COST-BENEFIT ANALYSIS

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1. INTRODUCTION AND OVERVIEW

OUTLINE

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1.1 WHAT IS COST-BENEFIT ANALYSIS?

- Cost-benefit analysis (CBA) is a public policy decision tool.
- In specific instances:
  - to assess whether or not the social benefits of a proposed policy or project outweigh its social costs.

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- More generally:
  - to facilitate the allocation of resources to their most valuable uses.
- An optimal allocation of resources can sometimes be achieved through markets: **price signals** direct resources to their most valued uses (the First Welfare Theorem).

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- So why is there a need for policy?
- Two rationale for policy intervention:
  - market failure (inefficiency)
  - wealth redistribution
- Purpose of CBA:
  - to assess the case for intervention, and guide that intervention.

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- CBA is importantly distinct from financial analysis.
- **Financial analysis** is a business tool that focuses on financial outlays and receipts associated with an investment.

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- In contrast, CBA is concerned with social costs and social benefits, and these can differ widely from simple financial outlays and receipts.

- Example:
  – environmental impacts impose a social cost but may have few associated financial implications.

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- We nonetheless must include a financial analysis as one component of the CBA for the purposes of identifying the impact of a project on government finances.

- Why? Any net outlays by government must be funded by other sources of government revenue (typically via taxation).

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1.2 THE MAIN STEPS OF A CBA

- Here we simply list the main steps. In section 1.3 following we provide an explanation of each step, along with some illustrative examples.

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1. Define the referent group.
2. Select the portfolio of project options.
3. Catalogue potential impacts and select measurement indicators.
4. Predict quantitative impacts over the life of the project relative to a well-defined base case.

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5. Monetize (attach monetary values to) all impacts.
6. Calculate the impact on government finances and the associated cost of funds.
7. Calculate the net present value.
8. Examine the distribution of costs and benefits.

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9. Examine the implications of uncertainty.
10. Report the results and make a recommendation.

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1.3 ILLUSTRATIVE EXAMPLES

STEP 1. DEFINE THE REFERENT GROUP

- The **referent group** is the set of people whose costs and benefits count for the purposes of the analysis.
- In legal terms, these people have **standing**.

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- For example, consider a highway construction project in British Columbia (BC) to be undertaken by the government of BC.
- Do travel-time savings that accrue to residents of Alberta, who sometimes visit BC, count as a benefit of the project?
- If the referent group is British Columbia, then the answer is “no”.

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- In contrast, if the referent group is Canada (or the entire global community), then the answer is “yes”.
- So who decides?

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- Standing is a political issue, and ideally should not be subject to the judgement of the analyst.
- The analyst should seek guidance from the client (typically, a government) as to the composition of the referent group.

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- When such guidance is unclear, it is best to consider (at least) two perspectives:
  - a “state perspective”, where the referent group is defined as the residents of the jurisdiction whose government is undertaking the project (in the highway example, BC residents); and
  - a “global perspective”, where the referent group includes all people, regardless of where they reside.

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- Throughout the course, we will often consider three referent groups (typically, at the level of province, nation, and globe).

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- We will later see (in Topic 2) that the definition of the referent group is crucial to the distinction between costs and benefits on one hand, and transfers on the other.

- A **transfer** is an exchange of money between members of the referent group, and so has no impact on the wealth of the referent group as whole.

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- For example, suppose a toll is placed on the highway to be constructed in BC.
- How are the toll revenues collected treated in the CBA?
- The answer depends on the referent group, in the following way.

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- If the referent group is BC then
  - tolls collected from BC residents are just a transfer; they are an exchange of money within the province
  - tolls collected from non-residents of BC are a benefit of the project; they constitute an inflow of wealth from outside the province

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- If the referent group is Canada then
  - tolls collected from Canadian residents are just a transfer; they are an exchange of money within the country
  - tolls collected from non-residents of Canada are a benefit of the project; they constitute an inflow of wealth from outside the country

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- If the referent group is global then
  - all tolls collected are just a transfer; they are an exchange of money within the global community

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- While transfers are neither a cost nor a benefit, they do typically have **distributional consequences** and so must be tracked and accounted for in Step 8 of the CBA.

- Transfers can also have an impact on **government finances** so they must also be tracked and accounted for in Step 6 of the CBA.

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- It is important to note that adopting a non-global referent group can lead to global inefficiency because costs and benefits from a project that fall outside the referent group are not counted.

- It is often nonetheless in the interests of any given jurisdiction to focus exclusively on its own costs and benefits.

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- Ideally, policy should be coordinated across jurisdictions via **cooperative agreements** between governments but these are often difficult to implement.

- We will revisit this issue in Topic 2-8.

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STEP 2. SELECT THE PORTFOLIO OF PROJECT OPTIONS

- At this step, we identify which project options will be examined.

- Examples for the highway project:
  - scale (four lanes vs. two vs. six); routing; pavement depth and type; toll or no toll.

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- In principle:
  - no limit to the options that could be considered.

- In practice:
  - a preliminary judgment must be made by the policy-maker or the analyst, and this is to some extent arbitrary because some options are ruled out of consideration without complete analysis.

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- For example, it seems perfectly reasonable to exclude a twenty-lane highway from consideration, but there is a (very small) chance that the optimal highway is in fact twenty lanes, but we will never know this because we did not consider that option.

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- Ultimately, we must be pragmatic, and recognize that we cannot consider every option but that necessarily means that mistakes can sometimes be made.

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STEP 3. CATALOGUE POTENTIAL IMPACTS AND SELECT MEASUREMENT INDICATORS

- **Impacts** are the inputs and outputs of the project.

- Inputs are usually costs (such as construction materials) while outputs are usually benefits (such as travel-time savings).

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- However, some “outputs” can be costs (such as the adverse environmental impacts of a highway), and so it is important to recognize that not all outputs are good.

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- It is also important to remember that we are concerned with all impacts regardless of whether or not they have financial implications.
- For example, travel-time savings for tourists using a highway may not save them money (whereas it might for commercial trucking), but those savings are nonetheless a benefit of the project.

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- In order to measure impacts of any kind, we first need to choose **indicators** that we can quantify.

- This is straightforward for some impacts (like hours of labour or tons of concrete) but it can be very challenging in the case of environmental impacts and other non-market impacts.

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- For example, if a highway project interferes with wildlife in the area, is “number of animals affected” a sufficient indicator, or do we need a more sophisticated indicator that captures genetic diversity within and across species?
- Issues of this nature are typically beyond the expertise of economists, and that is why good CBA requires input and advice from outside experts from a range of fields.

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STEP 4. PREDICT THE QUANTITATIVE IMPACTS OVER THE LIFE OF THE PROJECT

- The impacts of the project must be predicted relative to a well-defined base case.

- The **base case** (or **baseline**) is a forecast of what would happen in the absence of the project.

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- It is important to stress that the base case is not necessarily a description of the *status quo* since change may be expected even in the absence of the project (from income growth, population growth, technological change, climate change, etc.)

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- For example, a CBA for a safety-upgrade on an existing highway should use a base case for traffic volumes on the existing road that properly forecasts how that volume will change over time as the population in the area grows.
- Failure to do so would likely under-estimate the numbers of lives that will be saved over time by the safety-upgrade.

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- Both the base case and the impacts of the project relative to that base case are necessarily subject to uncertainty.

- This uncertainty means that the base case may have to be specified as **state-contingent**, where a number of possible base-case scenarios are specified with associated probabilities.

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- For example, the CBA for a highway project might specify a base case with three scenarios:
  – high population growth (with 20% probability)
  – medium population growth (with 60% probability)
  – low population growth (with 20% probability)
- In this example, these scenarios correspond to three different" states of nature"; hence, the term, "state contingent".

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STEP 5. MONETIZE ALL IMPACTS

- In this step we assign dollar values to all impacts.
- We will see that it is sometimes appropriate to use **market prices** to value inputs and outputs, but in many instances monetization is not nearly that simple, due to
  - market distortions (and “non-market” goods)
  - price effects from the project itself

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- In Topic 4, we will spend a lot time discussing how we calculate costs and benefits when faced with either (or both) of these complications.

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STEP 6. CALCULATE THE IMPACT ON GOVERNMENT FINANCES

- If the project requires a net financial outlay for government then we should take into account the cost of raising those funds, as measured by the **cost of funds** (discussed at length in Topic 4.12).

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- This means that we need to track all impacts of the project on government finances.

- For each period of the project we calculate financial outlays and financial receipts, and then calculate net financial outlays as the difference between the two.

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- For example, in a highway project, there will likely be substantial financial outlays associated with construction, but there may also be some longer-term financial receipts if a toll is imposed on the road.

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STEP 7. CALCULATE THE NET PRESENT VALUE

- Net present value (NPV) is the sum of discounted net benefits (benefits minus costs) over the life of the project.

- Future net benefits are discounted at the **public sector discount rate** (PSDR).

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- The PSDR is derived from the **rate of time preference** and the **investment rate of return** in the economy, which are determined jointly via borrowing and lending in the economy.

- All these concepts relating to NPV are discussed at length in Topic 5.

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STEP 8. EXAMINE THE DISTRIBUTION OF COSTS AND BENEFITS

- The NPV simply measures the difference between discounted benefits and costs; it provides no information about the distribution of those benefits and costs.

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- Distributional impacts are always a key concern to policy-makers in practice, and should be described in the CBA results
- The choice of "impact groups" – the groups whose costs and benefits will be measured and identified in detail – is ultimately a political one.

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- Commonly identified impact groups include
  - elderly residents
  - gender groups
  - households with young children
  - indigenous peoples
  - low-income households
  - producers
  - consumers

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- Throughout the course we will emphasize the importance of distributional analysis as a key part of any CBA.

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STEP 9. EXAMINE THE IMPLICATIONS OF UNCERTAINTY

- Most of the quantification and monetization of impacts in CBA is subject to considerable uncertainty.

- In principle, the calculation of costs and benefits should directly incorporate the implications of that uncertainty.

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- The economic theory underlying the valuation of impacts under uncertainty is complicated, and is very difficult to apply in most real-world policy settings.

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- In practice, we typically focus simply on **expected costs and benefits**, and then report the extent to which our results are subject to uncertainty.

- The goal is to convey to the policy-maker a sense of the risk associated with the project.

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- **Sensitivity-testing** is the simplest and most basic element of that reporting.
- A more comprehensive approach employs **simulation** via Monte Carlo experiments.
- We will briefly describe these techniques in Topic 6.

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STEP 10. MAKE A RECOMMENDATION

- The simple net present value rule:
  - accept the project option with the highest NPV if and only if that NPV>0; otherwise reject the project.
- In practice, our recommendation is also based on an assessment of the distributional impacts, and our level uncertainty about the results.

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- The final decision lies with the policy-maker; not the analyst.
- The role of CBA (and scientific study more generally) is to help inform policy decisions; not to make those decisions.

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- The policy-maker is ultimately subject to the judgement of the citizenry (via voting), whereas economists and scientists are not.
- Some might argue that this makes policy "political", but it is worth noting that "politics" is derived from the Greek word *politikos*, meaning "of, for, or **relating to citizens**".

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- Policy is – and should be – a political process.

- Our role as economists is to inform that process without taking a particular political position ourselves.

- We are analysts, not advocates.

END 59

TOPIC 1 REVIEW QUESTIONS

1. Which of the following is not one of the main steps of a CBA?

A. Select the portfolio of project options.

B. Monetize all impacts.

C. Balance the government budget to ensure that financial outlays equal financial receipts.

D. Examine the distribution of costs and benefits.

2. Suppose a study is conducted as part of the CBA of a proposed port expansion. The study determines that the port expansion will lead to a 50% increase in the amount of cargo-ship traffic in the surrounding waters. This study would form part of which step of the CBA?

A. Selection of project options (Step 2).

B. Cataloguing potential impacts (Step 3).

C. Quantifying potential impacts (Step 4).

D. None of the above.

3. Suppose a study is conducted as part of the CBA of a proposed airport expansion. The study identifies an increase in ambient noise as a likely consequence of the project. This study would form part of which step of the CBA?

A. Selection of project options (Step 2).

B. Cataloguing potential impacts (Step 3).

C. Quantifying potential impacts (Step 4).

D. None of the above.

4. Suppose a study is conducted as part of the CBA of a proposed mining project. The study identifies three possible approaches to mitigating the contamination of groundwater in the surrounding region, and determines that only two of those approaches are worthy of further consideration. This study would form part of which step of the CBA?

A. Selection of project options (Step 2).
B. Cataloguing potential impacts (Step 3).
C. Quantifying potential impacts (Step 4).
D. None of the above.

5. Suppose an analyst with a passion for wildlife photography is conducting a CBA of a proposed national park designation that will prohibit hunting in the designated area but enhance the opportunities for wildlife photography. He decides that the impact on hunters should not be included in the CBA despite his expectation that hunting groups will lobby the government against the proposal. This decision would form part of which step of the CBA?

A. Definition of the referent group (Step 1).
B. Cataloguing potential impacts (Step 3).
C. Reporting results and making a recommendation (Step 10).
D. None of the above.

6. One of the main steps in a cost-benefit analysis is to predict the quantitative impacts over the life of the project relative to a well-defined base case. In this context, a "state-contingent base case" refers to

A. the base case under a state perspective for the referent group.
B. a set of possible outcomes that can arise in the absence of the project, with an associated set of probabilities.
C. the project among the portfolio of projects that has the smallest net financial impact on government.
D. the most likely scenario among the possible outcomes if the project does not proceed.

7. Figure R1-1 depicts the incidence of opioid drug overdoses (measured as deaths per day) before and after a policy intervention. Which of the following is a plausible statement about the efficacy of the intervention?

A. It is clear from the data that the policy intervention had no effect on the incidence of overdoses because the incidence was leveling off anyway.

B. The efficacy of the policy must be judged against a counter-factual base case.

C. The efficacy of the policy must be judged against an estimated base case.

D. Both B and C

8. A proposed policy will impose a special yearly insurance surcharge on old diesel-powered vehicles in an attempt to get some of these old (and highly polluting) vehicles off the road. A base case has been specified that predicts the life of diesel-powered cars. This base case is described by a "survivor function", depicted in **Figure R1-2**. (A survivor function comes from biology; it plots the fraction of an original cohort that is still living at future points in time). Which of the following statements provide a correct interpretation of the properties of the survivor function in the context of diesel-powered vehicles?

A. The policy will cause the gradual retirement of diesel-powered cars, such that only 20% of cars on the road in the year 2028 will be diesel-powered.
B. The policy will cause the gradual retirement of diesel-powered cars, such that only 20% of diesel-powered cars that were new in 2010 will still be on the road in the year 2028.
C. In the absence of the policy, among the set of cars that were new in the year 2010, 80% will still be on the road after 18 years.
D. In the absence of the policy, among the set of cars that were new in the year 2010, 20% will still be on the road after 18 years.

9. Table R1-1 lists some of the benefits and costs associated with a proposal by the Province of British Columbia to build a new rock concert venue in Kelowna (in the interior of the province), under two different referent group scenarios.

Table R1-1

	Global	BC
Benefits		
Entertainment	125	105
Tax Revenue from Ticket Sales	0	7
Costs		
Construction	100	100

Which of the following is the best explanation for the difference between the values for "Tax Revenue" under the two scenarios?

A. Non-residents of BC are entitled to a rebate on BC taxes paid.

B. BC residents are exempt from paying BC taxes on educational experiences.

C. Some non-residents of BC are expected to buy tickets.

D. Taxes will not be charged on tickets for concerts by international artists.

10. In a liberal democracy, policy analysis should be based on science only, rather than on political considerations, because

A. science is objective.

B. economics is the only true science.

C. the people on whom policy is imposed should have no influence on that policy because they have vested interests.

D. None of the above.

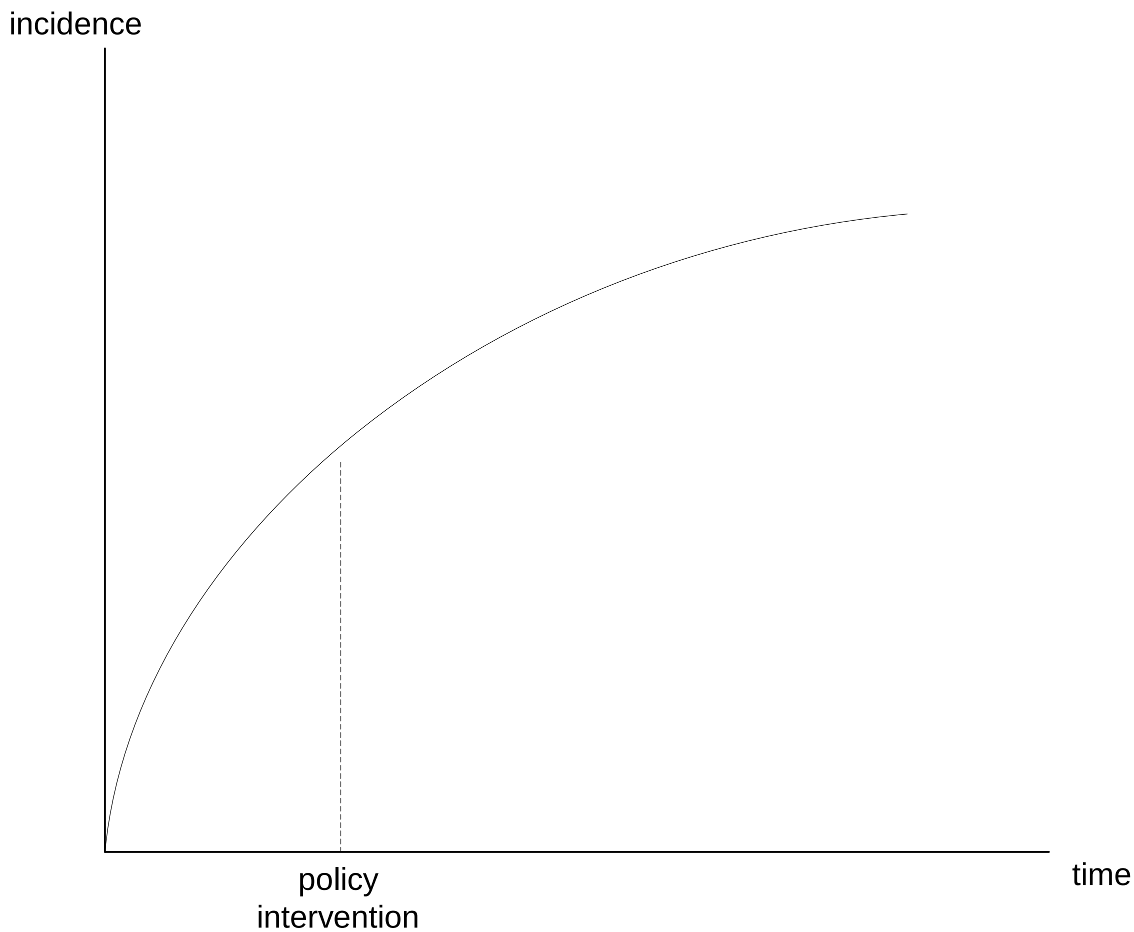

Figure R1-1

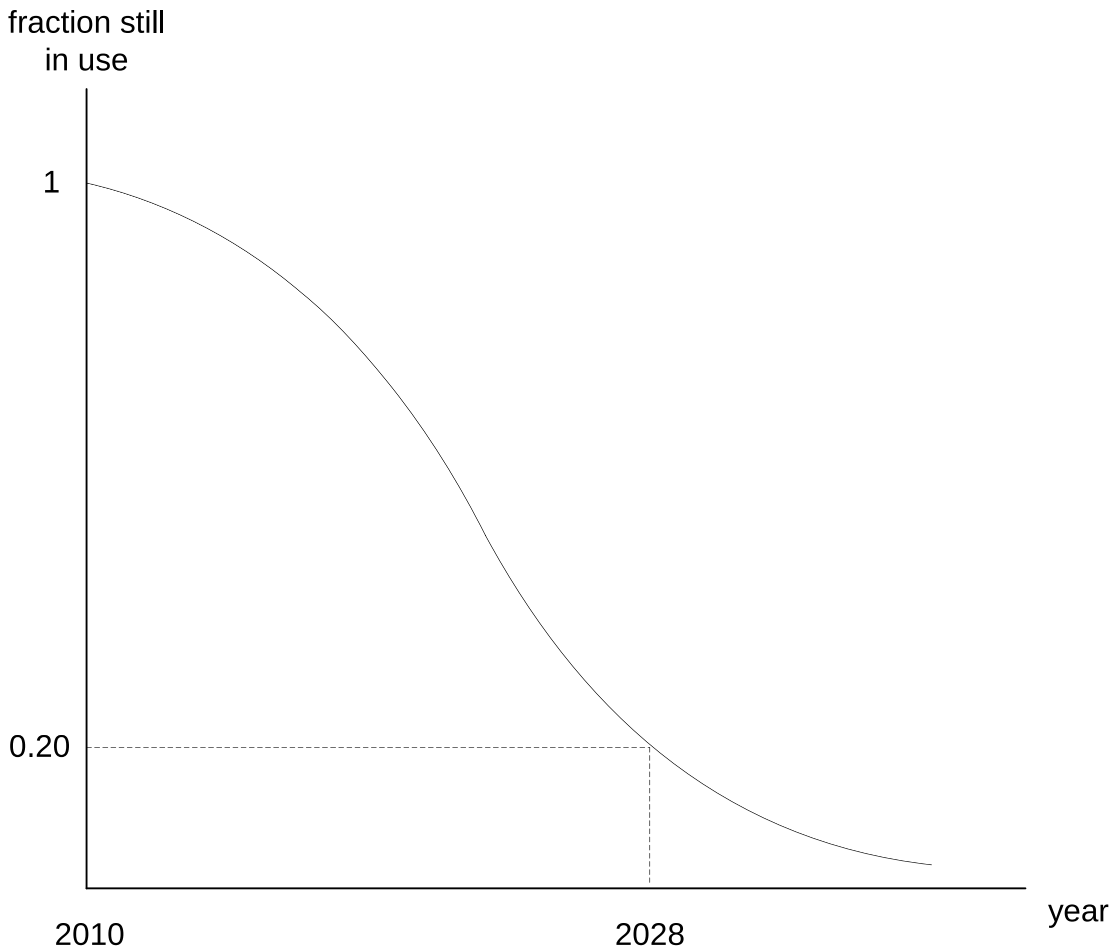

Figure R1-2

ANSWER KEY

1. C
2. C
3. B
4. A
5. D — The "analyst" in this case is imposing his own preferences on the assessment of the project, and in that sense is not actually conducting a CBA at all. The "analyst" should find a new career.
6. B
7. D — Response C is necessarily a correct response because a policy should always be assessed relative to an estimated base case. In most cases, we assess a policy before it is implemented and so the estimated base-case is a forecast of would we think would happen in the absence of the policy. In this particular case, the policy has already been implemented and so we are looking back in time, and asking what would have happened if this policy had not been implemented; this is what we mean by a "counter-factual": something that would have occurred but didn't because policy intervened. Thus, response C is also a correct response, and so D is the correct answer.
8. D
9. C
10. D — Response B is *of course* correct as a stand-alone statement, but the opening statement of the question is itself false in the context of a liberal democracy and so cannot be made true by any statement that follows it.

2. WELFARE FOUNDATIONS OF COST-BENEFIT ANALYSIS

OUTLINE

2.1 Introduction
2.2 Pareto Efficiency
2.3 "Social Preferences" and Arrow's Impossibility Theorem
2.4 Pareto Improvements
2.5 Potential Pareto Improvements and Social Surplus

2

3

2.1 INTRODUCTION

- In Topic 1 we said that the purpose of CBA is to facilitate, through public policy, the allocation of resources to their most valuable use.

- What determines "most valuable use"?

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- Ideally, the most valuable use is that which maximizes "social welfare".
- But how do we measure social welfare?

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- We will soon see that we cannot measure social welfare.
- However, we can say that some resource allocations are better than others, according to the Pareto criterion.

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2.2 PARETO EFFICIENCY

- An allocation of resources is **Pareto efficient** if it is not possible to reallocate those resources in a way that makes at least one person better-off and no person worse-off.
- An allocation is **inefficient** if and only if it is not Pareto efficient.

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- It is helpful to cast these definitions in terms of a closely-related concept.
- A **Pareto improvement** is a reallocation of resources that makes at least one person better-off and no person worse off.
- Thus, an allocation of resources is Pareto efficient if and only if there are no Pareto improvements available.

8

- We can also say that if moving from one allocation (**B**) to an alternative allocation (**A**) is a Pareto-improvement, then allocation **B** is **Pareto-dominated** by allocation **A**.

- Thus, an allocation is Pareto efficient if and only if it is not Pareto-dominated by an alternative allocation.

9

SOME RELATED TERMINOLOGY

- Two allocations **A** and **B** can be **Pareto-ranked** if and only if one Pareto-dominates the other.
- The **Pareto frontier** is the set of all Pareto efficient allocations.
- By definition, allocations on the Pareto frontier cannot be Pareto-ranked.

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- Relative to some existing allocation **B**, the **core with respect to B** is the set of Pareto-efficient allocations that Pareto-dominate **B**.

- Thus, the core is a subset of the Pareto frontier, and allocations in the core cannot be Pareto-ranked.

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- Consider an example.

- Suppose we have $100 to allocate between two persons.

- Any allocation that fully allocates the $100 between the two individuals – so no money is "left on the table" – is Pareto efficient; see Figure 2-1.

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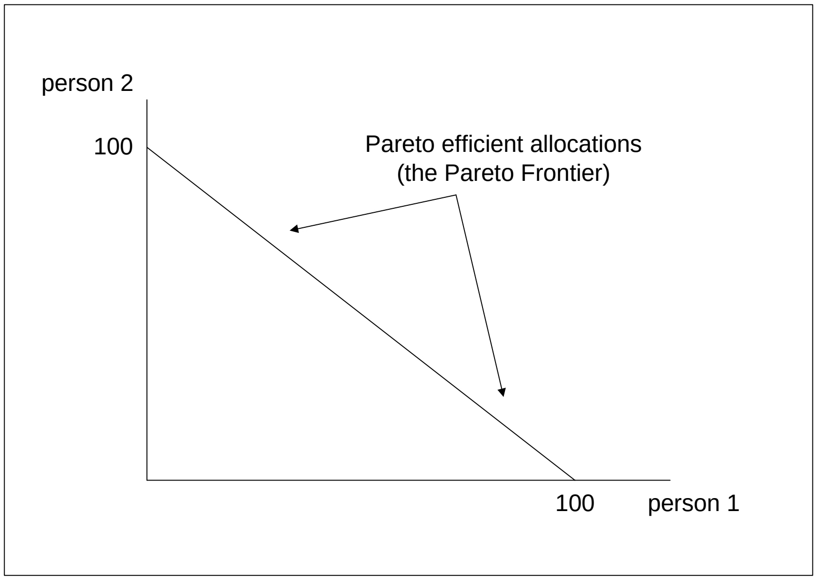

Figure 2-1

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- In contrast, an allocation in which each person has only $25 is inefficient, and is Pareto-dominated by any allocation in which neither person has less than $25 and at least one person has more than $25.

- See allocation **B** in Figure 2-2.

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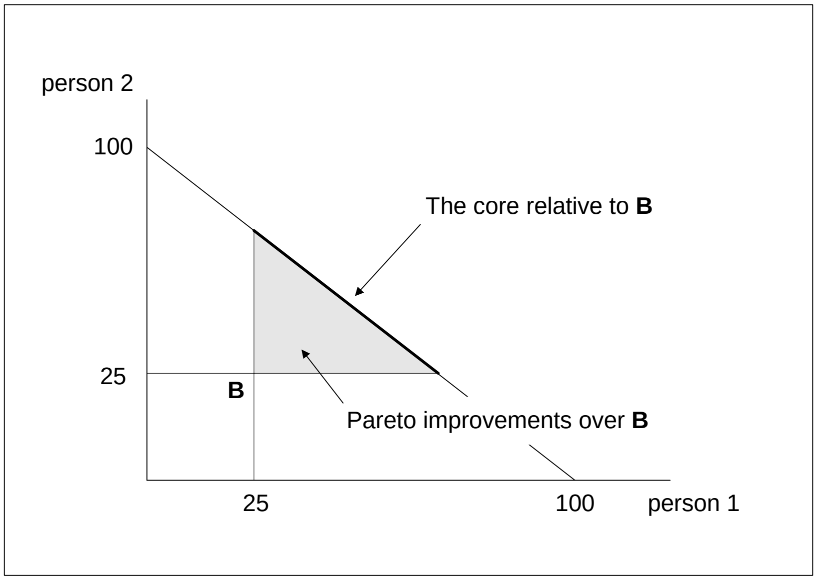

Figure 2-2

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2.3 "SOCIAL PREFERENCES" AND ARROW'S IMPOSSIBILITY THEOREM

- Can we rank Pareto-efficient allocations to determine which one has the highest social welfare?

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- Ideally we would like to construct “social preferences”, based on individual preferences, and use these social preferences to derive a social ranking or social choice rule.

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- This ideal is <u>not</u> possible.

- Roughly speaking, **Arrow’s Impossibility Theorem** tells us that it is not possible to derive a complete and consistent social choice rule derived exclusively from individual preferences, except dictatorship.

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ARROW'S IMPOSSIBILITY THEOREM

- No social choice rule for ranking alternative allocations can simultaneously satisfy the following five requirements:
  - no dictatorship
  - completeness, reflexivity, transitivity (CRT)
  - unrestricted domain (any set of individual preferences that are CRT is permissible)

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- – Pareto efficiency
- – independence of irrelevant alternatives: the social ranking over two allocations ***x*** and ***y*** is independent of individual rankings over ***x*** and ***z***, and ***y*** and ***z*** (where ***z*** is the "irrelevant alternative")

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- Interpretation of independence of irrelevant alternatives:
  - the preference ranking for an individual between ***x*** and ***z***, and ***y*** and ***z*** should be irrelevant for the social ranking of ***x*** and ***y***.
  - the only difference between rankings ***x*** > ***y*** > ***z***, and ***x*** > ***z*** > ***y*** is a difference in intensity of preference.

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- However, we cannot observe intensity of preference directly because individual utility cannot be measured cardinally.
- It is not possible to demonstrate, for example, that person **A** derives five units of happiness from a particular allocation, while person **B** derives only four units of happiness from that allocation.

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- This is sometimes called the impossibility of "interpersonal utility comparisons":
  - we cannot measure utility directly in any objective way that allows a comparison of utilities across different individuals
- This fundamental problem lies at the heart of the impossibility theorem.

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- Arrow's theorem means that there is no compelling social rule for ranking Pareto efficient allocations.
- It is generally not possible to identify a unique "best" allocation of resources that in any sense "maximizes social welfare".

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WHAT ABOUT ETHICS AND MORALS?

- A common response by some people to the impossibility theorem is that a “higher” criterion should be used for making social rankings, such as an “ethical” or “moral” criterion that transcends preferences.

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- This is a purely semantic argument: relabeling the preferences of some subset of individuals as “ethics” does not resolve the problem.

- An “ethical” solution is simply one based on the preferences of a subset of individuals (who effectively act as a collective dictatorship).

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- Note too that the possibility of a so-called "benevolent dictator" is eliminated by the impossibility theorem:
  - though possibly well-intentioned, the dictator is also faced with the impossibility of choosing an allocation based on the individual preferences of the subjects to whom she feels benevolent

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DOES VOTING SOLVE THE PROBLEM?

- Voting outcomes do not reflect preferences alone; they jointly reflect preferences <u>and</u> the structure of the voting rules in place (including the voting *agenda*).

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- That is, voting rules place a constraint on how many votes each individual has, and how those votes translate into direct influence over resource allocation.

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A VOTING EXAMPLE

- Example:
  - – three available allocations: *A*, *B* and *C*
  - – preference ordering for person 1: $A > B > C$
  - – preference ordering for person 2: $B > C > A$
  - – preference ordering for person 3: $C > A > B$
  - – what is the social preference ordering?

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- Consider a candidate social choice rule:
  - a two-step pair-wise majority voting rule
- Step 1: *A* vs. *B*
  - *A* wins by 2 votes to 1 and *B* is eliminated
- Step 2: *A* vs. *C*
  - *C* wins by 2 votes to 1

- Implied social ranking: $C > A > B$

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- But this outcome is **agenda-dependent.**

- In particular, suppose we reverse the steps.
- Step 1: *A* vs. *C*
  - *C* wins by 2 votes to 1 and *A* is eliminated
- Step 2: *C* vs. *B*
  - *B* wins by 2 votes to 1

- Implied social ranking: $B > C > A$

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- Thus, the social ranking over *A* and *B* is reversed if we choose a different agenda.

- So why not simply vote over the agenda?

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- Because a third agenda is also possible.
- Step 1: *B* vs. *C*
  - *B* wins by 2 votes to 1 and *C* is eliminated
- Step 2: *B* vs. *A*
  - *A* wins by 2 votes to 1

- Implied social ranking: $A > B > C$

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- Thus, the three different agenda yield three different social rankings (each one corresponding to the preference ordering of one of the three voters).
- This means that voting over the different agenda is equivalent to voting over the outcomes obtained under those agenda, and so we face the same problem all over again.

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WHAT ABOUT ALTRUISM?

- Can we identity a unique social optimum if people have altruistic preferences?
- In general, altruism eliminates some allocations that might otherwise be efficient, but it does not lead to a unique best allocation.

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- Unavoidable bottom line:
  - there is no way to derive a social ranking based on individual preferences alone.

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SOCIAL WELFARE FUNCTIONS

- Despite the impossibility of making inter-personal utility comparisons, many economists still sometimes construct "social welfare functions" that purport to assess aggregate welfare from individual preferences.

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- These "social welfare functions" can sometimes be useful for framing philosophical issues relating to social justice, but it is important to recognize that they can never be made operational for practical purposes.

- See Appendix 2-1 for some examples.

39

2.4 PARETO IMPROVEMENTS OVER THE BASE CASE

- Arrow's theorem tells us that we cannot identify a unique best allocation.

- However, policy is usually less ambitious; it is typically not revolutionary in scope, or in pursuit of a "new world order".

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- Consequently, our task as analysts is typically to determine whether a candidate policy is an improvement over the base case (rather than whether or not that policy implements the best possible allocation for society).

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- That task is relatively straightforward if the policy creates a Pareto improvement over the base case.

- For example, suppose the candidate policy (Policy 1) takes us from the base case at allocation **B** in Figure 2-3 to an alternative allocation **P1**.

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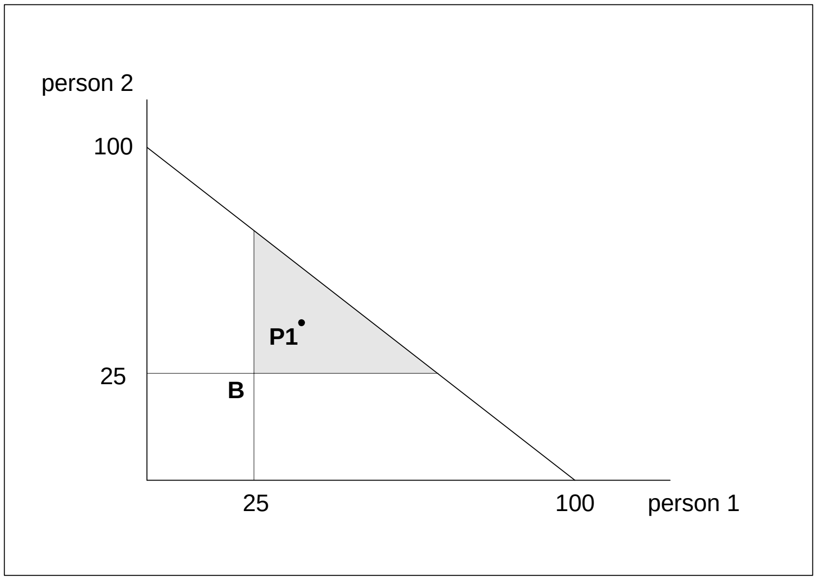

Figure 2-3

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- Policy 1 creates a Pareto improvement over the base case (since **P1** Pareto-dominates **B**), and would be sensibly viewed as a good policy.

44

- We can also sometimes rank competing policies based on the Pareto improvement criterion.

- For example, suppose a competing candidate policy (Policy 2) takes us from the base case at allocation **B** in Figure 2-4 to an alternative allocation **P2**.

45

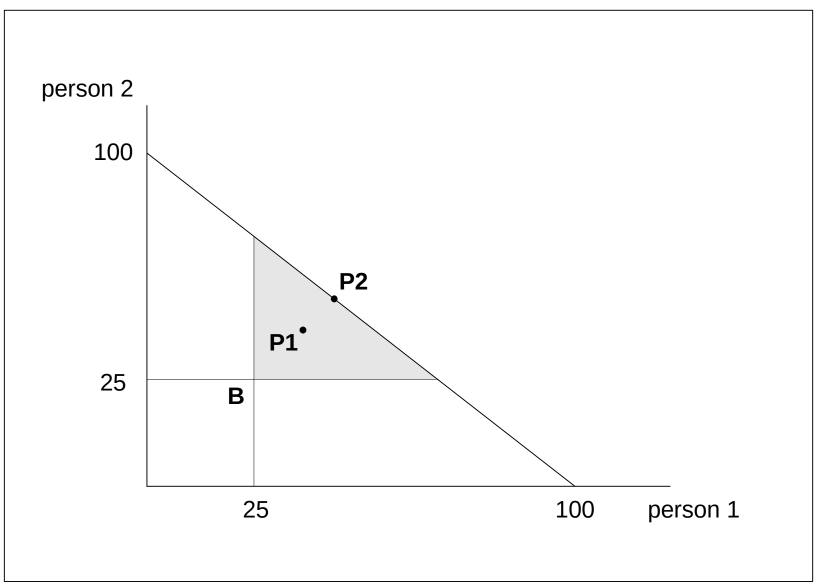

Figure 2-4

46

- Policy 2 creates a Pareto improvement over the base case (since **P2** Pareto-dominates **B**), and would sensibly be viewed as a good policy.

- Moreover, Policy 2 is a better policy than Policy 1 because **P2** Pareto-dominates **P1**.

- These two policies can be Pareto-ranked.

47

2.5 POTENTIAL PARETO IMPROVEMENTS AND SOCIAL SURPLUS

- In practice, most candidate policies do not create Pareto improvements; some people gain and some people lose.
- For example, a new water-pollution regulation might benefit water users but might also impose costs on regulated firms.

48

- How do we **aggregate** the gains and losses to determine whether the policy is a good policy?

- We use the potential Pareto improvement criterion.

49

- A policy creates a **potential Pareto improvement** (PPI) if the winners could in principle make a compensating payment to the losers such that the losers are at least as well off as in the base case, and the winners are still better off.

50

- Whether or not actual compensating payments are made from winners to losers is a decision to be made by policy-makers based on their distributional goals. (More on this later).

51

- The difference between the gains to the winners and the losses to the losers is the **net social benefit** of a policy, or the **social surplus** created by the policy.

- Thus, if a policy creates a PPI then it has a positive net social benefit, or equivalently, it creates social surplus.

52

- The potential Pareto improvement criterion is sometimes called the **Kaldor-Hicks criterion**, after the two economists who jointly proposed it.

- It is the central normative criterion in CBA, and in economic welfare analysis generally.

53

- Consider an example. Suppose a candidate policy (Policy 3) takes us from the base case at allocation **B** in Figure 2-5 to an alternative allocation **P3**.

54

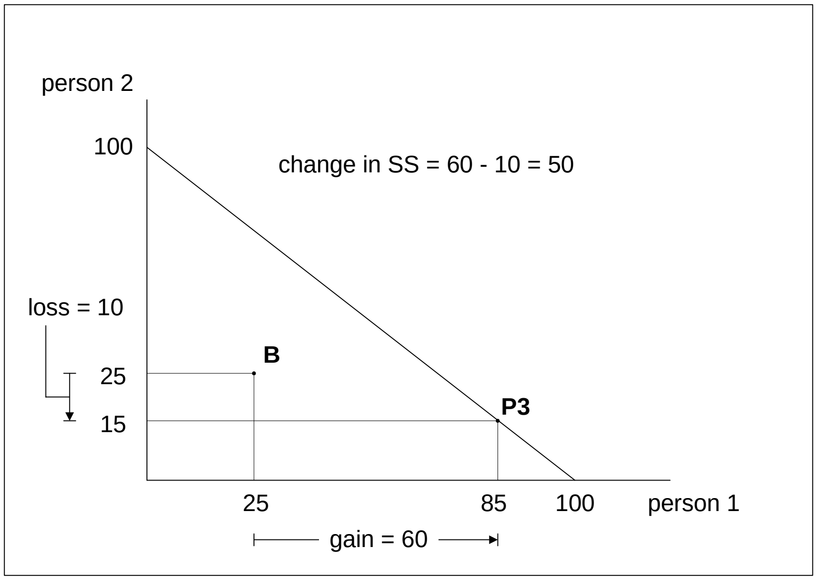

Figure 2-5

55

- The move from **B** to **P3** is not a Pareto-improvement because Person 2 is made worse-off.
- However, it is a PPI, because the gains to the winner are greater than the losses to the loser, by $50.
- Thus, the social surplus created is $50.

56

- The full set of feasible allocations that constitute a PPI over the base case is the shaded region in Figure 2-6, excluding the lower boundary identified by the dashed line.

- Note that this lower boundary is a line with slope negative one, passing through point **B**.

