate abatement is henceforth called the (net) gains from equiproportionate abatement (GFA) :

$$GFA(\theta) = -\left[\Delta^* D(\theta) + \Delta^* C_a(\theta) + \widetilde{\Delta} C_x(\theta)\right] > 0 \tag{33}$$

This is depicted in **Figure 5(b)**. Note that at some critical value of θ, labelled θ_0 in Figure 5(a), reduced damage just offsets increased abatement cost and so the gains from abatement (GFA) are just equal to the reduction in adaptation cost.

It is straightforward to show that GFA is increasing in θ despite the non-monotonic nature of some of its components. In the Barret way of thinking, this should mean that cooperation is easier to attain when θ is small. (Barrett 1994)

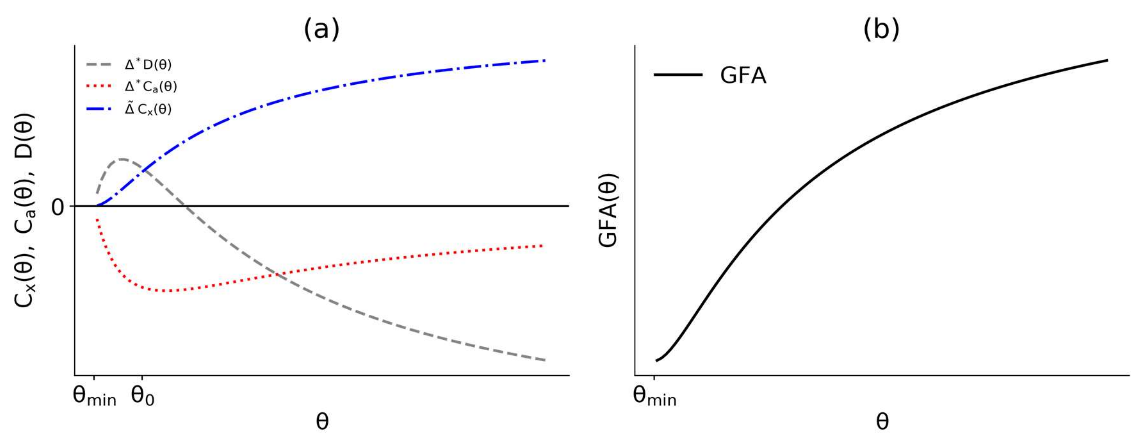

Figure 5: (Net) Gains from Equiproportionate Abatement

3.5 Gains from the Equalization of Marginal Abatement Costs (GFE)

The second step in our thought experiment involves reallocating emissions among countries to ensure that marginal abatement costs are equated across countries. This requires that each country emits at its MCS level $e_i(\theta)^{**}$. This movement from $\widetilde{e_i}(\theta)$ to $e_i(\theta)^{**}$ involves a change in abatement cost for country i equal to

$$\Delta^* c_{xi}(\theta) = \left[x_i(\theta)^{**^2} - \widetilde{x_i}(\theta)^2\right] y_i \tag{34}$$

It is straightforward to show that $\Delta^* c_{xi}(\theta) > 0$ for $y_i < \tilde{y}$ and $\Delta^* c_{xi}(\theta) < 0$ for $y_i > \tilde{y}$, where

$$\tilde{y} \equiv \frac{S}{Y} \tag{35}$$

That is, relatively small countries incur an increase in abatement cost, while relatively large countries enjoy a reduction in abatement cost, as illustrated in **Figure 6(a)** which plots the negative of (34). This reflects the fact that small countries were adapting way too much relative to abatement in the non-cooperative equilibrium (because of the scope of control effect).

Expressing S and Y in terms of mean and variance, we have:

$$\tilde{y} = \mu + \frac{\sigma^2}{\mu} \tag{36}$$

Thus, $\tilde{y}$ is greater than the mean.

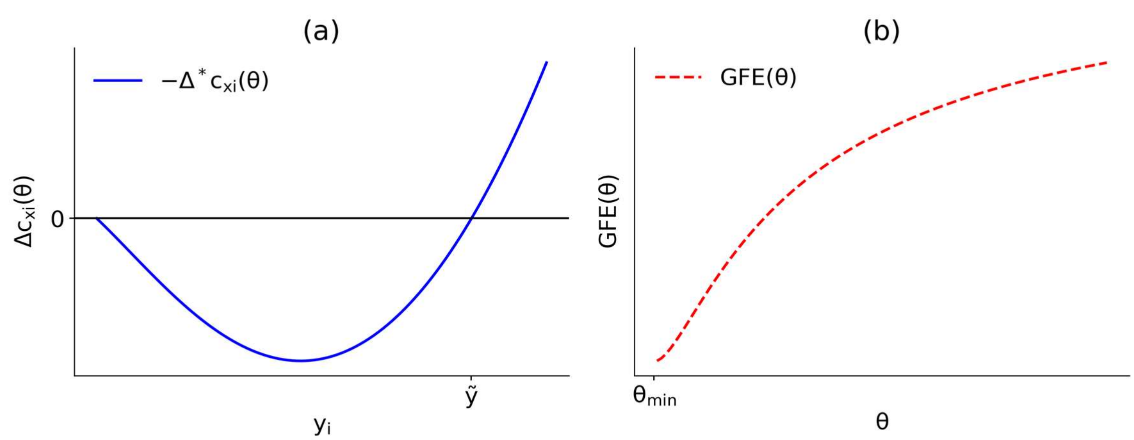

Figure 6: Gains from Equalization of Abatement Costs

Aggregating across countries in (34) yields the overall change in global abatement cost from the equalization of MACs. This reduction in global cost is henceforth called the gains from equalization (GFE) of MACS, and it can be expressed in terms of the scope-of-control effect:

$$GFE(\theta) = -\sum_{i=1}^{n} \Delta^{*} c_{xi}(\theta) = \phi(\theta)^2 \left(Q - \frac{S^2}{Y}\right)(1 - \beta(\theta)^{*})^2 > 0 \qquad (37)$$

This is increasing in $\phi(\theta)$; the steeper is the relationship between NCE abatement $\widehat{x_i}(\theta)$and GDP y_i, the greater is the difference between MACs at the non-cooperative level of emissions. This means, the higher the value of $\phi(\theta)$, the larger the gains from equalization of MACs. Moreover, when we force the countries to reduce their emissions equiproportionately from $\widehat{e_i}(\theta)$ to $\widetilde{e_i}(\theta)$, the differences between these MACs get amplified further.

$GFE(\theta)$ also depends on Q which depends on the skewness of GDP distribution; it is also increasing in θ because a higher cost of adaptation means more abatement which pushes the MACs in the NCE further apart. This is crucial because this very point drives the relationship between adaptation cost (θ) and the capacity of a treaty to reduce emissions discussed in section 5.

4. A SPAAR Treaty on Emissions

The modeling of cooperative action follows Kennedy (2016). In that paper Kennedy derives stability conditions for a coalition of any composition and any allowance allocation rule in a setting with heterogeneous countries but without adaptation. Here, we focus on a coalition that uses a single-parameter allowance-allocation rule (SPAAR)[13]. The rule allocates to member-country *i* an emissions allowance q_i based on some verifiable characteristic of that country, denoted κ_i. The rule specifies an allocation mechanism $m(\kappa_i, \omega)$ and a single allocation parameter ω that translates the rule into an actual allowance for member country *i*:

$$q_i(\omega) = m(\kappa_i, \omega)\ \forall i \in C$$

where C denotes the set of treaty members. Neither m(.) nor ω are country specific, the same rule applies to *all* treaty members and a country either accepts the allocation or remains outside the treaty.

[13] Hoel (1992), Eyckmans (1999), and Endres and Finus (2002) also study single parameter allocation rules

This approach is quite different from the standard approach where the coalition chooses emissions and adaptation levels to maximize the joint welfare of the coalition members, defined as the sum of private surplus across coalition members. Critically, that maximization takes as given the membership of the coalition. Yet we know that the emission and adaptation choices of the coalition affect incentives to join the coalition, and that the size of the coalition affects the payoff of its members. If the coalition truly wishes to maximize the welfare of its members, then it should take this into account. It should solve a constrained optimization problem, where the stability conditions are imposed as constraints on the welfare maximization.

Second, there are no transfers allowed within the coalition. This is justified on the basis of transaction costs but ideally these should be modeled and to the extent that these costs are a function of the coalition size, taken into account in the welfare maximization problem. None of this is easy.

Here we focus exclusively on the grand coalition treaty since analysis of a smaller treaty requires the actual distribution of income (& damage parameters in a setting with heterogeneous δ) and cannot produce more general results. We let the countries create a SPAAR treaty under which allowances are based on NCE emissions. In particular, the allowances for country i is

$$q_i(\omega) = (1-\omega)\ \widehat{e_i}(\theta) \tag{38}$$

where $\widehat{e_i}(\theta)$ is given by (13) and $\omega \in [0,1]$. Allowances may be traded among treaty members once a treaty is struck. Adaptation is not governed by the treaty[14].

Treaty composition is characterized in the standard way, following Barrett (1992). The game between countries involves a single coalition of member countries, and a group of non-member countries who act independently and non-cooperatively. Membership of the coalition is open to any country willing to meet the abatement requirements specified by the treaty. The game comprises two stages. In the first stage each country decides whether or not to join the treaty. The second stage involves abatement and adaptation by coalition members and non-members. The equilibrium requirement is sub-game perfection: each country correctly anticipates the equilibrium

[14] See (Breton and Sbragia 2019) for a numerical comparison of the performance & stability of three types of treaties where the members agree to cooperatively determine their- (i) abatement only, (ii) adaptation efforts only and (iii) both adaptation & emissions simultaneously.

in the second stage when deciding whether or not to join the treaty in the first stage. If a coalition exists in equilibrium, then it must exhibit internal and external stability, as proposed in the context of cartel formation by d'Aspremont et al. (1983): no member of the coalition wishes to leave it unilaterally, and no non-member wishes to join it unilaterally.

4.1 Emissions and Adaptation under the Grand Coalition Treaty

Aggregate global emissions under the Grand Coalition (GC) treaty are:

$$E(\omega) = \sum_{i=1}^{n} q_i(\omega) = (1-\omega)\left(\frac{2\theta(2Y-\delta S)}{4\theta-\delta^2 S}\right) \tag{39}$$

Recall that treaty members can trade allowances once a treaty is struck. Let p denote the market price of those allowances. Facing this price, member-country i solves the following cost-minimization problem:

$$\min_{a_i, x_i} x_i^2 y_i + \theta a^2 y_i + \delta\, E(\omega)(1-a_i)y_i - p[q_i(\omega) - (1-x_i)y_i] \tag{40}$$

where these four additive terms measure abatement cost, adaptation cost, domestic damage and proceeds from the sale of allowances respectively. The implied demand for emissions by member country i is

$$e_i(p) = \left(1 - \frac{p}{2}\right) y_i \tag{41}$$

and its adaptation choice is

$$a_i(\omega) = \frac{\delta E(\omega)}{2\theta} \tag{42}$$

Summing across i in (41) then yields aggregate demand for emissions by treaty members. By equating this aggregate demand to total emissions allowed for treaty members and solving for p, we can obtain the equilibrium price of allowances as a function of ω:

$$p = P(\omega) \equiv 2\left(1 - \frac{E(\omega)}{y}\right) \tag{43}$$

At this price, demand for allowances (and hence, emissions) by member country i is:

$$e_i(\omega) = \left(\frac{y_i}{Y}\right) E(\omega) \tag{44}$$

The abatement technology choice associated with this level of emissions is:

$$x_i(\omega) = 1 - \frac{E(\omega)}{Y} \tag{45}$$

The number of allowances sold by country i is:

$$s_i(\omega) = \frac{(1-\omega)\delta y_i\,(2\theta-\delta Y)\,(\tilde{y}-y_i)}{4\theta-\delta^2 S} \tag{46}$$

Thus, $\tilde{y}$ is the marginal seller. Recall that in the NCE, large economies have lower emissions intensities $\frac{e_i(\theta)}{y_i}$ than smaller economies. This relative difference is preserved when allowances are based on the equiproportionate reduction (EPR) rule. Allowance trading then brings emissions intensities into equality across member countries; thus, large economies are buyers while small economies are sellers. The only country that does not trade in allowances is one whose GDP happens to be just equal to $\tilde{y}$. The country with GDP closest to this level will play a key role in determining what an EPR treaty can achieve because this country reaps minimal gains from emissions trading under the treaty.

Total cost for member country i under the treaty is quadratic in ω, and takes the form:

$$c_i(\omega) = \tau_{2i}(\theta)\omega^2 + \tau_{1i}(\theta)\omega + \tau_{0i}(\theta) \tag{47}$$

where τ_{ji} coefficients are too complicated to be usefully reported here.

4.2 Membership Payoffs and Coalition Stability Conditions

A coalition must be internally and externally stable. External stability is guaranteed in a grand coalition, so we can restrict attention to internal stability. Internal stability requires that no coalition member can achieve a lower cost by leaving the treaty to act non-cooperatively.