57

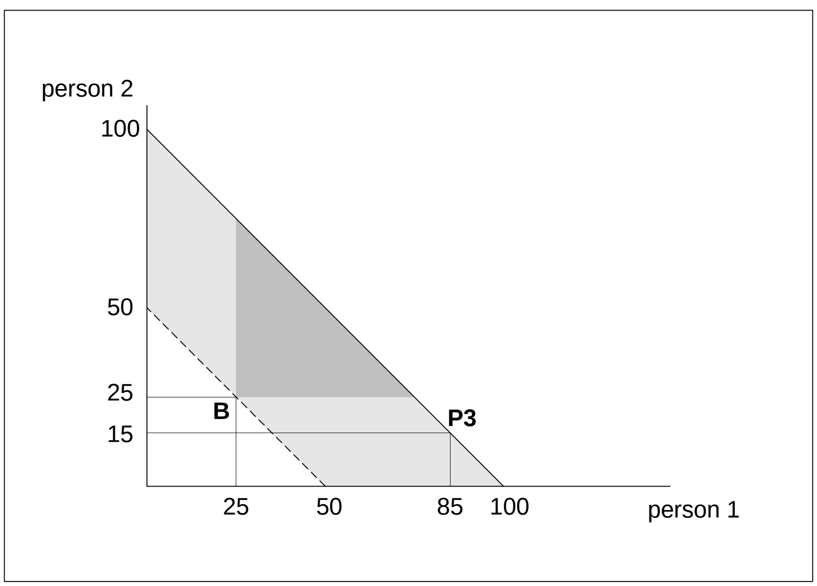

Figure 2-6

58

- Note too from Figure 2-6 that a subset of the allocations that are PPIs over the base case are also Pareto improvements over the base case (the dark shaded region).

- In general, every Pareto improvement is also a PPI (but the converse is not true).

59

SURPLUS MAXIMIZATION

- By definition, a PPI creates an increase in social surplus.

- It follows that social surplus is maximized at a given allocation if and only if there are no PPIs available at that allocation.

60

- In the context of our simple example, the set of allocations at which social surplus is maximized coincides with the Pareto frontier.

- Thus, in this example, Pareto efficiency and social-surplus-maximization (SSM) mean the same thing.

61

- In more general settings, this convenient correspondence between Pareto efficiency and surplus maximization typically does not hold.

- We will see a familiar example (monopoly) in a moment.

62

- An allocation at which social surplus is maximized must also be Pareto efficient but the converse is not true.

- In particular, an allocation at which social surplus is not maximized can nonetheless be Pareto efficient.

63

- For example, consider the standard monopoly problem illustrated in Figure 2-7.

64

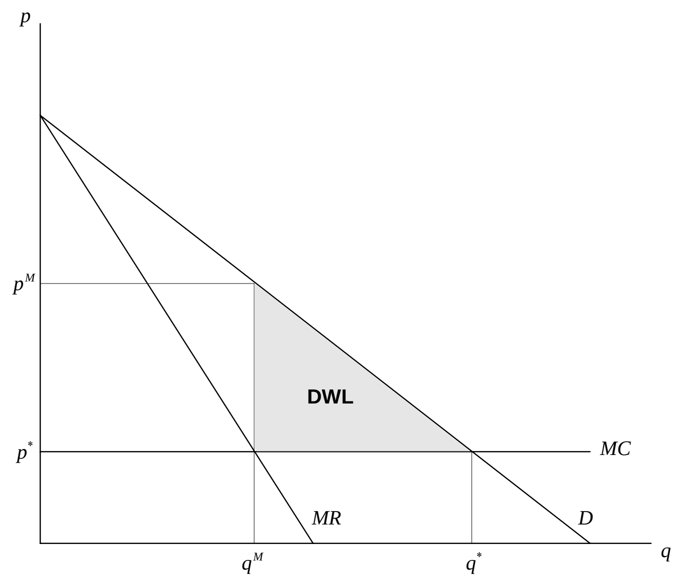

Figure 2-7

65

- The monopoly outcome (denoted $\boldsymbol{q^M}$ in Figure 2-7) is Pareto efficient: it is not possible to make consumers better off without making the monopolist worse off.
- However, the monopoly outcome does not maximize social surplus.
- Social surplus is maximized at $\boldsymbol{q^*}$.

66

- The deadweight loss (DWL) from monopoly (the shaded region in Figure 2-7) is the foregone surplus relative to the maximum possible surplus.

67

- Note that there is a unique SSM allocation in this monopoly setting, at $\boldsymbol{q}^*$.

- In contrast, there is a continuum of Pareto efficient allocations: the interval [$\boldsymbol{q}^M$, $\boldsymbol{q}^*$]. (At any $\boldsymbol{q} > \boldsymbol{q}^*$ the monopolist would make a loss and refuse to participate, so we exclude those values from consideration).

68

- The monopoly outcome is often described as "inefficient" because it does not maximize social surplus; this can be confusing and somewhat misleading .

69

- The term "inefficient" is often used to describe an outcome at which social surplus is not maximized even when that outcome is in fact Pareto efficient.

- This confusing terminology is especially common in textbook discussions of externalities, price controls, and other settings where DWLs arise.

70

- The confusion in terminology can also lead to misleading statements like "allocation A is <u>more efficient</u> than allocation B", when what is actually meant is that "allocation A has greater social surplus than allocation B" (and where both allocations could in fact be Pareto efficient).

71

- We will try to be more precise in our use of terminology here.

- We will use the term "inefficient" only to describe an allocation that is not Pareto efficient.

72

- If we wish to describe an allocation at which social surplus is not maximized, we will say exactly that.

- We will not describe one allocation as “more efficient” or “less efficient” than another.

73

2.6 WILLINGNESS-TO-TRADE

- The example from Figure 2-5 was very simple; the policy-induced changes were changes in monetary wealth.

74

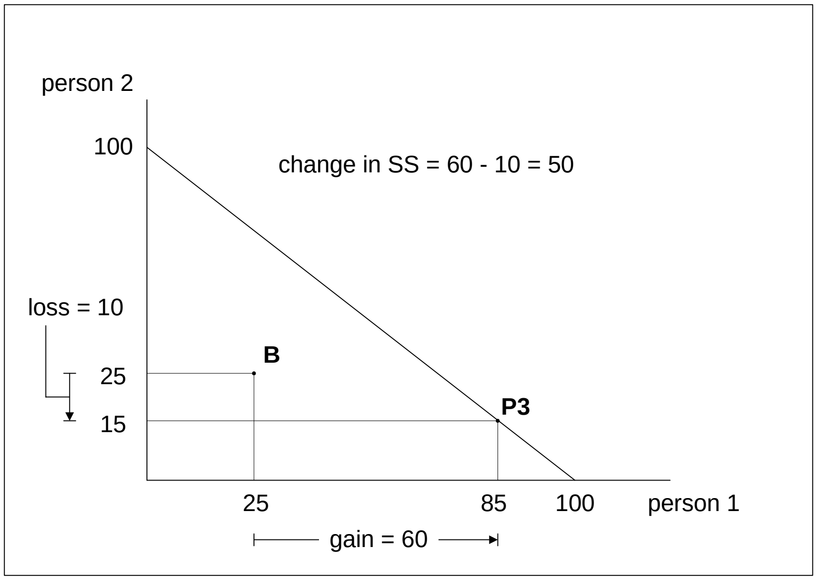

Figure 2-5 (repeat)

75

- Many policies create impacts that are not monetary, such as an increase in air quality or a reduction in the risk of death on a road.
- How do we convert these non-monetary impacts into a common unit of measure for the aggregation of gains and losses across individuals?

76

- In economics, we measure the benefit or cost of an impact to a person in terms of his or her **willingness-to-trade** (WTT).

77

- For example, suppose a project leads to a change in the quality of drinking water, ***q***.

- The value of that change for a given individual is measured in terms of his or her WTT changes in ***q*** for money (or some other numeraire good).

- This is the monetization step of the CBA.

78

- Note that using money as the unit of measure is only an accounting convenience; money is the unit of exchange in market economies, and so it is the natural unit of measure.

- WTT is fundamentally about tradeoffs, not about money *per se*.

79

- An individual's WTT can be measured by
  - willingness-to-pay (WTP); or
  - willingness-to-accept (WTA)

- Which is the correct measure? We will come to this question soon, but let us first define each of these measures using the water quality example.

80

- Suppose existing water quality is $\boldsymbol{q_0}$ and a project will raise it to $\boldsymbol{q_G} > \boldsymbol{q_0}$.

- The WTP for this gain by a person with income *y* is, by definition, an amount ***G***, such that

$$u(q_G, y - G) = u(q_0, y)$$

81

- That is, this person would be just indifferent between
  - making a payment ***G*** and receiving the higher water quality; and
  - not making that payment and retaining the original water quality.

82

- Suppose instead the project will reduce water quality to $\boldsymbol{q_L} < \boldsymbol{q_0}$.

- The WTA for this loss by a person with income *y* is, by definition, an amount ***L***, such that

$$u(q_L, y + L) = u(q_0, y)$$

83

- That is, this person would be just indifferent between
  - receiving a payment ***L*** and suffering the lower water quality; and
  - not receiving that payment and retaining the original water quality.

84

- It is clear from these definitions that when we say "WTP" we implicitly mean "maximum WTP".
- Similarly, when we say "WTA" we implicitly mean "minimum WTA".

85

WHICH IS THE CORRECT MEASURE?

- It is tempting to think that WTP is the correct measure of a gain, while WTA is the correct measure of a loss, as described in the water-quality example.
- However, it is not that simple.

86

- The correct measure to use depends on the circumstances, and in particular, on the implied assignment of **property rights** in those circumstances.

87

- The default practice is to assume that the *status quo* determines the assignment of property rights.

- This is usually the correct approach in the assessment of market transactions where a person currently in legal possession of an object – such as a car or a house – does typically have property rights over that object.

88

- However, in many instances where public policy is involved, the assignment of property rights is often unclear, and the *status quo* may not necessarily reflect the implicit assignment of rights reflected in the policy itself.

89

- For example, does a firm that is currently discharging pollution into a river necessarily have a right to do so?

- If a new regulation requires the firm to stop polluting – without compensation – then the terms of the regulation implicitly reveal that that the firm does not have that right.

90

- Similarly, the *status quo* does not necessarily reflect the rights of water users.
- For example, the regulation itself might implicitly recognize that water users have a right to some higher quality (perhaps at some historical level before the water became polluted).

91

- Textbooks are often surprisingly fuzzy on these issues, and they have not even been fully resolved at a theoretical level.
- In this course, we will typically follow common practice and use the *status quo* as the determinant of property rights.
- That is, we will use WTA as the measure of a loss, and WTP as the measure of a gain.

92

- Appendix 2-2 provides a more thorough treatment of the issue, and proposes a "purpose-based" approach to the measurement of value, where the *status quo* does not play such a dominant role in the determination of which valuation measure should be used.

93

APPLICATION OF WTP AND WTA

- We now want to interpret point **P3** in Figure 2-5 as a "monetization" of the policy-induced impacts on persons 1 and 2 using WTP and WTA as measures of value; see Figure 2-8.

94

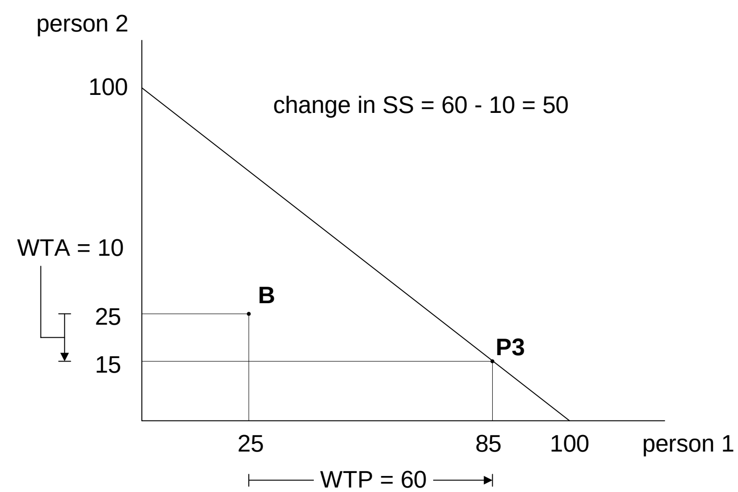

Figure 2-8

95

- For example, suppose the policy is a regulation that requires Person 2 to reduce pollution, thereby improving water quality for Person 1.

- If property rights are implicitly determined by the *status quo*, then the loss to Person 2 is measured by her WTA, and the gain to Person 1 is measured by his WTP.

96

- In the example in Figure 2-8, WTA = 10, and WTP = 60.
- Thus, the social surplus created by the policy (the net-benefit of the policy) is 50.
- This monetization of physical impacts is precisely what we do in Step 5 of the CBA.

97

SOCIAL SURPLUS AND GAINS FROM TRADE

- It is useful to relate the social surplus created by a policy-induced change to the gains from trade that arise through a voluntary trade between individuals.

98

- Suppose Person 2 currently possesses an object that could be traded.
  - her valuation of the object is her WTA to part with it.

- Suppose Person 1 would like to own that object.
  - his valuation of the object is his WTP to obtain it.

99

- If $WTA < WTP$ then these agents can make a mutually beneficial trade at some price p such that

$$WTA \leq p \leq WTP$$

100

- The private surplus captured by the seller in the trade (Person 2) is

$$PS_S = p - WTA$$

- The private surplus captured by the buyer in the trade (Person 1) is

$$PS_B = WTP - p$$

101

- The social surplus created by the trade (assuming that no other parties are affected by the trade) is

$$\begin{aligned} SS &= PS_B + PS_S \\ &= (WTP - p) + (p - WTA) \\ &= WTP - WTA \end{aligned}$$

102

- The social surplus created by the trade is called the **gains from trade**.
- Note that the gains from trade are independent of the price at which the trade takes place.
- In contrast, the distribution of those gains does depend on the trading price.

103

- Now suppose that the buyer and seller are forced to trade at price $p = 0$.
- The social surplus created (the gains from trade) is still equal to

$$SS = GFT = WTP - WTA$$

104

- The forced trade at $p = 0$ creates a winner and a loser.
- The "buyer" is the winner, and his gain is his WTP.
- The "seller" is the loser, and her loss is her WTA.

105

- We can think of a policy-induced reallocation, where there are winners (who do not have to pay for the win) and losers (who are not compensated for the loss), as like a forced trade under which no price is paid between the "buyers" (the winners) and the "sellers" (the losers).

106

- Of course, the policy-maker may choose to make an actual compensation to potential losers and thereby offset – at least partially – their losses.
- Similarly, the policy-maker may require some payment from the winners, and thereby moderate their gains.

107

- In particular, suppose the loser (the "seller" in our forced trade) receives a payment ***s***, and the winner (the "buyer") makes a payment ***b***.

108

- The private surplus for the "seller" is now

$$PS_S = s - WTA$$

- The private surplus for the "buyer" is now

$$PS_B = WTP - b$$

109

- Any difference between ***s*** and ***b*** must be funded (or received) by taxpayers, so we now have to consider the surplus to that group as well, which is

$$PS_T = b - s$$

110

- The social surplus created by the forced trade (assuming that no other parties are affected by the trade) is now

$$SS = PS_B + PS_S + PS_T$$
$$= (WTP - b) + (s - WTA) + (b - s)$$
$$= WTP - WTA$$

111

- Thus, the social surplus created is unchanged by the payments to and from the “seller” and “buyer”.

- Of course, the distribution of that surplus does depend on those payments, and the distributional goals of the policy-maker will determine those payments.

112

2.7 WTA AND OPPORTUNITY COST

- It is useful to think of WTA in terms of opportunity cost.

- The **opportunity cost** of allocating a resource to a particular use is the value of that resource in its next most valuable alternative use.

113

- The existing user of a resource would be willing to give up that resource if and only if she receives a payment that is at least as great as its current value to her.

- That is,

WTA = opportunity cost

114

- In a “perfectly competitive” market, the opportunity cost of an input is equal to its market price. (More on this later).
- In the presence of “market failure”, opportunity cost and market price can be very different. (More on this later too).

115

2.8 SOME IMPORTANT DISTRIBUTIONAL ISSUES

- We have seen that the social surplus created by a policy is simply equal to the difference between the benefits and costs of that policy, regardless of the distribution of those benefits and costs across individuals within the referent group.

116

- However, the distribution of benefits and costs is almost always important to the policy-maker.

- Hence, we need to present a distributional impact accounting as part of the CBA (in Step 8).

117

- An additional and separate set of distributional issues arise with respect to how we actually calculate costs and benefits.

- There are three issues of importance here:
  - the referent group and external impacts
  - global transfers as local costs or benefits
  - WTP/ WTA and the distribution of wealth

118

2.8-1 THE REFERENT GROUP AND EXTERNAL IMPACTS

- The client government usually defines the referent group (eg. a CBA for the national government typically takes all residents of that nation as the referent group).

119

- This means that costs and benefits from a project that fall outside the referent group – that is, costs and benefits that are external to the referent group – are not counted.

- This state perspective can lead to global inefficiency.

120

- For example, suppose the net domestic benefit of tropical rainforest protection to the host nation is negative $100m.

- If the host nation is the referent group then the rainforest area would be cleared and used for another purpose (like agriculture) rather than protected.

121

- Now suppose that the collective WTP of foreigners to protect the rainforest is $150m.

- From a global perspective, protection has a net benefit of $50m, and the rainforest should be protected.

122

- However, the host nation has no incentive to account for the WTP of foreigners because there are few effective mechanisms through which the host country can capture that foreign WTP.

- The rainforest has elements of a global public good and so its protection tends to be under-provided by the host nation (see Topic 3 for a discussion of public goods).

123

- Ideally, policy should be coordinated across jurisdictions via **cooperative agreements** between governments so that external impacts are fully internalized into decision-making.

- In the absence of a global government, striking these cooperative agreements can be very difficult.

124

- There is a large body of economic literature on **strategic interaction** among policy-makers across jurisdictions and the obstacles to striking cooperative agreements, especially in the context of tax-competition, trade policy, financial regulation, and environmental policy.

125

- We will not pursue this issue further here; it lies outside the usual scope of a cost-benefit analysis course (and requires the use of some game-theoretic modeling).

- However, it is important to recognize that a policy that is optimal from the perspective of the referent group may not be optimal from a global perspective.

126

2.8-2 GLOBAL TRANSFERS AS LOCAL COSTS OR BENEFITS

- A second issue that arises when we do not take a global perspective is the treatment of global transfers from a local perspective.

127

- Recall the highway example from Topic 1, where the toll revenue received from travelers within the referent group was treated as a transfer – neither a cost nor a benefit to the referent group as a whole – but toll revenue received from travelers outside the referent group was treated as a benefit to the referent group.

128

- In general,
  - flows of money from outside the referent group into the referent group are benefits from the perspective of the referent group
  - flows of money from the referent group to outside the referent group are costs from the perspective of the referent group.

- See Figure 2.9.

129

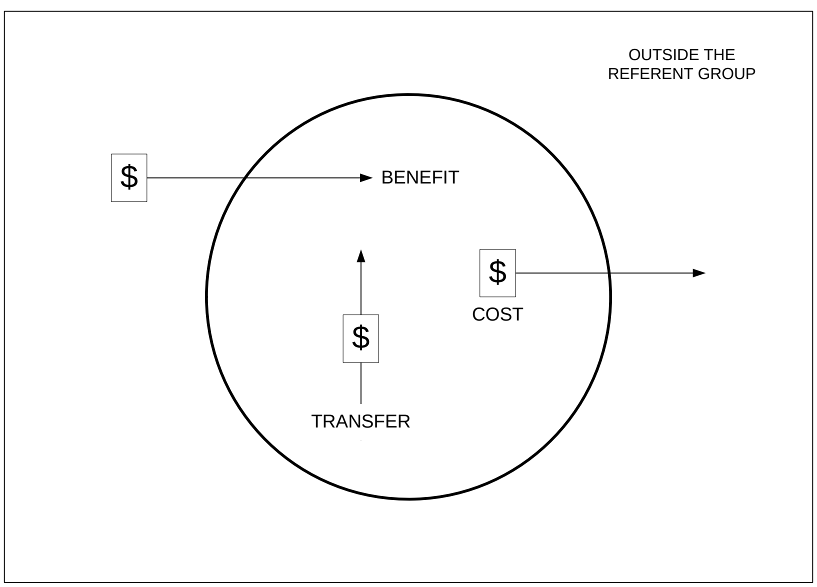

Figure 2-9

130

- Note that if we take a global perspective then by definition there can be no flows of money either out of or into the referent group.

- All flows of money in that case are transfers.

131

- Consider an example.
  - The City of Victoria asks 10 of its employees to each work 20 hours of overtime to complete an urgent water main repair.

  - These employees would otherwise have spent that time in leisure, which they value at $15 per hour.

132

- Union rules require that the workers are paid $30 per hour for overtime work.
- The workers will have to pay tax on their overtime: $3 per hour in Provincial tax plus $6 per hour in Federal tax.
- What is the labour-cost of this project?

133

- There are three natural (nested) referent groups to consider:
  - the City
  - the Province
  - the Country
- See Figure 2-10.

134

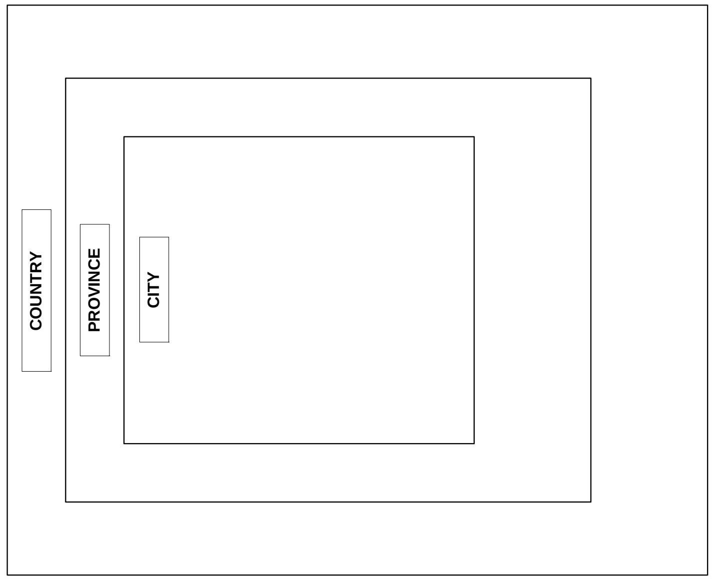

Figure 2-10

135

- In all three referent-group scenarios, the only resource used is the time of the workers, and we know that its opportunity cost is $15 per hour.
- Thus, the value of the real resources used is 10*20*15 = $3000.
- However, we also need to track inflows and outflows of wealth.

136

- If <u>Victoria is the referent group</u> then there are three components to the labour-cost:
  - the opportunity cost of the time
  - the outflow of Federal income tax
  - the outflow of Provincial income tax
- The wage payments are a transfer.
- See Figure 2-11.

137

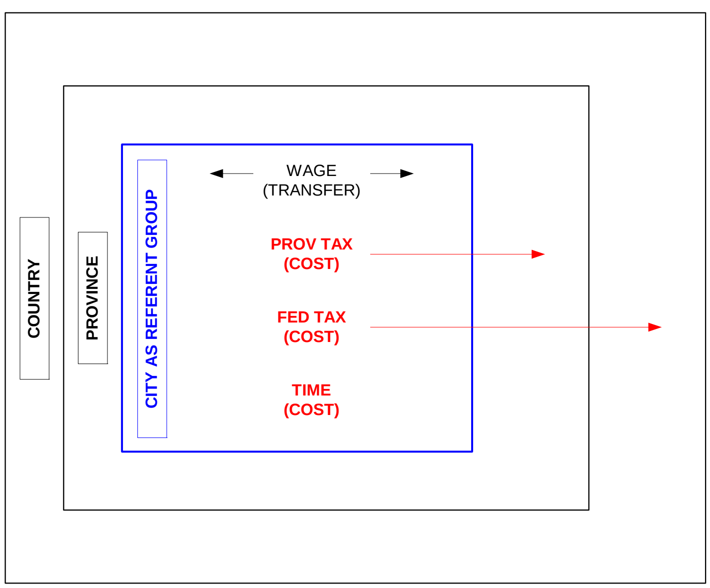

Figure 2-11

138

- Thus, if Victoria is the referent group, then the labour-cost of the project is
  - cost = 3000 + 10*20*(6 + 3) = $4800

139

- If <u>BC is the referent group</u> then there are two components to the labour-cost:
  - the opportunity cost of the time
  - the outflow of Federal income tax
- The wage payments and the Provincial income taxes are transfers.
- See Figure 2-12.

140

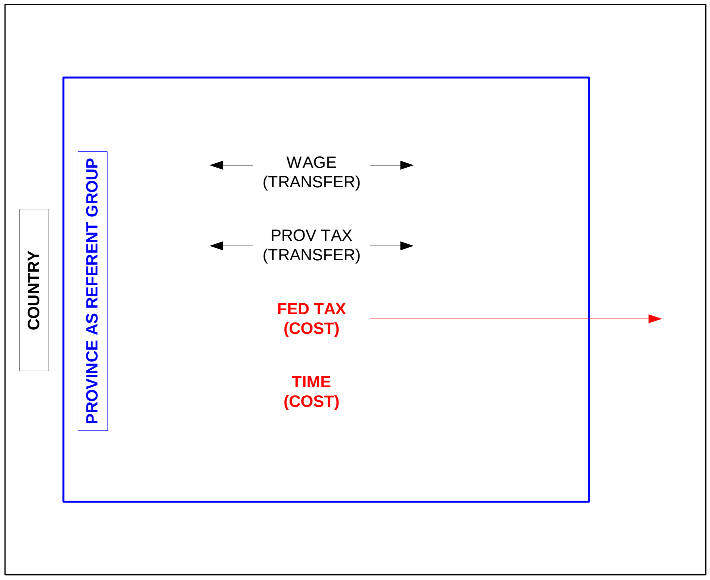

Figure 2-12

141

- Thus, if BC is the referent group, then the labour-cost of the project is

  cost = 3000 + 10*20*6 = $4200

142

- If Canada is the referent group then there is only one component to the labour-cost:
  - the opportunity cost of the time
- The wage payments, the Provincial income taxes, and the Federal income taxes are transfers.
- See Figure 2-13.

143

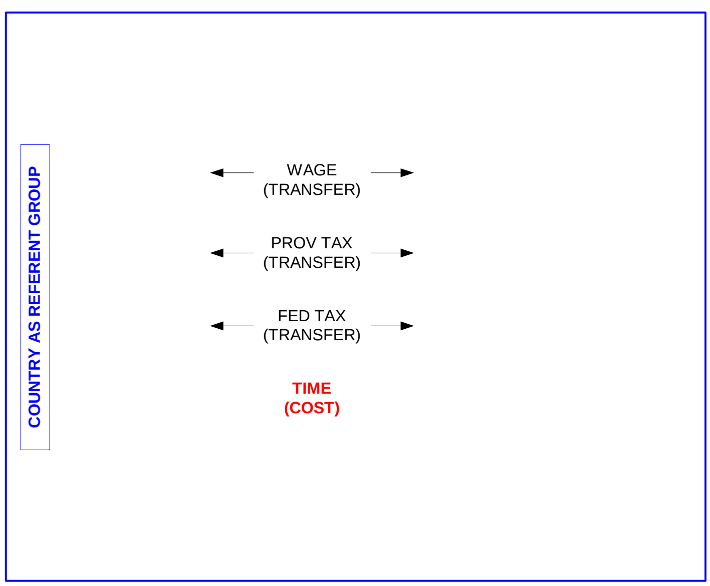

Figure 2-13

144

- Thus, if Canada is the referent group, then the labour-cost of the project is

  cost = 10*20*15 = $3000

145

2.8-3 WTP/ WTA AND THE DISTRIBUTION OF WEALTH

- WTP and WTA are functions of an individual's level of wealth.

- If the good being valued is a <u>normal</u> good, then both WTP and WTA are <u>increasing</u> in wealth.

146

- If the good being valued is an inferior good then both WTP and WTA are decreasing in wealth.

- This means that our calculation of costs and benefits is contingent on a particular distribution of wealth across individuals.

147

- A significant redistribution of wealth in society would typically change our measures of costs and benefits, and potentially change the sign of the net benefit from a particular policy (from positive to negative or vice versa).

148

- Consider an example. Suppose we have two individuals with wealth $\boldsymbol{w_1}$**=100** and $\boldsymbol{w_2}$**=200** respectively.

- Suppose a project will have a negative impact on person 1 and a positive impact on person 2, and that those impacts are valued at $\mathbf{WTA_1}$**= 4** and $\mathbf{WTP_2}$**=12** respectively.

149

- The social surplus created by the project is

$$\Delta SS = WTP_2 - WTA_1 = 8$$

150

- Now suppose we assess this same project but under a hypothetical different distribution of wealth, where $\boldsymbol{w_1}$**=150** and $\boldsymbol{w_2}$**=150** respectively.

- Thus, person 1 is richer by 50 and person 2 is poorer by 50 than in the first distribution.

151

- Under this different distribution of wealth, we might now observe that $\mathbf{WTA_1 = 9} > \mathbf{4}$ (because person 1 is now richer) and that $\mathbf{WTP_2 = 7 < 12}$ (because person 2 is now poorer).

- The social surplus created by the project is now

$$\Delta SS = WTP_2 - WTA_1 = -2$$

152

- Which is the <u>correct</u> assessment of the project?
- An egalitarian non-economist might argue that the second distribution of wealth is more "equitable", and that it should therefore be the basis for the assessment.
- Under that assessment, the project would not proceed because $\Delta SS < 0$.

153

- However, suppose the <u>actual</u> distribution of wealth is the first (less equitable) one.
- Then by not proceeding with the project, we forgo social surplus.
- In particular, we forgo the opportunity to make both individuals better off by $4 by proceeding with the project and making a transfer of $8 from person 2 to person 1.

154

- Even our egalitarian non-economist might agree that the sensible thing to do is to make person 1 better off by $4 if we can, even if he believes that the current distribution of wealth is inequitable.

155

- In general, costs and benefits must be calculated on the basis of the actual distribution of wealth, and not on some hypothetical alternative distribution of wealth even if many people believe that the hypothetical alternative distribution is better.

- Policy should not be based on wishful thinking.

END

156

APPENDIX A2-1
SOCIAL WELFARE FUNCTIONS

The general form of a social welfare function is

(2.1) $$W = W(u_1, u_2, \ldots, u_m)$$

where $u_1, u_2, \ldots, u_m$ are the utilities of the *m* individuals in the economy. It must be stressed that such a function is an entirely artificial construct. It is not possible to measure *W* for any given specification of the function *W(.)* because its arguments, the utility of individuals, are not measurable in a cardinal way.

Consider three specific social welfare functions.

1. THE UTILITARIAN (OR BENTHAMITE) WELFARE FUNCTION

This is often associated with Jeremy Bentham, a nineteenth century philosopher.

(2.2) $$W = \sum_{i=1}^{m} u_i$$

This welfare function reflects the **utilitarian ethic**: everyone's utility should count equally regardless of their level of utility.

2. THE RAWLSIAN WELFARE FUNCTION

This was proposed by John Rawls, in *A Theory of Justice*, (1971)

(2.3) $$W = \min(u_1, u_2, \ldots, u_m)$$

This reflects the **Rawlsian** ethic: the welfare of society is equal to that of its least well-off member. It can be derived as the allocation rule preferred by infinitely risk averse agents choosing between different rules from behind a "veil of ignorance".

In some sense the Rawlsian ethic is at the opposite end of the concern-for-distribution spectrum to the utilitarian ethic. Somewhere in the middle is the weighted utilitarian function.

3. WEIGHTED UTILITARIAN WELFARE FUNCTION

(2.4) $$W = \sum_{i=1}^{m} \alpha_i u_i$$

where the weight α_i reflects the "importance" of individual *i* to overall social welfare. The usual interpretation is that changes in the utility of poor people carry more weight in determining a change in social welfare than do changes in the utility of wealthy people.

It is worth reiterating that none of these welfare functions can be made operational for practical purposes because there is no way to measure *W* for any of the three functions.

APPENDIX 2-2
WTP vs. WTA

- In this Appendix, we will argue that the correct valuation measure ultimately depends on the **purpose** of our measurement.

A PURPOSE-BASED APPROACH

- To see how our choice of valuation measure would depend on the purpose of the measurement, consider again water-quality example from the main slides.
- We will first consider a project that reduces water quality (creating a loss for users), and then consider one that raises it (creating a gain).

3

VALUATION OF A LOSS

- Suppose the project reduces water quality from $\boldsymbol{q_0}$ to $\boldsymbol{q_L} < \boldsymbol{q_0}$.

- The way we measure this loss to a water user should depend on whether or not she will be compensated for the loss.

4

A COMPENSATED LOSS

- First consider the case where the water user will be compensated for the loss (with an actual money payment).

- In this case our purpose for making the valuation is to calculate the correct value of the compensation.

5

- Accordingly, we measure this **compensated loss** using the WTA measure we defined in the main slides, and repeated here as

$$u(q_L, y + L_C) = u(q_0, y)$$

but where we have now added the "C" subscript to denote a compensated loss.

6

- The underlying logic of this measurement:
  - since the water user will be compensated for the loss then by implication she must implicitly have a right to the existing water quality.
- That is, she has been deemed to effectively "own" the existing water quality, and so she will be compensated for the reduction.

7

- This implicit assignment of property rights – as implied by the fact that an actual compensating payment will be made – effectively makes $\boldsymbol{q_0}$ the "reference point" in our calculation.
- Thus, $u(\boldsymbol{q_0}, y)$ is the reference utility (on the RHS of the equation on s.6 in this Appendix).

8

AN UNCOMPENSATED LOSS

- Now consider the case where the individual who incurs the loss of water quality will not be compensated.

- In this case our purpose is to measure the value of this uncompensated loss.

9

- Accordingly, we should measure this **uncompensated loss** using her WTP to avoid that loss, denoted $\boldsymbol{L_U}$, and defined by:

$$u(q_0, y - L_U) = u(q_L, y)$$

where the "U" subscript denotes an uncompensated loss.

10

- That is, this person would be just indifferent between
  - making a payment $\boldsymbol{L_U}$ and retaining the existing water quality; and
  - not making that payment but incurring the lower water quality.

- The underlying logic of this measurement:
  - if she will not be compensated then by implication she has no right to the existing water quality.

- That is, she has been deemed to not “own” the existing water quality, and so she will not be compensated for the reduction.

- This implicit assignment of property rights – as implied by the fact that no compensating payment will be made – effectively makes $\boldsymbol{q_L}$ the "reference point" in our calculation.

- Thus, $u(\boldsymbol{q_L}, y)$ is the reference utility (on the RHS of the equation on s.10 in this Appendix).

13

VALUATION OF A GAIN

- Now suppose the project will raise water quality to $\boldsymbol{q_G} > \boldsymbol{q_0}$.

- The value of this gain to a water user depends on whether or not he will have to pay for that gain.

14

A COMPENSATED GAIN

- First consider the case where the water user will have to make an actual payment in return for the gain in water quality.
- In this case our purpose for making the valuation is to calculate the correct value of this payment.

15

- Accordingly, we measure this **compensated gain** using the WTP measure we defined in the main slides, and repeated here as

$$u(q_G, y - G_C) = u(q_0, y)$$

where we have now added the "C" subscript to denote a compensated gain.

16

- The underlying logic of this measurement:
  - since he will actually have to pay for the gain, then he must have no right to that gain.
- That is, he has been deemed to effectively "own" only the existing water quality, and so he must pay to get the higher water quality.

- This implicit assignment of property rights – as implied by the fact that an actual payment will be made – effectively makes $\boldsymbol{q_0}$ the "reference point" in our calculation.
- Thus, $u(\boldsymbol{q_0}, y)$ is the reference utility (on the RHS of the equation on s.16 in this Appendix).

AN UNCOMPENSATED GAIN

- Now consider the case where the individual who receives the higher water quality will not have to pay for that gain.

- In this case our purpose is to measure the value of this uncompensated gain.

19

- Accordingly, we measure this **uncompensated gain** using his WTA to forgo that gain, denoted $\boldsymbol{G_U}$, and defined by:

$$u(q_0, y + G_U) = u(q_G, y)$$

where the "U" subscript denotes an uncompensated gain.

20

- That is, this person would be just indifferent between
  - receiving a payment $\boldsymbol{G_U}$ and forgoing the improved water quality; and
  - not receiving that payment but enjoying the higher water quality.

21

- The underlying logic of this measurement:
  - since he will not actually have to pay for the gain, then he must have an implicit right to that gain.

- That is, he has been deemed to effectively "own" the higher water quality, and so he will not have to pay to get it.

22

- This implicit assignment of property rights – implied by the fact that no actual payment will be made – effectively makes $\boldsymbol{q_G}$ the "reference point" in our calculation.

- Thus, $u(\boldsymbol{q_G}, y)$ is the reference utility (on the RHS of the equation on s.20 in this Appendix).

- It might seem odd to suppose that an individual facing an existing water quality $\boldsymbol{q_0}$ has a "right" to a higher quality than that.

- However, in many settings water quality has declined over time – due to pollution, for example – and current water users may be deemed to have a right to the <u>restoration</u> of water quality to some historical higher level.

WHO DECIDES WHERE PROPERTY RIGHTS LIE?

- Since the appropriate measure of gains and losses depends critically on the assignment of property rights, who makes the decision as to where property rights lie?

25

- Not economists.
- In principle, the judiciary decides where property rights lie (at least in the context of a liberal democracy).
- However, in practice there is often no formal resolution of property rights relating to the impact of public policies and projects.

26

- Consequently, property rights are often never made explicit, and so as economists we must infer the implicit assignment of property rights, as revealed by whether or not actual payments will be made, which is in turn decided by policy-makers.

27

- Given this implicit determination of property rights, our role as economists is to choose the valuation measure (WTP or WTA) that is consistent with that implicit determination.

- That is, we should choose the valuation measure that is consistent with the purpose of our measurement.

28

- It should be stressed that there is still much debate on this issue in the economics literature, and that there is no consensus that the approach outlined here is the only "correct approach".

- Nonetheless, in practice we must by necessity adopt a clear method and use it consistently.

TOPIC 2 REVIEW QUESTIONS

1. The Pareto frontier is defined as "the set of allocations in which social surplus is maximized".

A. True.

B. False.

2. If a project creates social surplus than it must also create a Pareto improvement.

A. True.

B. False.

3. If a project has a positive net social benefit then it must create social surplus.

A. True.

B. False.

4. If allocation **A** Pareto-dominates allocation **B** then allocation **B** cannot lie on the Pareto frontier.

A. True.

B. False.

5. Figure R2-1 depicts a setting in which a total of *m* resources is available for allocation between two persons. The base case (the current endowment) is at point B, where person 1 has b_1 and person 2 has b_2, and where $b_1 + b_2 = b$. A candidate policy would move this economy to point P, where person 1 would have p_1 and person 2 would have p_2, where $p_1 + p_2 = m$. The proposed policy

A. is not a Pareto improvement.

B. creates social surplus.

C. has a positive net benefit.

D. All of the above.

6. Consider again the setting described in **Question 5** above. An alternative candidate policy would move this economy to point Q, where person 1 would have q_1 and person 2 would have q_2. Which of the following statements is true?

A. Policy Q Pareto-dominates the base case.

B. Policy Q does not Pareto-dominate policy P.

C. Policy P creates more social surplus than policy Q.

D. All of the above.

7. Consider again the setting described in **Question 5** above. The set of feasible allocations that Pareto-dominate allocation P is

A. described by the triangular area p_2Pm, including its boundaries.

B. described by the line *AC*, including its endpoints.

C. described by the line *mm*, including its endpoints.

D. empty.

8. Consider again the setting described in **Question 5** above. Relative to the base case, policy Q has a net social benefit equal to

A. $q_1 + q_2 - b$

B. $(b_1 - q_1) + (b_2 - q_2)$

C. $(m - b_1 + q_1) - (m - b_2 + q_2)$

D. There is not enough information to make a determination.

9. Consider again the setting described in **Question 5** above. The line *AC* (including its endpoints) identifies

A. a strict subset of the Pareto-efficient allocations.

B. the set of allocations that constitute a Pareto improvement over the base case.

C. the set of allocations that Pareto-dominate the base case.

D. All of the above.

10. In the context of a social choice rule, "independence of irrelevant alternatives" requires that the social ranking over two allocations *x* and *z* is independent of individual rankings over *x* and *y*, and *z* and *y*.

A. True.

B. False.

11. One implication of Arrow's Impossibility Theorem is that a benevolent dictator can maximize social welfare if and only if she has complete knowledge of the individual preferences of all citizens.

A. True.

B. False.

12. Consider a setting in which three individuals have the following preference rankings over three candidates *X*, *Y* and *Z*:

Person 1: $X > Y > Z$

Person 2: $X > Z > Y$

Person 3: $Y > X > Z$

A two-step pair-wise majority voting rule in this setting will produce *X* as the winning candidate regardless of the voting agenda.

A. True.

B. False.

Questions 13 – 17 relate to the following proposed project, to be conducted by the Province of British Columbia.

- The project will hire 100 workers who are currently unemployed residents of BC, for 1000 hours each.
- Let *OCL* (opportunity cost of labour) denote the value per hour of the activity in which these workers are currently engaged. Assume that *OCL* is the same for all of these workers.
- The workers will each be paid $25 per hour.

- Each worker will lose his current welfare payments for the period of employment. These payments would have been $8000 each. These payments are currently made by the Province.
- The project will be subsidized by the Government of Canada. In particular, the Government of Canada will pay the Province $10 per hour for every worker hired.
- The project has no other costs.

13. The net private surplus to each worker hired for this project is

A. $(17,000 – 1000*OCL*)

B. $(25,000 – 1000*OCL*)

C. $(17,000 + 1000*OCL*)

D. $(25,000 + 1000*OCL*)

14. The net financial outlay for the Provincial government is

A. $1,700,000

B. $1,500,000

C. $700,000

D. $800,000

15. The net financial outlay for the Federal government is

A. $1,000,000

B. $800,000

C. $0

D. None of the above.

16. If the referent group is the Province, then the cost of the project is

A. $700,000

B. $1,000,000

C. $(100,000)*OCL* – $1,000,000

D. $(100,000)*OCL* + $1,000,000

17. If the referent group is Canada, then the cost of the project is

A. $(100,000)*OCL*

B. $1,000,000

C. $800,000

D. $0

Questions 18 – 23 relate to the following candidate project, to be conducted by the City of Victoria.

- The project will require 10 employees currently working for the City to each work a total of 100 hours of overtime over the course of 10 weeks.
- These workers are currently paid $30 per hour but union rules specify an overtime wage of $40 per hour. (These workers are free to decline the overtime work but all have agreed to do it).
- Due to the higher wage during overtime, each worker will pay $2 per hour more in income taxes to the Federal Government, and $1 per hour more in income taxes to the Provincial Government. Thus, their after-tax overtime wage is only $37 per hour.
- Let *OCL* (opportunity cost of labour) denote the value per hour of the activity in which these workers would be engaged if not working overtime (for example, watching a movie or playing with their children). Assume that *OCL* is the same for all of these workers.
- The project has no other costs.

18. If the project proceeds, the highest possible OCL for these workers is

A. $30

B. $37

C. $40

D. There is not enough information to make a determination.

19. The net private surplus to each worker from being hired for this project is

A. $3,700

B. $(3,700 – 100*OCL*)

C. $(4,000 – 100*OCL*)

D. $4,000

20. The net financial outlay for the City of Victoria is

A. $37,000

B. $40,000

C. $43,000

D. None of the above.

21. If the referent group is the City of Victoria, the cost of the project is

A. $(3000 + 1000*OCL*)

B. $(40,000 + 1000*OCL*)

C. $(37,000 + 1000*OCL*)

D. $(40,000 – 1000*OCL*)

22. If the referent group is British Columbia, the cost of the project is

A. $1000*OCL*

B. $37,000

C. $40,000

D. $(2000 + 1000*OCL*)

23. If the referent group is Canada, the cost of the project is

A. $1000*OCL*

B. $37,000

C. $40,000

D. $(2000 + 1000*OCL*)

Questions 24 – 32 relate to the following candidate project, to be conducted by the Government of Canada. The project will construct a pipeline from the oil sands in Alberta to the coast of British Columbia. It will carry diluted bitumen (a product refined from tar extracted from the oil sands) for loading into foreign tankers which will ship the bitumen to foreign export markets. It will then be refined further into a fuel that will eventually be burned to provide energy. Suppose we have the following information (all of which is made up).

- There will be 100 megalitres of bitumen produced and transported each year. In the absence of the pipeline project, this bitumen would have remained in the ground forever.
- The cost incurred by Alberta to extract the bitumen and pump it through the pipeline is 10 cents per litre. The price received by Alberta from the exporters is 20 cents per litre.
- Greenhouse gases are released during the bitumen-extraction process in Alberta and the estimated associated climate-change cost to global society beyond Canada is 10 cents per litre.
- Miraculously, Canada is immune from the effects of climate change.
- There is a risk of an oil spill in BC waters. The probability-weighted cost of a spill implies that the environmental damage cost of the pipeline to BC is 5 cents per litre.
- The cost of refining the bitumen into usable fuel – to be undertaken in foreign countries – is 5 cents per litre. Shipping costs from BC are 5 cents per litre. These costs are incurred by foreigners.
- The retail price of the refined fuel is 40 cents per litre, and this price is a true measure of value to the foreign consumers of the fuel.
- Combustion of the refined fuel will create greenhouse gases. The estimated associated climate-change cost to global society beyond Canada is 10 cents per litre.

Assume that there are no other costs or benefits associated with the pipeline project beyond those listed above.

24. The annual net benefit to Alberta is

A. $5m

B. $7m

C. $10m

D. $12m

25. The annual net benefit to British Columbia is

A. $3m

B. $1m

C. – $3m

D. – $5m

26. The annual net benefit to Canada is

A. $5m

B. $7m

C. $10m

D. $12m

27. The annual net benefit to global society as a whole is

A. $3m

B. $1m

C. – $3m

D. – $5m

We now wish to conduct a distributional analysis where we identify four impact groups: Albertans; British Columbians; Canadians outside Alberta and British Columbia; and non-Canadians.

28. The annual net gain to Albertans is

A. $3m

B. $5m

C. $8m

D. $10m

29. The annual net gain to British Columbians is

A. $1m

B. – $2m

C. – $5m

D. – $7m

30. The annual net gain to Canadians outside Alberta and British Columbia is

A. $4m

B. $3m

C. $1m

D. $0

31. The annual net gain to non-Canadians is

A. – $10m

B. – $7m

C. – $5m

D. $0

32. The sum of the correct answers to Qs. 28 – 31

A. must be less than the correct answer to Q.27

B. must be more than the correct answer to Q.27

C. must be equal to the correct answer to Q.27

D. could be more or less than the correct answer to Q.27

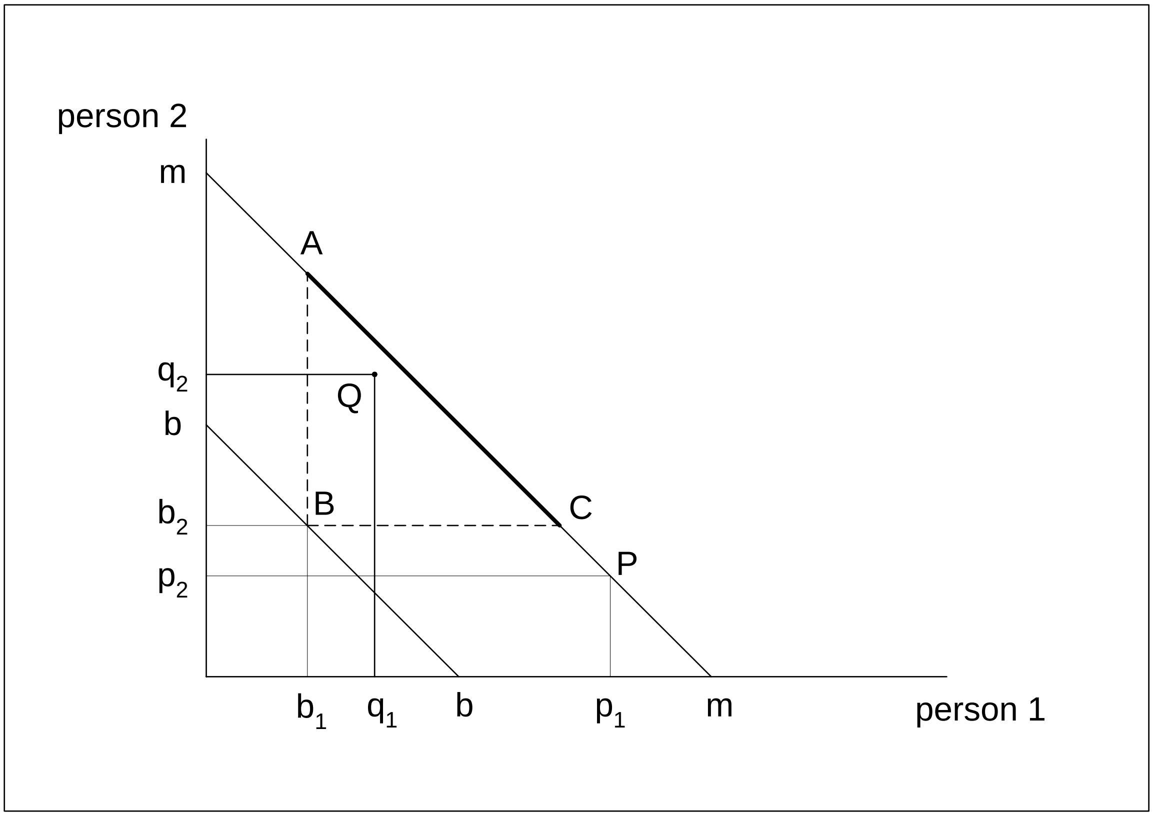

Figure R2-1

ANSWER GUIDE

1. B
2. B
3. A
4. A
5. D
6. D
7. D
8. A
9. A

<u>All</u> points in the triangle BAC (including its boundaries but excluding B itself) Pareto-dominate B. (Similarly, a move from B to any of those points in the triangle is a Pareto-improvement over B). The interval AC is only a subset of the that triangular set. Responses *B* and *C* to the question suggest incorrectly that AC is <u>the</u> set that Pareto-dominates B; it is not. It is only a subset of the points that Pareto-dominate B.

10. A
11. B
12. A
13. A 1000(25 – *OCL*) – 8000 = 17,000 – 1000*OCL*
14. C 100(1000(25 – 10) – 8000) = 700,000
15. A 100(1000(10))) = 1m
16. C 100(1000(*OCL* – 10))) = (0.1*OCL* – 1)m
17. A
18. B They would not take on the additional work if OCL exceeds the after-tax wage of $37.
19. B
20. B $10(100(40))) = $40,000
21. A
22. D $10(100(*OCL* + 2)) = $(2000 + 1000*OCL*)
23. A $10(100(*OCL*)) = $(1000*OCL*)

24. C $100(0.2 – 0.1)m = $10m (see Figure R2-2 below)

25. D – $100(0.05)m = – $5m

26. A The sum of 24C and 25D = $5m

27. D $100(0.4 – (0.1 + 0. 1 + 0.05 + 0.05 + 0.05 + 0.1))m = – $5m

28. D Same as 24C

29. C Same as 25D

30. D No impact on Canadians outside BC and Alberta

31. A $100(0.4 – (0.2 + 0.1 + 0.05 + 0.05 + 0.1))m = – $10m

32. C The aggregation of gains and losses to the members of the referent group (in this case, global society) must always equal the net benefit to the group as a whole.

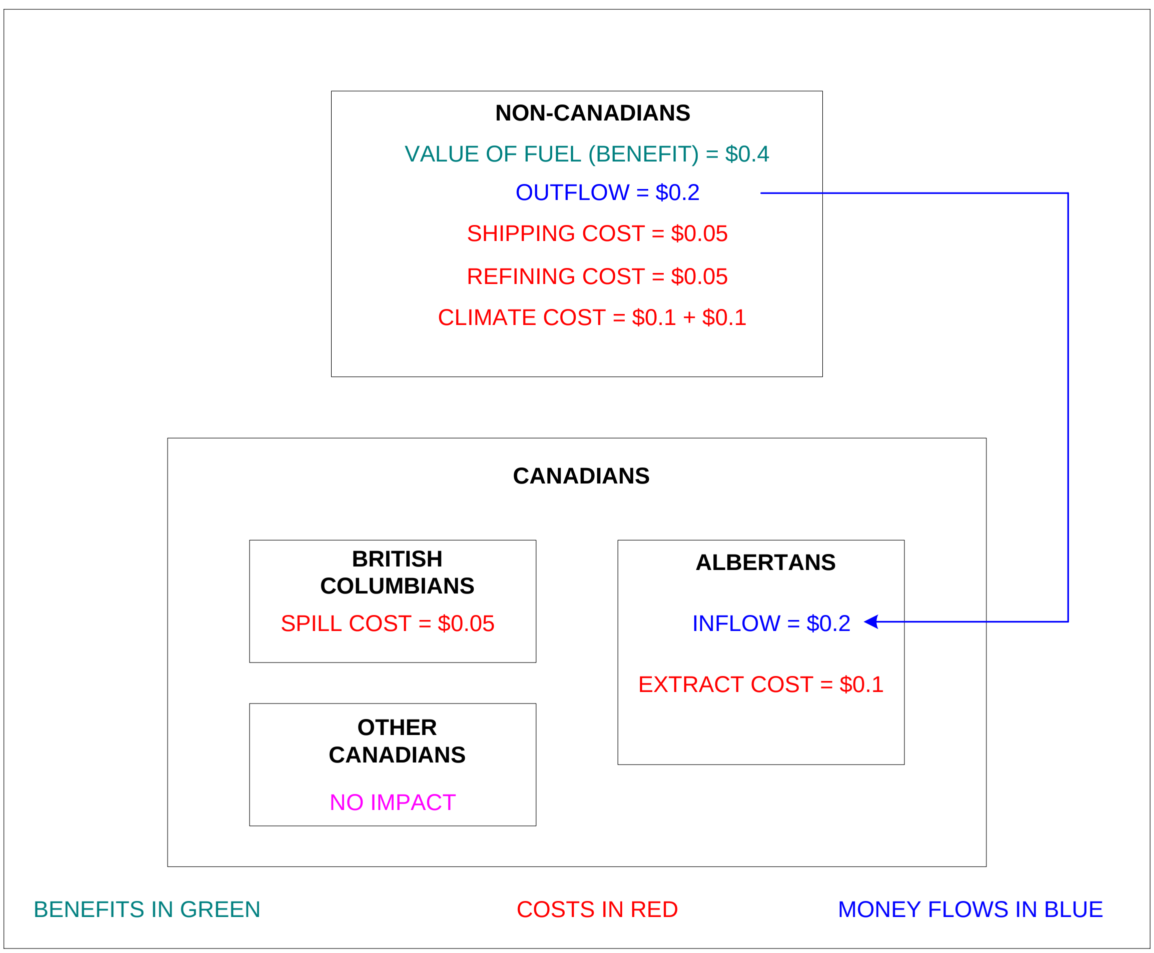

Figure R2-2

3. RATIONALE FOR POLICY INTERVENTION

OUTLINE

3.1 The First Welfare Theorem
3.2 Redistribution
3.3 Market Failure
3.4 Externalities
3.5 Reciprocal Externalities
3.6 Market Power

2

3

3.1 THE FIRST WELFARE THEOREM

- In an economy in which
  - all agents are price-takers
  - there are no externalities
  - there are no scale economies
  - information is symmetric between buyers and sellers

  market allocations are Pareto efficient.

4

- An economy satisfying these conditions is called “perfectly competitive”.

5

- If market outcomes have this desirable efficiency property, where is the need for policy intervention?

- There are two rationale for intervention:
  - wealth redistribution
  - market failure

6

3.2 REDISTRIBUTION

- Recall from Topic 2 that Pareto efficiency generally does not identify a unique allocation of resources.
- Different efficient allocations correspond to different distributions of wealth; see Figure 3.1.

7

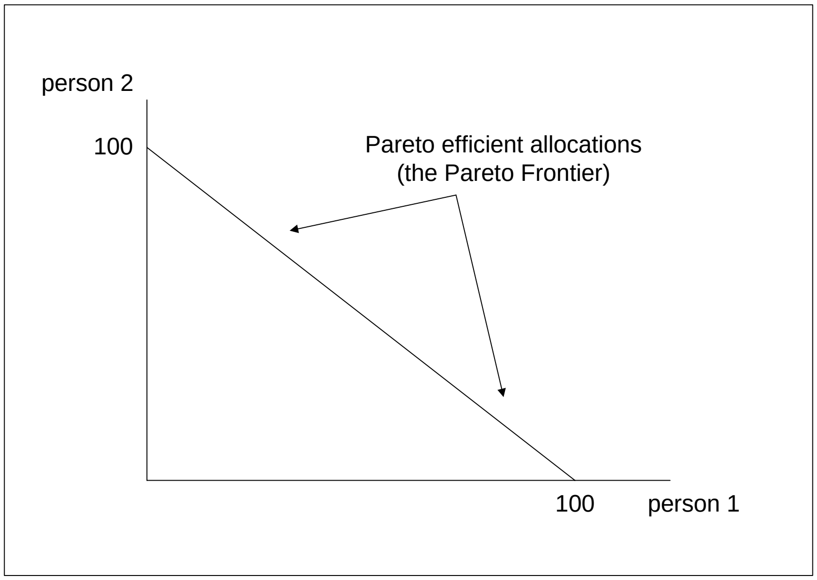

Figure 3-1

8

- Similarly, the particular allocation that arises from a perfectly competitive market is a function of the underlying distribution of wealth.

- If the distribution of wealth is uneven then the associated market allocation will also tend to be uneven (though nonetheless efficient).

9

- A society may collectively decide (through voting, for example) that some efficient allocations are better than others in terms of distribution, and so that society may support policy intervention to achieve a "more equitable" outcome.

10

- There are two ways of doing this:
  - abandon the market mechanism entirely and make centrally-planned resource allocations
  - intervene with wealth redistribution policies and allow the market to achieve a new, “more equitable” outcome based on the revised distribution of wealth.
- The foundation for the second approach is the second welfare theorem.

11

THE SECOND WELFARE THEOREM

- Any Pareto efficient allocation can be supported as an equilibrium of a perfectly competitive economy with appropriate lump-sum transfers (that is, with appropriate transfers of wealth).

12

- By definition, lump-sum transfers do not depend on the behaviour of the individuals involved in that transfer, and so they do not create incentives for those individuals to change their behaviour.

- Problem in practice:
  - if the redistribution is based on existing wealth, then lump-sum transfers are generally not available for that purpose because wealth depends at least in part on behaviour.

13

INCENTIVES AND THE CONFLICT BETWEEN EFFICIENCY AND REDISTRIBUTION

- In practice, if transfers are based on existing wealth, then incentives to create wealth are typically distorted.

- Example:
  - income taxation and the distortion of the work-leisure choice.

14

- Thus, there arises a potential conflict between efficiency and “equity”:
  - in trying to distribute wealth more evenly, the total amount of wealth available is often reduced, due to the adverse impact on incentives.
- This does not necessarily mean that redistribution is a bad idea; it means that there is often a cost to redistribution.

15

- An important role for economic analysis is to identify least-cost (or “cost effective”) redistribution policies.
- Such policies are designed to achieve a given redistributive goal with minimum adverse impact on the creation of wealth.
- Designing such policies is the subject of **optimal tax theory**.

16

IS CONFLICT INEVITABLE?

- Is there always a conflict between redistribution and wealth creation?
- Not necessarily. Wealth redistribution might sometimes increase wealth by reducing
  - crime and insurrection
  - other external effects of poverty

17

3.3 MARKET FAILURE

- The market is said to "fail" if the equilibrium in that market yields an outcome that does not maximize social surplus.
- Market failure means that prices are distorted and so they send the wrong signals with respect to the allocation of resources.

18

- There are three main sources of market failure:
  - Externalities (including public goods)
  - Market power and scale economies
  - Asymmetric information
- We will briefly review each in turn.

19

3.4 EXTERNALITIES

- An **externality** (or external effect) is an impact associated with an action that is external to the agent taking that action.
- Externalities can be positive (an external benefit) or negative (an external cost).

20

- The standard textbook example of a negative externality is unpriced pollution:
  - pollution has an effect on other agents for which the polluting agent does not have to account.

- If an action has an associated externality then the privately optimal action typically does not maximize social surplus.

21

THE PRIVATE OPTIMUM

- Let ***PB*(*z*)** denote private benefit as a continuous function of some action ***z*** (eg. benefit from driving a car ***z*** kilometers).

- Let ***PC*(*z*)** denote the private cost as a continuous function of that action (eg. fuel and maintenance costs).

22

- The **net private benefit** (or private surplus) from the activity is the difference between private benefit and private cost:

$$NPB(z) = PB(z) - PC(z)$$

- The **private optimum** is the level of z that maximizes net private benefit.

23

- If private benefit and private cost are both continuously differentiable in z then we can define the **marginal private benefit** of z as

$$MPB(z) \equiv \partial PB(z)/\partial z$$

and the **marginal private cost** of z as

$$MPC(z) \equiv \partial PC(z)/\partial z$$

24

- Moreover, if *MPB* is non-increasing in z and *MPC* is non-decreasing in z then the private optimum is $\hat{z}$ such that

$$MPB(\hat{z}) = MPC(\hat{z})$$

- See Figure 3-2 (drawn for the standard case where *MPB* is decreasing in z, and *MPC* is increasing in z).

25

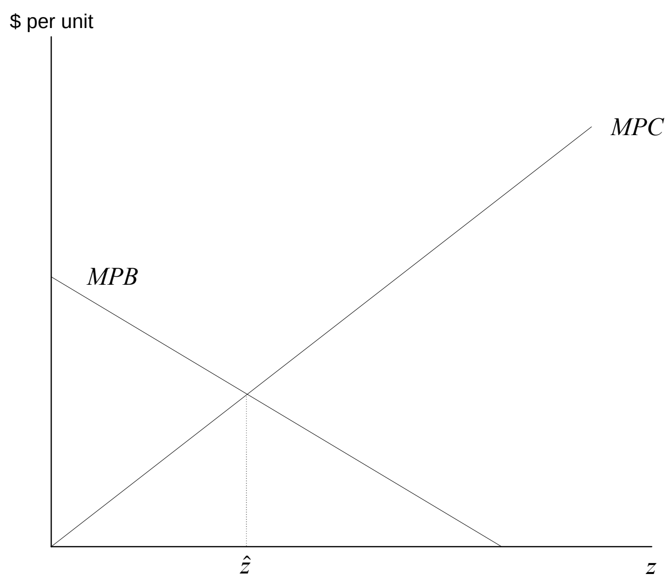

Figure 3-2

26

- Private benefit at the private optimum is the area under *MPB*(z) up to $\hat{z}$.

- See Figure 3-3.

27

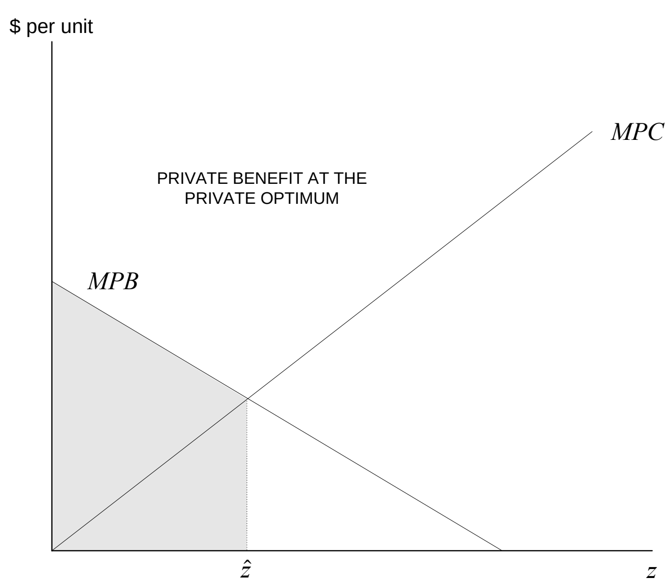

Figure 3-3

28

- Private cost at the private optimum is the area under $MPC(z)$ up to $\hat{z}$.

- See Figure 3-4.

29

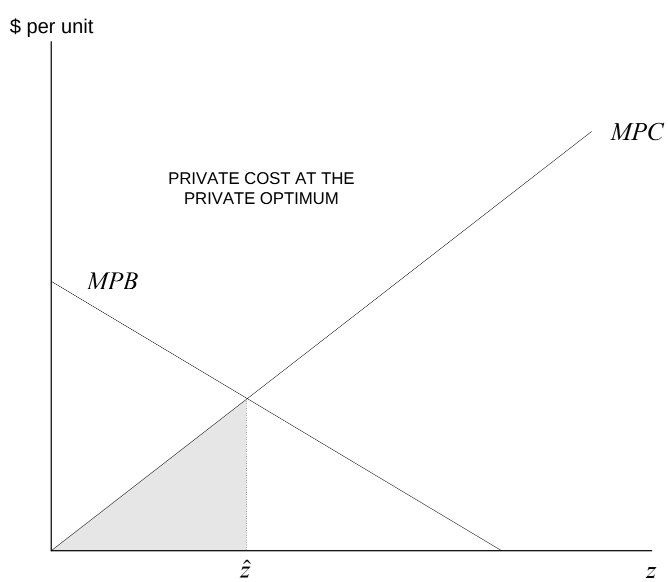

Figure 3-4

30

- Net private benefit (private surplus) at the private optimum is the area between *MPB*(z) and *MPC*(z) up to $\hat{z}$.

- See Figure 3-5.

31

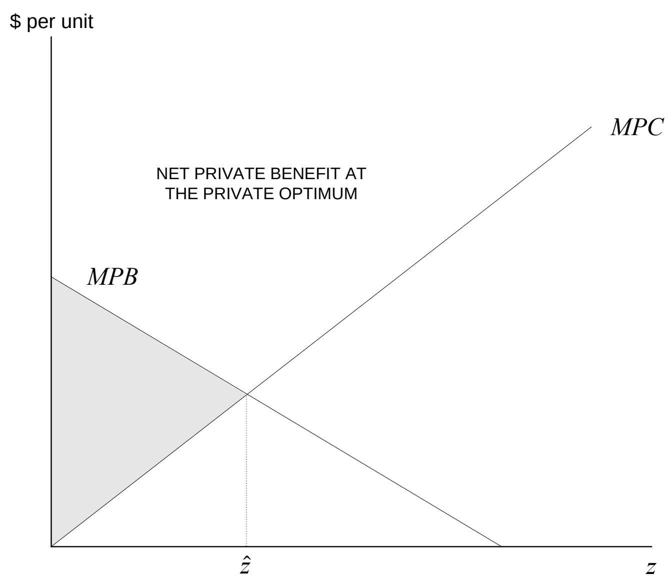

Figure 3-5

32

THE SOCIAL OPTIMUM

- We will use the term "**social optimum**" to identify the allocation that maximizes social surplus.

- It is important to remember that this social optimum does not maximize "social-welfare", which we know from Topic 2 cannot even be defined.

33

- Suppose action ***z*** potentially imposes an external cost ***D*(*z*)** and an external benefit ***G*(*z*).**

- Then we define the **social cost** of ***z*** as

$$SC(z) = PC(z) + D(z)$$

and the **social benefit** of ***z***:

$$SB(z) = PB(z) + G(z)$$

34

- The **net social benefit** (or social surplus) from the activity is the difference between social benefit and social cost:

  $NSB(z) = SB(z) - SC(z)$

- The **social optimum** is the level of z that maximizes net social benefit.

35

- Let us retain our earlier assumptions on private benefit and private cost, and assume in addition that

  $D(z)$ is continuously differentiable and that $\partial D(z)/\partial z$ is non-decreasing in z; and

  $G(z)$ is continuously differentiable and that $\partial G(z)/\partial z$ is non-increasing in z.

36

- Under these assumptions we can define the social optimum by $\boldsymbol{z}^*$ such that

  $MSB(z^*) = MSC(z^*)$

  where $MSB(z) \equiv \partial SB(z)/\partial z$ is **marginal social benefit**, and $MSC(z) \equiv \partial SC(z)/\partial z$ is **marginal social cost**.

37

TANGENT: IS THE SOCIAL OPTIMUM UNIQUE?

- Social benefit and social cost are both functions of the distribution of wealth.

- Hence, the social optimum is unique for any given distribution of wealth but it is not invariant to a change in the distribution of wealth.

38

A POSITIVE EXTERNALITY

- Figure 3.6 illustrates a setting where there is an external benefit but no external cost; that is, $G(z)>0$ but $D(z)=0$.

- The vertical difference between *MSB* and *MPB* is **marginal external benefit**, $MEB(z) \equiv \partial G(z)/\partial z$.

39

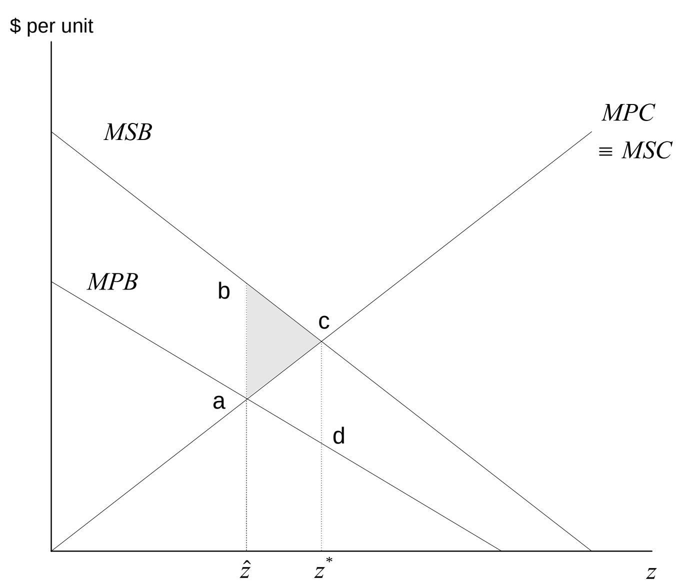

Figure 3-6

40

- Figure 3-6 is drawn for the case where *MEB* is decreasing in z; thus, the vertical difference between *MPB* and *MSB* becomes smaller as z increases.

41

- The presence of the external benefit means

$$\hat{z} < z^*$$

  – that is, the privately optimal level of activity is lower than the socially optimal level.

- Intuition: the source agent does not take into account the external benefit she bestows on others when she chooses her action, and so her chosen level of that action is too low from a social perspective.

42

- To see this, consider the implications of a forced move from $\hat{z}$ to z^*, without compensation to the source agent.

- There is a gain to external agents, equal to

  $area(abcd)$

  but a loss to the source agent, equal to

  $area(acd)$

43

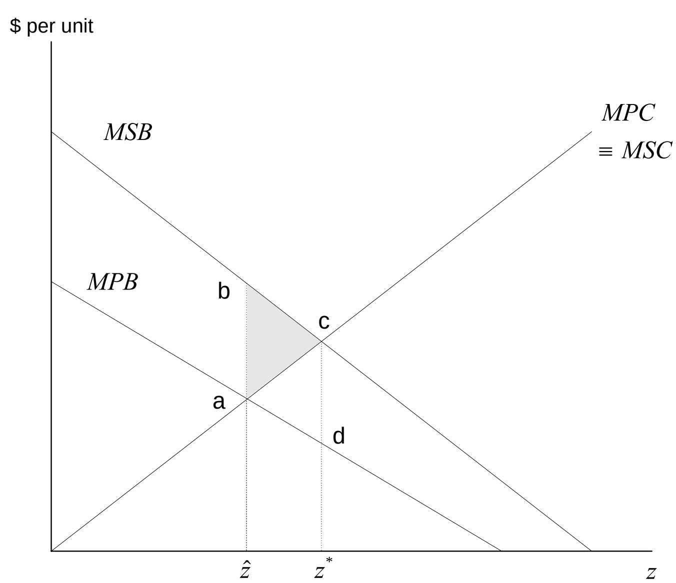

Figure 3-6 (repeat)

44

- Thus, the reallocation is not a Pareto improvement (because the source agent is made worse off).

45

- However, it is be a potential Pareto improvement:
  - the gain to the external agents is greater than the loss to the source agent, so the external agents could in principle compensate the source agent for her loss and still be better off.

- In particular, social surplus would increase by *area*(*abc*), the shaded area in Figure 3-6.

46

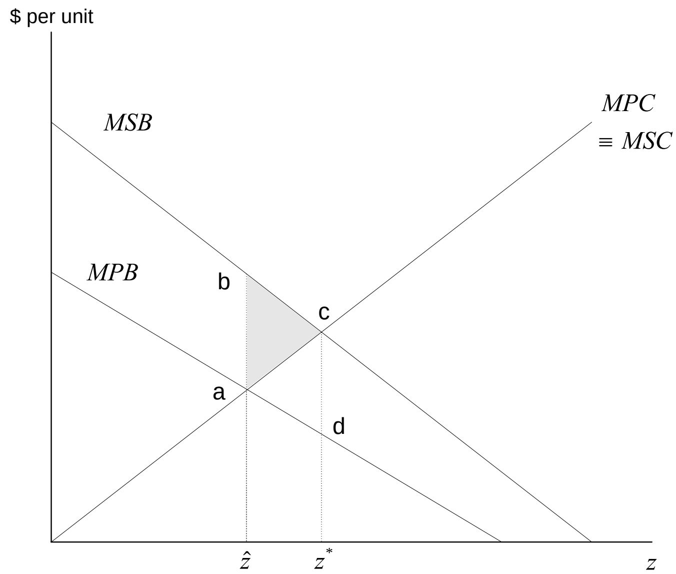

Figure 3-6 (repeat)

47

A NEGATIVE EXTERNALITY

- Figure 3.7 illustrates a setting where there is an external cost but no external benefit; that is, $D(z)>0$ but $G(z)=0$.

- The vertical difference between *MSC* and *MPC* is **marginal external cost**, $MEC(z) \equiv \partial D(z)/\partial z$.

48

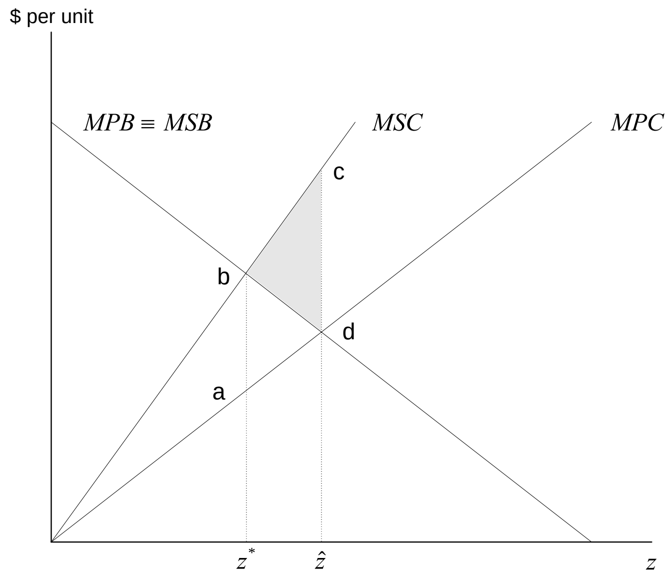

Figure 3-7

49

- Figure 3-7 is drawn for the case where *MEC* is increasing in z; thus, the vertical difference between *MPC* and *MSC* becomes larger as z increases.

50

- The presence of the external cost means

$$\hat{z} > z^*$$

  – that is, the privately optimal level of activity is higher than the socially optimal level.

- Intuition: the source agent does not take into account the external cost she imposes on others when she chooses her action, and so her chosen level of that action is too high from a social perspective.

51

- To see this, consider the implications of a forced move from $\hat{z}$ to z^*, without compensation to the source agent.

- There is a gain to external agents, equal to

  area(*abcd*)

  but a loss to the source agent, equal to

  area(*abd*)

52

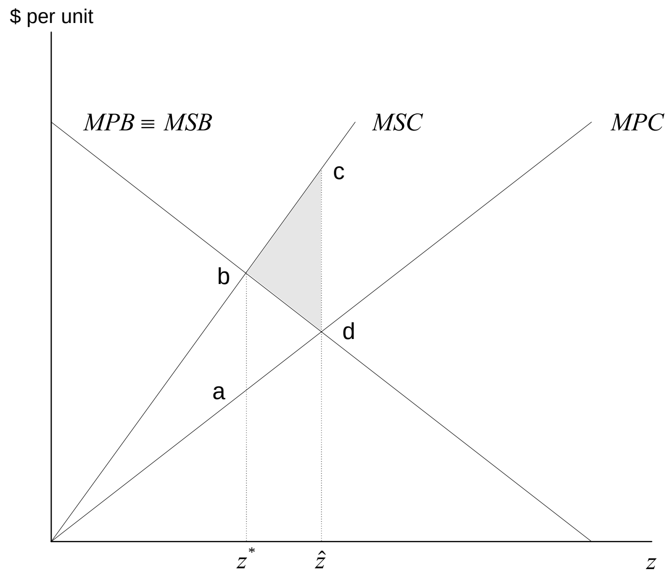

Figure 3-7 (repeat)

53

- Thus, the reallocation is <u>not</u> a Pareto improvement (because the source agent is made worse off).

54

- However, it is be a potential Pareto improvement:
  - the gain to the external agents is greater than the loss to the source agent, so the external agents could in principle compensate the source agent for her loss and still be better off.

- In particular, social surplus would increase by *area*(*bcd*), the shaded area in Figure 3-7.

55

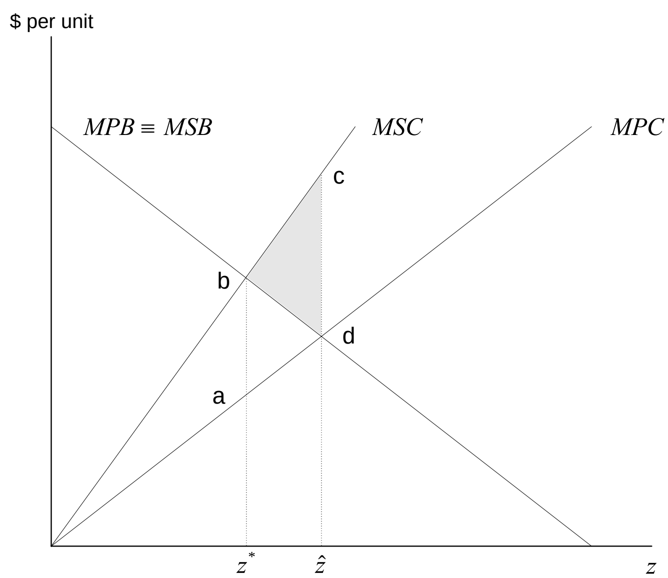

Figure 3-7 (repeat)

56

3.5 RECIPROCAL EXTERNALITIES

- Our characterization of externalities in the previous section relates to **unilateral externalities**, where the external effect runs in only one direction: from source agent to external agent.

57

- In contrast, some important types of externality are **reciprocal**, where each external agent is also a source agent.

- To model this formally requires a game-theoretic approach and that is beyond our scope here.

58

- We will instead discuss informally two important types of reciprocal externality:
  - open-access resources; and
  - public goods

59

OPEN ACCESS RESOURCES

- An **open access resource** is one to which many agents have free access.

- Examples:
  - unregulated fisheries or grazing lands; the assimilative capacity of the environment.

60

- When one agent exploits the resource (for example, by harvesting fish) and thereby draws down the stock of the resource, she does not take into account the cost she imposes (in terms of reduced fishing productivity) on other agents who are also exploiting the resource.

61

- Consequently, each agent harvests too much from a social perspective, and so the aggregate social surplus captured from the resource is lower than it could be.

62

- Moreover, the excessive harvesting can lead to the depletion (and possible extinction) of the resource.
- This potential outcome is sometimes called the “tragedy of the commons”.

63

- In principle the problem can be solved through the assignment of property rights (for example by using tradeable fishing licenses and tradeable timber licenses).

64

- However, in many instances, no government has jurisdiction over the resource – as with open-ocean fisheries and the global atmosphere – and so regulated use can only be achieved through a negotiated treaty.

- Designing such treaties is the subject of **coalition theory**.

65

PUBLIC GOODS

- Public goods are characterized by two key features:
  - joint consumption possibilities
  - high exclusion costs

66

- **Joint consumption possibilities** means that the benefits of the good can be enjoyed by more than one agent simultaneously
  - eg. a park, a lighthouse beam, a radio signal

- In this sense, public goods are sometimes described as “non-rival” goods.

67

- **High exclusion costs** means that it is very costly to prevent agents from consuming the good once it is provided
  - eg. it is costly to build a fence around a national park or to prevent a ship from seeing a lighthouse beam.

68

- Public goods are a special kind of positive reciprocal externality:
  – the provision of the good by one agent bestows a benefit on other agents who are then able to use the good too.

69

- Public goods can be roughly categorized on the basis of congestibilty and excludability.

- **Congestible (or impure) public goods** are subject to congestion; the benefits of consumption decline as more agents consume the good (eg. roads, the radio spectrum, a wilderness area).

70

- **Pure public goods** are not subject to congestion; they are perfectly non-rival (eg. radio signals, a lighthouse beam, knowledge).

- **Club goods** are congestible public goods that have relatively low exclusion costs (eg. a swimming pool, a restaurant).

71

- The market provision of public goods is typically inefficient.

- Why? The external benefit bestowed on others when one agent contributes to the provision of the good cannot be internalized via pricing because the good is non-excludable.

72

- Each agent can **free-ride** on the contributions that other agents make, and so each agent has an incentive to let other agents provide the good.

- The relative excludability of club goods means that they are less subject to the free-rider problem, and so they are typically provided efficiently by the market.

73

3.6 MARKET POWER

- An agent (a person or a firm) has market power when the actions of that agent have a significant effect on the market price.
- Extreme examples: monopoly (single seller); monopsony (single buyer).

74

- Market power stems from **barriers to entry**.

- Barriers to entry can be due to
  – economies of scale
  – network effects
  – proprietary technological advantages
  – regulatory barriers (including patents)

75

- Market power causes a distortion of prices (unless the firm can perfectly price discriminate):
  – price does not reflect marginal cost

- This distortion of prices in turn distorts the allocation of resources, and consequently, social surplus is not maximized.

76

- The standard textbook example is monopoly, as depicted in Figure 3-8, where the DWL is the social surplus foregone.
- The study of market power is primarily the subject of **industrial organization** theory.

77

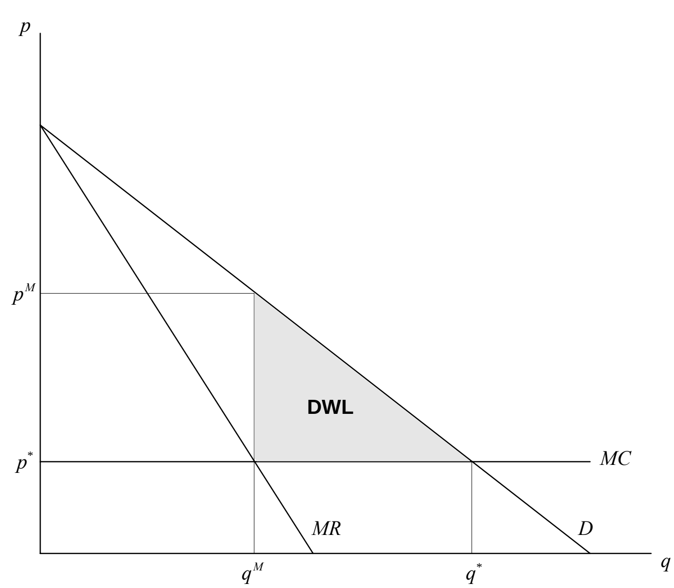

Figure 3-8

78

3.7 ASYMMETRIC INFORMATION

- Asymmetric information arises when the buyer and seller in an exchange have different information.
- There are two broad classes of asymmetric information problem:
  - adverse selection
  - moral hazard

79

ADVERSE SELECTION

- Adverse selection arises from asymmetric information about agents' <u>characteristics</u>.
- Consider a market for goods of variable quality where the seller of a particular good knows its quality but potential buyers do not.