Recall that under this SPAAR treaty, the allowance allocations to the remaining treaty members do not change; each country still receives an unchanged fraction of its NCE emissions.[15]

However, *actual* emissions from the residual coalition will rise if the defecting country was a buyer of allowances within the coalition because the residual coalition members must have been emitting less than their collective allowance in that case. Conversely, actual emissions from the residual coalition will fall if the defecting country was a seller of allowances within the coalition because

[15] Kennedy (2016) examines a modified version of the simple SPAAR treaty in which the allocation rule is coverage-contingent; if a country exists the treaty, then the reductions undertaken by the remaining treaty members are reduced, thereby diminishing the gains to the exiting country.

the residual coalition members must have been emitting more than their collective allowance in that case. Let $E_{-i}(\omega,-i)$ denote collective emissions from the residual coalition, where the "$-i$" inside the brackets indicates that country i is not a member of the coalition. Then

$$E_{-i}(\omega,-i) = E(\omega) - q_i(\omega) \tag{48}$$

where $q_i(\omega)$ is allowance for country i as a member of the treaty, given by (38) above. In response to these emissions from the residual coalition, the defecting country adopts the non-cooperative strategies described in (8) and (9) for adaptation and abatement technology respectively.

Making the substitution for E_{-i} from (48) in (9) yields the abatement technology choice for the defecting country, from which we can derive its equilibrium emissions. This can be expressed in terms of its treaty quota:

$$e_i(\omega,-i) = q_i(\omega) + \omega\left(\frac{2\theta(2-\delta y_i)y_i}{4\theta-\delta^2 y_i^2}\right) \tag{49}$$

where the "$-i$" here again indicates that country i is not a member of the treaty. Since $y_i < Y$, it follows from (6) ISCs that $e_i(\omega,-i) > q_i(\omega)$; the defecting country emits more than its quota inside the treaty. However, it may not emit more than it would actually emit inside the treaty if it would have been a buyer of allowances inside the treaty. The threshold value of GDP above which a defecting country would reduce emissions does not have a closed-form solution but its lower bound (as $\theta \mapsto \infty$) is

$$y_L = \tilde{y} + \frac{\omega(2Y-\delta S)}{\delta Y} > \tilde{y} \tag{50}$$

Any country with $y_i < y_L$ (which includes all allowance sellers) emits more outside the treaty than it would inside it.

The collective response by the residual coalition to the defection of country i depends simply on whether country i is a buyer or seller of allowances inside the treaty. In particular, emissions from the residual coalition change by $s_i(\omega)$ from (46), where $s_i(\omega) < 0$ if defecting country i is an allowance seller inside the treaty, and where $s_i(\omega) > 0$ if defecting country i is an allowance buyer inside the treaty. In both cases the defection causes overall global emissions to rise, where the new level of global emissions is:

$$E(\omega, -i) = E(\omega) + \omega \left(\frac{2\theta(2-\delta y_i)y_i}{4\theta-\delta^2 y_i^2}\right) \tag{51}$$

where $E(\omega, -i)$ denotes global emissions if country i leaves the treaty, and $E(\omega)$ denotes global emissions under the grand coalition treaty given by (39) above.

By setting $E_{-i} = E(\omega, -i) - e_i(\omega, -i)$ in (8) and (9) we can usefully express the defecting country's adaptation and abatement technology choices in terms of the level of global emissions that its defection induces. These are:

$$x_i(\omega, -i) = \frac{\delta[2\theta - \delta E(\omega,-i)]y_i}{4\theta} \tag{52}$$

and

$$a_i(\omega, -i) = \frac{\delta E(\omega,-i)}{2\theta} \tag{53}$$

respectively.

We can now construct the payoff to country i from remaining outside the treaty. Its total cost outside the treaty (the sum of abatement cost, adaptation cost & domestic damage) reduces to

$$c_i(\omega, -i) = \eta_{2i}(\theta)\omega^2 + \eta_{1i}(\theta)\omega + \eta_{0i}(\theta) \tag{54}$$

where the $\{\eta_{ji}\}$ coefficients are too complicated to be usefully reported here. This cost is quadratic in ω but it declines monotonically in ω for $\omega \in [0,1]$. That is, the greater the reduction in emissions by treaty members, the lower the cost for a non-treaty member because damage is declining in ω. Its total cost inside the treaty is $c_i(\omega)$, as given by (47) above.

Internal stability of the grand coalition requires that the payoff from remaining in the treaty is non-negative for all countries; that is,

$$\pi_i(\omega, \theta) \equiv c_i(\omega, -i) - c_i(\omega) \geq 0 \;\forall i \tag{55}$$

This GC payoff is quadratic in ω and can be expressed as:

$$\pi_i(\omega, \theta) = -\lambda_i(\theta)\omega^2 + \gamma_i(\theta)(1-\omega)^2 \tag{56}$$

where

$$\lambda_i(\theta) = \frac{\theta(2-\delta y_i)^2 y_i}{4\theta-\delta^2 y_i^2} > 0 \tag{57}$$

and

$$\gamma_i(\theta) = y_i(y_i - \tilde{y})^2\phi(\theta)^2 \geq 0 \tag{58}$$

where $\phi(\theta) > 0$ is the scope of control term identified in (12). The second term in (56) is the gains from trade that country i captures by being able to trade in allowances as a member of the treaty. To see this, recall that country i is allocated an allowance $q_i(\omega) = (1-\omega)\ \widehat{e_i}(\theta)$ as a member of the treaty.

Now suppose that member countries are not allowed to trade allowances. Then country i can meet its allowance under the treaty only by choosing a technology that induces that allowance:

$$x_i^0(\omega) = 1 - \frac{q_i(\omega)}{y_i} \tag{59}$$

where the "0" superscript indicates that there is no allowance trading. Its choice of adaptation is unaffected by the absence of trading, and so is still given by (42). Aggregate emissions are also unchanged and given by (39). Total cost for country i under the treaty without emissions trading is therefore given by:

$$c_i^0(\omega) = x_i^0(\omega)^2 y_i + \theta a_i(\omega)^2 y_i + \big(1 - a_i(\omega)\big)\delta_i y_i E(\omega) \tag{60}$$

Total cost when emissions trading is allowed is given by (47). Trade therefore reduces the cost of membership for country i by:

$$GFT_i(\omega,\theta) \equiv c_i^0(\omega) - c_i(\omega) = \gamma_i(\theta)(1-\omega)^2 \tag{61}$$

Thus, $\gamma_i(\theta)(1-\omega)^2$ represents the gains from trade (GFT) for country i as a member of the treaty. Note that these GFT are decreasing in ω because the gap between allowances and post-trade emissions declines (towards zero) as $\omega \mapsto 1$. Aggregating across i in (61) yields aggregate gains from trade for the coalition as a whole:

$$GFT(\omega,\theta) = \phi(\theta)^2\left(Q - \frac{S^2}{Y}\right)(1-\omega)^2 \tag{62}$$

Setting $\omega = \beta(\theta)^*$ yields $GFT(\beta(\theta)^*,\theta) = GFE(\theta)$ from (37). That is, the GFT in a treaty that sets ω such that aggregate emissions are equal to its MCS level, are equal to the gains from the equalization of marginal abatement costs (GFE) identified in section 3.5 above. These gains from trade for country i are critical to its membership decision.