80

- The potential buyers will base their initial valuation of the good on the market-wide expected quality.

81

- The seller of a high quality good – who cannot credibly convince the buyer that it is high quality, and thereby charge a high price – may decide to retain the good rather than sell it an average-quality price.

- Conversely, the seller of a low quality good will be happy to sell it at an average-quality price.

82

- Thus, the market adversely selects the lowest quality goods for sale, even though there may be buyers and sellers who would mutually benefit from the sale of the high quality good.

- Adverse selection therefore generates a loss of social surplus:
  - potential gains from trade go unrealized

83

- In extreme cases, low quality can drive out good quality until the entire market collapses.

84

MORAL HAZARD

- Moral hazard arises from asymmetric information about agents' actions.

- Consider an insurance market where a risk-averse agent faces some uncertainty (such as the possibility of a house fire) and buys insurance from a firm.

85

- If the agent buys full insurance (to completely cover all loss) and her actions are unobservable to the firm, then she has no incentive to take precautionary action to reduce the likelihood of a loss (because that action is costly to her).

- Thus, the purchase of insurance effectively raises the probability of a loss, and the probability of payout by the insurance firm.

86

- In response to this moral hazard problem the insurance firm will offer only partial insurance (a deductible is required), and so the agent is exposed to some risk.

- This creates an incentive for the insured agent to take precautionary action but it also exposes her to a risk that she would prefer to insure away.

87

- The insured agent would be better-off by taking the precautionary action and obtaining full insurance to eliminate the residual risk, but the moral hazard problem makes this impossible.

- Thus, there is a loss of social surplus:
  - potential gains from trade (via a full insurance contract) are unrealized

88

- The same type of problem arises in any **principal-agent problem** where the payoff to the principal depends in an uncertain way on the action of an agent contracted to perform that action, but where payment for the agent's services can be based only on the realized outcome and not on the action itself (because the action itself is not observable).

89

- However, if the payment is tied exclusively to the realized outcome, the agent may face too much risk and therefore be unwilling to enter the contract.

- The principal will therefore have to offer some fixed component to the payment to reduce that risk.

90

- But this fixed payment reduces the incentives of the agent to undertake effort, and this reduces the payoff to the principal.

- The optimal contract must balance the creation of incentives with the creation of risk.

91

- There is a large theoretical literature on optimal contract design, but it is generally not possible to eliminate the agency problem entirely; there is always a loss of social surplus relative to a full-information setting.

- The construction of these optimal contracts is called **mechanism design**.

92

3.8 MARKET FAILURE AND TRANSACTION COSTS

- We have seen that market failure leads to a loss of social surplus, but does the presence of market failure necessarily justify government intervention?

93

- To consider this issue, recall the simple unilateral negative externality problem, as depicted in Figure 3-7.

94

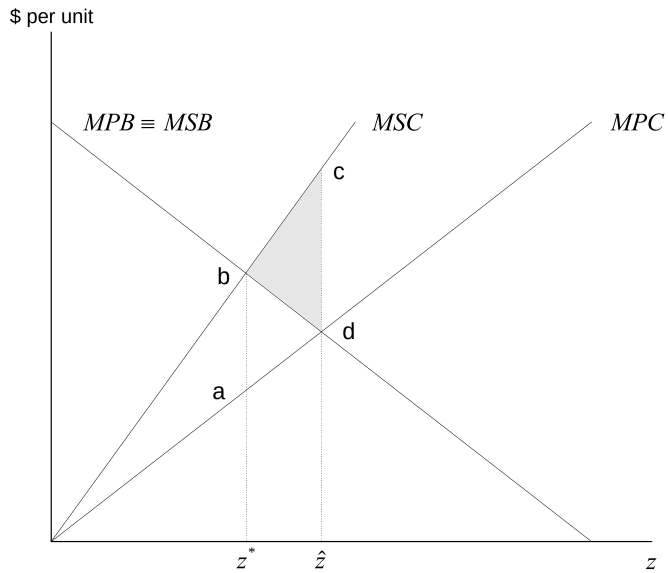

Figure 3-7 (repeat)

95

- The source agent and the external agents in this setting could all be better-off at the social optimum if the external agents compensate the source agent for the change in her behaviour, and then share some of the remaining surplus with her as well; that is, give her *area*(*abd*) plus a share of *area*(*bcd*).

- That is, there are potential gains from trade here.

96

- Why don’t these agents simply write a private contract to capture these gains from trade? Where is the need for government intervention?

97

- One extreme view: there is <u>no role</u> for government intervention here except to assign and enforce **property rights**.

- In particular, if rights are clearly defined then they can be traded via an enforceable private contract.

98

- But why might a private contract <u>not</u> be able to realize the gains from trade even if property rights are assigned?
  - the potential obstacle is **transaction costs**

99

- The cost of negotiating and writing a contract can sometimes more than offset the gains from that contract.
- That is, once the costs of constructing the contract are taken into account, there may be no actual gains from trade left to be captured.

100

- This still does not mean that government should necessarily intervene; it too will face costs when trying to reallocate resources.

- The argument for government intervention on efficiency grounds (as opposed to redistributional grounds) must rest on the possibility of an **institutional advantage** over private contracts.

101

- That is, in some cases government intervention may be able to achieve a surplus-enhancing reallocation of resources at lower transaction cost than private contracts.
  - eg. a global climate-change agreement

102

- However, it is important to recognize that the mere existence of externalities and other types of market failure is not enough to justify government intervention.

- Any proposed government intervention on efficiency grounds should be subject to the question:
  – can the government do better than the market?

103

- There is no universal answer to this question; it depends on the situation under consideration.

- In particular, an ideologically based response – always “yes” or always ‘no” – is unhelpful.

- Our role as analysts is to consider the question on a case-by-case basis, <u>without</u> ideological bias.

104

3.9 THE PROBLEM OF SECOND-BEST

- Suppose a good case is made for government intervention in response to an instance of market failure.
- Will that one intervention necessarily improve things if other market failures remain uncorrected?

105

- The **problem of second-best**:
  - correcting a distortion in one part of an economy will not necessarily increase overall social surplus, and could actually reduce it, if there remain uncorrected distortions in other parts of the economy.

106

- For example, suppose a government policy places a restriction on the use of a polluting input in a production process, but does not restrict the use of an even-more-damaging substitute input.

- The overall environmental harm could be worse after the policy intervention, and social surplus may actually be lower.

107

- That is, a well-intentioned policy could have costly unintended consequences that more than offset the benefits of the policy.

108

- This points to a problem with conducting policy in a piecemeal fashion.
- Ideally, we would like to implement all corrective policies simultaneously, in one coordinated intervention.
- Of course, this is almost never feasible.

109

- Does this mean that piecemeal policy intervention should not be undertaken at all?
- Not necessarily.

110

- We should take a long-run view of policy and recognize that a given policy implemented today may not be perfect but will nonetheless be beneficial overall when other complementary policies are introduced in the future.

- However, this takes careful and thoughtful long-term planning (which is not always consistent with short electoral cycles).

111

- The extent to which we should be concerned by the problem of second-best also depends on the scale of the policy intervention under consideration.

- If a single proposed intervention is on a fairly small scale, and at a localized level, then it may be reasonably safe to proceed with such a policy in isolation.

112

- If instead a project or policy will generate significant effects in other areas of the economy – as a large carbon tax or a free-trade agreement would – then a **general equilibrium analysis** is necessary.
- Such an analysis would trace through the impact of the policy on all markets and account for any distortions in those markets.

113

- Rough threshold rule:
  – if the proposed policy is likely to cause price changes in excess of 10% in other parts of the economy then a general equilibrium analysis should ideally be conducted.

114

- In this course we will confine consideration to the analysis of policies and projects that do <u>not</u> have significant general equilibrium effects.

- The mathematical and modeling tools needed to undertake general equilibrium analysis are beyond our scope here.

END

115

TOPIC 3 REVIEW QUESTIONS

1. Which of the following is not a condition of the First Welfare Theorem?

A. All agents are price-takers.

B. There are no externalities.

C. There are no scale economies.

D. Information is perfect among buyers and sellers.

2. "The Second Welfare Theorem states that any Pareto efficient allocation can be supported as an equilibrium of a perfectly competitive economy with appropriate lump-sum transfers".

A. True.

B. False.

3. Which of the following is the best explanation for the tradeoff between wealth creation and the redistribution of wealth?

A. The impossibility theorem implies that an ideal distribution of wealth cannot be derived from preferences alone, and must therefore be based on willingness-to-trade measures which are themselves a function of the distribution of wealth.

B. The Second Welfare Theorem is wrong because lump-sum transfers are unavailable in practice.

C. The redistribution of wealth from the rich to the poor typically requires the use of policy instruments that distort incentives to create wealth.

D. Extremely high levels of inequality can lead to the pursuit of rent-seeking behavior – such as crime and revolution – that requires the redirection of resources away from wealth-creation and towards defensive activity.

Questions 4 – 8 relate to **Figure R3-1**. It depicts a setting where an action undertaken by a single source agent imposes an external cost on other agents. There are no fixed costs or fixed benefits associated with this action, and there are no external benefits.

4. "The triangular area $0az^*$ measures the social cost at the social optimum". True or False?

A. True.

B. False.

5. "The triangular area 0*eb* measures social surplus at the social optimum". True or False?

A. True.

B. False.

6. Suppose a regulatory intervention requires the source agent to reduce her activity from $\hat{z}$ to z^*. Which of the following measures the gain to the external agents?

A. the trapezoidal area $\hat{z}adz^*$.

B. the trapezoidal area *abcd.*

C. the triangular area *abc.*

D. the triangular area *acd.*

7. A regulatory intervention requiring the source agent to reduce her activity from $\hat{z}$ to z^* without compensation will

A. create social surplus by an amount equal to the triangular area *abd*

B. raise social welfare by an amount equal to the triangular area *bcd*

C. create a Pareto improvement.

D. None of the above.

8. The external cost at the private optimum is

A. the trapezoidal area *abcd.*

B. the triangular area 0*ec.*

C. the triangular area 0*cd.*

D. the trapezoidal area $0ecz^*$.

Questions 9 – 13 relate to **Figure R3-2**. It depicts a setting where an action undertaken by a single source agent bestows an external benefit on other agents. There are no fixed costs or benefits associated with this action, and there are no external costs.

9. "The triangular area $0a\hat{z}$ measures the social cost at the private optimum". True or False?

A. True.

B. False.

10. "The trapezoidal area $0eb\hat{z}$ measures the private benefit to the source agent at the private optimum". True or False?

A. True.

B. False.

11. Suppose a regulatory intervention requires the source agent to increase her activity from $\hat{z}$ to z^*. Which of the following measures the gain to the external agents?

A. the trapezoidal area $\hat{z}adz^*$.

B. the trapezoidal area *abcd.*

C. the triangular area *abc.*

D. the triangular area *acd.*

12. A regulatory intervention requiring the source agent to increase her activity from $\hat{z}$ to z^* without compensation will

A. create social surplus.

B. improve social welfare.

C. create a Pareto improvement.

D. All of the above.

13. The external benefit at the social optimum is

A. the trapezoidal area *abcd.*

B. the triangular area 0*ec.*

C. the triangular area *ecd.*

D. the trapezoidal area 0ec z^* .

14. Education has an associated positive externality because

A. highly educated people earn more than less educated people.

B. a more educated population tends to have less inequality.

C. educated workers typically choose to work with other educated workers.

D. the education of any given worker often boosts the productivity of her co-workers.

15. Open access resources (like international fisheries) are often subject to over-use relative to the social optimum because

A. these resources usually do not regenerate at a rate fast enough to offset harvesting by users.

B. technological developments have made harvesting too easy.

C. users of the resource do not have to account for the negative impact that their use of the resource has on the productivity of other users of the resource.

D. users of the resource usually do not have full information about the rate of regeneration of the resource.

16. A public good is characterized by two key features:

A. joint production possibilities and high exclusion costs.

B. congestion and high exclusion costs.

C. joint consumption possibilities and low exclusion costs.

D. None of the above.

Questions 17 – 19 relate to **Figure R3-3**. It depicts a setting in which a monopoly firm sells to all consumers at the same price.

17. The monopoly will sell

A. an amount *f* at price *c*

B. an amount *f* at price *b*

C. an amount *g* at price *c*

D. an amount *a* at price *a*

18. The deadweight loss associated with the monopoly outcome is

A. area *kde*

B. area *hke*

C. area *gdef*

D. area *hde*

19. The monopoly outcome is inefficient because

A. social surplus is not maximized.

B. there is a deadweight loss.

C. an increase in the amount produced and sold would yield gains to consumers that exceed the losses to the firm.

D. None of the above.

20. Consider the following scenario. In an effort to retain customers in the face of new competition, a home insurance company announced an expansion of its insurance policy to include optional coverage of asbestos removal if the home-owner discovers its presence in the home. (Asbestos was commonly used as a component of insulation during the 1960s and 1970s. It is a carcinogenic material, and its removal is very expensive due to the safety precautions that must be taken). This optional coverage required an additional premium of $100 per year. About one in every 100 homes is thought to contain asbestos, and the insurance company has a very large portfolio of homes covered by its policies. In the first year of offering the new policy coverage, 1000 home-owners purchased the additional coverage, and all of them made a claim within a month of becoming insured.

This scenario is an example of a

A. a moral hazard problem.

B. an adverse selection problem.

C. a public good problem.

D. an open access problem.

21. Recall **Figure R3-2**, which depicts a setting with a positive externality. Suppose there is a single source agent and a single external agent in this setting. Suppose further that these two agents can write an enforceable contract under which the source agent increases her activity from $\hat{z}$ to z^* in return for a payment P from the external agent. The cost of writing and enforcing this contract is T, and this cost would be paid by the external agent. Which of the following statements are true?

A. The contract will increase social surplus if and only if $P > T$.

B. If P < area(*acd*) then the source agent will agree to the contract.

C. If $P + T$ < area(*abcd*) then the external agent will agree to the contract.

D. Both A and C.

22. Consider the following scenario. The residents of a city who commute by car do not currently pay for the congestion they cause on city roads. This causes more people to commute by car than is socially optimal. (An externality problem). The city residents who commute by bicycle do not currently pay for the medical services they use if they crash while riding because they are covered by universal medicare. This causes cyclists to be less careful than is socially optimal. (A moral hazard problem). Government has announced a policy to impose road tolls on car commuters so as to put a price on the congestion externality. The theory of second-best tells us that this policy will lead to an increase in social surplus.

A. True.

B. False.

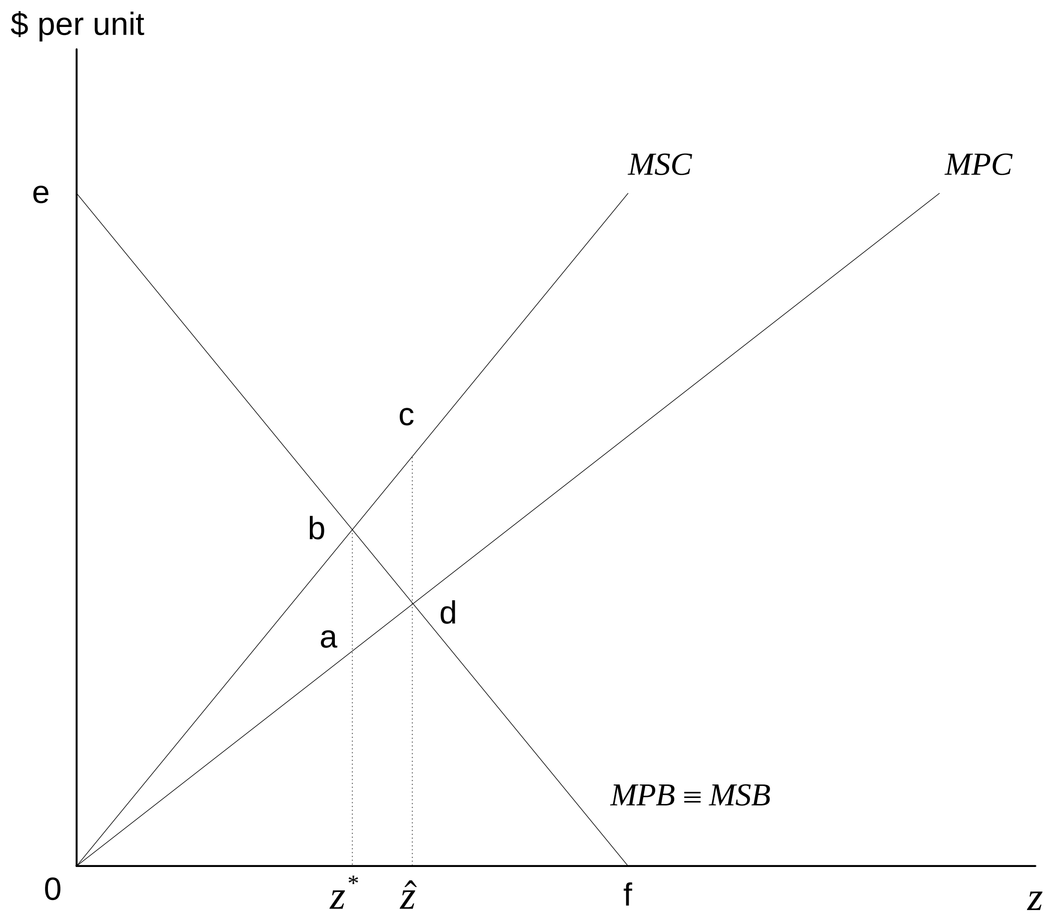

Figure R3-1

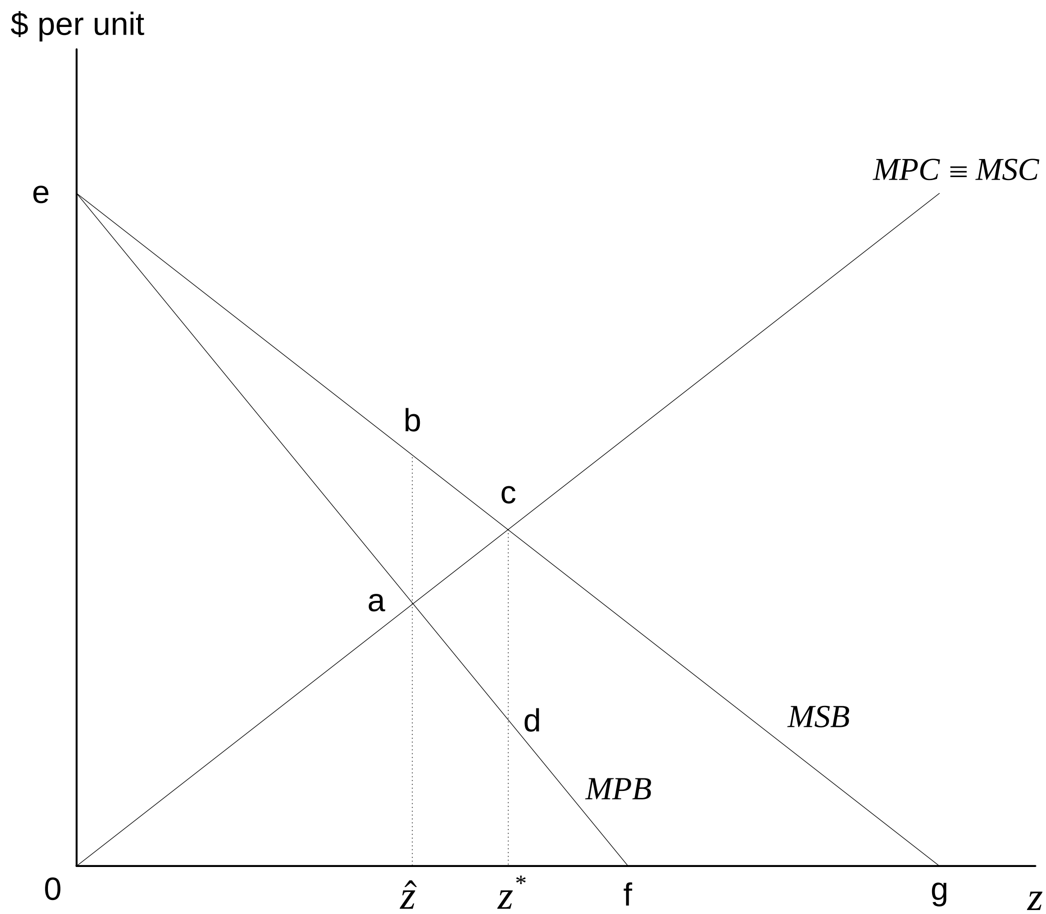

Figure R3-2

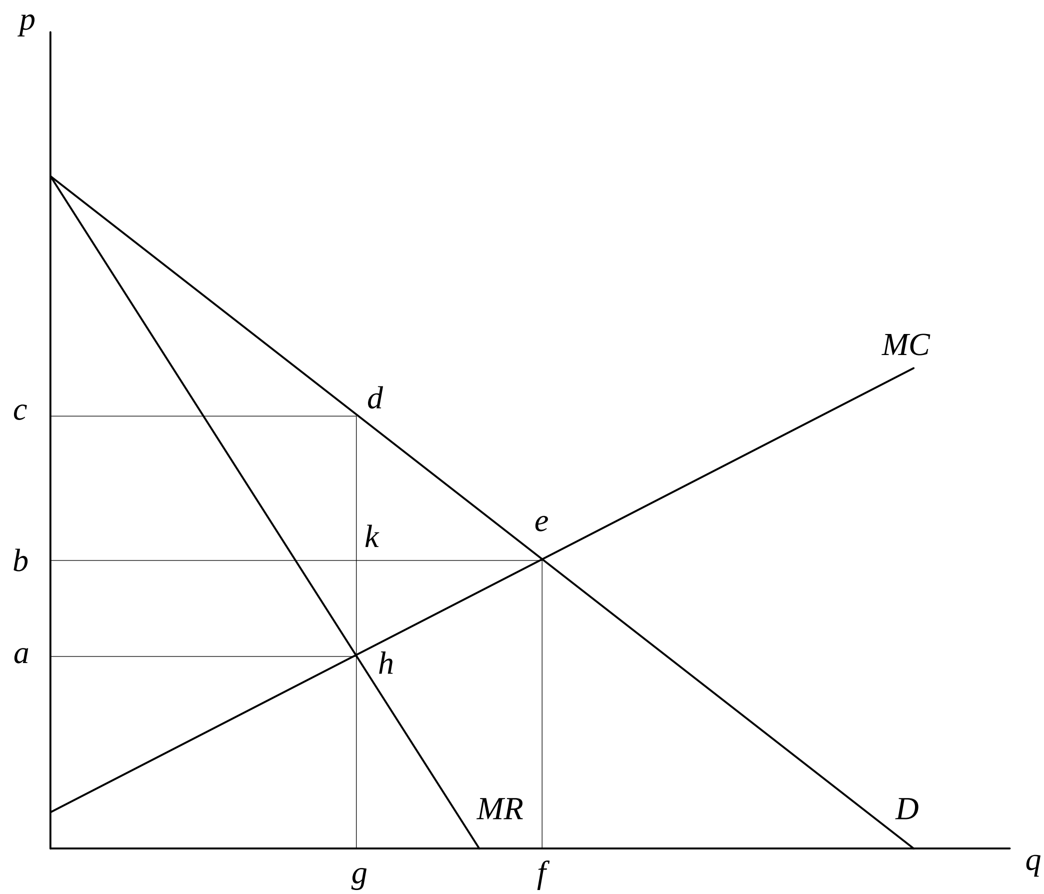

Figure R3-3

ANSWER GUIDE

1. D
2. A
3. C
4. B
5. A
6. B
7. D
8. C
9. A
10. B
11. B
12. A
13. C
14. D
    It is worth noting that reducing inequality could have associated external benefits by potentially reducing anti-social behaviour sometimes associated with poverty. However, education may not necessarily reduce inequality. If income depends on innate ability only when that innate ability is fostered via education, and there is a distribution of innate ability, then education could actually exacerbate inequality.
15. C
16. D
17. C
18. D
19. D
20. B
21. C
22. B

4. CALCULATING COSTS AND BENEFITS

OUTLINE

* Advanced Topic

4.7 Calculating Costs when there are Price Effects

4.8 Calculating Benefits when there are Price Effects

4.9 Calculating Costs when Input Markets are Distorted

4.10 Calculating Benefits when Output Markets are Distorted

3

4.11 Using Shadow Prices from Secondary Sources

4.12 Calculating the Cost of Funds

4

4.1 INTRODUCTION

- A key question in CBA:
  - does a policy or project lead to an increase in social surplus?
- This requires the calculation of surplus changes in all the markets in which the project has an impact, together with any non-market effects (treated in Topic 7).

5

- We begin by reviewing the calculation of market surplus in a perfectly competitive market, using the standard measures of consumer surplus and producer surplus.

6

4.2 CONSUMER SURPLUS

- The ordinary (or Marshallian) inverse demand curve for an individual can be interpreted as measuring his **WTP** for the marginal unit consumed.
- That is, at his current level of consumption in response to the market price, we know that he is just willing to pay that price on the last unit consumed.

7

- We can show this formally from the utility-maximization problem but the intuition is straightforward:
  - if the market price is less than his WTP for one more unit, then he would choose to buy that additional unit, and so consume more than his current level
  - if the market price is more than his WTP for the marginal unit consumed, he would choose not to consume it, and so consume less than his current level

8

- We can think about this in terms of marginal costs and benefits:
  - the consumer will buy an amount up to the point where the marginal private benefit from consumption (as measured by marginal WTP) is just equal to the marginal private cost of that unit of consumption (which is the market price).

- See Figure 4-1.

9

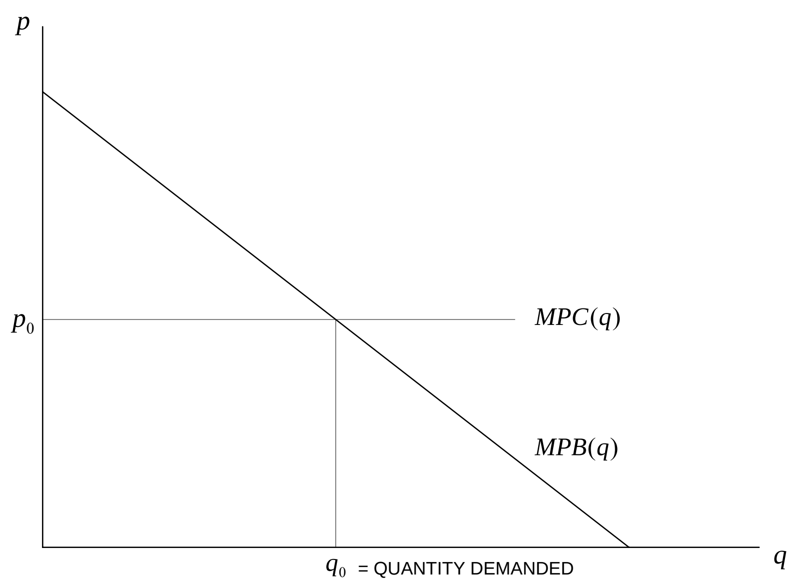

Figure 4-1

10

- We can therefore use the ordinary inverse demand curve as a way to approximate the marginal private benefit from consumption, and to use the <u>area</u> under that curve to approximate the <u>total</u> benefit from consumption.

- See Figure 4-2.

11

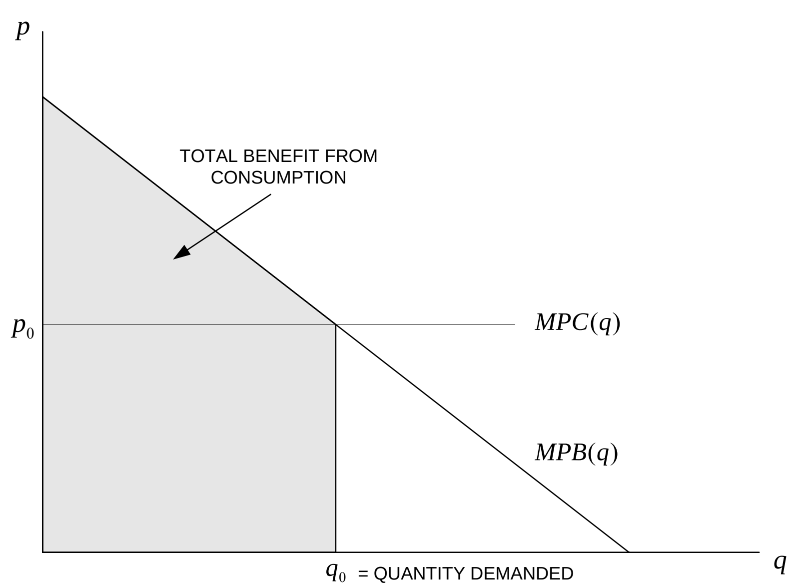

Figure 4-2

12

- Why is the area under the inverse ordinary demand only an approximation of the total private benefit?

- The reason relates to the existence of income effects.

13

- Recall that the response to a price change along the ordinary demand curve can be decomposed into two parts:
  – a substitution effect
  – an income effect

14

- The **substitution effect** (SE) is negative (except in the special case where goods are perfect complements, when it is zero).

- That is, the SE causes consumption to fall when the price rises.

15

- This negativity of the substitution effect reflects **preference for variety** (or strict convexity of preferences), as reflected in the usual "bowed-in" shape of indifference curves.

16

- The **income effect** is, by definition, negative for normal goods and positive for inferior goods (and zero for income-neutral goods).

- That is, if the price rises then real income falls, and so consumption falls for a normal good and rises for an inferior good (and is unchanged for a neutral good).

17

- The presence of non-zero income-effects complicates the interpretation of the area under the ordinary inverse demand curve.

- In particular, if a consumer actually paid what he is willing to pay for an extra unit of the good then his disposable income would be lower than otherwise, and so his demand curve would effectively pivot around the vertical intercept (down for a normal good and up for an inferior good).

18

- To see this another way, recall that we derive an ordinary demand based on the assumption that the consumer pays the same price for each unit.

- If instead he has to pay a higher price on the initial units consumed, then he will not be able to afford as many units as the ordinary demand curve predicts.

19

- This means that the area under the ordinary inverse demand curve overstates the true WTP in the case of a normal good, and understates it in the case of an inferior good.

- It is an exact measure only for an income-neutral good.

20

- The true WTP is measured as the area under an inverse **Hicksian** (or compensated) demand curve, which strips out the income effect of a price change.

- This hypothetical demand curve cannot be observed directly and so it does not provide a very practical way to measure the benefit from consumption.

21

- For our purposes in CBA, the approximate measurement of benefit that the ordinary demand curve provides is good enough.

- Appendix A4-1 provides more detail on the theoretically correct measures.

22

- We now want to use the ordinary inverse demand curve to construct a measure of the **net benefit** from consumption, called consumer surplus.

23

- **Consumer surplus** (CS) measures the difference between the total benefit from consumption and the amount actually paid for the amount consumed.

- Thus, CS measures the net private benefit from consumption.

- See Figure 4-3.

24

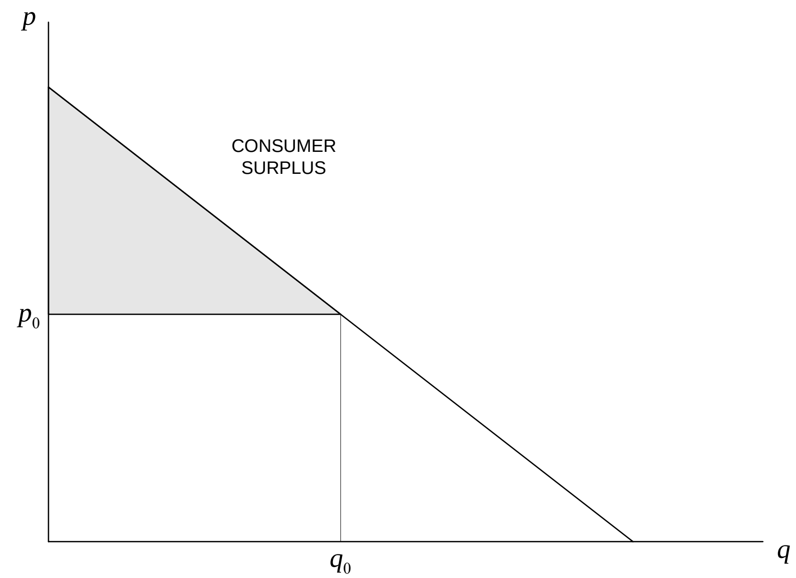

Figure 4-3

25

- A change in the price of a good leads to a change in the CS from that good.
- See Figure 4-4 and Figure 4-5.

26

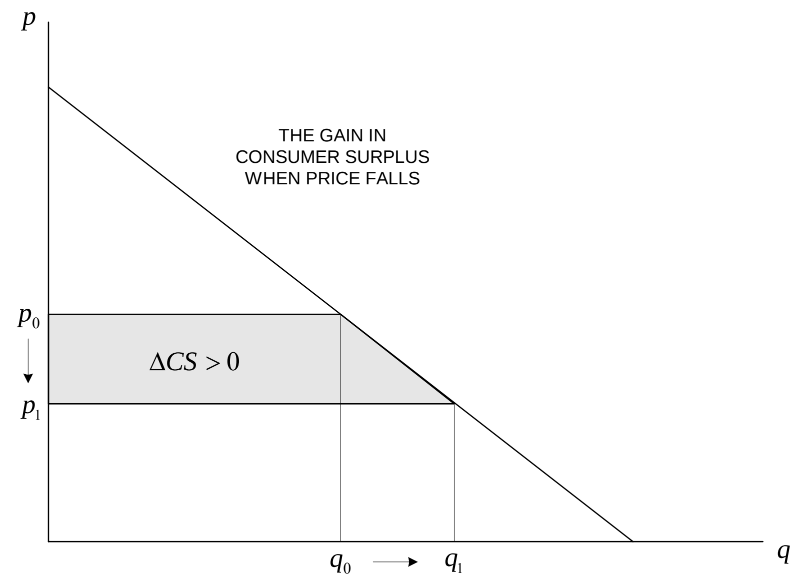

Figure 4-4 27

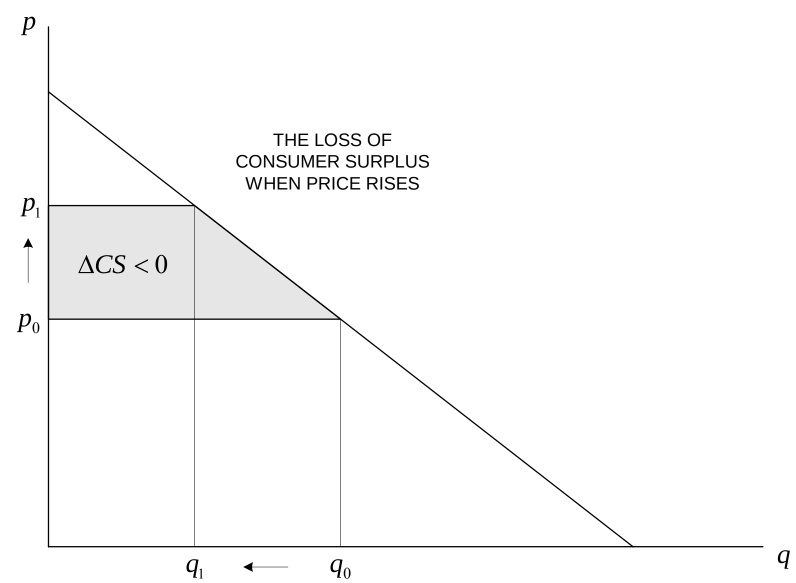

Figure 4-5 28

- We have so far worked only with the demand curve for an individual consumer.

- An **aggregate demand curve** can be constructed by summing across individual demand curves (via “horizontal summation”).

29

- The consumer surplus measure under the aggregate inverse demand curve reflects the aggregate net benefit to the consumers of the good.

30

4.3 PRODUCER SURPLUS

- The ordinary inverse supply curve for a supplier measures her **marginal WTA** to provide the good or service (and hence measures marginal private opportunity cost).
- In the case of a price-taking firm, that marginal private opportunity cost is the marginal cost of production.

31

- In the case of an individual supplier of labour, the marginal private opportunity cost is the value of that labour in its next best alternative use (which might be a leisure activity or a household-production activity).

32

- In either case, at her current level of supply in response to the market price (or wage), we know that the supplier is just willing to accept that price on the last unit supplied.

33

- In the case of a price-taking firm, we can show formally from the profit-maximization problem that
  - if the marginal cost of production is lower than the product price, then the firm will raise production and thereby earn more profit
  - if the marginal cost of production is higher than the product price, then the firm will reduce production and thereby earn more profit

34

- In the case of labour supply, we can show formally from the utility-maximization problem that:
  - if she is willing to accept <u>less</u> than the market wage, then she would supply more labour at that wage
  - if her willingness-to-accept is <u>more</u> than the market wage, then that market wage is not high enough to warrant her current level of supply and she would supply less labour at that wage

35

- In both cases, if the supplier is behaving optimally, then at any given price (or wage) she will supply an amount up to the point where the marginal cost of supply is just equal to the price (or wage) because price is the marginal benefit of that unit supplied.

- See Figure 4-6.

36

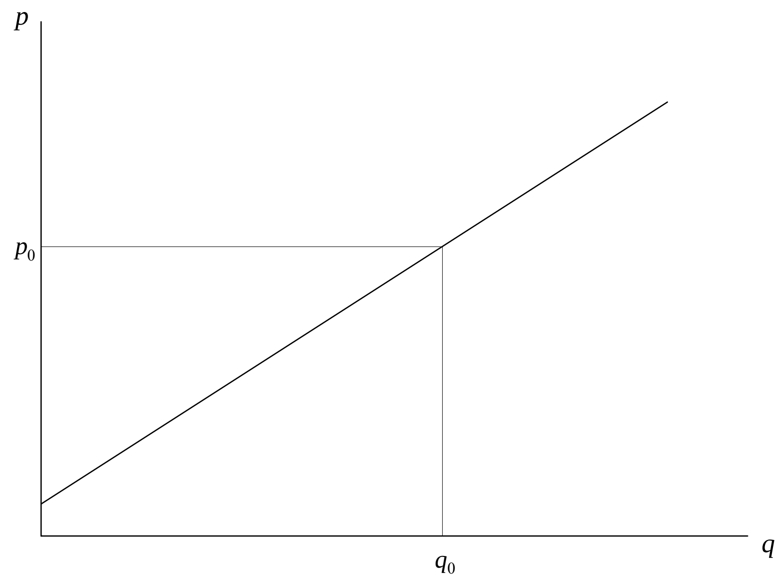

Figure 4-6

37

- We can therefore interpret the ordinary inverse supply curve as a way to measure the **marginal private cost** of supply.

38

- This in turn means that we can interpret the area under the ordinary inverse supply curve over a given range of quantity as the total variable cost of supplying that amount.

- See Figure 4-7.

39

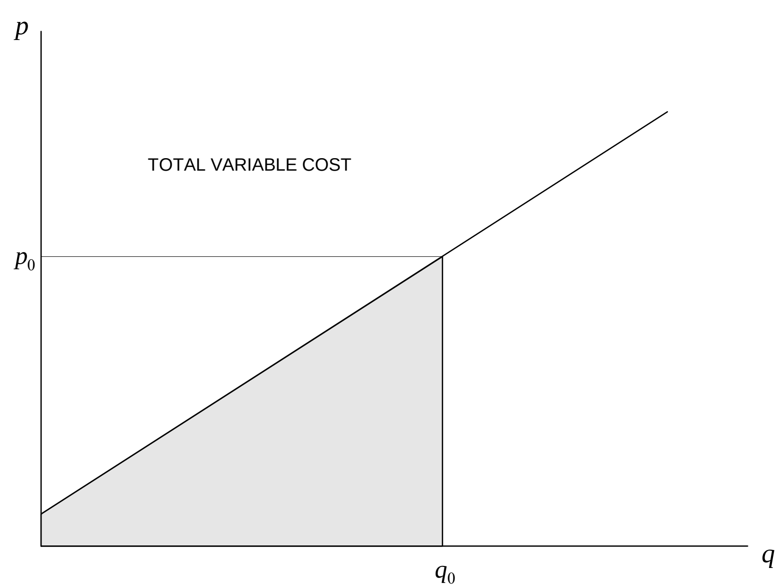

Figure 4-7

40

- Note that any fixed costs required for supply are not included in this area since these are not relevant to the marginal supply decision except on the very first unit (where the decision is a binary one: incur the fixed costs or supply nothing at all).

41

- We need one qualification in the case of labour supply.

- The substitution and income effects discussed in the case of consumer demand also arise for labour-supply, and so the area in Figure 4-7 is only an approximation of total variable cost in that case.

42

- We can in principle measure the area under an inverse Hicksian labour supply curve to get the correct measure of cost, but as with the demand side, it is usually sufficient in CBA to work with the ordinary supply curve.

43

- We now want to use the inverse ordinary supply curve to construct a measure of the **net benefit** from supply, called producer surplus.

44

- **Producer surplus** (PS) measures the difference between the amount actually received for the amount supplied and the total variable cost of supply.

- Thus, PS measures the net private benefit from supply.

- See Figure 4-8.