5. Properties of the Grand Coalition Treaty

Setting $\pi_i(\omega, \theta) = 0$ and solving for ω yields the highest value of ω that will just keep country i in the GC treaty. Let $\widetilde{\omega_i}$ denote this limiting value for member country i.

Proposition 2: The highest percentage equiproportionate reduction (ω) that country i is willing to tolerate to join the GC treaty is given by:

$$\widetilde{\omega_i}(\theta) = \frac{\gamma_i(\theta) + I\,\Phi_i(\theta)}{\gamma_i(\theta) - \lambda_i(\theta)} \tag{63}$$

where

$$\Phi_i(\theta) = \phi(\theta)(2 - \delta y_i)(\tilde{y} - y_i)y_i \left(\frac{\theta}{4\theta - \delta^2 y_i^2}\right)^{\frac{1}{2}} \tag{64}$$

and $I = -1$ if $y_i < \tilde{y}$ and $I = 1$ otherwise.

Proof: See the appendix.

This threshold value of $\omega(\theta)$ is different for different countries, hence for stability of the Grand Coalition, we choose the minimum of these $\widetilde{\omega_i}$. Hence,

$$\omega^{GC}(\theta) = min\left(\widetilde{\omega_i}\,(\theta)\right)$$

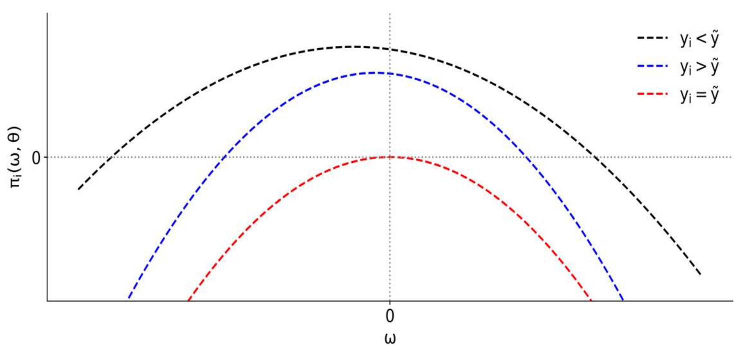

Figure 7: Relationship between the payoff from remaining in the treaty to country i with ω

Figure 7 plots $\pi_i(\omega, \theta)$ against $\widetilde{\omega_i}\,(\theta)$. It is evident that, if there is any country whose GDP = $\tilde{y}$, then the GC cannot achieve any reduction in emissions, although the equalization of MACs would still generate global surplus.

Proposition 3: $\widetilde{\omega_i}\,(\theta)$ is decreasing in y_i for $y_i < \tilde{y}$ and increasing in y_i for $y_i > \tilde{y}$.

Proof: See the appendix.

Figure 8 plots $\widetilde{\omega_i}\,(\theta)$ against y_i. Note that $\widetilde{\omega_i}\,(\theta) = 0$ at $y_i = \tilde{y}$; a country with exactly this GDP received no gains from trade (GFT) as a member of a treaty and is therefore unwilling to join a treaty with $\omega > 0$. Thus, if there does exist a country with $y_i = \tilde{y}$ then the GC cannot achieve *any* reduction in emissions. A country with $y_i = \tilde{y}$ does not buy or sell allowances in the equilibrium. In contrast, countries with $y_i > \tilde{y}$ buy allowances and countries with $y_i < \tilde{y}$ sell allowances and hence gain from the treaty membership. Note also that $\widetilde{\omega_i}\,(\theta)$ is decreasing in y_i for $y_i < \tilde{y}$ and increasing in y_i for $y_i > \tilde{y}$. This reflects the fact that GFT (as a fraction of GDP) are highest for the smallest and largest economies; in particular $\frac{\gamma_i(\theta)}{y_i}$ is increasing in $(y_i - \tilde{y})^2$.

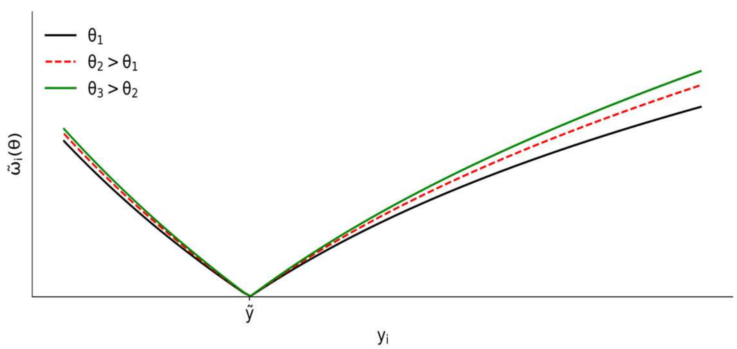

Figure 8: Relationship between $\tilde{\omega}_i(\theta)$ with y_i

Proposition 4: The largest percentage reduction in emissions, that country *i* is willing to accept to join a stable grand coalition treaty is increasing in the relative cost of adaptation, i.e.,

$$\widetilde{\omega_i}\,(\theta) \text{ is increasing in } \theta \text{ for all } y_i \neq \tilde{y}$$

Proof: See the appendix.

The higher is the cost of adaptation, the larger is the percentage reduction that country *i* is willing to tolerate to join the GC treaty. When adaptation cost is higher, NCE abatement is higher; thus difference among the MACs are higher leading to higher gains from trade. Increasing adaptation cost means higher gains from trade. Hence, the GC treaty can achieve a higher % reduction in emissions.

An increase in θ causes the arms of **Figure 8** to rise keeping the pivot point fixed at $\tilde{y}$. But this rise is steeper for countries with GDP $y_i > \tilde{y}$ as depicted in **Figure 9**. It is important to note that this grand coalition treaty would be stable as the internal stability condition for all countries is satisfied by $\omega^{GC}(\theta)$.

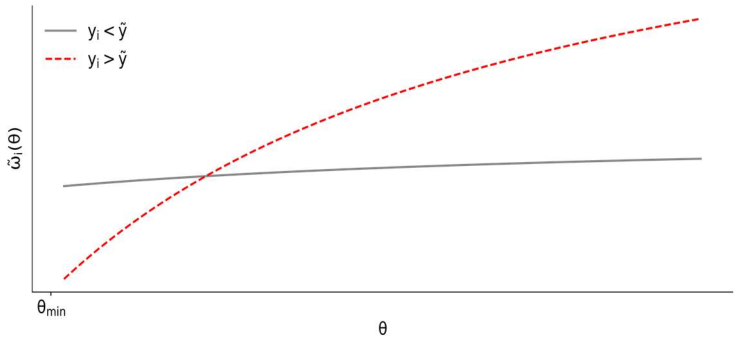

Figure 9: Relationship between $\widetilde{\omega_i}\,(\theta)$ with θ

6. Extension to Heterogeneous Damage

Introduction of heterogeneity in the damage parameters, does not change the qualitative implication of any of the propositions discussed above. However, it implies that the scope of control effect and the non-trading GDP is now co-determined by the country i's susceptibility to climate damage along with the aggregate quantities. The scope of control effect is now given by:

$$\phi_i(\theta) = \frac{\delta_i(4\theta - M) - \delta_i^2(2Y - K)}{2(4\theta - M)} \tag{65}$$

where

$$M \equiv \sum_{i=1}^{n} \delta_i^2 y_i^2 \ and \ K \equiv \sum_{i=1}^{n} \delta_i y_i^2 \tag{66}$$

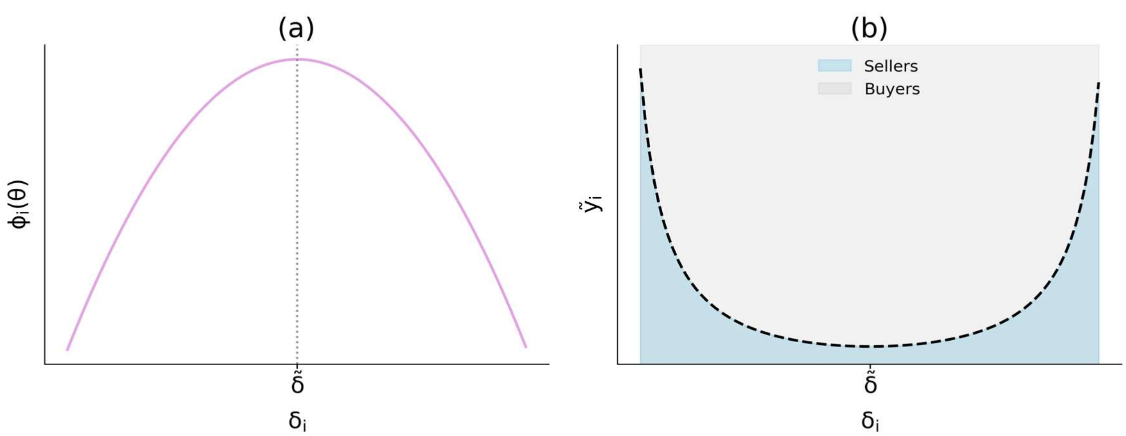

Figure 10: Scope of Control Effect & Non-trading GDP

Hence, the particular value of δ_i would play an important role in terms of determining the slope of $\widehat{x_i}(\theta)$ and y_i. While the scope of control effect that larger economies control a higher fraction of global emission and chooser cleaner technologies in the non-cooperative equilibrium still holds true, introduction of heterogeneity in damage parameters now imply that for a given value of GDP, country i's choice of abatement technology is increasing in δ_i if $\delta_i < \tilde{\delta}$ and decreasing in δ_i if $\delta_i > \tilde{\delta}$ where,

$$\tilde{\delta} = \frac{4\theta - M}{2(2Y - K)} \tag{67}$$

Figure 10 (a) shows the relationship between $\phi_i(\theta)$ and δ_i. This means that it is possible that a relatively smaller GDP country's choice of technology is cleaner than a relatively larger GDP country's choice of technology, depending on the combination of their GDP (y_i) and damage parameter (δ_i).

The size of $\phi_i(\theta)$ determines how differences in y_i translates into differences in $\widehat{x_i}(\theta)$ and it is the differences in $\widehat{x_i}(\theta)$ that determines the differences in marginal abatement costs & hence the Gains from Equalization of MACs. Thus, the distribution of damage parameters would play in important role in terms of determining the size of Gains from Trade through its influence on determining the size of $\phi_i(\theta)$.

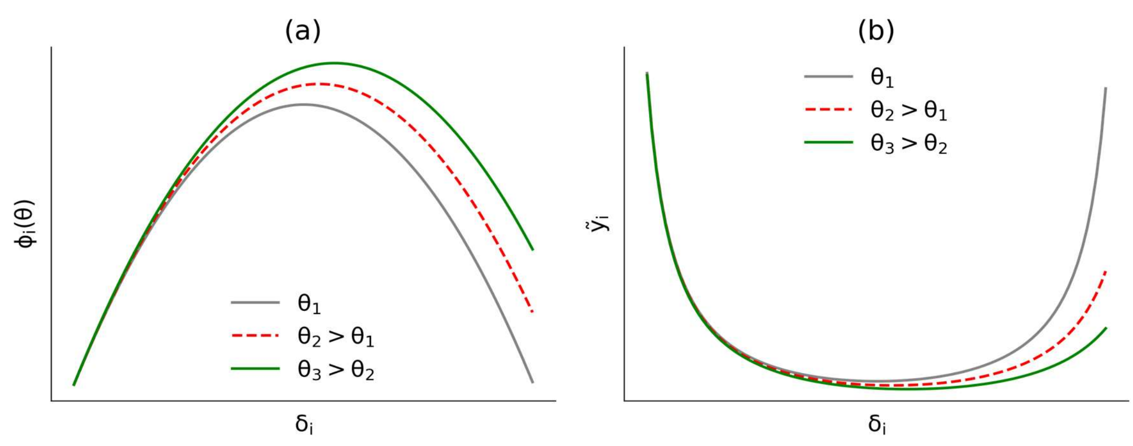

Figure 11: Relationship of $\phi_i(\theta)$ and $\tilde{y}$ with δ_i

In this setting, for every value of δ_i there will be a GDP such that any country with that particular GDP and damage parameter doesn't trade allowances in the treaty, which is given by:

$$\widetilde{y_i} = \frac{2(2K\theta - M\ \ )}{Y\delta_i(K\delta_i - M - 2Y\delta_i + 4\theta)} \tag{68}$$

Figure 10 (b) partitions the $(\delta_i, \widetilde{y_i})$ space into two regions: buyers & sellers of allowances. It is straightforward to show, the non-trading GDP $\widetilde{y_i}$ is increasing in δ_i if $\delta_i > \tilde{\delta}$ and vice-versa, where $\tilde{\delta}$ is given by equation (67).