45

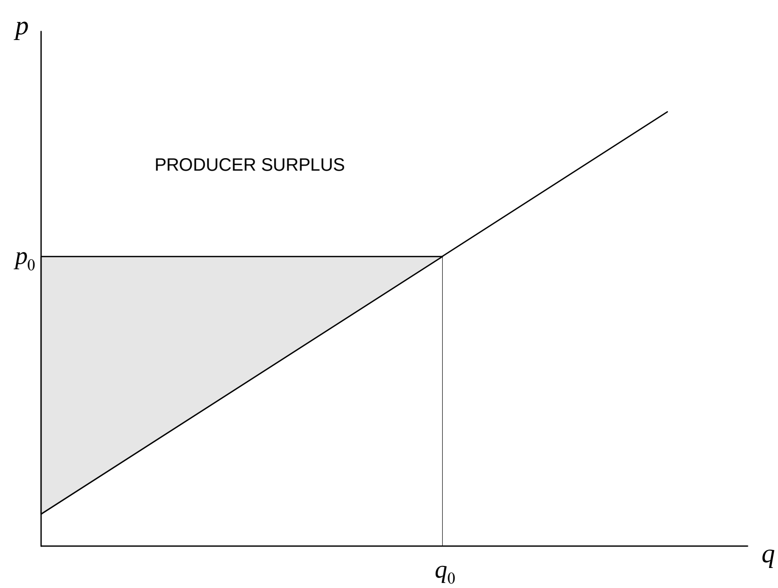

Figure 4-8

46

- PS measures the difference between the payment actually received by the supplier and the minimum payment she would be willing to accept.
- PS is sometimes called **economic rent**.
- In the case of a price-taking firm, **profit** is the difference between PS and fixed costs.

47

- A change in the price of a good leads to a change in PS for the supplier of that good.
- See Figure 4-9 for the case of a price reduction.

48

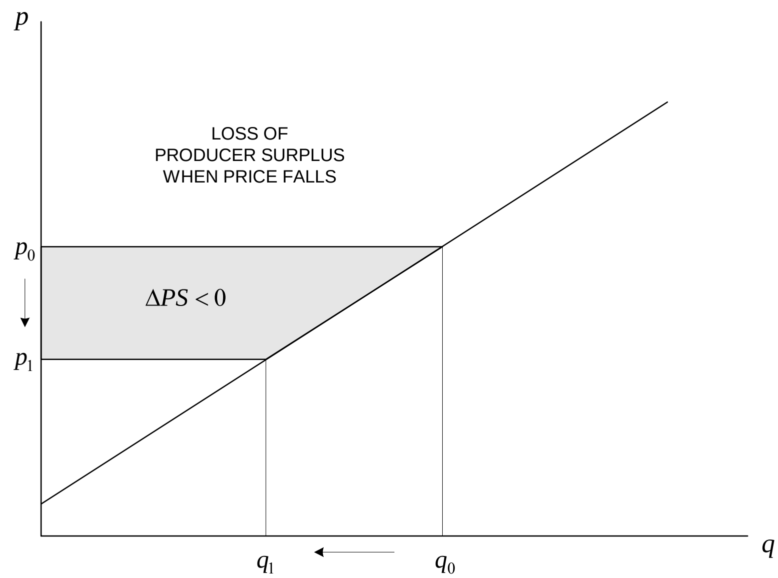

Figure 4-9

49

- We have so far worked only with the supply curve for an individual supplier.

- An **aggregate supply curve** can be constructed by summing across individual supply curves (via "horizontal summation").

50

- The producer surplus measure constructed from the inverse aggregate supply curve reflects the aggregate net benefit to the suppliers of the good.

51

4.4 USING ELASTICITIES TO CALCULATE SURPLUS CHANGES*

- If we assume that demand and supply are both linear, then we can calculate producer and consumer surplus changes as simple functions of the elasticities of supply and demand.

* Advanced Topic

52

- Let ε_0 denote our estimate of demand elasticity, evaluated at the current price and quantity.

- Let η_0 denote our estimate of supply elasticity, evaluated at the current price and quantity.

53

- It can be shown that the change in consumer surplus is

$$\Delta CS = q_0\left(\frac{|\Delta p|\varepsilon_0}{2p_0} - 1\right)\Delta p$$

- See Appendix A4-2

54

- It can be shown that the change in producer surplus is

$$\Delta PS = q_0\left(\frac{|\Delta p|\eta_0}{2p_0}+1\right)\Delta p$$

- See Appendix A4-2

55

4.5 MARKET EQUILIBRIUM AND SOCIAL SURPLUS

- Equilibrium in a perfectly competitive (PC) market occurs where the price equates supply and demand.

- Social surplus in a PC market
  = consumer surplus + producers surplus

56

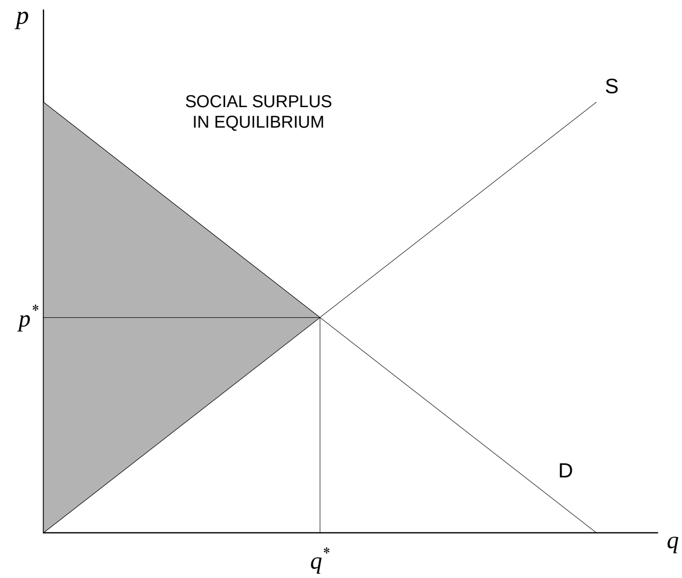

Figure 4-10

57

- Social surplus is maximized at the competitive market equilibrium:
  – the value of the marginal unit is just equal to its opportunity cost
- A production level below or above the equilibrium quantity leads to a loss of surplus.

58

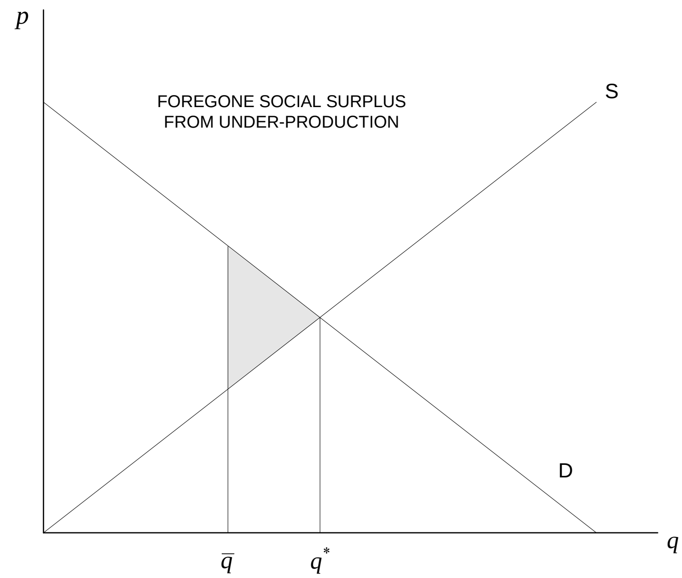

Figure 4-11

59

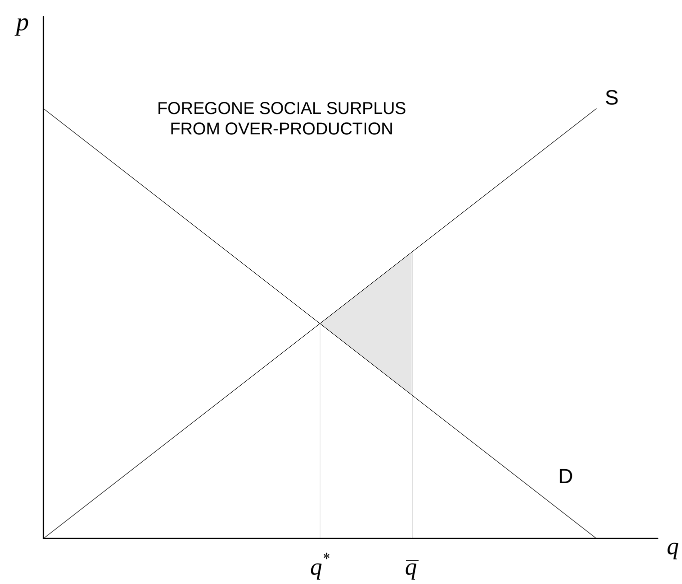

Figure 4-12

60

4.6 USING MARKET PRICES IN CBA

- To recap, the equilibrium market price in the PC market simultaneously measures:
  - the social benefit of the marginal unit
  - the opportunity cost of the marginal unit

61

- Thus, for CBA we can generally use market prices to value benefits and costs if
  - the relevant markets are perfectly competitive; and
  - the policy does not cause prices to change.
- Under these conditions:
  - the marginal social cost of an input is equal to its market price
  - the marginal social benefit of an output is equal to its market price

62

- It is generally <u>not</u> appropriate to use market prices when:
  - the policy has price effects; or
  - markets are distorted (or non-existent)

63

- Thus, there are four cases we need to consider:
  - calculating costs when there are price effects
  - calculating benefits when there are price effects
  - calculating costs when input markets are distorted
  - calculating benefits when output markets are distorted

64

- Consider the first of these four cases:
  - **calculating costs when there are price effects**
  - calculating benefits when there are price effects
  - calculating costs when input markets are distorted
  - calculating benefits when output markets are distorted

65

4.7 CALCULATING COSTS WHEN THERE ARE PRICES EFFECTS

- Suppose a project uses a large amount of a particular input (for example, steel for a bridge project).
- The project thereby **augments** the existing private demand for that input by a significant amount (Figures 4-13 & 4-14).

66

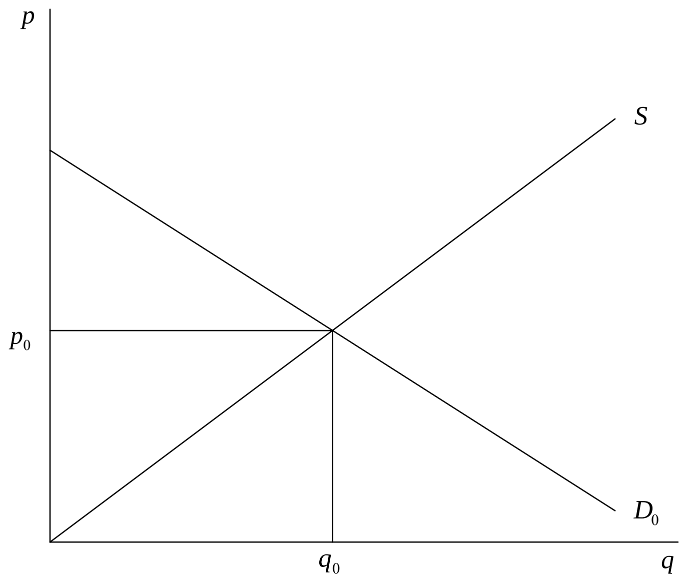

Figure 4-13 67

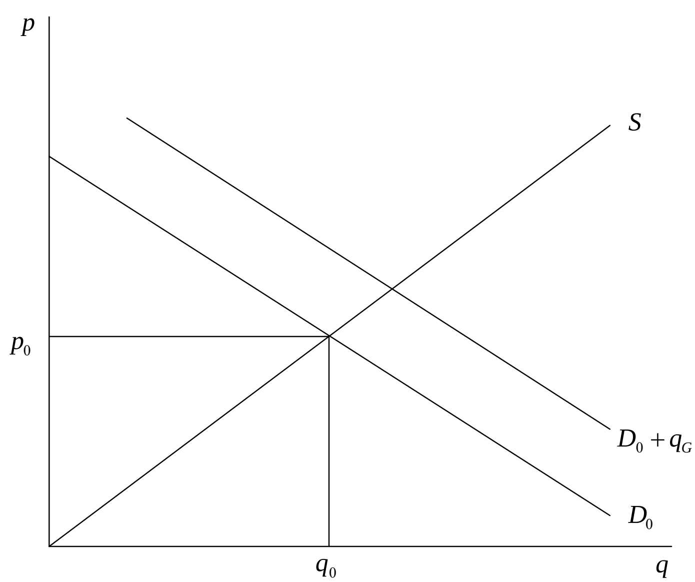

Figure 4-14 68

- Suppose the market for the input is competitive but supply is upward-sloping, as in Figure 4-14.

- Then the market price of the input must rise in order for the additional demand to be met; see Figure 4-15.

69

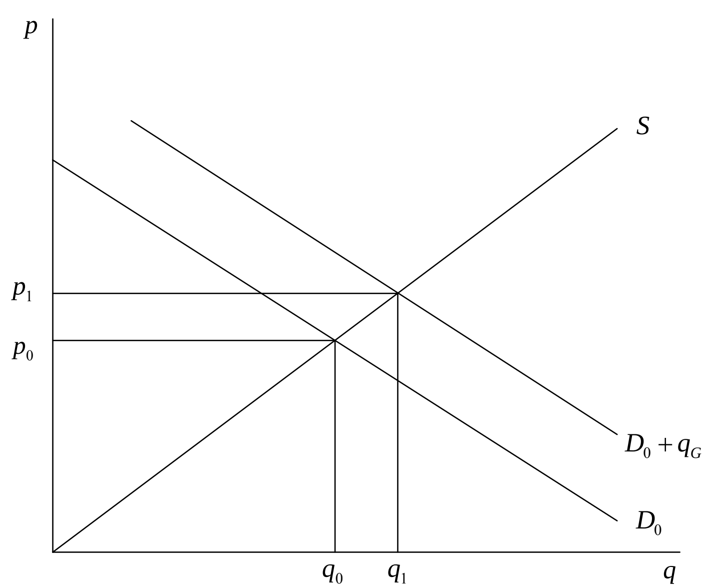

Figure 4-15

70

- The project therefore causes a **crowding-out** of existing private demand; see Figure 4-16.

71

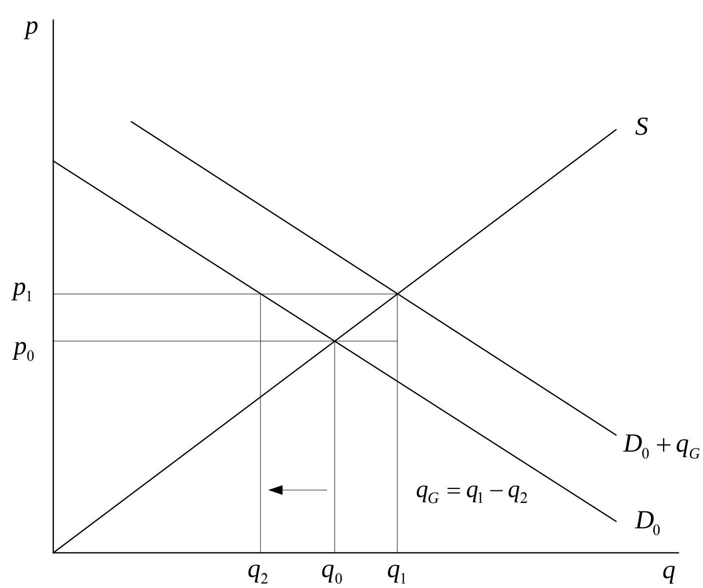

Figure 4-16

72

- Thus, the input used for the project comes
  - partly from an expansion of supply: $q_0 \rightarrow q_1$
  - and partly from a reduction in the quantity purchased privately: $q_0 \rightarrow q_2$

73

- What is the cost of the input under this scenario?

- There are two equivalent ways to think about this:
  - calculate the surplus changes for all affected parties
  - think in terms of overall social opportunity cost

74

METHOD 1: SURPLUS CHANGES

- There are three parties affected:
  - taxpayers (via impact on government finances)
  - existing private consumers
  - suppliers

- The surplus changes for each group are illustrated in Figures 4-17 to 4-19.

75

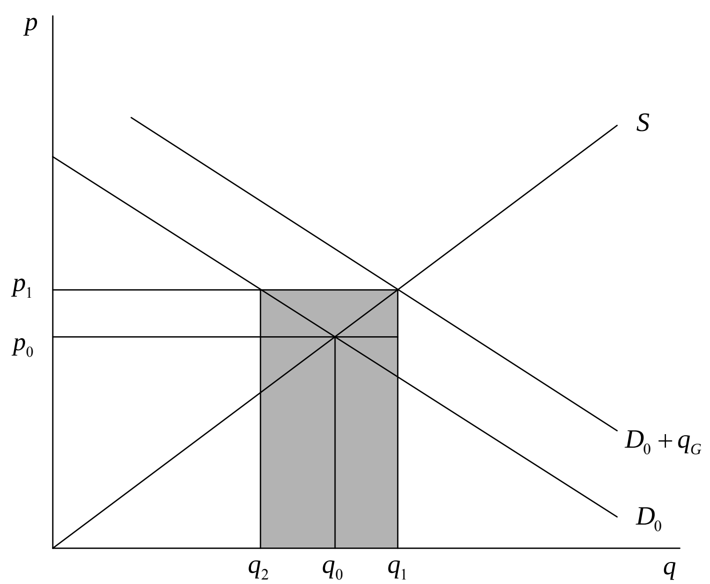

Figure 4-17

76

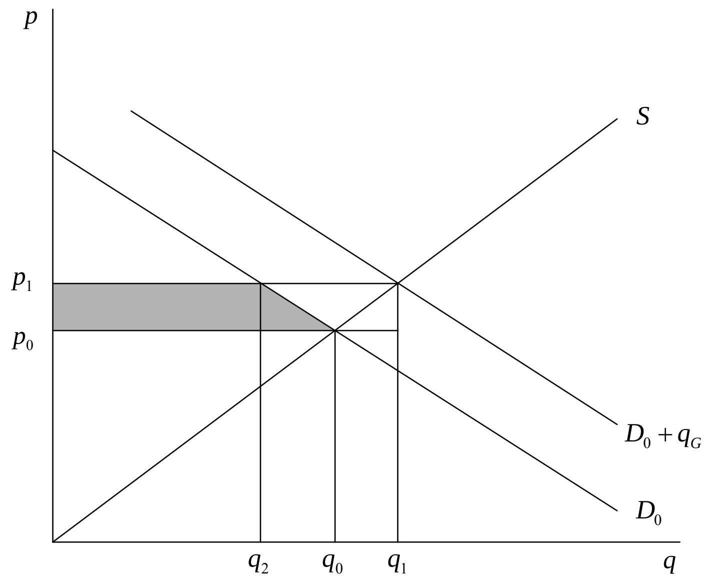

Figure 4-18

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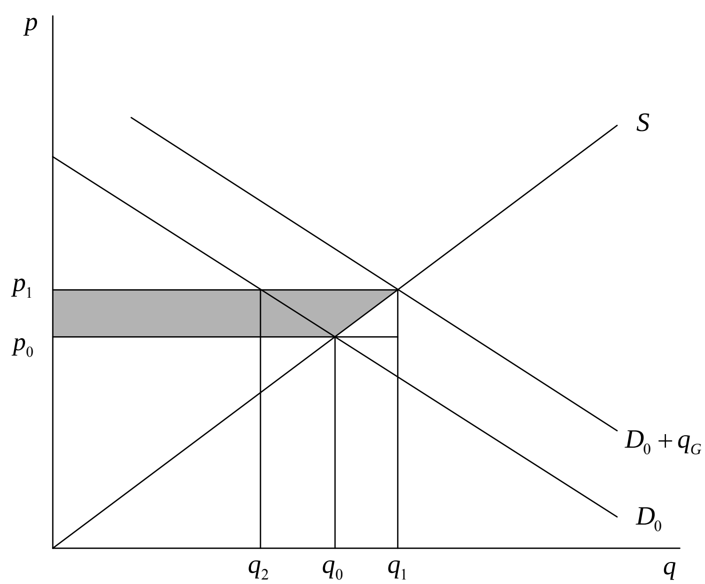

Figure 4-19

78

- The net surplus change is illustrated in Figure 4-20 and summarized in Table 4-1.

79

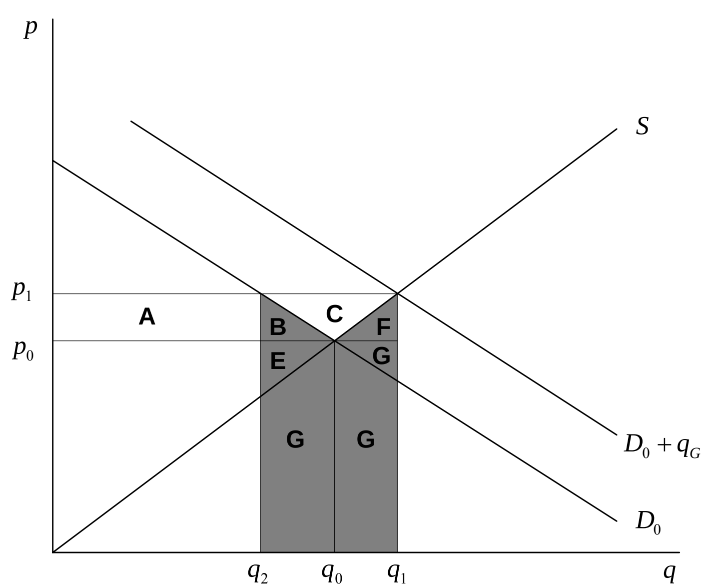

Figure 4-20

80

	Gains	Losses
Existing buyers		A + B
Sellers	A + B + C	
Government agency		B + C + G + E + F
Social cost		B + G + E + F

Table 4-1

81

- Note that the true cost of the input is less than the financial outlay by government, by area C.

- Why? Area C is a <u>transfer</u> from taxpayers to producers, and hence not a cost.

82

METHOD 2: SOCIAL OPPORTUNITY COST

- Recall that the project usage of the input is drawn partly from an increase in supply and partly from displaced consumption by existing buyers.
- Figures 4-21 and 4-22 illustrate the cost of each part.

83

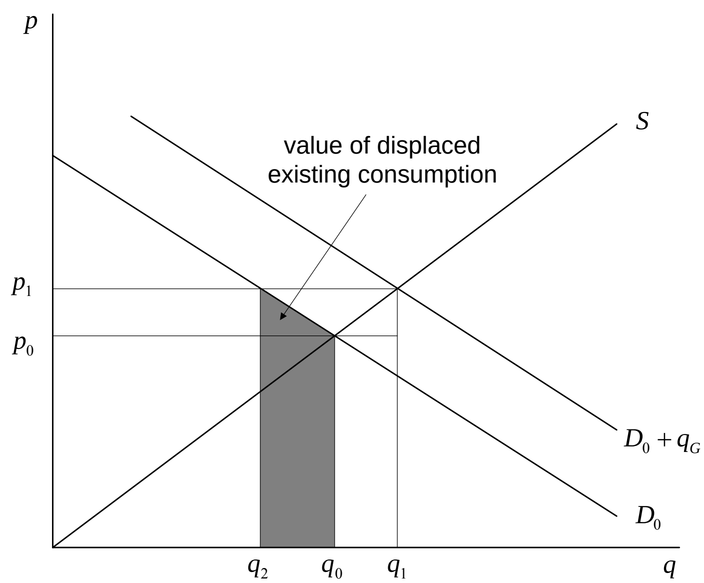

Figure 4-21

84

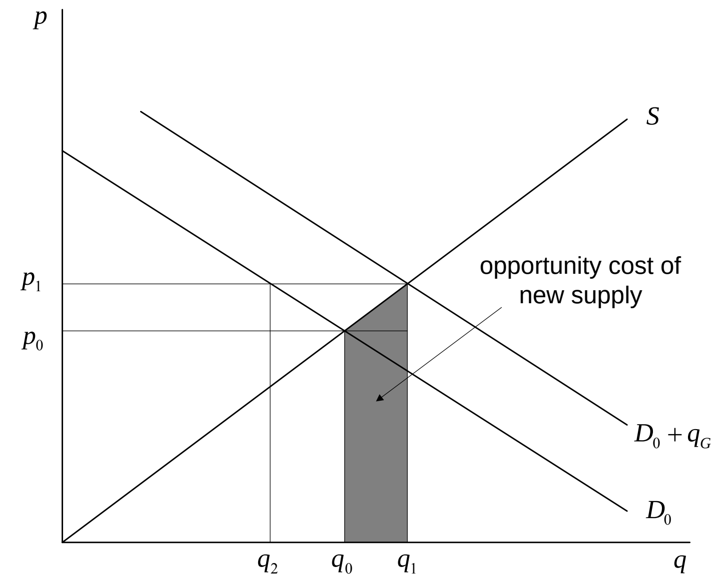

Figure 4-22

85

- The sum of these costs is illustrated in Figure 4-23.

86

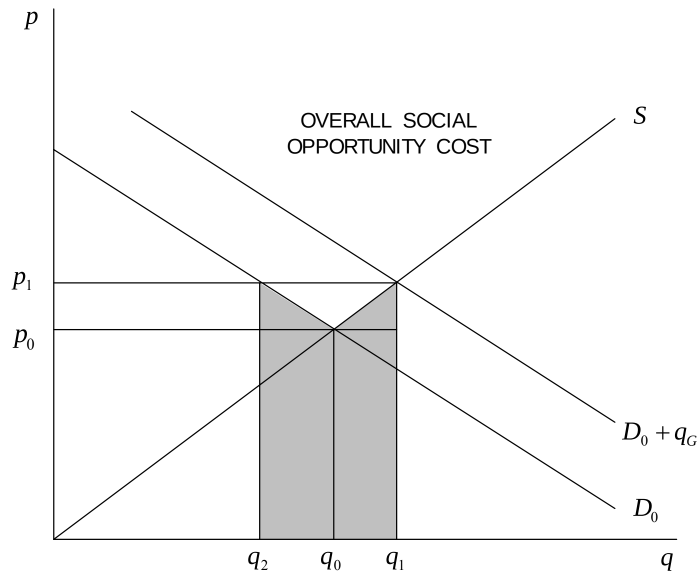

Figure 4-23

87

- A reasonable approximation to this overall social opportunity cost can be obtained by using an average of the pre-project and (predicted) post-project prices.

88

- That is,

$$\text{opportunity cost } = q_G \left(\frac{p_0 + p_1}{2} \right)$$

- This approximation is exact if supply and demand are both linear.

89

USING ELASTICITIES TO PREDICT PRICE AND SURPLUS CHANGES*

- How do we predict the new price, p_1?

- If we assume that demand and supply are both linear, then we can calculate the new price as a function of the elasticities of supply and demand.

* Advanced Topic

90

- Let ε_0 denote our estimate of demand elasticity, evaluated at the current price and quantity.

- Let η_0 denote our estimate of supply elasticity, evaluated at the current price and quantity.

91

- Then it can be shown that the predicted new price is

$$p_1 = p_0\left(1 + \frac{q_G}{q_0(\eta_0 - \varepsilon_0)}\right)$$

- See Appendix A4-3.

92

- We can then use this predicted price change to predict the changes in consumer and producer surplus (as identified in Figures 4-18 and 4-19 respectively) using the formulae from Topic 4.4.

- These estimates will provide a close approximation even if demand and supply are not linear.

93

- Next consider the second of our four cases:
  - calculating costs when there are price effects
  - **calculating benefits when there are price effects**
  - calculating costs when input markets are distorted
  - calculating benefits when output markets are distorted

94

4.8 CALCULATING BENEFITS WHEN THERE ARE PRICE EFFECTS

- We will consider two cases:
  - – a direct addition to supply
  - – a cost reduction for private supply
- In both cases we will assume that the markets are competitive (free of distortions).

95

4.8-1 A DIRECT ADDITION TO SUPPLY

- Suppose the project produces a large increase in the supply of a good that is already supplied in the market.
- The project thereby **augments** the existing private supply of that good by a significant amount (Figures 4-24 and 4-25).

96

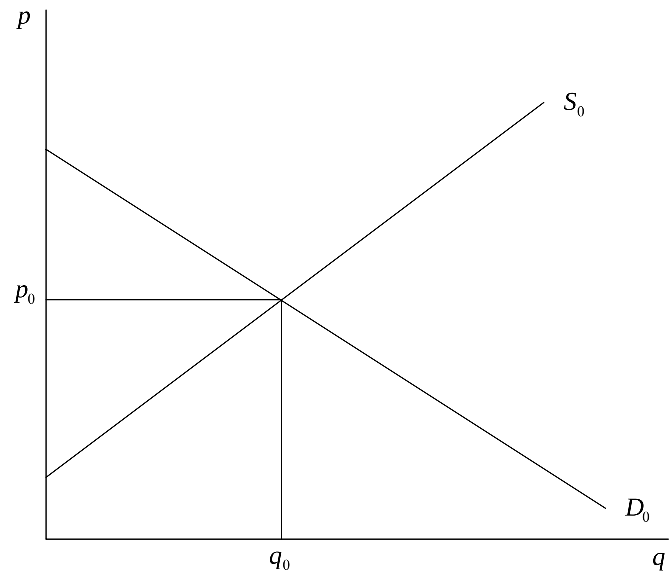

Figure 4-24 97

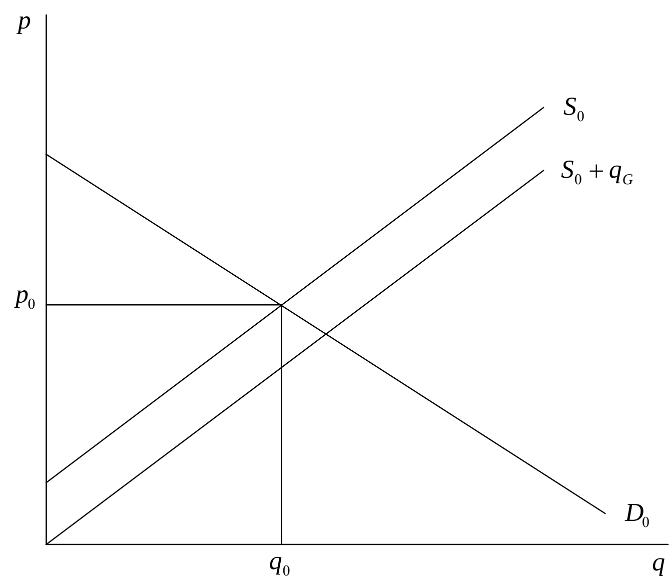

Figure 4-25 98

- Suppose also that the market for the good is competitive but demand is downward-sloping.

- Then the market price of the good must fall in order for the additional supply to be absorbed (Figure 4-26).

99

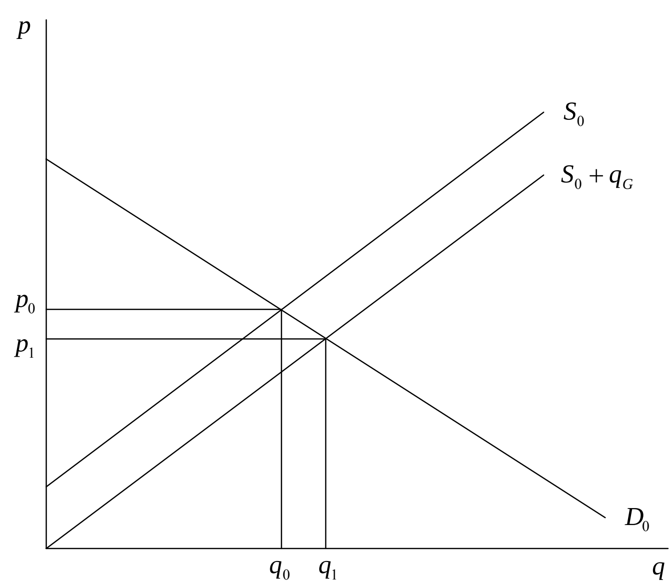

Figure 4-26

100

- The project therefore causes a **crowding-out** of existing private supply (Figure 4-27).

101

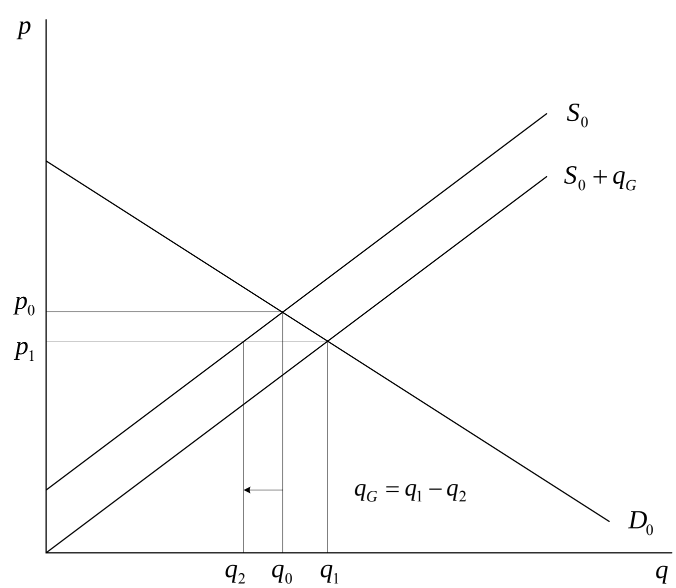

Figure 4-27

102

- Thus, the amount supplied by the project is absorbed
  - partly by an increase in the quantity demanded: $q_0 \rightarrow q_1$
  - and partly by a reduction in the quantity supplied privately: $q_0 \rightarrow q_2$

103

- What is the benefit of the project output under this scenario?

- There are two equivalent ways to think about this:
  - calculate the surplus changes for all affected parties
  - think in terms of overall social benefit

104

METHOD 1: SURPLUS CHANGES

- There are three parties affected:
  - – taxpayers (via impact on government finances)
  - – consumers
  - – existing private suppliers
- The surplus changes for each group are illustrated in Figures 4-28 to 4-30.

105

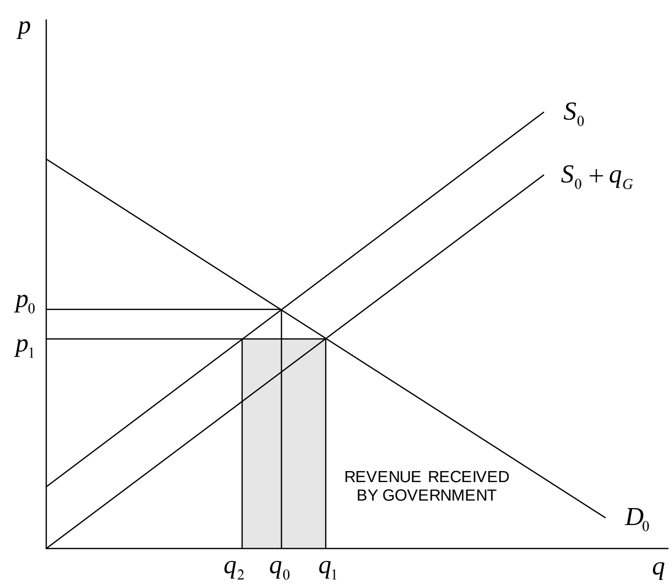

Figure 4-28

106

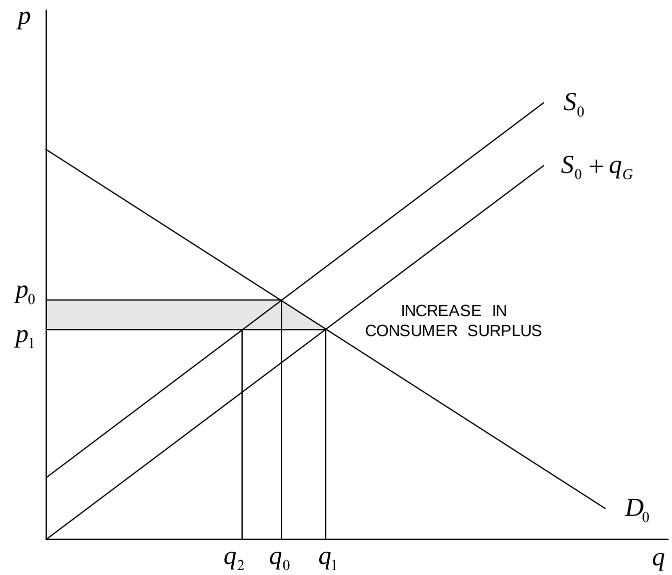

Figure 4-29 107

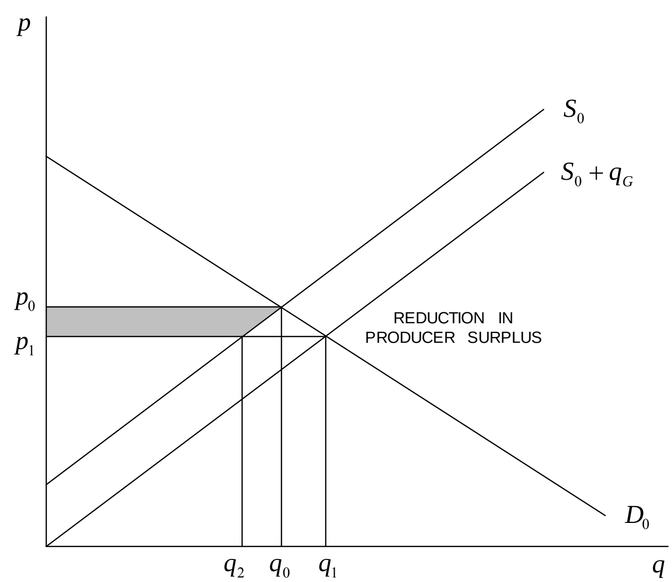

Figure 4-30 108

- The overall change in social surplus due to the project supply is the sum of the surplus changes for the three groups affected (where the change is negative for existing producers).

- This is the social benefit of the project supply (Figure 4-31).

109

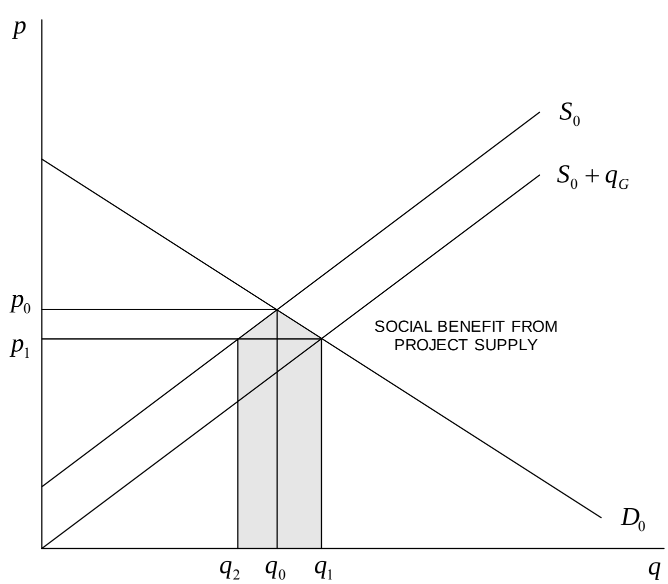

Figure 4-31

110

- Note that the social benefit from the project supply is somewhat greater than the financial value of that supply (as measured at the new market price).
- The difference relates to the project-induced price change, and is best understood in terms of increased consumption and displaced production, as follows.

111

METHOD 2: OVERALL SOCIAL BENEFIT

- Recall that the project output is absorbed partly by an increase in the quantity consumed and partly by displaced production by existing suppliers.
- Figures 4-32 and 4-33 illustrate the benefits of each component.

112

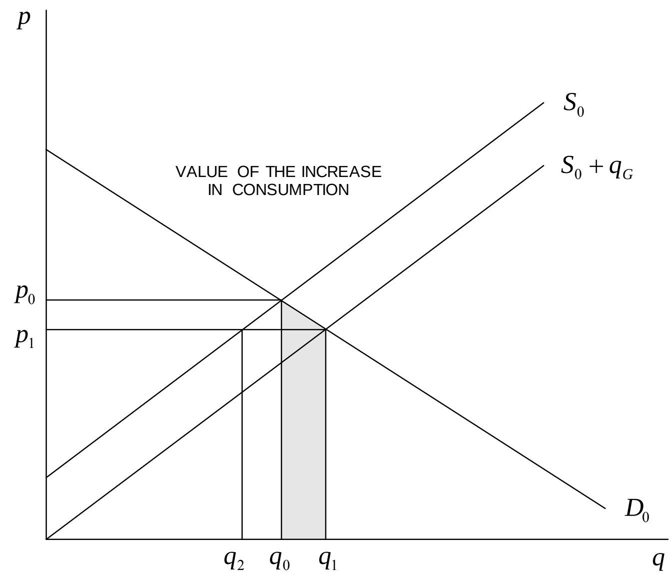

Figure 4-32 113

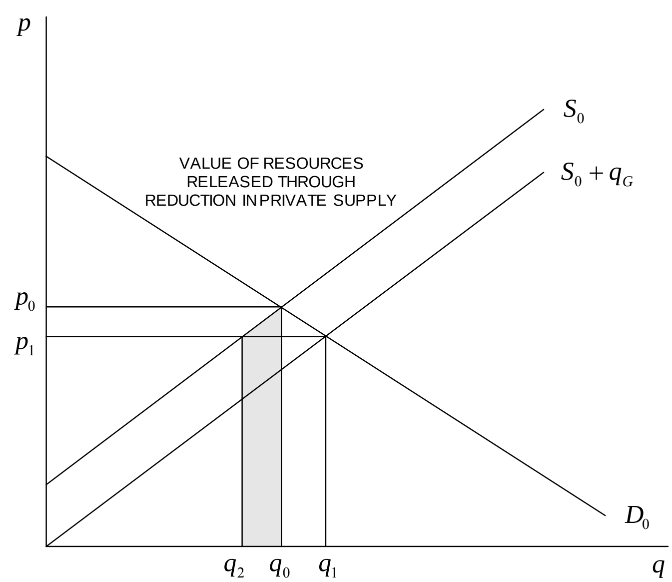

Figure 4-33 114

- Figure 4-34 illustrates the sum of these components.