$\phi_i(\theta)$ is increasing in θ. This retains the fact that an increase in the cost of adaptation causes abatement to rise in the NCE thus magnifying the differences in MACs. On the other hand, $\widetilde{y_i}$ is decreasing in θ. An increase in θ hence implies that the space of possible allowance buyer GDP is enlarged while that of allowance seller GDPs decrease, as shown in **Figure 11 (b)**. The high δ countries are over-adapting in the NCE and since emission allowances are allocated equiproportionately to their NCE emissions, the higher δ countries get a larger allocation due to higher emissions in the NCE. But, as θ increases, adaptation becomes expensive and so the countries substitute into abatement. Hence, the extent of over-adaptation must shrink as θ rises. In fact, it vanishes as θ goes to infinity. It is this narrowing of over-adaptation that increases the high δ countries' demand for allowances in the treaty and hence turning them from sellers to buyers.

7. Conclusion

What this paper looks at is the performance of a treaty in terms of what it achieves for aggregate emissions relative to the non-cooperative equilibrium. Our results show that an increase in the cost of adaptation (relative to abatement) increases the highest percentage equiproportionate reduction in emissions that a SPAAR treaty can achieve. This means that efforts to reduce abatement costs globally (thus making adaptation relatively expensive) can help increase the prospects of a stable treaty achieving some reductions in global emissions over the non-cooperative equilibrium. Policies directed towards reducing the costs of advanced biofuels and non-emitting sources of electricity like wind, nuclear power & solar can be such examples.

Our results also show that incorporating a system of transfer among the countries, like an emissions trading program, increases global surplus, even if a treaty does not achieve any reduction in global emissions. An emissions trading program simply eases the process of capturing the potential gains from trade and hence facilitates cooperation.

One important way countries interact among one another is through international trade. The concern for countries like the US in particular, is that engaging in a treaty that does not assign them a large allocation, will impact their competitiveness in terms of international trade and that may well completely dominate any gains from emissions trading associated with the treaty. Hence incorporating the potential loss of competitiveness in international trade from restricting one's emissions may be an interesting extension to our model.

Asymmetry across countries in our model is introduced through economic size (GDP). Hence the small economies are not necessarily poor and the large economies are not necessarily rich. Allowing the adaptation cost parameter to vary according to GDP per capita, while preserving the central scope-of-control effect might be another extension worth pursuing.

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A Appendix

The following provides the model solutions keeping heterogeneity in the damage parameters. We introduce the following quantities from (66):

$$M \equiv \sum_{i=1}^{n} \delta_i^2 y_i^2 \ and \ K \equiv \sum_{i=1}^{n} \delta_i y_i^2 \tag{66}$$

And

$$V \equiv \sum_{i=1}^{n} \delta_i y_i \ and \ W \equiv \sum_{i=1}^{n} \delta_i^2 y_i \tag{69}$$

The interior solution conditions (ISCs) will then be as follows:

$$\theta > \frac{\max\{\delta_i\}(2Y-K)+M}{4} \equiv \theta_{min} \ and \ 2Y > K \ and \ V > 2 \tag{70}$$

The Non-Cooperative Equilibrium (NCE)

Solving the first order conditions obtained from (7) yields the following best-response functions:

$$a_i(E_{-i}) = \frac{\delta_i(2y_i-\delta_i y_i^2+2E_{-i})}{4\theta-\delta_i^2 y_i^2} \tag{71}$$

and

$$x_i(E_{-i}) = \frac{\delta_i(2\theta-\delta_i y_i+\delta_i E_{-i})\, y_i}{4\theta-\delta_i^2 y_i^2} \tag{72}$$

Using equation (1) we can get the best response function in terms of emissions:

$$e_i(E_{-i}) = \frac{(4\theta-2\theta\delta_i y_i+\delta_i^2 E_{-i} y_i) y_i}{4\theta-\delta_i^2 y_i^2} \tag{73}$$

Substituting $E_{-i} = E - e_i$ in (73) and rearranging to make e_i the subject, we get

$$e_i = y_i + \frac{(\delta_i^2 y_i E-2\theta\delta_i y_i) y_i}{4\theta} \tag{74}$$

Summing across *i* allows us to solve for equilibrium E:

$$\widehat{E(\theta)} = \frac{4\theta Y-2\theta\sum_{i=1}^{n}\delta_i y_i^2}{4\theta-\sum_{i=1}^{n}\delta_i^2 y_i^2} = \frac{2\theta(2Y-K)}{4\theta-M} \tag{75}$$

Setting $E = \widehat{E(\theta)}$ in (74) we can then solve for equilibrium $\widehat{e_i}$:

$$\widehat{e_i}(\theta) = y_i - \phi_i(\theta) y_i^2 \tag{76}$$

Where $\phi_i(\theta)$ is given by (65).

We can then solve for Nash Equilibrium Abatement & Adaptation as:

$$\widehat{x_i}(\theta) = \phi_i(\theta) y_i \tag{77}$$

And

$$\widehat{a_i}(\theta) = \frac{\delta_i - 2\phi_i(\theta)}{\delta_i} \tag{78}$$

Setting $\delta_i = \delta$, we can then get the solutions presented in Section 3.1

Minimum Cost Solution (MCS)

The first order conditions for x_i and a_i are respectively,

$$2x_i y_i = y_i \sum_{j=1}^{n} \delta_j y_j (1 - a_j) \quad \forall i \tag{79}$$

$$2\theta a_i y_i = y_i \delta_i \sum_{j=1}^{n} y_j (1 - x_j) \quad \forall i \tag{80}$$

From (79) we obtain,

$$x_i = \frac{1}{2} \sum_{j=1}^{n} \delta_j y_j (1 - a_j) \equiv x \quad \forall i \tag{81}$$

Thus, abatement in the MCS is identical across countries. Substituting x from (81) for x_j in (80) and rearranging, we obtain

$$a_i = \frac{\delta_i Y(1-x)}{2\theta} \tag{82}$$

Substituting a_i from (82) for a_j in (81) and solving for x yields:

$$x_i^{**} = x^{**} = \frac{2\theta V - YW}{4\theta - YW} \tag{83}$$

Finally, substituting x^{**} for x in (82) yields

$$a_i^{**} = \frac{\delta_i Y(2-V)}{4\theta - YW} \tag{84}$$

Thus, adaptation in the MCS will vary among countries as per their vulnerability. Substituting $x_i = x$ in equation (1) and summing across *i* yields:

$$E = Y(1 - x) \tag{85}$$

Setting $x = x^{**}$ and solving yields equilibrium emissions in the MCS:

$$E(\theta)^{**} = \frac{2\theta Y\,(2-V)}{4\theta - YW} \tag{86}$$

Setting $\delta_i = \delta$, we can then get the solutions in Section 3.2 We consider $\delta_i = \delta\ \forall i$ from this point onwards.

Proof of Proposition 1

We denote this gap as follows:

$\beta(\theta)^* = 1 - \frac{E(\theta)^{**}}{\bar{E}(\theta)}$

Differentiating with respect to θ yields:

$$\frac{\partial \beta(\theta)^*}{\partial \theta} = \frac{2\,\zeta\,Y\,\delta\,(2 - \delta Y)}{(4\theta - \delta^2 Y^2)^2}$$

where the term $\zeta = \frac{2\,\delta\,(Y^2 - S)}{2Y - \delta S}$

From the ISCs these terms are positive. Moreover, this is increasing at a decreasing rate.

$$\frac{\partial^2 \beta(\theta)^*}{\partial \theta^2} = \frac{16\,\zeta\,Y\,\delta\,(\delta Y - 2)}{(4\theta - \delta^2 Y^2)^3}$$

where the term $\delta Y - 2$ is negative by the ISCs.

Proof of Proposition 2

Setting $\pi_i(\omega, \theta)$ from (56) equal to zero and solving for ω yields two solutions:

$$\omega_i^-(\theta) = \frac{\gamma_i(\theta) - \sqrt{\gamma_i(\theta)\lambda_i(\theta)}}{\gamma_i(\theta) - \lambda_i(\theta)} \tag{87}$$

and

$$\omega_i^+(\theta) = \frac{\gamma_i(\theta) + \sqrt{\gamma_i(\theta)\lambda_i(\theta)}}{\gamma_i(\theta) - \lambda_i(\theta)} \tag{88}$$

Both of these roots are real since $\gamma_i(\theta) > 0$ and $\lambda_i(\theta) > 0$ when ISCs hold. Evaluating $\frac{\partial \pi_i(\omega,\theta)}{\partial \omega}$ at each root reveals that $\pi_i(\omega, \theta)$ is increasing in ω at $\omega = \omega_i^+(\theta)$ and decreasing in ω at $\omega = \omega_i^-(\theta)$. In particular,

$$\frac{\partial \pi_i(\omega,\theta)}{\partial \omega}\Big|_{\omega = \omega_i^-(\theta)} = -2\sqrt{\gamma_i(\theta)\lambda_i(\theta)} \tag{89}$$

Thus the relevant root is $\omega_i^-(\theta)$; at any $\omega > \omega_i^-(\theta)$, $\pi_i(\omega,\theta)$ becomes negative and member *i* leaves the treaty. Taking the square root of the term under the radical in $\omega_i^-(\theta)$ yields two branches, one for $y_i - \tilde{y} < 0$ and one for $y_i - \tilde{y} > 0$. These two branches of the solution are reported as $\widetilde{\omega_i}(\theta)$ in (63) with $I = -1$ and $I = 1$ respectively. Since stability of grand coalition requires $\omega^{GC}(\theta) \leq \widetilde{\omega_i}(\theta)\ \forall i$, it follows that $\omega^{GC}(\theta) = min\left(\widetilde{\omega_i}(\theta)\right)$.

Proof of Proposition 3

Differentiate $\widetilde{\omega_i}(\theta)$ with respect to y_i to obtain

$$\frac{\partial \widetilde{\omega_i}(\theta)}{\partial y_i} = (2\theta - \delta Y)(4\theta - \delta^2 S)\left(2\theta - \delta Y + \delta \frac{Y(2-\delta y_i)^2 y_i + 2(Y-y_i)(2Y-\delta S)}{2(2Y-\delta S)}\right) A_i \tag{90}$$

where the three bracketed terms are positive (by the ISCs) and A_i is a term with no roots in θ (too complicated to usefully report here).

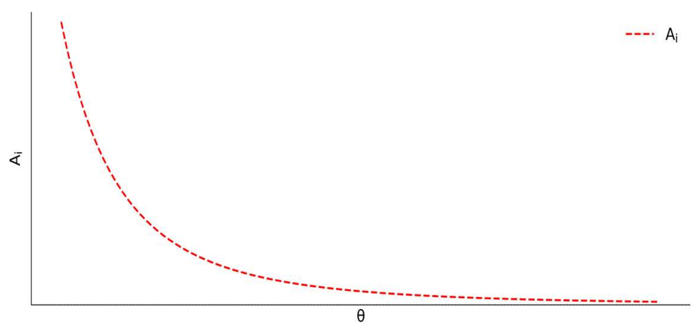

Figure 12: Relationship between A_i with θ

Thus $\frac{\partial \widetilde{\omega_i}(\theta)}{\partial y_i}$ is monotonic in θ for $\theta > \frac{\delta Y}{2}$. Now differentiate $\frac{\partial \widetilde{\omega_i}(\theta)}{\partial y_i}$ with respect to θ and evaluate this derivative at $\theta = \frac{\delta Y}{2}$ to obtain

$$\frac{\partial^2 \widetilde{\omega_i}(\theta)}{\partial y_i \partial \theta}\Big|_{\theta = \frac{\delta Y}{2}} = 2\sqrt{2}\left(\frac{Y(2-\delta y_i)^2 y_i + 2(Y-y_i)(2Y-\delta S)}{Y(2Y-\delta S)(2-\delta y_i)^2\sqrt{Y(2-\delta y_i)^2}}\right) I \tag{91}$$

where $I = -1$ if $y_i < \tilde{y}$ and $I = 1$ otherwise. The term inside the brackets is positive (by the ISCs). Thus, at any $\theta > \frac{\delta Y}{2}$, $\frac{\partial \widetilde{\omega_i}(\theta)}{\partial y_i} < 0$ for $y_i < \tilde{y}$ and $\frac{\partial \widetilde{\omega_i}(\theta)}{\partial y_i} > 0$ for $y_i > \tilde{y}$.

Proof of Proposition 4

Differentiate $\widetilde{\omega_i}\,(\theta)$ with respect to θ to obtain

$$\frac{\partial \widetilde{\omega_i}\,(\theta)}{\partial \theta} = (y_i - \tilde{y})(2 - \delta y_i)\, B_i \tag{92}$$

where $(2 - \delta y_i) > 0$ by the ISCs and B_i is a term with no roots in y_i in the interval $y_i \in \left(0, \frac{2}{\delta}\right)$. Thus $\frac{\partial \widetilde{\omega_i}\,(\theta)}{\partial \theta}$ is monotonic in y_i for $y_i < \tilde{y}$ and for $y_i > \tilde{y}$.