115

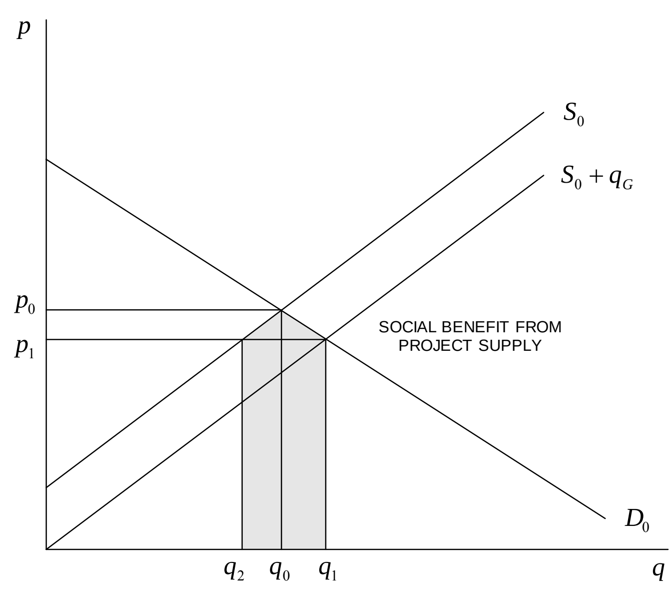

Figure 4-34

116

- A reasonable approximation to this social benefit can be obtained by using an average of the pre- and (estimated) post-project prices to calculate value.

117

- That is,

$$\text{social benefit } = q_G \left(\frac{p_0 + p_1}{2} \right)$$

- This approximation is exact if supply and demand are both linear.

118

USING ELASTICITIES TO PREDICT PRICE AND SURPLUS CHANGES*

- How do we predict the new price, p_1?
- If we assume that demand and supply are both linear, then we can calculate the new price as a function of the elasticities of supply and demand.

* Advanced Topic

119

- Let ε_0 denote our estimate of demand elasticity, evaluated at the current price and quantity.
- Let η_0 denote our estimate of supply elasticity, evaluated at the current price and quantity.

120

- Then it can be shown that the predicted new price is

$$p_1 = p_0\left(1-\frac{q_G}{q_0(\eta_0-\varepsilon_0)}\right)$$

- See Appendix A4-3.

121

- We can then use this predicted price change to predict the changes in consumer and producer surplus (as identified in Figures 4-29 and 4-30 respectively) using the formulae from Topic 4.4.

- These estimates will provide a close approximation even if demand and supply are not linear.

122

4.8-2 A COST REDUCTION FOR PRIVATE SUPPLY

- Important example: public infrastructure investment can reduce the costs of private sector production.
- The cost reduction is reflected in a downward shift of the market supply curve (see Figure 4-35) that may not be parallel.

123

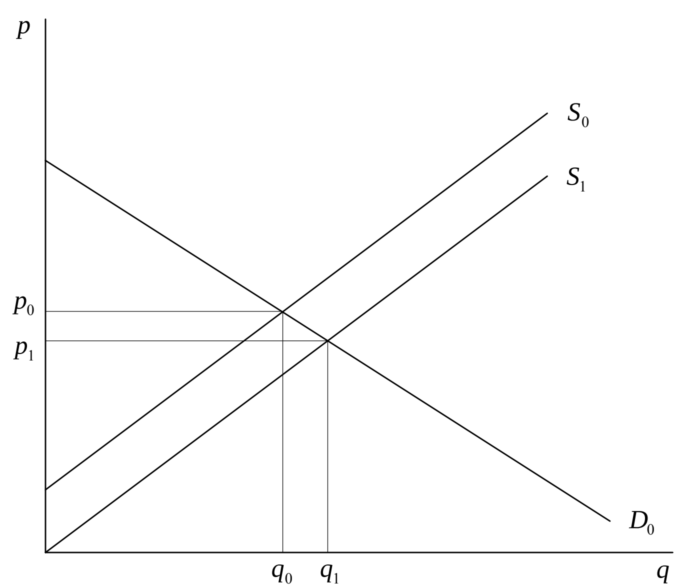

Figure 4-35

124

- What is the benefit of the project under this scenario?
- There are two equivalent ways to think about this:
  - calculate the surplus changes for all affected parties
  - calculate the cost reduction for existing supply plus the net surplus from new supply

125

METHOD 1: SURPLUS CHANGES

- There are two parties affected:
  - consumers
  - suppliers
- See Figures 4-36 and 4-37.

126

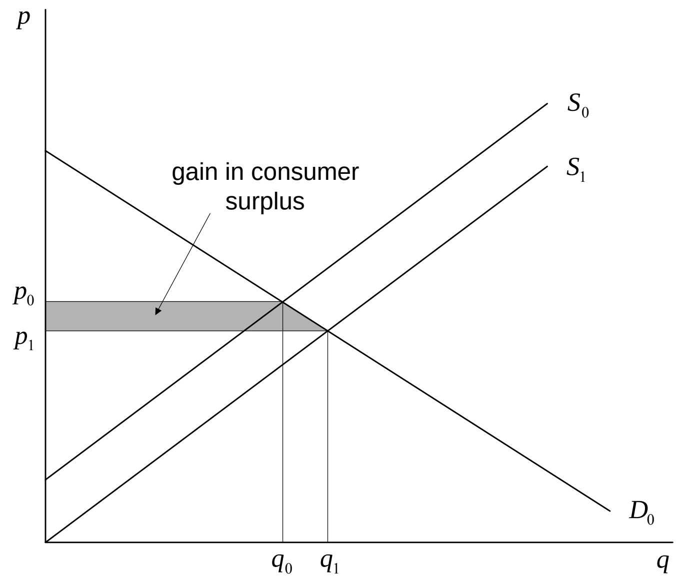

Figure 4-36 127

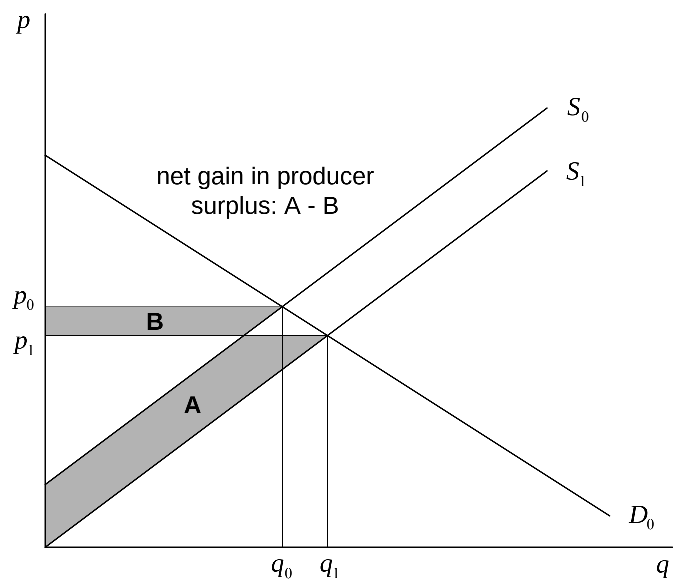

Figure 4-37 128

- Note that the cost to taxpayers from the infrastructure project is treated separately and does not feature in our the calculation of surplus changes here.

- The sum of the consumer and producer surplus changes is illustrated in Figure 4-38.

129

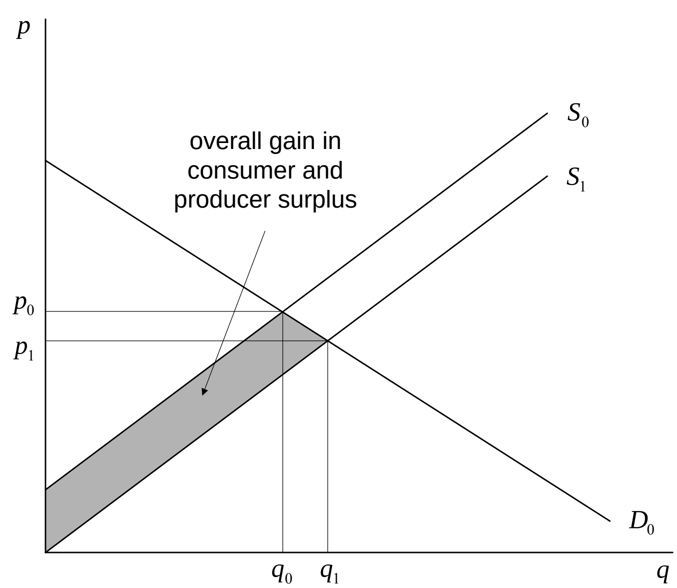

Figure 4-38

130

METHOD 2: COST REDUCTION PLUS NEW NET SURPLUS

- We can also calculate the benefit as the sum of two components:
  - cost reduction for existing supply
  - net surplus from new supply
- See Figure 4-39.

131

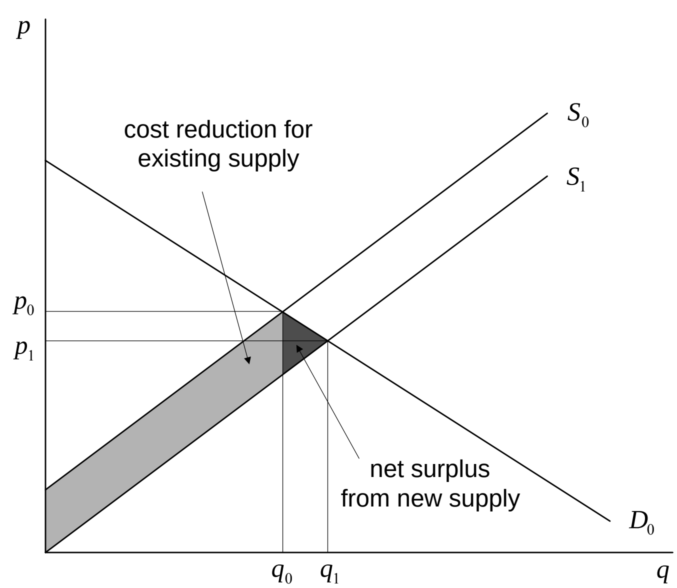

Figure 4-39

132

- Next consider the third of our four cases:
  - calculating costs when there are price effects
  - calculating benefits when there are price effects
  - **calculating costs when input markets are distorted**
  - calculating benefits when output markets are distorted

133

4.9 CALCULATING COSTS WHEN INPUT MARKETS ARE DISTORTED

- If markets are distorted then the equilibrium market price will generally not reflect the true marginal social cost or benefit of the resource.
- Main sources of distortion:
  - market failure
  - government intervention (eg. taxes, price controls)

134

- An input drawn from a distorted market should be valued at its true social opportunity cost or **shadow price**.

- We will examine three examples:
  - taxes on inputs
  - using unemployed labour
  - pollution generated in the production of an input

- We will assume no price effects.

135

4.9-1 TAXES ON INPUTS

- Consider an input drawn from a market in which there is a sales tax.
- Example:
  - price before tax: $20 per unit
  - price after tax: $25 per unit

- Which is the appropriate shadow price?

136

- The answer depends on whether or not the tax revenue collected stays within the referent group, as determined by which government is collecting the tax.

137

- If the tax revenue stays within the referent group then

  shadow price = before-tax price ($20)

- If the tax revenue flows outside the referent group then

  shadow price = after-tax price ($25)

138

4.9-2 USING UNEMPLOYED LABOUR

- Unemployment (an excess supply of labour at the prevailing wage) occurs because some market friction or government regulation stops the wage from falling to equate supply and demand.

139

- This means that in the presence of unemployment, the prevailing market wage overestimates the true opportunity cost of using that unemployed labour.

- So what is the correct opportunity cost?

140

- The problem we face is that the true opportunity cost is not observable.
- We will consider this issue in the context of a very simple model of a distorted labour market, as depicted in Figure 4-40.
- We will assume that all workers are identical in terms of skill and productivity.

141

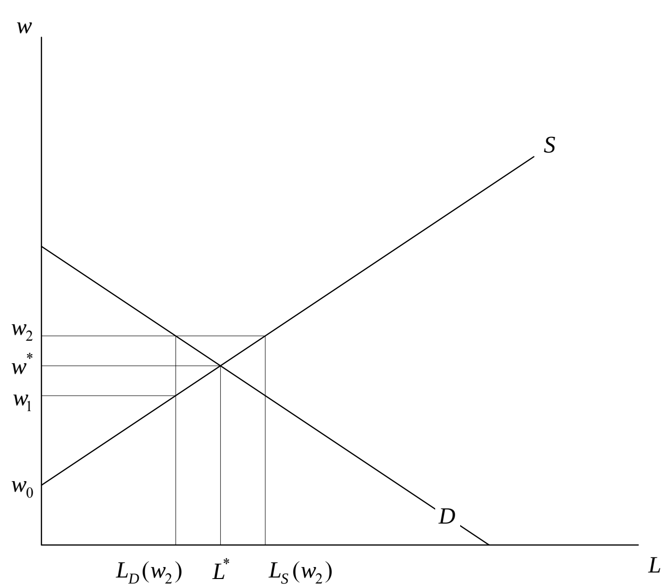

Figure 4-40

142

- The labour supply schedule (*S*) measures the opportunity cost of labour (OCL) for potential workers.
- At any given wage *w*, only those with OCL no greater than *w* are willing to work.

143

- The labour demand schedule (*D*) measures the value marginal product of labour (which is the physical marginal product of labour weighted by the price of output).
- Firms are indifferent between individual workers because all workers are equally productive.

144

- The market-clearing wage is w^*, and at that wage there would be L^* workers employed.

- A market friction of some sort prevents the wage from falling to w^*, and instead the wage is fixed at w_2.

145

- At w_2, the number of workers willing to work is $L_S(w_2)$ but firms are only willing to hire $L_D(w_2)$ workers at that wage.

- Thus, there is unemployment equal to

$$u = L_S(w_2) - L_D(w_2)$$

146

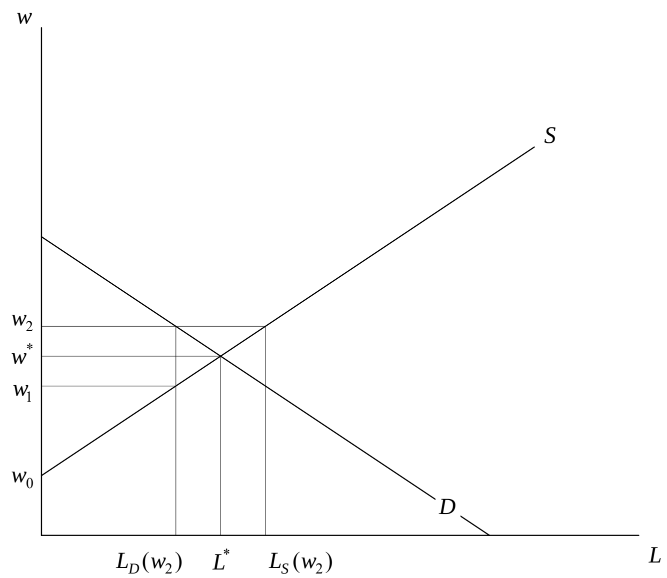

Figure 4-40 (repeat)

147

- It is important to recognize that $L_D(w_2)$ is the number of employed workers, but we cannot know from the information in Figure 4-40 who those employed workers are.

- In particular, we cannot say that the employed workers are all drawn that part of the labour supply between w_0 and w_1.

148

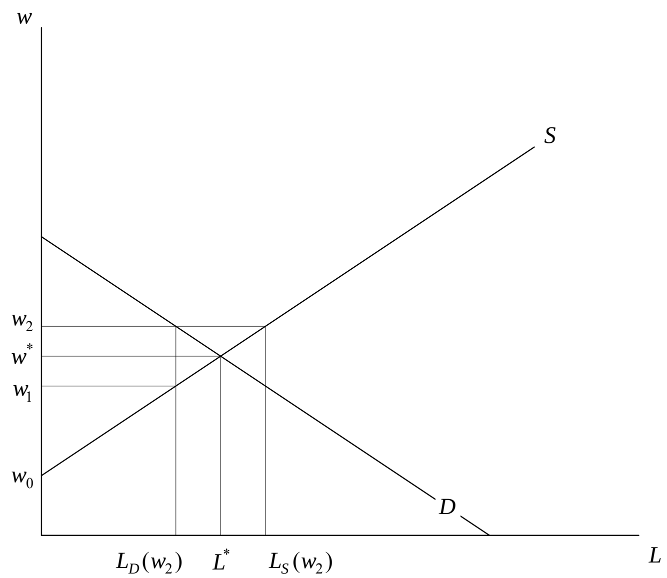

Figure 4-40 (repeat)

149

- Why not? All potential workers with OCL between w_0 and w_2 are willing to work at the prevailing wage, and firms hire from this set of workers <u>at random</u> because all the workers are the same from their perspective.
- The firms have no reason to hire workers on the basis of their OCL even if they could observe it.

150

- Similarly, we cannot say that unemployed workers are all in that part of the labour supply between w_1 and w_2.
- We can say only that all potential workers who are willing to work at wage w_2 – whether they are employed or unemployed – come from the labour supply curve between w_0 and w_2.

151

- This means that if we hire unemployed workers for a project, we know only that their OCL is between w_0 and w_2.
- So what value do we assume for the opportunity cost of labour in our project?

152

- If we assume that the distribution of workers along the supply curve between w_0 and w_2 is a uniform distribution (and it can be shown that it must be if the supply curve is linear) then the **average worker** between w_0 and w_2 has OCL equal to

$$\hat{w} = \frac{w_0 + w_2}{2}$$

153

- Thus, on average the workers we hire for the project will have OCL equal to $\hat{w}$.

- Our next problem is that we cannot observe w_0, so we have to make an estimate of that.

154

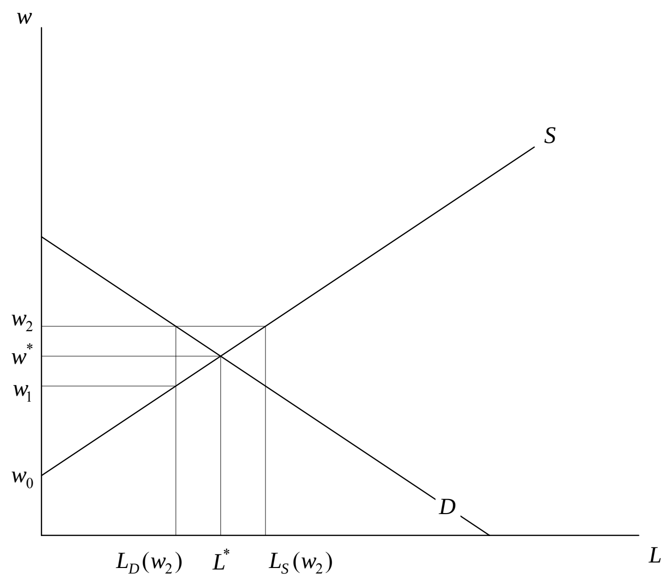

Figure 4-40 (repeat)

155

- One possibility is to assume that $w_0 = 0$.
- This is almost surely an under-estimate of its true value but it is the simplest solution.
- Under that assumption, the OCL for the labour used in our project is simply one-half the market wage:

$$OCL = \frac{w_2}{2}$$

156

- There are two other complicating issues with respect to the calculation of the cost of labour:
  - the treatment of taxes
  - the treatment of unemployment insurance payments

157

- **Treatment of income tax and payroll tax**:
  - calculate shadow price using the after-tax wage (that is, the net wage received by the worker)
  - include any tax payments that flow outside the referent group as an additional cost (tax payments that remain inside the referent group are just transfers).

158

- **Treatment of unemployment insurance payments**:
  - if referent government makes UI payments then UI payments saved are just transfers (because they are gained by the government but lost by the newly employed workers)
  - if UI payments are made by a government outside the referent then UI payments given up by the newly employed workers are a cost of the project (because they no longer flow into the referent group).

159

4.9-3 POLLUTION GENERATED IN THE PRODUCTION OF AN INPUT

- Suppose the production of an input generates pollution, and that pollution is not properly regulated.

- The market price of the input will not include the social cost of the pollution; it is external to the supplier of the input.

160

- The costing of the project should include that pollution cost.
- There are two ways we can report it.

161

- The first approach is to cost the input at its true shadow price, where

shadow price
= market price of the input
+ marginal pollution cost

162

- The alternative approach is to use the market price of the input and then include pollution costs in a separate "environmental costs" category in the CBA.

163

- Next consider the fourth of our four cases:
  – calculating costs when there are price effects
  – calculating benefits when there are price effects
  – calculating costs when input markets are distorted
  – **calculating benefits when output markets are distorted**

164

4.10 CALCULATING BENEFITS WHEN OUTPUT MARKETS ARE DISTORTED

- An important rationale for policy intervention is to supply goods that are not provided optimally by the market (that is, whose equilibrium market quantity does not maximize social surplus).

165

- Important examples:
  - childhood nutrition
  - childhood education
  - health services
  - transportation services
  - some public utilities (like electricity, water, sewer)
  - environmental protection

166

- We will examine intervention in the supply of a good with a positive externality (like childhood nutrition).

167

4.10-1 THE SUPPLY OF A GOOD WITH A POSITIVE EXTERNALITY

- Consider a government program to subsidize market provision (for example, via vouchers for childhood nutrition).

- The subsidy effectively reduces the price paid by consumers.

168

- The case illustrated in Figures 4-41 to 4-43 is an <u>optimal</u> subsidy; it implements the social optimum as a corrected equilibrium.

169

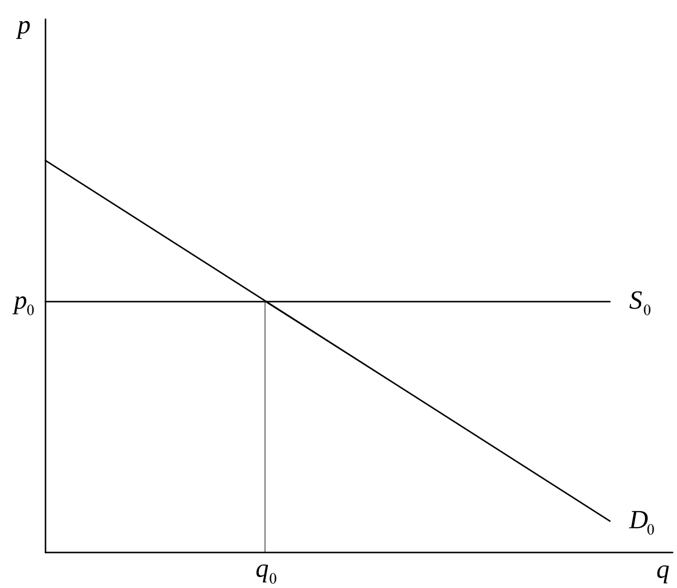

Figure 4-41

170

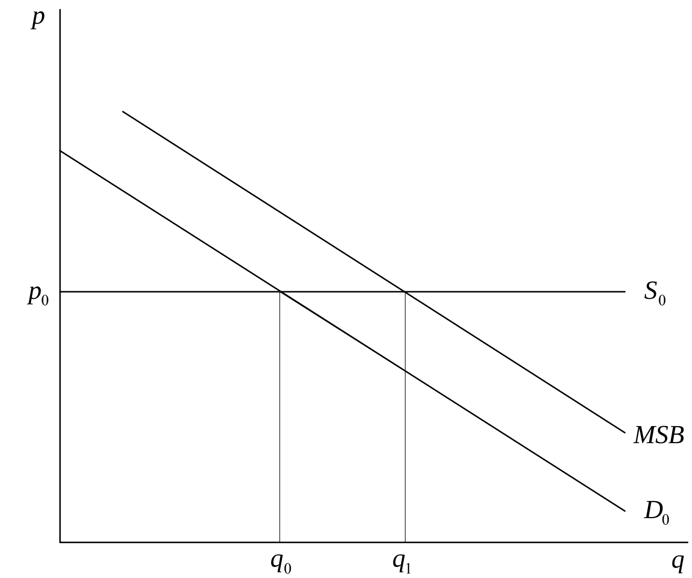

Figure 4-42 171

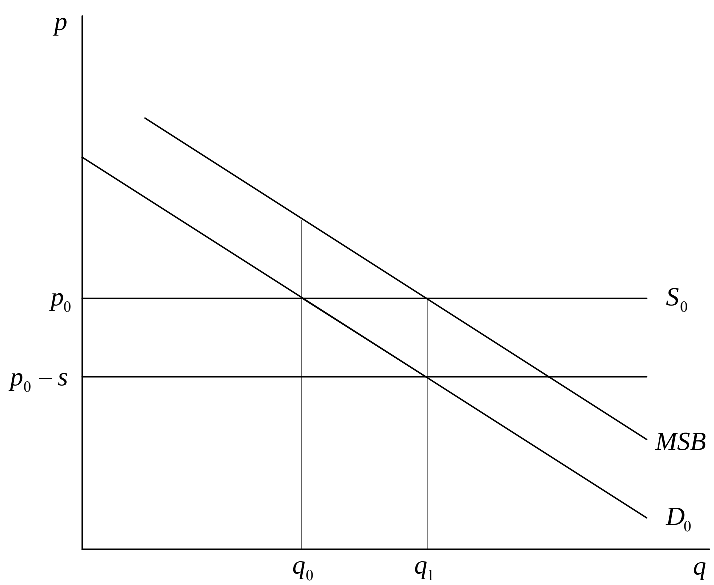

Figure 4-43 172

- What is the benefit of the subsidy policy?
- There are two equivalent ways to think about this:
  - calculate the surplus changes for all affected parties
  - think in terms of overall social benefit and social cost

173

METHOD 1: INDIVIDUAL SURPLUS CHANGES

- There are three parties affected:
  - consumers
  - taxpayers (via impact on government finances)
  - external agents
- Figures 4-44 to 4-46 illustrate the surplus changes for each group.

174

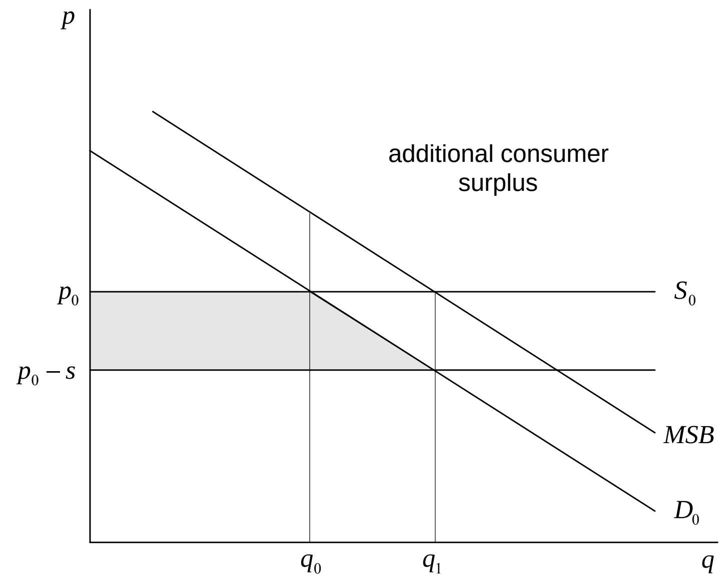

Figure 4-44

175

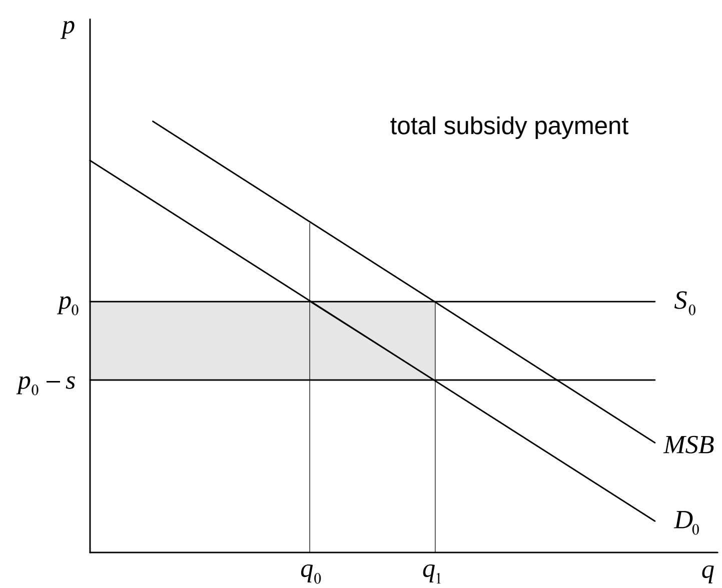

Figure 4-45

176

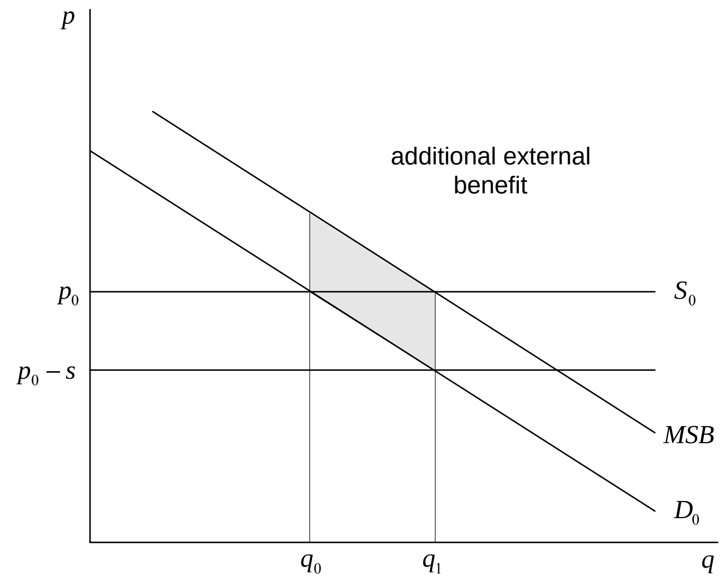

Figure 4-46

177

- Figure 4-47 illustrates the sum of these surplus changes.

178

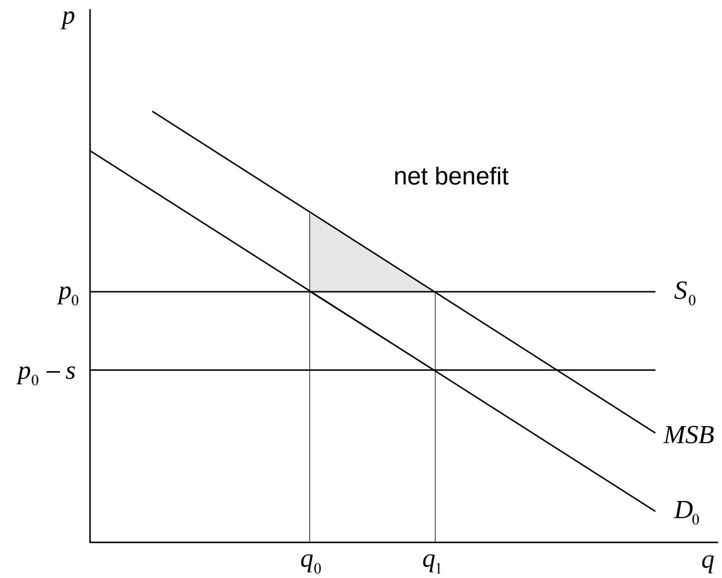

Figure 4-47

179

- Note that there is no change in producer surplus for the suppliers here because supply is assumed to be perfectly elastic here.
- If supply is upward-sloping then there will be a change in producer surplus.

180

METHOD 2: SOCIAL BENEFIT AND SOCIAL COST

- We can also calculate the net benefit as the difference between the total social benefit of the additional consumption and its total social cost (Figures 4-49 and 4-49).

181

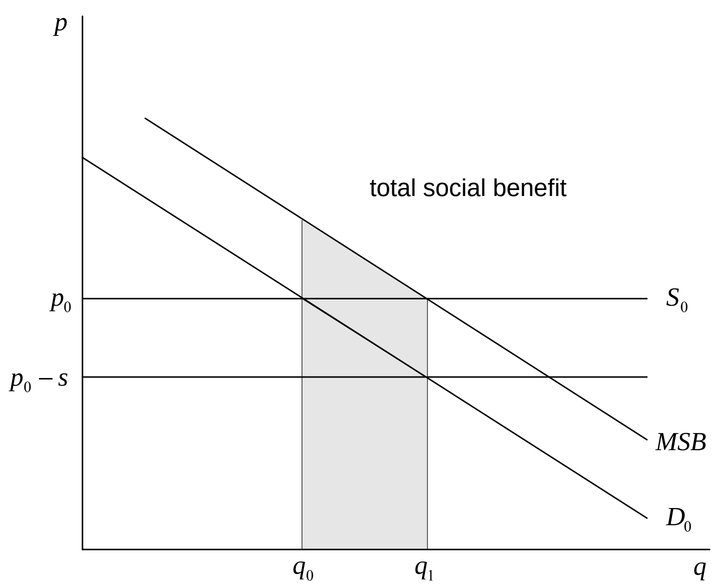

Figure 4-48

182

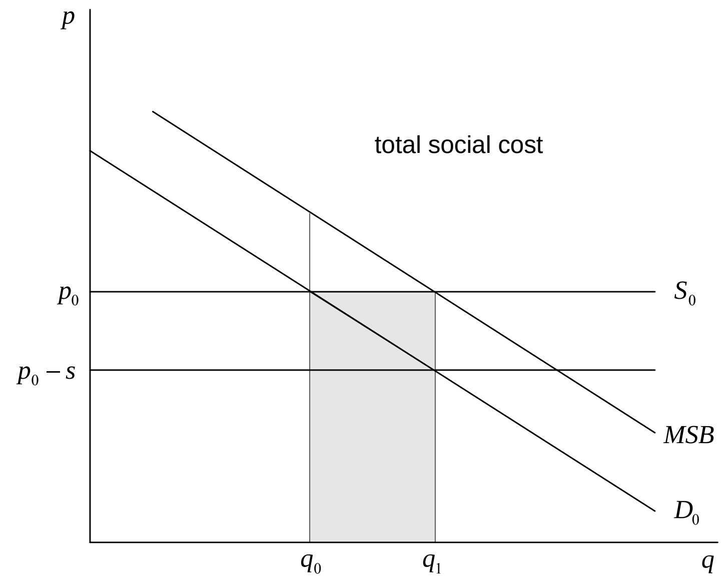

Figure 4-49

183

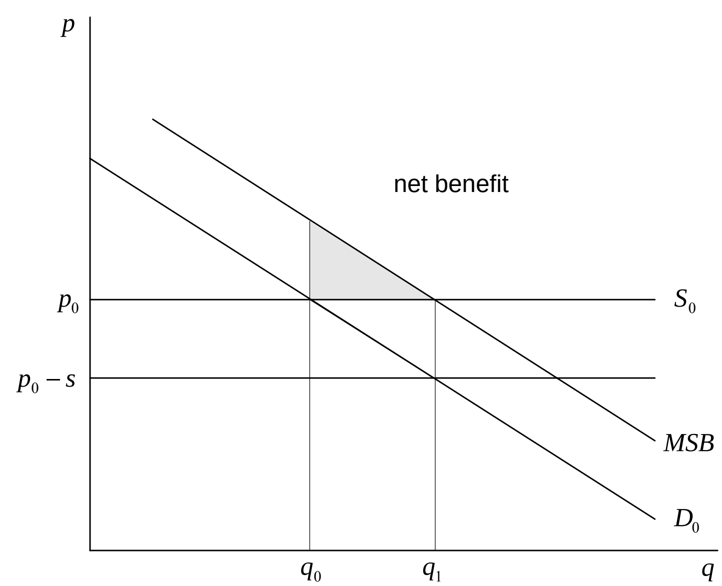

Figure 4-47 (repeat)

184

4.11 USING SHADOW PRICES FROM SECONDARY SOURCES

- It is often not necessary (or possible) to calculate all shadow prices relevant to a CBA from first principles for each analysis.

185

- It is sometimes legitimate to use shadow prices from secondary sources.

- This methodology this is sometimes called – rather oddly – “ benefits transfer”.

186

- The legitimacy of using a number from an existing study depends on the similarity between the proposed application and the setting from which the number was derived.
- At a minimum, it is usually necessary to make an adjustment for differences in per-capita income.

187

- For example, if a value is derived in a US context and then used in a Canadian context then it would be weighted by the ratio of per-capita GDP in the two countries.

188

- Common instances where values are taken from secondary sources:
  - morbidity costs
  - time costs
  - environmental impacts
  - the value of a "statistical life"

189

THE VALUE OF A "STATISTICAL LIFE"

- A **specific life** is the life of an identifiable individual.

- Placing a value on a specific life is beyond the realm of CBA.

190

- In most policy settings, lives saved (or lost) are anonymous; these are **statistical lives**.

- Example:
  - a road upgrade will reduce traffic deaths from 4 in 100,000 trips to 3 in 100,000 trips
  - based on 1 million trips per year, there are an average of 10 statistical lives saved per year

191

- The value of a statistical life is measured in terms of the value people place on small changes in the risk of death.

Example:
  - suppose a person has a WTP of $50 to reduce the risk of death on the road by 1 in 100,000
  - the implied value of their statistical life is $50 * 100,000 = $5m

192

- A variety of **non-market valuation techniques** can be used to estimate WTP values for reduced risk of death.

- The same sort of techniques are used to calculate the value of environmental amenities (see Topic 7).

193

4.12 THE COST OF FUNDS

- Public projects and policies are usually funded by taxes or debt (deferred taxation).
- Raising revenue via taxes is typically costly, for two reasons:
  – administrative cost; and much more importantly
  – loss of social surplus due to the distortion of incentives ("deadweight losses").

194

- Each dollar of revenue raised through (non-Pigouvian) taxes takes more than one dollar of surplus out of the private economy.
- The amount of surplus removed when one extra dollar of revenue is raised is called the **marginal cost of funds** (MCF).
- The likely MCF for Canada: 1.2 - 1.4

195

- Figures 4-50 to 4-55 illustrate how an **excise tax** creates a deadweight-loss.

196

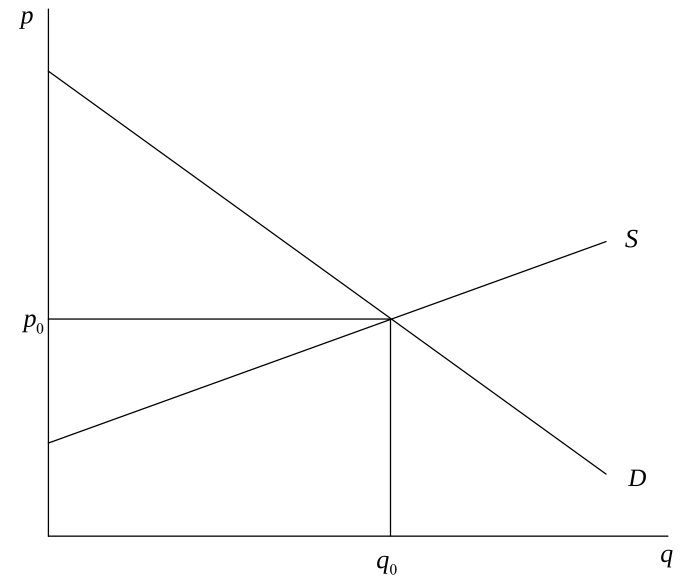

Figure 4-50

197

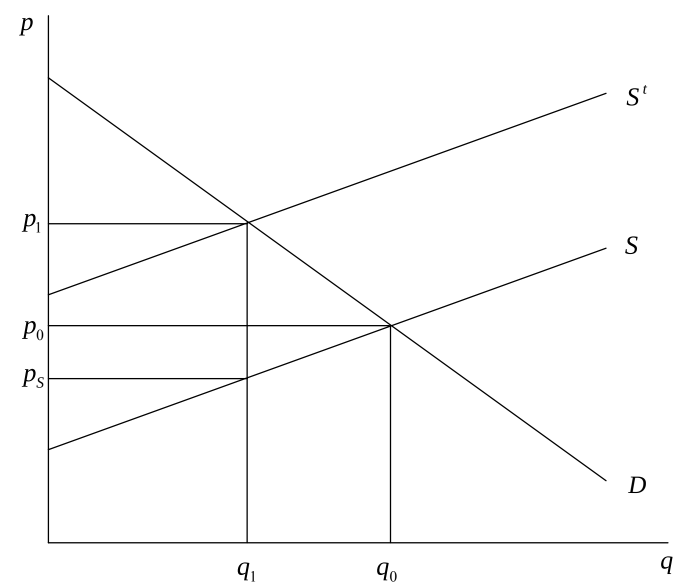

Figure 4-51

198

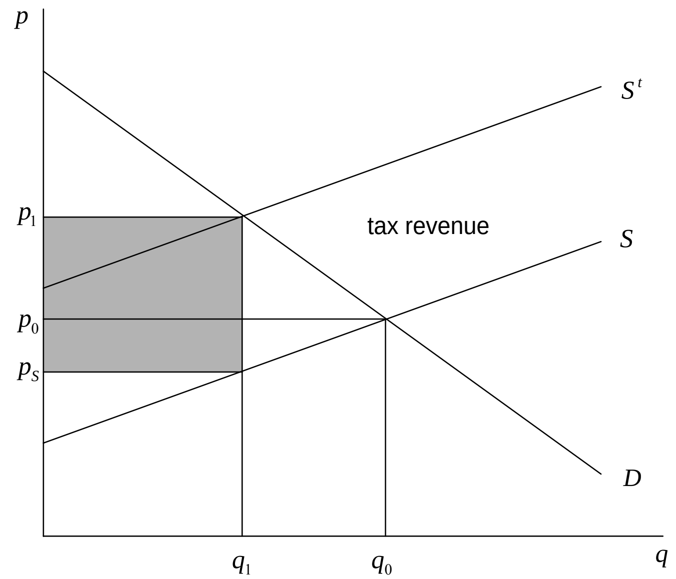

Figure 4-52 199

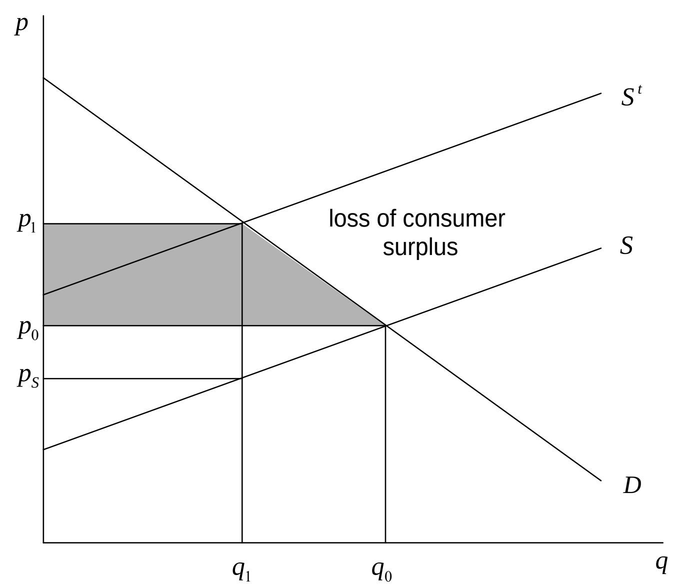

Figure 4-53 200

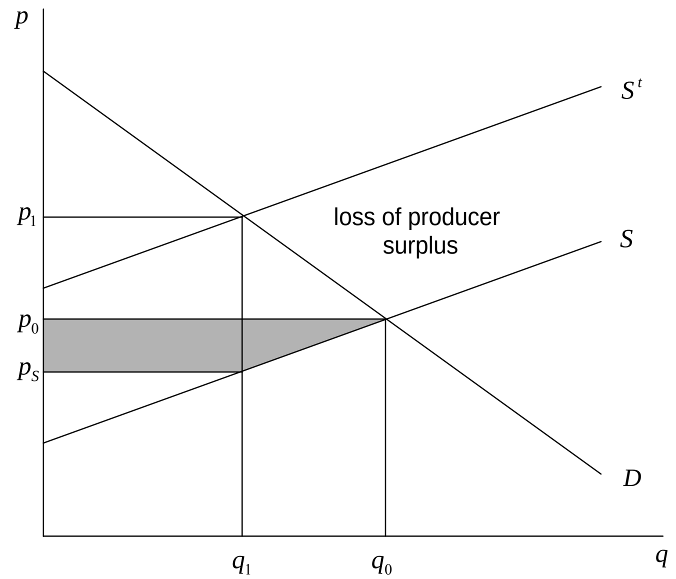

Figure 4-54

201

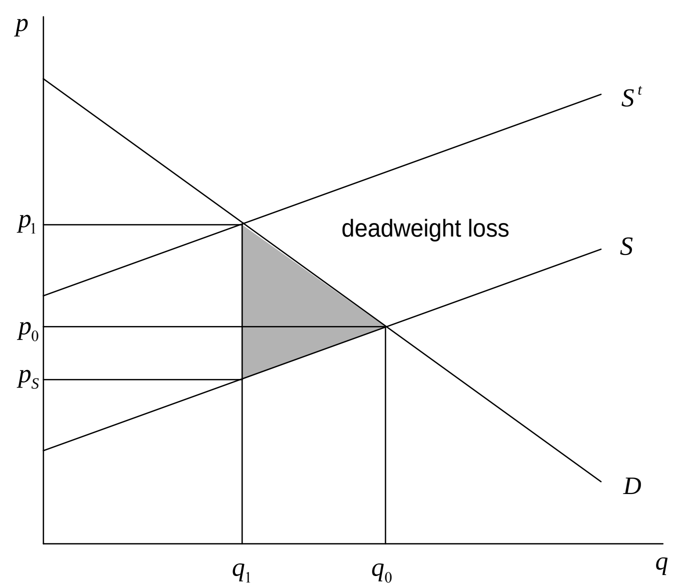

Figure 4-55

202

- These surplus changes are summarized in Figure 4-56 and Table 4-2.