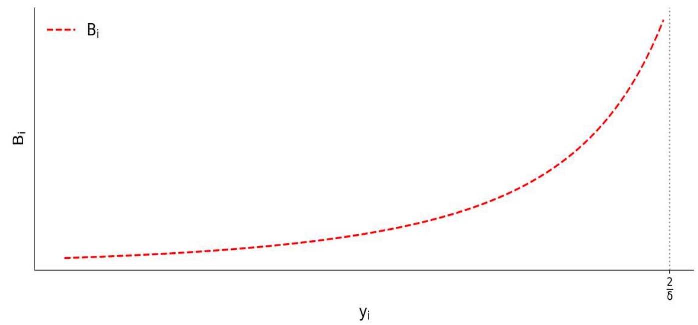

Figure 13: Relationship between B_i with y_i

We now show that $\frac{\partial \widetilde{\omega_i}\,(\theta)}{\partial \theta} \equiv W_1$ has no real roots in θ in the relevant parameter space and that it is indeed positive in that range.

At first, we identify any real roots of W_1 in θ, given as follows:

$$RT_1 = \frac{12 y_i Y \;-\; 2\,\delta y_i\, S \;+\; 2\,\delta^2 y_i \sqrt{(2Y - \delta S)(18 Y y_i^2 \;-\; 16 Y S \;-\; y_i^2 \delta S)}}{2(8\,\delta\, y_i^2 \;-\; 16\,\delta\, S \;+\; 32\, Y)} \tag{93}$$

$$RT_2 = \frac{12 y_i Y \;-\; 2\,\delta y_i\, S - \; 2\,\delta^2 y_i \sqrt{(2Y - \delta S)(18 Y y_i^2 \;-\; 16 Y S \;-\; y_i^2 \delta S)}}{2(8\,\delta\, y_i^2 \;-\; 16\,\delta\, S \;+\; 32\, Y)} \tag{94}$$

The term under the radical is convex quadratic in y_i. This term is positive only for $y_i > y_R$ where y_R is given by:

$$y_R = \frac{4\sqrt{(18\,Y - \delta\,S)Y\,S}}{(18\,Y - \delta\,S)} \tag{95}$$

Thus, W_1 has no real roots in θ for y_i less than y_R. The following graph of RT_1 and RT_2 shows that both real roots of W_1 in θ are less than θ_{min} at any $y_i < Y$.

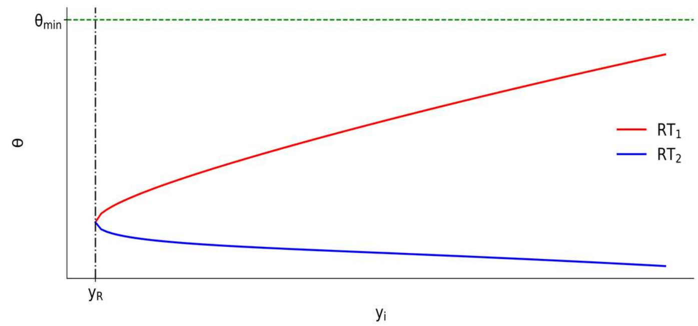

Figure 14: Plot of real roots of $\frac{\partial \widetilde{\omega_\iota}\,(\theta)}{\partial \theta}$ in θ

RT_1 crosses θ_{min} at $y_i = \frac{\sqrt{2\delta Y}}{\delta}$ which is greater than Y, because squaring this term yields $\frac{2Y}{\delta}$ and we know that $\frac{2}{\delta} > Y$. Now we show that RT_1 does cross θ_{min} from below, as it is increasing in y_i.

$$D_1 \equiv \frac{\partial R\ _1}{\partial y_i} = \frac{\delta^2(2Y-\delta S)\left(d_m+6Yy_id_j+8\delta S^2Y+y_i^2S^2\delta^2-y_i\delta Sd_j\right)}{2\left(\delta y_i^2-2\delta S+4Y\right)^2 d_j} \tag{96}$$

where,

$$d_j = \sqrt{(2Y-\delta S)(18Yy_i^2-16YS-y_i^2\delta S)}$$

and

$$d_m = 36Y^2y_i^2 - 16Y^2S - 16\delta y_i^2YS$$

This has no roots is y_i. Thus, it is either positive everywhere or negative everywhere. The denominator is real (the term under the radical is the same convex quadratic term we had in (93) & (94)) and hence positive. Now we show that the numerator of D_1 is positive. Evaluating the numerator at $y_i = y_R$ yields:

$$\frac{8\delta^2(2Y-\delta S)YS(6Y-\delta S)^2}{18Y-\delta S}$$

This is positive since $2Y > \delta\, S$. Hence $D_1 > 0$ at $y_i = y_R$, hence it is positive everywhere and crosses θ_{min} from the below. Hence $RT_1 < \theta_{min}$ for $y_i < Y$.

Now, we show that the second root of W_1 is also less than θ_{min}. The difference between RT_1 and RT_2 is given by:

$$J_1 \equiv RT_1 - RT_2 = \frac{\delta^2 y_i \sqrt{(2Y-\delta S)\left(18Yy_i^2 - 16YS \quad {}_i^2 \delta S\right)}}{4\left(\delta y_i^2 - 2\delta S + 4Y\right)} \tag{97}$$

The numerator is positive as shown above and the denominator is also positive by the ISCs. Hence, $RT_1 < RT_2 < \theta_{min}$. Thus, we have shown that W_1 has not real roots in the relevant range of parameters. Hence, if it is positive anywhere in this range, then it is positive everywhere in our relevant parameter range.

Since we know that $W_1 = 0$ at $y_i = \tilde{y}$, we now show that W_1 is increasing in $y_i = \tilde{y}$.

$$\frac{\partial W_1}{\partial y_i} \Big|_{y_i = \tilde{y}} = \frac{\delta^2 (32 \quad {}^3\theta^2 - 16Y^2\theta^2\delta S - 12\delta^2\theta S^2 Y + \delta^4 S^3 Y + 8\delta\theta^2 S^2 + 2\delta^3 S^3 \theta)}{2\,(4\theta - \delta^2 S)^2\, \theta^2\, (2Y - \delta S)\, Y\, j_k} \tag{98}$$

where

$$j_k = \sqrt{\frac{4\,Y^2\,\theta - \delta^2\,S^2}{Y^2\,\theta}}$$

The denominator is real and positive by the ISCs. The numerator is a convex quadratic in θ with a strictly positive minimum. Thus, the numerator is always positive and hence W_1 is increasing in y_i everywhere in our relevant range of parameters.