203

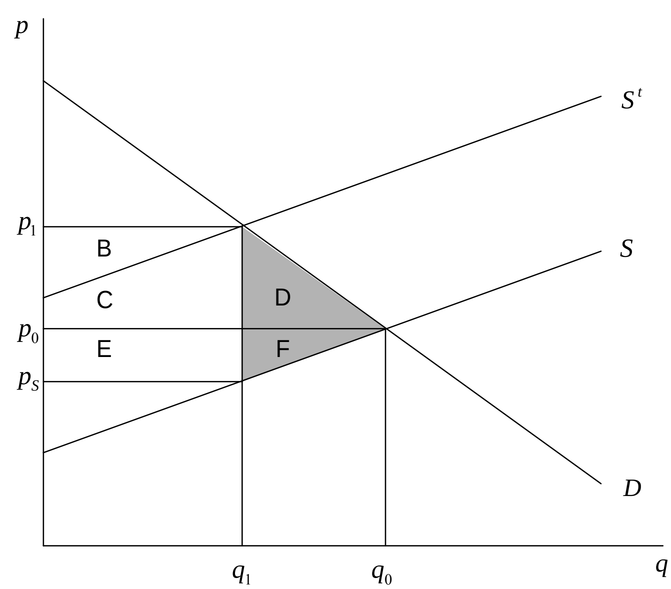

Figure 4-56

204

	Benefits	Costs
Foregone consumer surplus		B + C + D
Foregone producer surplus		E + F
Tax revenue raised (R)	B + C + E	
DWL		D + F

Table 4-2

205

- The MCF is the amount by which (*R*+*DWL*) rises when an extra dollar of revenue is raised.

206

- The cost of funds should be included as a cost of any public project or policy, and is calculated as

  Cost of funds
  = (MCF-1)*(net financial outlay for govt.)

- This is often not done in practice.

END

APPENDIX A4-1

COMPENSATING VARIATION AND EQUIVALENT VARIATION

If we could measure utility directly then we could measure welfare change for an individual in terms of a utility difference. However, we cannot measure utility. Instead we must frame the answer in terms of WTP or WTA. These could be expressed in terms of any numeraire good, but it is most convenient to use a money metric.

Two alternative money metric measures of welfare change for an individual are **compensating variation** (CV) and **equivalent variation** (EV).

The difference between EV and CV relates to the choice of reference point for measuring the welfare change. The EV uses the new allocation as the reference point while the CV uses the initial allocation as the reference point.

We can construct CV and EV measures for any type of change that has an impact on individual welfare but here we will focus on price changes. (In Topic 7-2 we briefly describe the use of CV and EV to measure the value of environmental change).

A4-1-1. COMPENSATING VARIATION

CV is the (negative of the) amount of money that would have to be given to an individual after some change in conditions (like a price change) to enable her to attain the same level of utility she enjoyed before the change. Note that the reference point is the initial level of utility: we are measuring the compensation needed to achieve the pre-change level of utility given the post-change conditions.

By convention, we state CV as the negative of the compensation required to offset the change in conditions. Thus,

- $CV < 0$ if the agent is made worse-off by the change (such as a price rise).
- $CV > 0$ if the agent is made better-off by the change (such a s price fall).

It is useful to interpret CV in terms of WTP and WTA:

- if the individual is made worse-off by the change in conditions (as from a price rise) then $|CV|$ measures her WTA for that change in conditions.
- if the individual is made better-off by the change in conditions (as from a price fall) then $|CV|$ measures her WTP to obtain that change in conditions.

We could also consider the converse: her WTP to avoid a change that would make her worse-off, and her WTA to forego a change that would make her better-off. This alternative perspective is the basis of the equivalent variation.

A4-1-2. EQUIVALENT VARIATION

EV is the (negative of the) amount of money that would have to be taken away an individual in the absence of the change in conditions (like a price change) to leave her with the same level of utility she enjoys after the change. Note that the reference point is post-change utility: we are measuring the equivalent change in income needed to achieve the post-change level of utility given the pre-change conditions.

By convention, we EV as the negative of the income that would have to be taken away to achieve the equivalent welfare impact as the change in conditions.

- $EV < 0$ if the agent is made worse-off by the change (such as a price rise).
- $EV > 0$ if the agent is made better-off by the change (such a s price fall).

It is useful to interpret EV in terms of WTP and WTA:

- if the individual is made worse-off by the change in conditions (as from a price rise) then $|EV|$ measures her WTP to avoid the change in conditions.
- if the individual is made better-off by the change in conditions (as from a price fall) then $|EV|$ measures her WTA to forego that change in conditions.

A4-1-3. WHICH MEASURE SHOULD WE USE?

EV and CV are generally not equal because the reference point for measuring the welfare change is different in each case. In fact, the difference between EV and CV could even be infinite: WTP is bounded by wealth (and must be finite) but WTA is not bounded by wealth and could in principle be infinite. For example, consider the WTP not to die and the WTA for dying.

The difference between WTP and WTA is a function of the elasticity of substitution between money and the change in conditions. No amount of money can substitute for the loss of some things (such as life itself for many people).

So which measure should we use? The conventional answer that it depends on the assignment of **property rights** implicit in the analysis:

- if the individual is deemed to have a right to the benefit of the change in conditions, or a right not to be harmed by the change in conditions, then we should use WTA

  ⇒ use EV if she gains from the change, and CV if she loses from the change.
- if the individual is deemed to have no right to the benefits of the change in conditions, or no right not to be harmed by the change in conditions, then we should use WTP

  ⇒ use CV if she gains from the change, and EV if she loses from the change.

This conventional answer is not very satisfactory because property rights are often not defined in the context of many changes induced by policy or by the behaviour of other agents. For example, do you have a right to less polluted air, or does a car driver have a right to drive her car, and pollute the air as a consequence?

A potentially better approach is to first ask what purpose we have in mind for the measurement of the welfare impact. If our purpose is to calculate the payment that will <u>actually</u> be made to compensate a damaged individual, then we should use CV because it is based on WTA in that setting. Similarly, if our purpose is to calculate the payment that a beneficiary will <u>actually</u> make in return for a change in conditions, then we should use

CV because it is based on WTP in that setting. This ensures that the actual property-rights assignment implied by the payments is consistent with the welfare measure used.

Conversely, if our purpose is to calculate the loss that a change in conditions will impose on an individual who will not actually be compensated for the change, then we should use EV because it is based on her WTP in that context. Similarly, if our purpose is to calculate the gain that a beneficiary will receive without having to actually pay for that gain, then we use EV because it measures WTA in that context. Again, this ensures that the actual property-rights assignment implied by the absence of payments is consistent with the welfare measure used.

To summarize, if actual payments will be made then we should use CV to calculate those payments. If no actual payments will be made, then we should use EV to measure the gains and losses that will arise precisely because compensating payments were not actually made.

A4-1-4. GRAPHICAL REPRESENTATION OF CV AND EV

Suppose An individual experiences a set of price changes, causing her switch her optimal consumption bundle from x^0 to x', as illustrated in Figure A4-1.

The associated CV is illustrated in Figure A4-2 and the associated EV is illustrated in Figure A4-3. The convention is to represent CV and EV graphically in terms of the good measured on the vertical axis, rather than in money terms. That is, we represent CV and EV graphically as the amount of x_2 that the dollar amounts would buy.

Compensated Demand Curves

The EV and the CV can be interpreted as areas beneath a compensated demand curve.

Recall that the ordinary or Marshallian demand curve represents the relationship between price and the quantity demanded with income held constant. A change in income is reflected in a shift of the demand curve.

The Marshallian demand response to a price change can be decomposed into an income effect and a substitution effect. Figure A4-4 depicts the demand response to a fall in the price of x_1 when income remains unchanged.

The **substitution effect** is defined as the demand response associated with the price change given that the individual is compensated with income and restored back to her initial level of utility. By definition this income compensation is the $|CV|$ for the price change.

The **income effect** is the demand response due to the change in real income associated with the price change. Note that Figure A4-4 is drawn for the case of a normal good: the income effect and substitution effect work in the same direction. For an inferior good the income effect for a price fall is negative..

The **compensated demand curve** (or **Hicksian demand curve**) measures only the substitution effect associated with a price change. That is, it represents the relationship between price and quantity demanded when utility is held constant via an income compensation.

The relationships between the ordinary and compensated demand curves are illustrated in Figures A4-5 through A4-7.

In Figure A4-5, the compensated demand curve labeled $H(u^0)$ is drawn for the initial level of utility, u^0. It is steeper than the ordinary demand when the good in question is normal. (For an inferior good the compensated demand is flatter). The area beneath $H(u^0)$ bounded by p_1^0 and p_1' is the $|CV|$ associated with the price change $p_1^0 \rightarrow p_1'$.

In Figure A4-6, the compensated demand curve labeled $H(u^0)$ is drawn for the post-change level of utility, u^1. The area beneath $H(u')$ bounded by p_1^0 and p_1' is the $|EV|$ associated with the price change $p_1^0 \rightarrow p_1'$.

Note that $H(u')$ does *not* measure the substitution effect for the price change $p_1^0 \rightarrow p_1'$. However, $H(u')$ does measure the substitution effect of the reverse price change, viz., $p_1' \rightarrow p_1^0$. Thus, the $|EV|$ associated with the price change $p_1^0 \rightarrow p_1'$ is equivalent to the $|CV|$ associated with the price change $p_1' \rightarrow p_1^0$, and vice versa. More generally, $|WTP(x^0 \rightarrow x')| \equiv |WTA(x' \rightarrow x^0)|$.

The CV and EV for the price fall $p_1^0 \rightarrow p_1'$ are illustrated together in Figure A4-7. Note that the EV is larger than the CV in the case illustrated. (The WTA to forego the price fall exceeds the WTP to have the price fall).

In general,

- price fall for a normal good: $|EV| > |CV|$
- price rise for a normal good: $|CV| > |EV|$
- price fall for an inferior good: $|CV| > |EV|$
- price rise for an inferior good: $|EV| > |CV|$

A4-1-5. RELATIONSHIP TO CONSUMER SURPLUS

The change in consumer surplus (ΔCS) associated with a price change can be thought of as an approximation of the EV and CV for that price change. The relationship between EV, CV and ΔCS for a price fall for a normal good is illustrated in Figure A4-8. The shaded area is ΔCS .

ΔCS is always bounded by EV and CV:

- if $|EV| > |CV|$ then $|EV| > |\Delta CS| > |CV|$
- if $|CV| > |EV|$ then $|CV| > |\Delta CS| > |EV|$

If all goods whose prices have changed are income-neutral, then $|CV| = |\Delta CS| = |EV|$.

Path Dependency

If more than one price changes then the CV and EV can be calculated as the sum of the CVs and EVs associated with the individual price changes taken sequentially. The value of the total CV or EV is invariant to the order of the calculation. That is, the sequence of price changes assumed for the calculation is irrelevant.

This is not true of ΔCS . It is not invariant to the sequence of changes assumed for the calculation when more than one price changes or if prices and income change. This property of ΔCS is called **path dependency**.

Path dependency reflects the presence of income effects in ordinary demand curves, which means that cross-price effects are generally not symmetric (unless preferences are homothetic). Thus, it matters which demand curve is allowed to shift first for the purposes of measuring areas.

In contrast, EV and CV are path <u>independent</u> because they are measured under the compensated demand curves which are free from income effects by definition, and so have symmetric cross-price effects.

In the special case where all goods whose prices have changed are income-neutral, then ΔCS is path <u>in</u>dependent, and in that special case, $|CV| = |\Delta CS| = |EV|$.

Despite its shortcomings, ΔCS is a reasonable approximation for measuring welfare change. The theoretical appeal of EV and CV is likely to be overshadowed in practice by difficulties associated with estimating compensated demand curves.

APPENDIX A4-2

USING ELASTICITIES TO CALCULATE SURPLUS CHANGES

A4-2-1. CONSUMER SURPLUS

The change in consumer surplus when demand is linear is illustrated in Figure A4-9, for the case of a price fall. The sum of the two shaded areas is

(A4-1) $$\Delta CS = -q_0 \Delta p + \frac{|\Delta p| \Delta q}{2}$$

Note that constructing the expression this way – using absolute value – means that this expression applies to the case of a price fall and to the case of a price rise. In particular, if $\Delta p < 0$ then $\Delta q > 0$, and so both terms in (A4-1) are positive; consumer surplus rises. Conversely, if $\Delta p > 0$ then $\Delta q < 0$, and so both terms in (A4-1) are negative; consumer surplus falls.

Now let us now express ΔCS in terms of the elasticity of demand. Suppose demand is linear and that the elasticity of demand at the current price and quantity is given by

(A4-2) $$\varepsilon_0 = \frac{\Delta q_D}{q_0} \bigg/ \frac{\Delta p}{p_0}$$

Rearranging this expression tells us the change in quantity demanded for any change in price:

(A4-3) $$\Delta q_D = \left(\frac{\varepsilon_0 q_0}{p_0} \right) \Delta p$$

Making this substitution for Δq in (A4-1) yields

(A4-4) $$\Delta CS = -q_0 \Delta p + \left(\frac{|\Delta p| \varepsilon_0 q_0}{2 p_0} \right) \Delta p = q_0 \left(\frac{|\Delta p| \varepsilon_0}{2 p_0} - 1 \right) \Delta p$$

A4-2-2. PRODUCER SURPLUS

The change in producer surplus when demand is linear is illustrated in Figure A4-10, for the case of a price rise. The sum of the two shaded areas is

(A4-5) $$\Delta PS = q_0 \Delta p + \frac{|\Delta p| \Delta q}{2}$$

Note that constructing the expression this way – using absolute value – means that this expression applies to the case of a price rise and to the case of a price fall. In particular, if $\Delta p > 0$ then $\Delta q > 0$, and so both terms in (A4-5) are positive; producer surplus rises. Conversely, if $\Delta p < 0$ then $\Delta q < 0$, and so both terms in (A4-5) are negative; producer surplus falls.

Now let us now express ΔPS in terms of the elasticity of supply. Suppose supply is also linear and that the elasticity of supply at the current price and quantity is given by

(A4-6) $$\eta_0 = \frac{\Delta q_S}{q_0} \bigg/ \frac{\Delta p}{p_0}$$

Rearranging this expression tells us the change in quantity supplied for any change in price:

(A4-7) $$\Delta q_S = \left(\frac{\eta_0 q_0}{p_0} \right) \Delta p$$

Making this substitution for Δq in (A4-5) yields

(A4-8) $$\Delta PS = q_0 \Delta p + \left(\frac{|\Delta p| \eta_0 q_0}{2 p_0} \right) \Delta p = q_0 \left(\frac{|\Delta p| \eta_0}{2 p_0} + 1 \right) \Delta p$$

APPENDIX A4-3

USING ELASTICITIES TO PREDICT PRICE AND SURPLUS CHANGES

The predicted price and surplus changes after a project-induced augmentation of demand (or a project-induced augmentation of supply) are based on our beliefs about demand and supply elasticities. These beliefs may in turn be based on rigorous econometric analysis, or they may be based on sophisticated guesses. (We will discuss the treatment of uncertainty and the formation of beliefs in Topic 6).

We will first consider the case of a project-induced augmentation of demand, and then consider the case of a project-induced augmentation of supply (a scenario we will examine in Topic 4.9).

A4-3-1. A PROJECT-INDUCED AUGMENTATION OF DEMAND

We know from (A4-3) and (A4-7) that the changes in quantity demanded and quantity supplied after a price change can be expressed in terms of the elasticities of demand and supply respectively. We also know that the changes in quantity demanded and quantity supplied must together clear the market after a project-induced augmentation of demand. In particular, we know that adjustment to the new equilibrium requires

(A4-9) $$\Delta q_S - \Delta q_D = q_G$$

where $\Delta q_D < 0$ because some private demand has been crowded out.

Equations (A4-3), (A4-7) and (A4-9) can now be solved simultaneously to yield price and quantity changes as a function of the project-induced augmentation:

(A4-10) $$\Delta p = \frac{q_G}{q_0}\left(\frac{p_0}{\eta_0 - \varepsilon_0}\right) > 0$$

(A4-11) $$\Delta q_D = q_G\left(\frac{\varepsilon_0}{\eta_0 - \varepsilon_0}\right) < 0$$

(A4-12) $$\Delta q_S = q_G\left(\frac{\eta_0}{\eta_0 - \varepsilon_0}\right) > 0$$

We can then calculate the new price as

(A4-13) $$p_1 = p_0 + \Delta p = p_0\left(1 + \frac{q_G}{q_0(\eta_0 - \varepsilon_0)}\right)$$

We can also substitute Δp from (A4-10) in our expressions for ΔCS and ΔPS from (A4-4) and (A4-8) in Appendix A4-2 to yield expressions for the surplus changes in terms of elasticities and the size of the demand augmentation.

A4-3-2. A PROJECT-INDUCED AUGMENTATION OF SUPPLY

The calculation here follows the same steps as in A4-2-1 above, except that in this case we know that adjustment to the new equilibrium requires

(A4-14) $$\Delta q_D - \Delta q_S = q_G$$

where $\Delta q_S < 0$ because some private supply has been crowded out.

Equations (A4-3), (A4-7) and (A4-14) can now be solved simultaneously to yield price and quantity changes as a function of the project-induced augmentation:

(A4-15) $$\Delta p = \frac{q_G}{q_0}\left(\frac{p_0}{\varepsilon_0 - \eta_0}\right) < 0$$

(A4-16) $$\Delta q_D = q_G\left(\frac{\varepsilon_0}{\varepsilon_0 - \eta_0}\right) > 0$$

(A4-17) $$\Delta q_S = q_G\left(\frac{\eta_0}{\varepsilon_0 - \eta_0}\right) < 0$$

We can then calculate the new price as

(A4-18) $$p_1 = p_0 + \Delta p = p_0\left(1 - \frac{q_G}{q_0(\eta_0 - \varepsilon_0)}\right)$$

We can also substitute Δp from (A4-18) in our expressions for ΔCS and ΔPS from (A4-4) and (A4-8) in Appendix A4-1 to yield expressions for the surplus changes in terms of elasticities and the size of the supply augmentation.

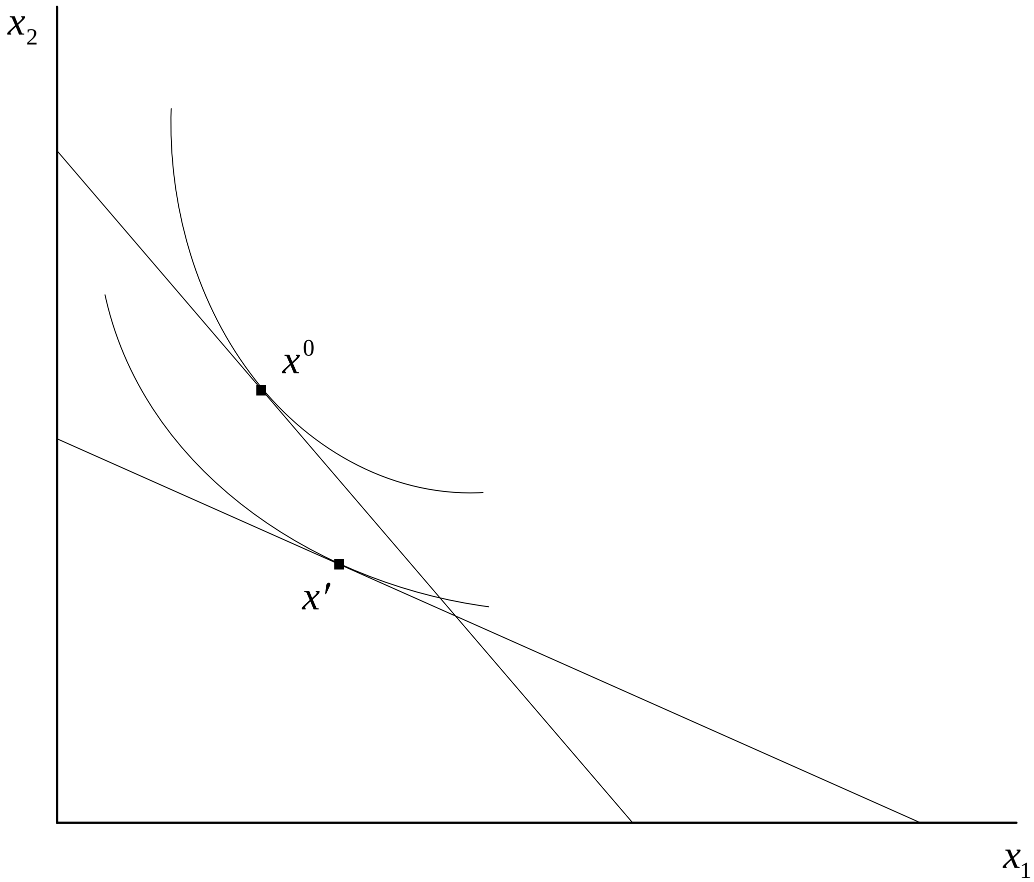

Figure A4-1

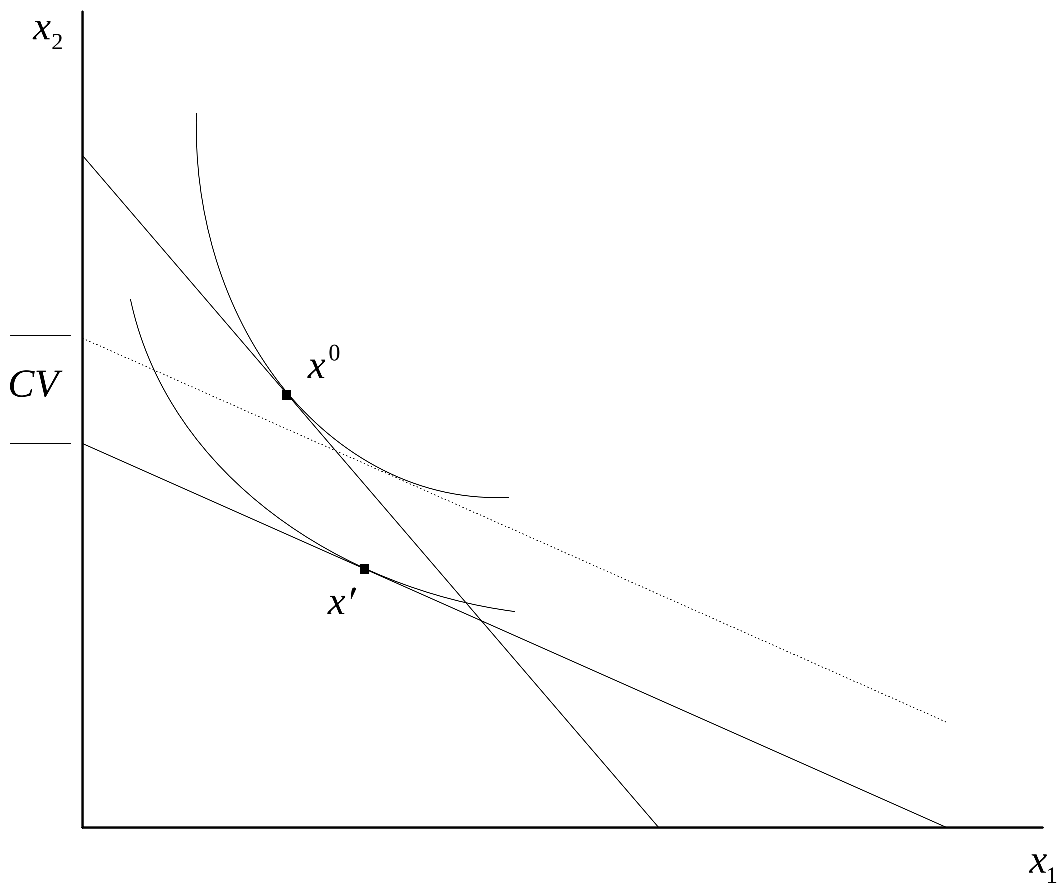

Figure A4-2

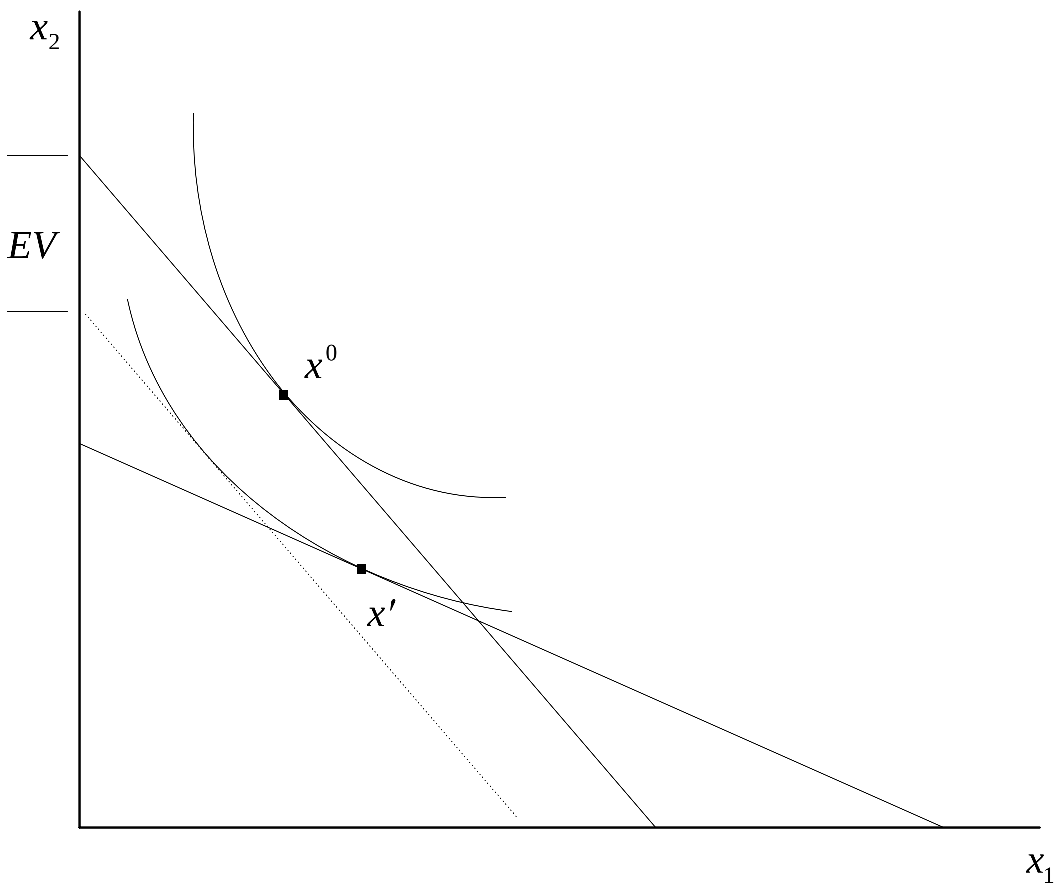

Figure A4-3

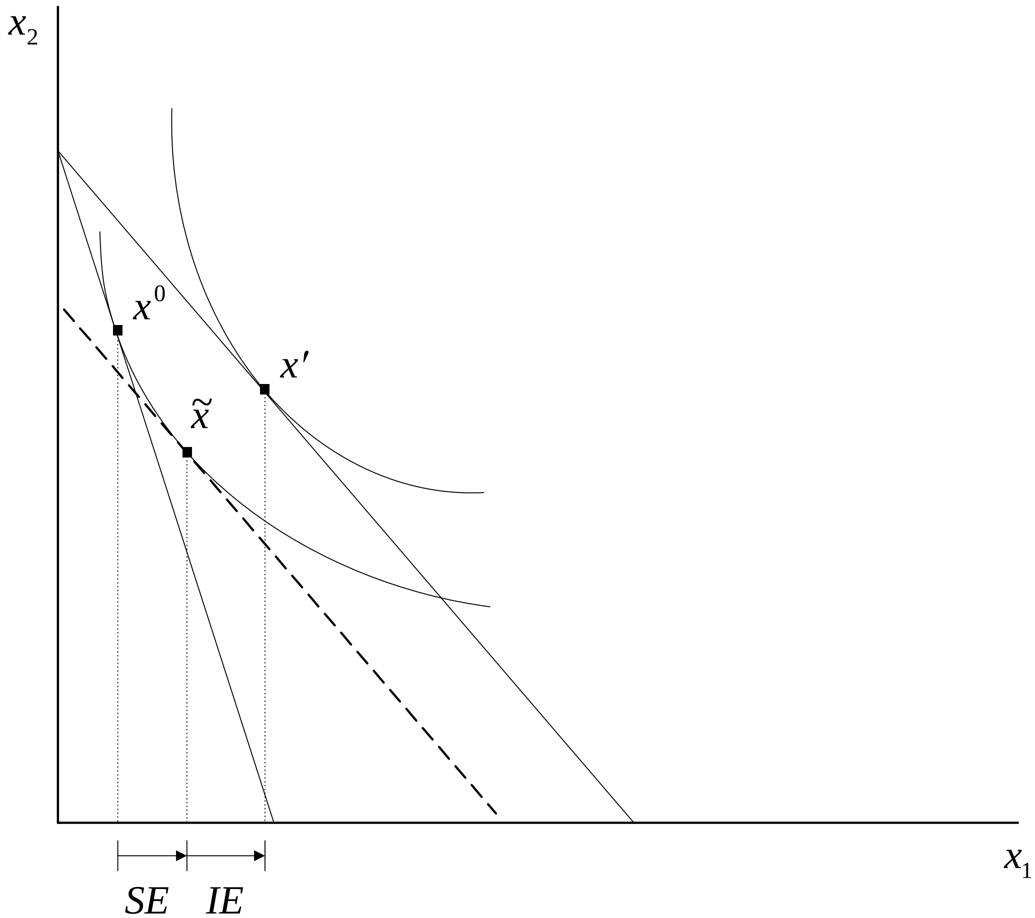

Figure A4-4

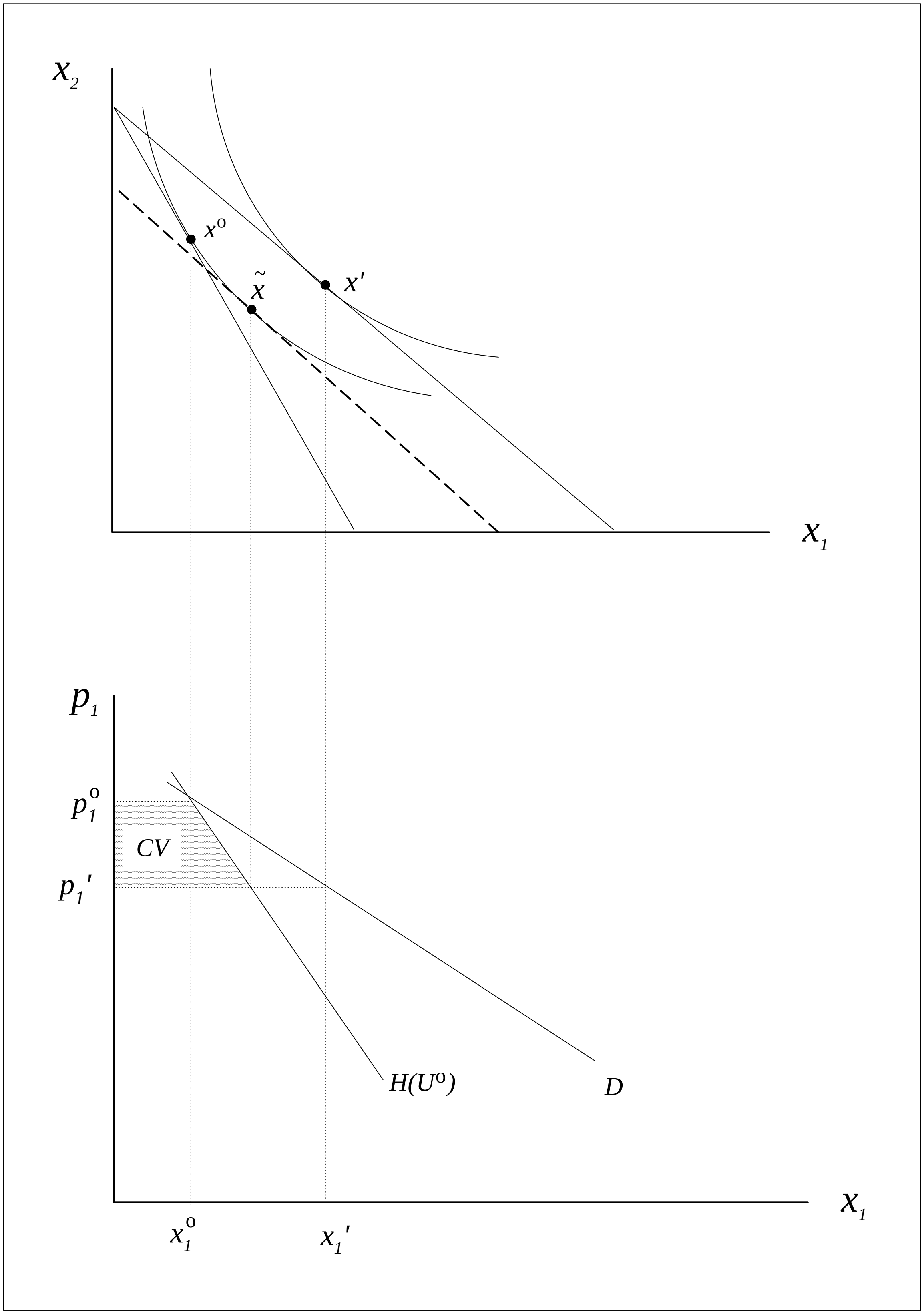

Figure A4-5

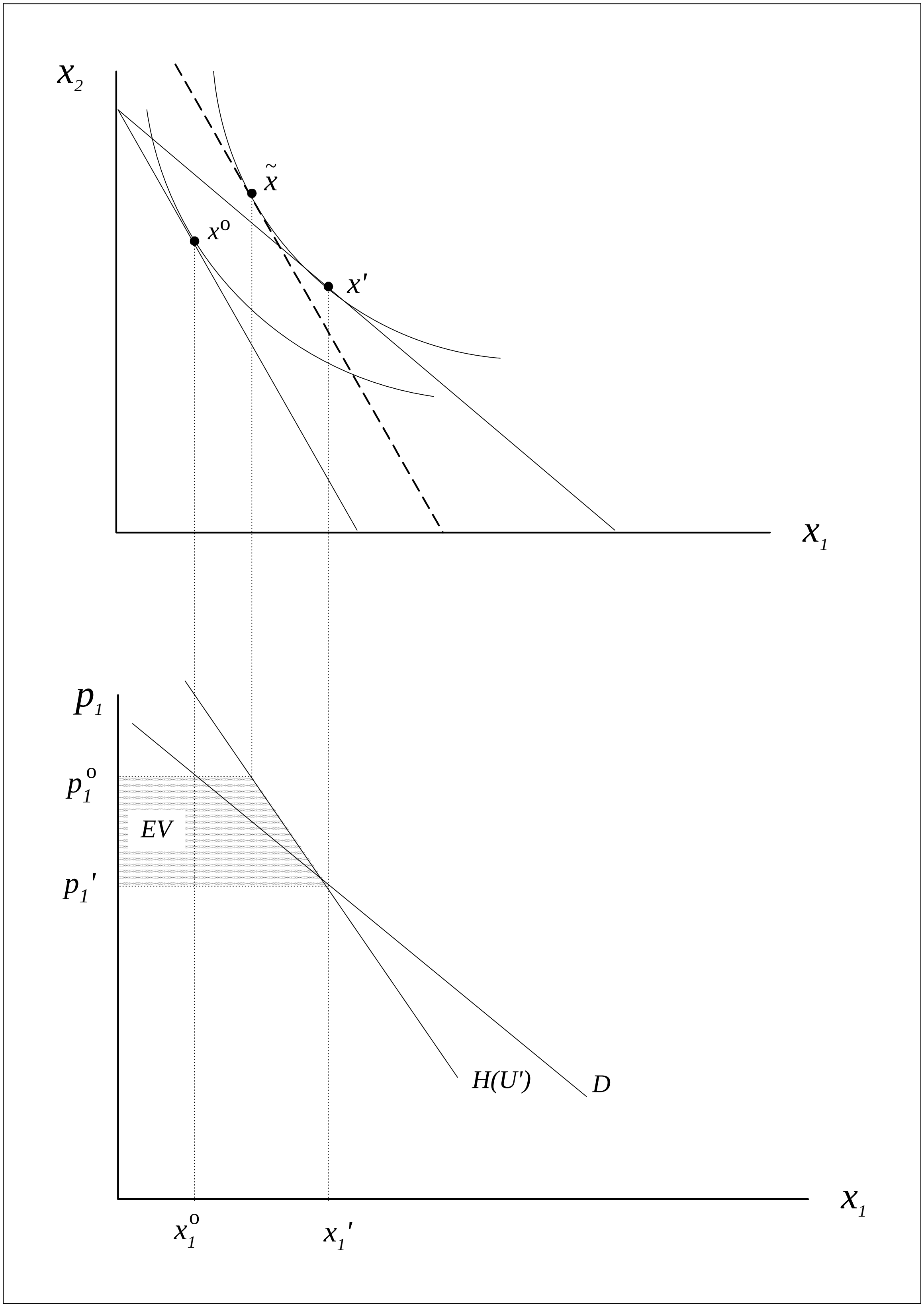

Figure A4-6

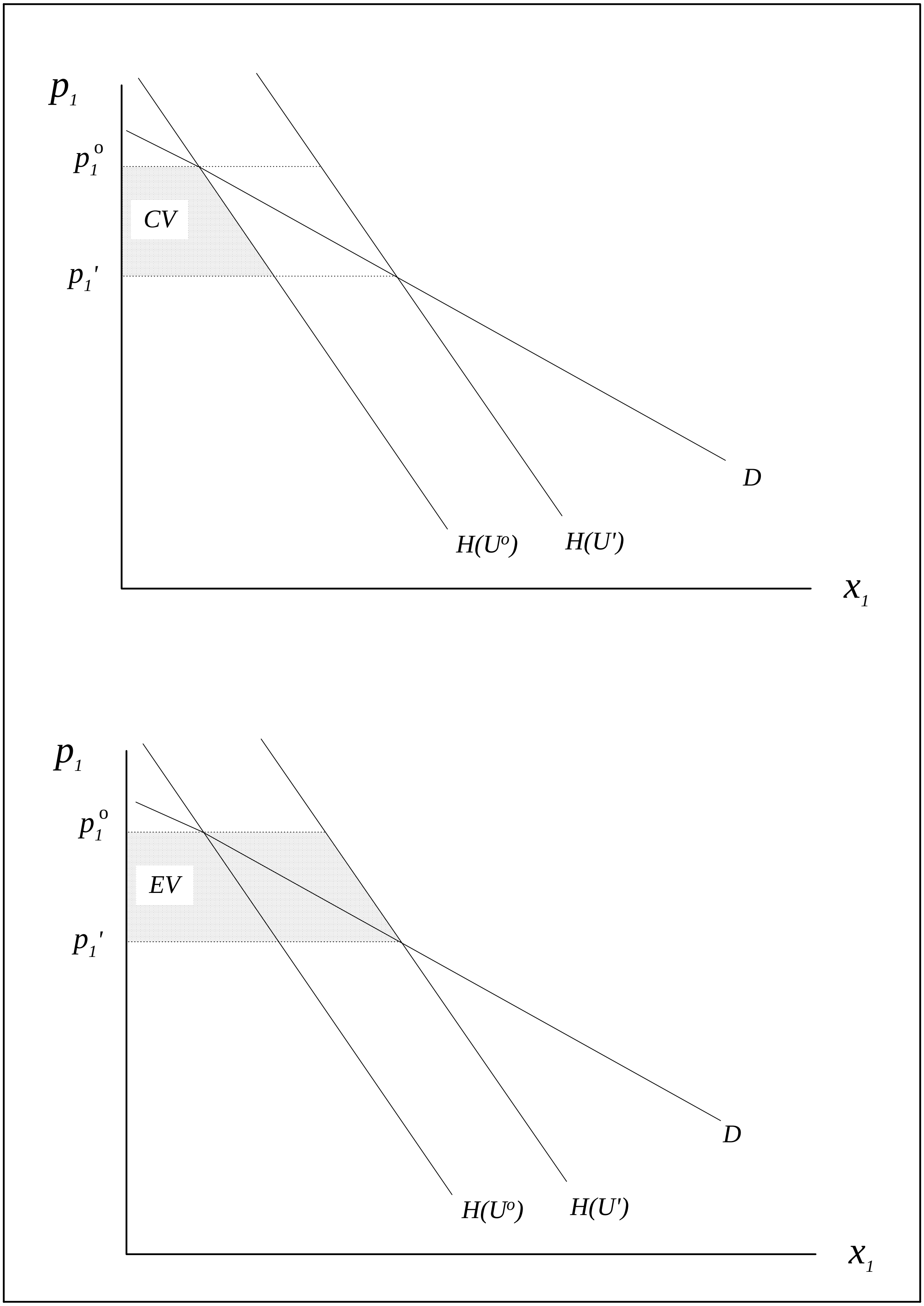

Figure A4-7

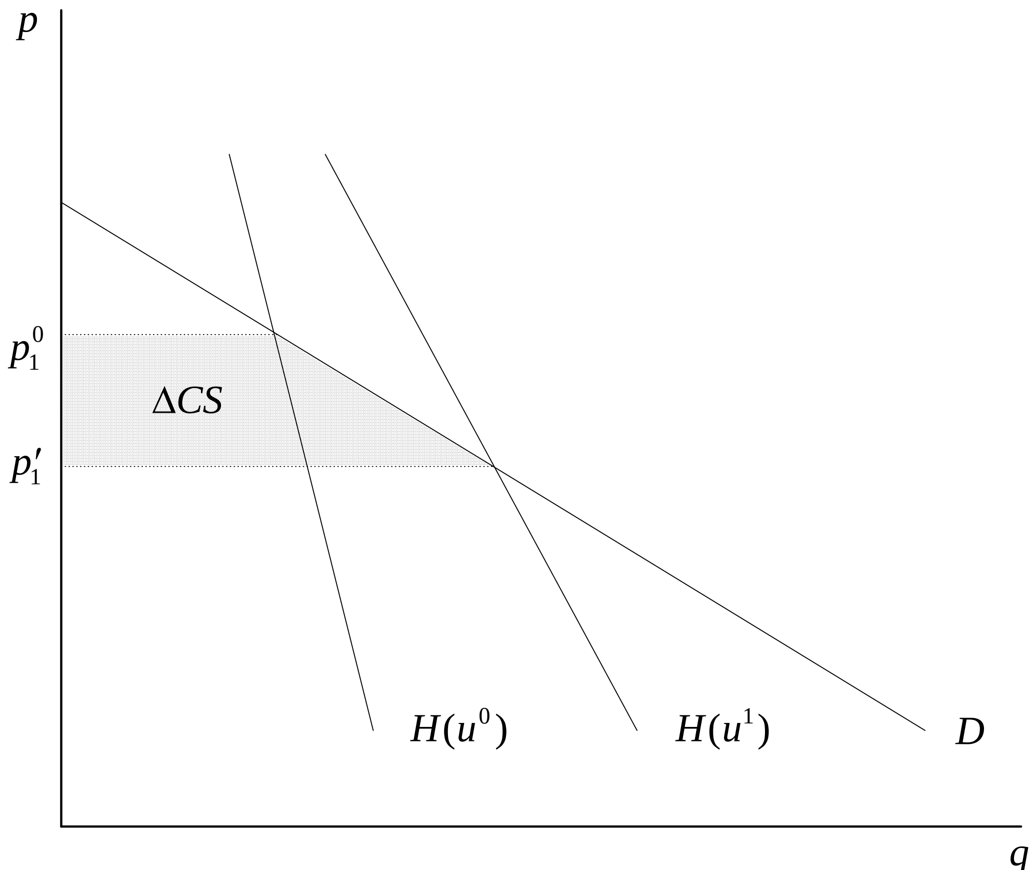

Figure A4-8

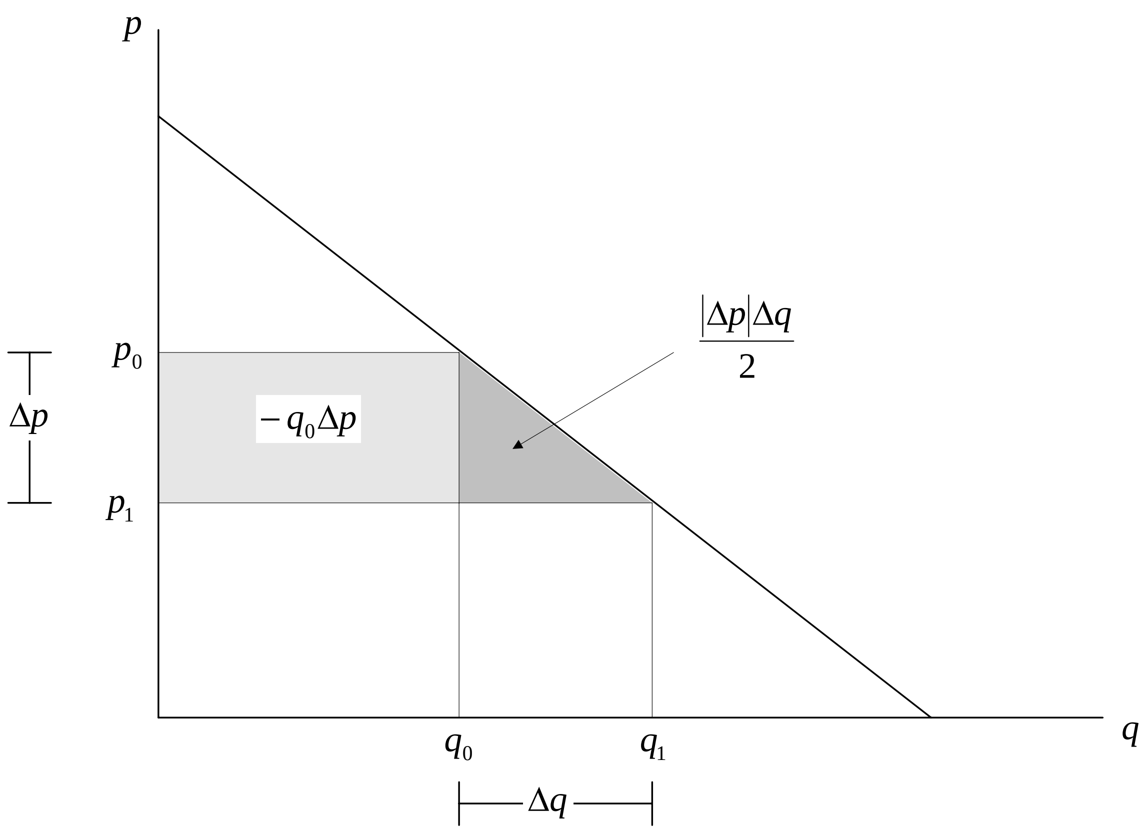

Figure A4-9

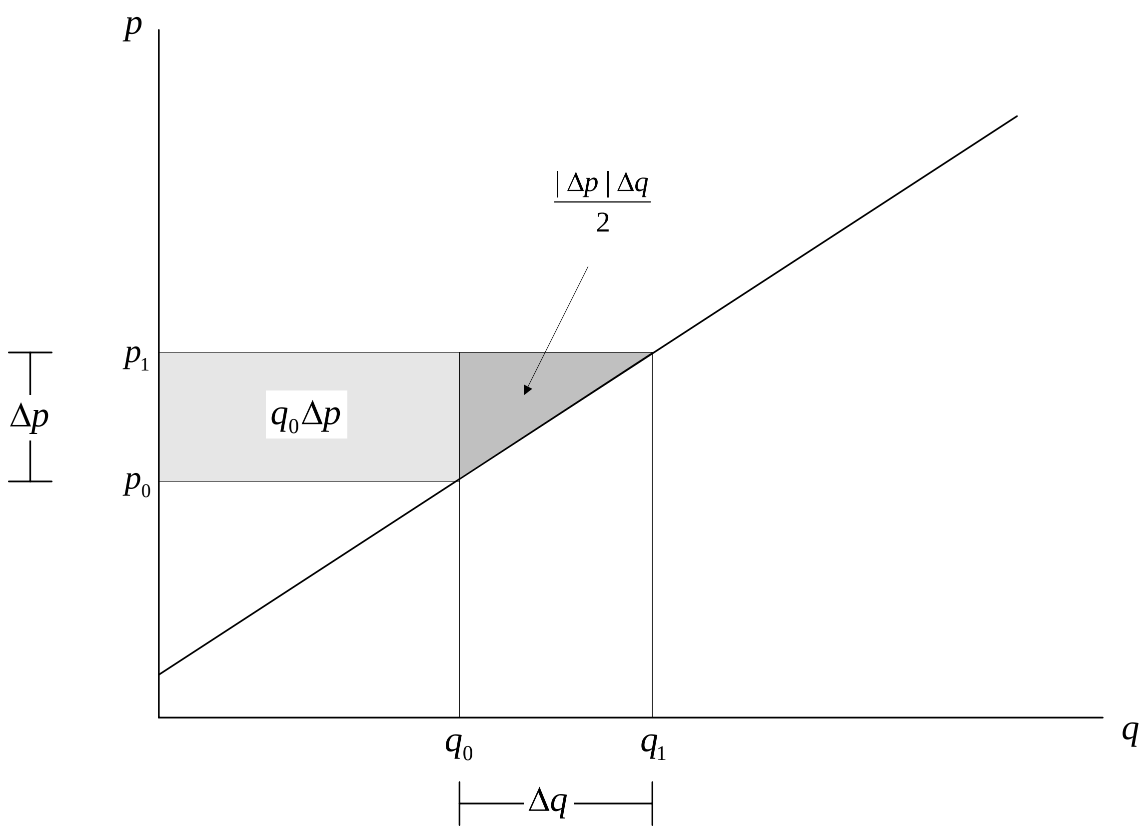

Figure A4-10

TOPIC 4 REVIEW QUESTIONS

Questions 1 – 11 relate to the following information.

Consider a setting where a government project draws an input from a competitive market in an amount sufficiently large to cause the market price to change. **Figure R4-1** provides the relevant data (where supply and demand are both linear).

1. Use of the input in this project causes a shift in the private demand for this input.

A. True.

B. False.

2. The input used in the project is supplied partly through

A. an increase in supply via a phenomenon known as "crowding out".

B. a reduction in the amount privately consumed.

C. an increase in price.

D. Both A and B.

3. The quantity of input used in the project is

A. 10

B. 5

C. 15

D. None of the above.

4. The change in consumer surplus for private consumers of the input is

A. $47.5

B. $45

C. – $45

D. – $47.5

5. The change in producer surplus for suppliers of the input is

A. $52.5

B. –$52.5

C. –$55

D. $45

6. The financial outlay for government is

A. –$110

B. –$100

C. $100

D. $110

7. The economic cost of the input for use in the CBA is

A. $110

B. $100

C. $105

D. None of the above.

8. The value of the displaced private consumption is

A. $47.5

B. $45

C. $52.5

D. $105

9. The cost of the net increase in supply is

A. $52.5

B. –$52.5

C. –$55

D. $45

10. “The strong relationship between your answers to Q5 and Q9 is a coincidence specific to this example”.

A. True.

B. False.

11. The project results in a transfer from taxpayers to

A. suppliers, equal to $5

B. consumers, equal to $5

C. suppliers, equal to $7.5

D. consumers, equal to $7.5

Questions 12 – 22 relate to the following information.

Consider a setting where a government project draws an input from a competitive market in an amount sufficiently large to cause the market price to change. **Figure R4-2** provides the relevant data (where supply and demand are both linear).

12. Use of the input in this project augments the demand for this input.

A. True.

B. False.

13. “Crowding out” in the context of this project refers to

A. a contraction in supply in the market into which output from the project is sold.

B. a reduction in private consumer surplus in the input market.

C. an increase in producer surplus in the input market.

D. None of the above.

14. The quantity of input used in the project is

A. 10

B. 20

C. 15

D. 25.

15. The change in consumer surplus for private consumers of the input is

A. $20

B. $22.5

C. –$20

D. –$22.5

16. The change in producer surplus for suppliers of the input is

A. –$32.5

B. $32.5

C. $22.5

D. $45.5

17. The change in surplus for taxpayers is

A. $220

B. –$200

C. –$220

D. $200

18. The economic cost of the input for use in the CBA is

A. $215

B. $220

C. $200

D. None of the above.

19. The value of the displaced private consumption is

A. $221.5

B. $157.5

C. $52.5

D. $152.5

20. The cost of the net increase in supply is

A. $221.5

B. $157.5

C. $52.5

D. $152.5

21. Calculate the average of the post-project and pre-project prices, and multiply this price by the amount of input purchased by government. Let *V* denote the result. Is the relationship between *V* and your answer to Q18 a coincidence specific to this example?

A. Yes.

B. No.

22. The project results in a transfer from taxpayers to

A. suppliers, equal to $5

B. consumers, equal to $5

C. suppliers, equal to $10

D. consumers, equal to $10

Questions 23 – 33 relate to the following information.

Consider a setting where a government project makes a direct addition to the supply of a product in a competitive market in an amount sufficiently large to cause the market price to change. **Figure R4-3** provides the relevant data (where supply and demand are both linear).

23. The output from the project causes a shift in the supply of this product.

A. True.

B. False.

24. The quantity supplied by the project is absorbed partly through

A. a reduction in the amount privately supplied via a phenomenon known as "crowding out".

B. a reduction in the amount privately consumed.

C. a reduction in price.

D. Both A and C.

25. The quantity supplied by the project is

A. 5

B. 10

C. 15

D. 20

26. The actual increase in consumption is

A. 10

B. 15

C. 5

D. None of the above.

27. The change in consumer surplus for buyers of this product is

A. $40

B. $42.5

C. – $40

D. – $42.5

28. The change in producer surplus for suppliers of this product is

A. – $37.5

B. $37.5

C. $27.5

D. – $27.5

29. The financial receipt for government is

A. – $120

B. $120

C. – $100

D. $110

30. The economic benefit from the project output is

A. $115

B. $120

C. $110

D. None of the above.

31. The value of the increase in consumption is

A. $47.5

B. $52.5

C. $57.5

D. $72.5

32. The value of resources released is

A. $47.5

B. $52.5

C. $57.5

D. $72.5

33. Calculate the average of the post-project and pre-project prices, and multiply this price by the amount of output produced by government. Let *V* denote the result. Is the relationship between *V* and your answer to Q30 a coincidence specific to this example?

A. Yes.

B. No.

Questions 34 – 44 relate to the following information.

Consider a setting where a government project makes a direct addition to the supply of a product in a competitive market in an amount sufficiently large to cause the market price to change. **Figure R4-4** provides the relevant data (where supply and demand are both linear).

34. The output from the project augments the private supply of this product.

A. True.

B. False.

35. The quantity supplied by the project is absorbed partly through

A. a reduction in the amount privately supplied.

B. an increase in the amount consumed.

C. a reduction in price.

D. Both A and B.

36. The quantity supplied by the project is

A. 5

B. 8

C. 12

D. 20

37. The actual increase in consumption is

A. 2

B. 3

C. 8

D. 12

38. The change in consumer surplus for buyers of this product is

A. $18

B. –$18

C. $23

D. $27

39. The change in producer surplus for suppliers of this product is

A. –$17

B. $21

C. $27

D. None of the above.

40. The change in surplus for taxpayers is

A. –$40

B. $40

C. $32

D. $36

41. The economic benefit from the project output is

A. $32

B. $36

C. $40

D. $44

42. The value of the increase in consumption is

A. $6

B. $9

C. $18

D. $27

43. The value of resources released is

A. $9

B. $18

C. $27

D. $36

44. Compare your answer to Q41 with the sum of your answers to Q42 and Q43. Is the relationship between these values a coincidence specific to this example?

A. Yes.

B. No.

Questions 45 – 55 relate to the following information.

Consider a setting where a government infrastructure project causes a reduction in the marginal cost of production in a competitive market by an amount sufficiently large to cause the market price to change. **Figure R4-5** provides the relevant data (where supply and demand are both linear, and the marginal-cost reduction is a constant).

45. "The project augments the private supply of this product".

A. True.

B. False.

46. The quantity supplied in the market rises because

A. government has added to demand.

B. more firms enter the market, thereby creating more competition and lower costs.

C. production costs have fallen.

D. the price falls.

47. At any given level of output, marginal cost falls by

A. \$2 per unit

B. \$1 per unit

C. 50%

D. Both A and C

48. The change in equilibrium price is less than the change in equilibrium marginal cost because

A. there is not enough competition among firms in this market.

B. the level of production rises.

C. the quantity demanded rises.

D. None of the above.

49. The change in consumer surplus for buyers of this product is

A. $52

B. –$52

C. $42.5

D. $37

50. The change in producer surplus for suppliers of this product is

A. $52

B. –$52

C. $42.5

D. None of the above.

51. The change in surplus for taxpayers in this market is

A. $495

B. $480

C. $15

D. $0

52. The economic benefit from the project in this market is

A. $85

B. $125

C. $97.5

D. None of the above.

53. The cost reduction for existing supply is

A. $2

B. $80

C. $90

D. $116

54. The net surplus from the additional quantity supplied is

A. $57.5

B. $52.5

C. $42.5

D. $5

55. Compare your answer to Q52 with the sum of your answers to Q53 and Q54. Is the relationship between these values a coincidence specific to this example?

A. Yes.

B. No.

Questions 56 – 64 relate to the following information.

Consider a setting where a government infrastructure project causes a reduction in the marginal cost of production in a competitive market by an amount sufficiently large to cause the market price to change. **Figure R4-6** provides the relevant data (where supply and demand are both linear, and the marginal-cost reduction is a constant).

56. “The project shifts the private supply of this product”.

A. True.

B. False.

57. The quantity demanded in the market rises because

A. government has added to demand.

B. more firms enter the market, thereby creating more competition and lower costs.

C. the quantity supplied rises.

D. the price falls.

58. At any given level of output, marginal cost falls

A. by $1 per unit

B. by $\$\frac{4}{3}$ per unit

C. to zero

D. Both B and C

59. The change in equilibrium price is determined at least partly by

A. the change in marginal cost.

B. the slope of the demand schedule.

C. the slope of the supply schedule.

D. All of the above.

60. The change in consumer surplus for buyers of this product is

A. $23

B. $-\$17$

C. $32

D. $11

61. The change in producer surplus for suppliers of this product is

A. $32

B. $16

C. $\$7\frac{2}{3}$

D. None of the above.

62. The economic benefit from the project in this market is

A. $21

B. $\$27\frac{1}{3}$

C. $\$33\frac{2}{3}$

D. None of the above.

63. The cost reduction for existing supply is

A. $29 $\frac{1}{3}$

B. $36 $\frac{2}{3}$

C. $41

D. $45

64. The net surplus from the additional quantity supplied is

A. $16

B. $12

C. $9 $\frac{2}{3}$

D. $1 $\frac{1}{3}$

Questions 65 – 70 relate to the following information.

Consider a setting where a government project uses unemployed labour as an input. The labour is drawn from a very simple market in which all workers have identical productivity. Workers differ only according to their private opportunity cost of labour (POCL), as reflected in a positively-sloped supply schedule. The unemployment is due to some wage friction which keeps the market wage above the market-clearing wage. There are no other distortions in the market. **Figure R4-7** provides the relevant data (where supply and demand are both linear). The project will use 5 workers.

65. The number of unemployed workers is

A. 10

B. 20

C. 30

D. 40

66. The range of POCL among employed workers is

A. 4 – 14

B. 4 – 12

C. 12 – 14

D. 0 – 14

67. The range of POCL among unemployed workers is

A. 4 – 14

B. 4 – 12

C. 12 – 14

D. None of the above.

68. If we know all of the data in **Figure R4-7**, can we calculate the exact cost of using the labour of the unemployed workers?

A. Yes.

B. No.

69. Suppose we believe that labour supply is linear but we can only observe the current wage and the number of employed and unemployed workers. Then a reasonable estimate for the cost of the labour used in the project is

A. $25

B. $35

C. $30

D. $20

70. Based on your estimate from Q69, the total producer surplus captured by the newly employed workers is

A. $25

B. $35

C. $30

D. $30

Questions 71 – 75 relate to the following information.

Consider a setting where a government project uses unemployed labour as an input. The labour is drawn from a very simple market in which all workers have identical productivity. Workers differ only according to their private opportunity cost of labour (POCL), as reflected in a positively-sloped supply schedule. The unemployment is due to some wage friction which keeps the market wage above the market-clearing wage. There are no other distortions in the market. **Figure R4-8** provides the relevant data (where supply and demand are both linear). The project will use 3 workers.

71. The number of employed workers is

A. 8

B. 20

C. 22

D. 28

72. The range of POCL among employed workers is

A. $\frac{4}{3}$ – 5

B. $\frac{4}{3}$ – 6

C. 0 – 5

D. 0 – 6

73. The range of POCL among "workers" who are neither employed nor unemployed is

A. $\frac{4}{3}$ – 5

B. $\frac{4}{3}$ – 6

C. above 5

D. above 6

74. If we know all of the data in **Figure R4-8**, but nothing more, what is a reasonable estimate for the cost of the labour used in the project?

A. $7.5

B. $9

C. $11

D. $9.5

75. Based on your estimate from Q74, the total producer surplus captured by the newly employed workers is

A. $7

B. $9

C. $11

D. $18

Questions 76 – 84 relate to the following information.

Consider a setting where government introduces a subsidy for the purchase of a good whose consumption creates a positive externality. In particular, each unit of the good consumed bestows an external benefit of $10 per unit. There are no other distortions in the market. **Figure R4-9** provides the relevant data on supply and demand (which are both linear).

76. The optimal subsidy in this setting is

A. $5 per unit

B. $10 per unit

C. $15 per unit

D. None of the above.

77. The change in consumer surplus under the optimal subsidy is

A. $500

B. $700

C. $250

D. $150

78. The change in producer surplus under the optimal subsidy is

A. $750

B. $500

C. $250

D. Zero

79. The change in surplus for taxpayers is

A. – $500

B. – $750

C. $750

D. $500

80. The change in surplus for the external beneficiaries is

A. $850

B. $750

C. $250

D. None of the above.

81. The total social benefit from the subsidy-induced increase in consumption is

A. $500

B. $750

C. $1000

D. $1250

82. The total social cost of the subsidy-induced increase in production is

A. $500

B. $750

C. $1000

D. $1250

83. The net benefit of the subsidy policy is

A. $850

B. $750

C. $250

D. None of the above.

84. Compare your answer to Q83 with the sum of your answers to Q77 – Q 80. Is the relationship between these values a coincidence specific to this example?

A. Yes.

B. No.

Questions 85 – 93 relate to the following information.

Consider a setting where government introduces a subsidy for the purchase of a good whose consumption creates a positive externality. In particular, each unit of the good consumed bestows an external benefit of $4 per unit. There are no other distortions in the market. **Figure R4-10** provides the relevant data on supply and demand (which are both linear).

85. The optimal subsidy in this setting is

A. $2 per unit

B. $6 per unit

C. $12 per unit

D. None of the above.

86. The change in consumer surplus under the optimal subsidy is

A. $16

B. $32

C. $64

D. $128

87. The change in producer surplus under the optimal subsidy is

A. Zero

B. $16

C. $32

D. $64

88. The financial outlay for government is

A. $80

B. $32

C. – $32

D. – $80

89. The change in surplus for the external beneficiaries is

A. $16

B. $32

C. $64

D. None of the above.

90. The net benefit of the subsidy policy is

A. $32

B. $64

C. $128

D. None of the above.

91. The total social benefit from the subsidy-induced increase in consumption is

A. $32

B. $48

C. $64

D. $128

92. The total social cost of the subsidy-induced increase in production is

A. $32

B. $48

C. $64

D. $128

93. Compare your answer to Q86 with your answer to Q91. Is the relationship between these values a coincidence specific to this example?

A. Yes.

B. No.

Questions 94 – 101 relate to the following information.

Consider a setting where government introduces a subsidy for the purchase of a good whose consumption creates a positive externality. In particular, each unit of the good consumed bestows an external benefit of $10 per unit. There are no other distortions in the market. **Figure R4-11** provides the relevant data on supply and demand (which are both linear).

94. The optimal subsidy in this setting is

A. $5 per unit

B. $7 per unit

C. $10 per unit

D. $17 per unit

95. The change in consumer surplus under the optimal subsidy is

A. $130 $\frac{1}{2}$

B. $262 $\frac{1}{2}$

C. $320

D. $366

96. The change in producer surplus under the optimal subsidy is

A. $130 $\frac{1}{2}$

B. $262 $\frac{1}{2}$

C. $320

D. $366

97. The financial outlay for government is

A. $650

B. $850

C. $950

D. $250

98. The change in surplus for the external beneficiaries is

A. $850

B. $750

C. $250

D. None of the above.

99. The total social benefit from the subsidy-induced increase in consumption is

A. $260 $\frac{1}{2}$

B. $282 $\frac{1}{2}$

C. $362 $\frac{1}{2}$

D. $487 $\frac{1}{2}$

100. The total social cost of the subsidy-induced increase in production is

A. $\$260\frac{1}{2}$

B. $\$282\frac{1}{2}$

C. $\$362\frac{1}{2}$

D. $\$487\frac{1}{2}$

101. The net benefit of the subsidy policy is

A. $125

B. $255

C. $750

D. None of the above.

Questions 102 – 107 relate to the following information.

Consider a setting where government introduces an excise tax into an otherwise undistorted market. **Figure R4-12** provides the relevant data on supply and demand (which are both linear).

102. The tax rate is

A. $10 per unit.

B. $5 per unit.

C. 142.9%

D. 41.7%

103. The loss of consumer surplus is

A. $112.5

B. $117.5

C. $137.5

D. $160

104. The loss of producer surplus is

A. $112.5

B. $117.5

C. $137.5

D. $160

105. The revenue raised by the tax is

A. $45

B. $150

C. $200

D. $400

106. The deadweight loss from the tax is

A. $225

B. $175

C. $150

D. $125

107. The average cost of funds (ACF) for a tax is defined as the loss of market surplus per dollar of tax revenue raised. For this tax, the AFC is approximately

A. 0.833

B. 1.833

C. 0.546

D. None of the above.

Questions 108 – 113 relate to the following information.

Consider a setting where government introduces an excise tax into an otherwise undistorted market. **Figure R4-13** provides the relevant data on supply and demand (which are both linear).

108. The tax rate is

A. $3 per unit.

B. $4 per unit.

C. $1 per unit.

D. 60%

109. The loss of consumer surplus is

A. $57

B. $42

C. $37

D. $21

110. The loss of producer surplus is

A. $19

B. $37

C. $42

D. $45

111. The revenue raised by the tax is

A. $48

B. $64

C. $88

D. None of the above.

112. The deadweight loss from the tax is

A. $36

B. $24

C. $12

D. $6

113. The average cost of funds (ACF) for a tax is defined as the loss of market surplus per dollar of tax revenue raised. For this tax, the AFC is approximately

A. 1.188

B. 1.822

C. 0.477

D. None of the above.

*** Questions 114 – 119 relate to the following information.**

Consider a setting where government introduces an *ad valorem* tax into an otherwise undistorted market. **Figure R4-14** provides the relevant data on supply and demand (which are both linear).

114. The tax rate is

A. $1 per unit.

B. $2 per unit.

C. $\frac{2}{11}$

D. $\frac{1}{12}$

115. The loss of consumer surplus is

A. $17

B. $37.5

C. $57.5

D. $61

116. The loss of producer surplus is

A. $17

B. $37.5

C. $57.5

D. $61

117. The revenue raised by the tax is

A. $40

B. $35

C. $70

D. $82.7

118. The deadweight loss from the tax is

A. $5

B. $7

C. $11

D. $21

119. The average cost of funds (ACF) for a tax is defined as the loss of market surplus per dollar of tax revenue raised. For this tax, the AFC is approximately

A. 0.467

B. 1.071

C. 1.277

D. None of the above.

*** Questions 120 – 125 relate to the following information.**

Consider a setting where government introduces an *ad valorem* tax into an otherwise undistorted market. **Figure R4-15** provides the relevant data on supply and demand (which are both linear).

120. The tax rate is

A. 100%

B. 50%

C. $4 per unit.

D. None of the above.

121. The loss of consumer surplus is

A. $57

B. $42

C. $37

D. $21

122. The loss of producer surplus is

A. $19

B. $37

C. $42

D. $45

123. The revenue raised by the tax is

A. $48

B. $64

C. $88

D. None of the above.

124. The deadweight loss from the tax is

A. $36

B. $24

C. $12

D. $6

125. The average cost of funds (ACF) for a tax is defined as the loss of market surplus per dollar of tax revenue raised. For this tax, the AFC is approximately

A. 1.188

B. 1.822

C. 0.477

D. None of the above.

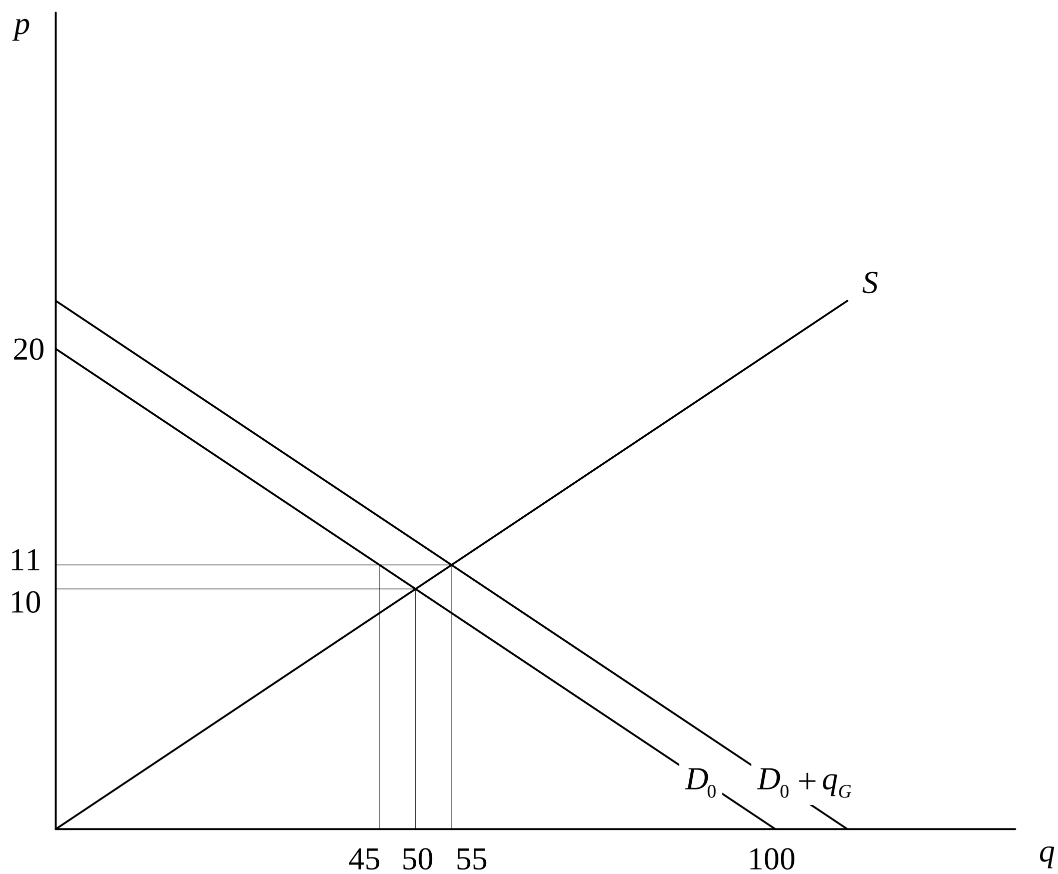

Figure R4-1

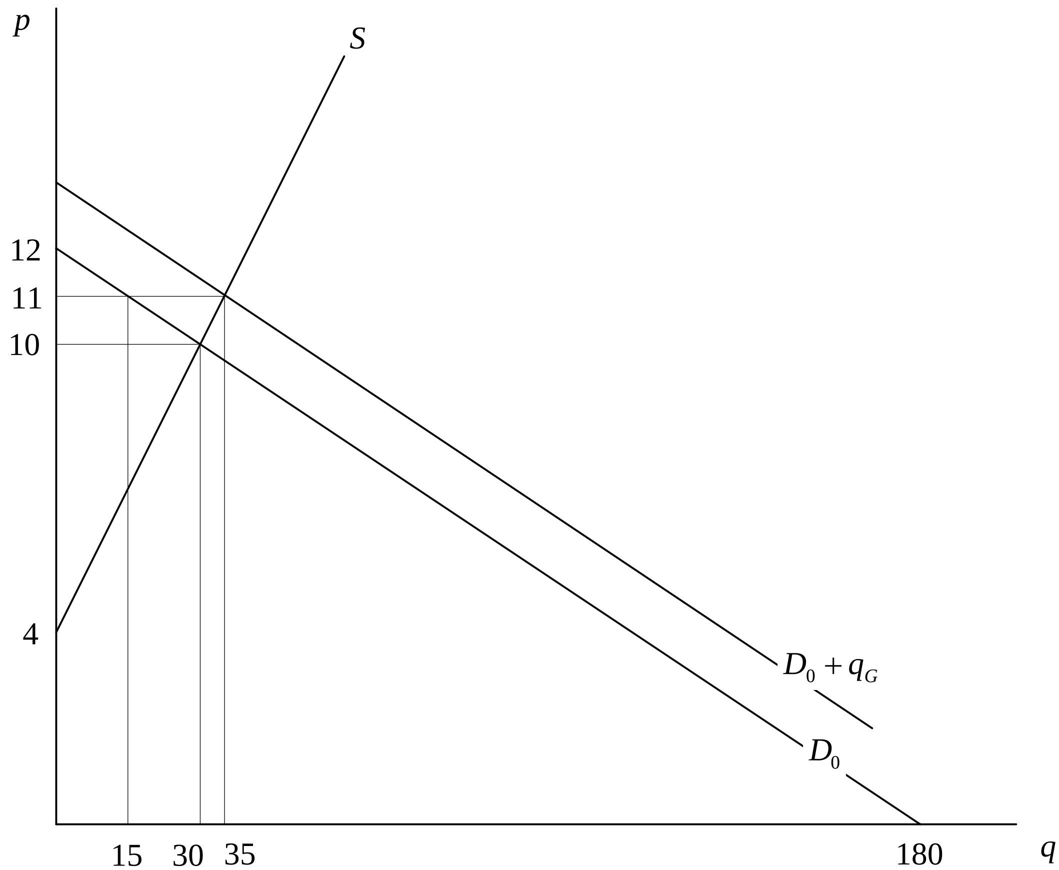

Figure R4-2

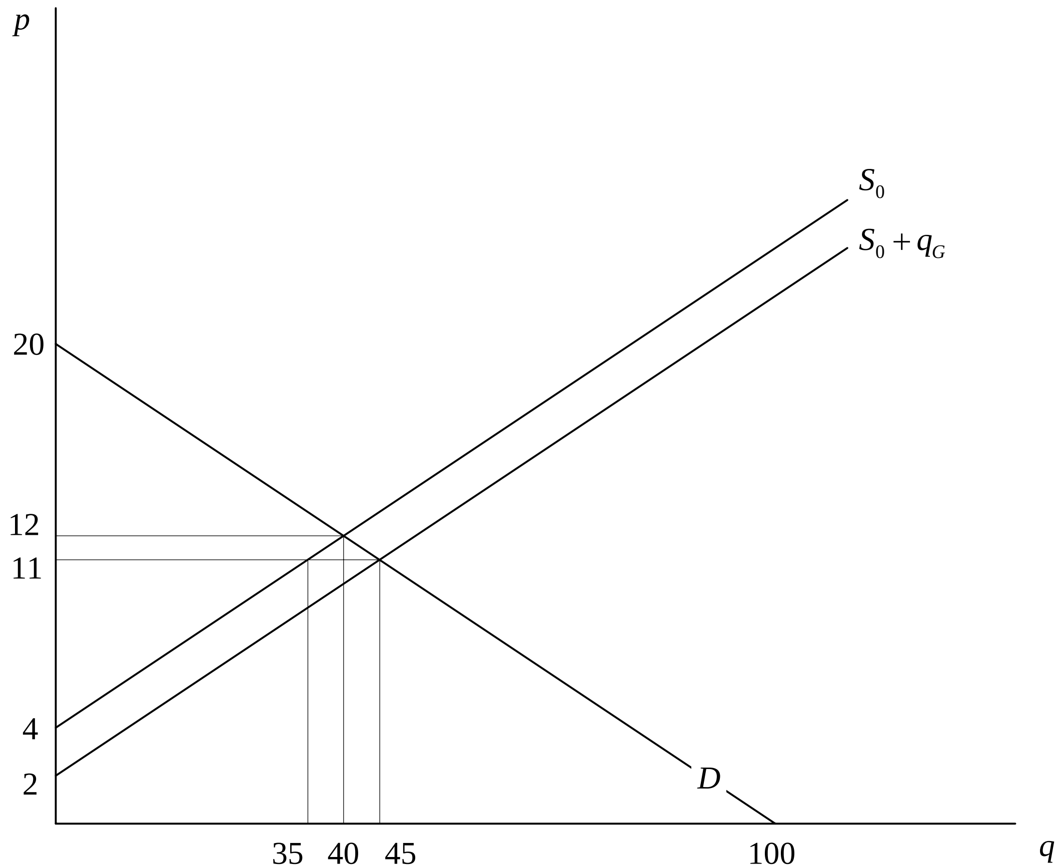

Figure R4-3

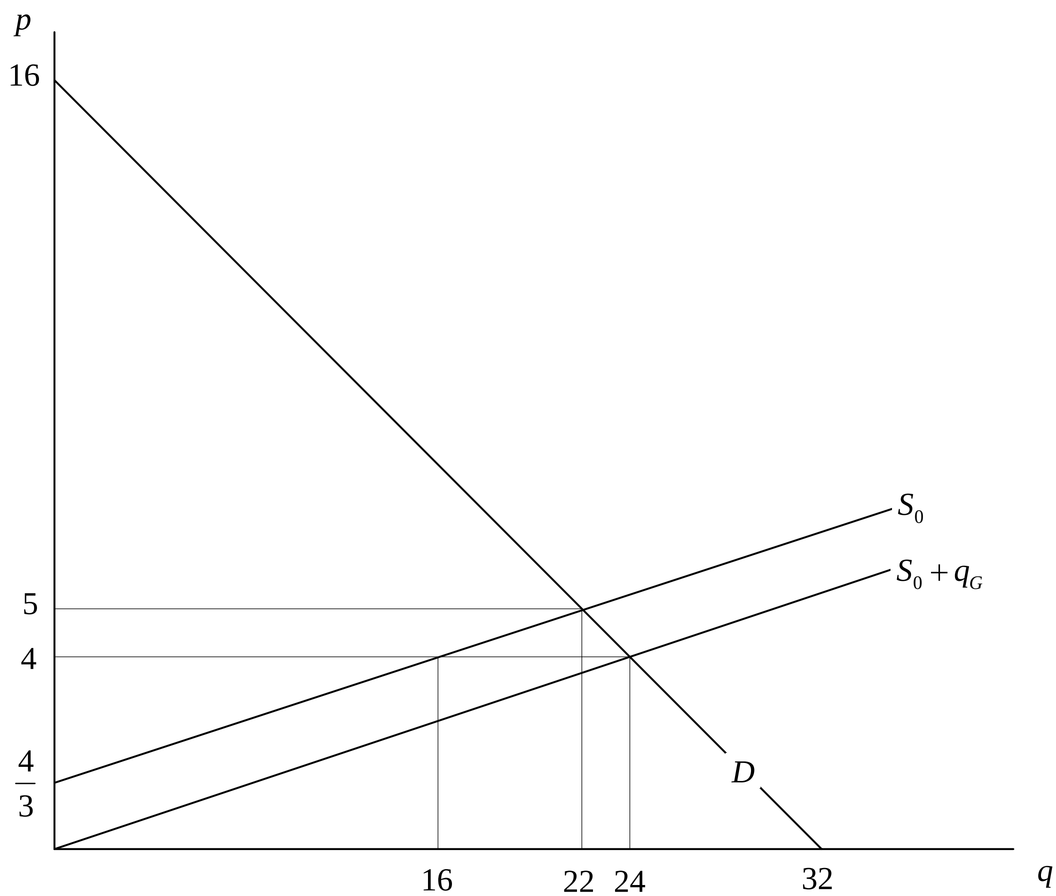

Figure R4-4

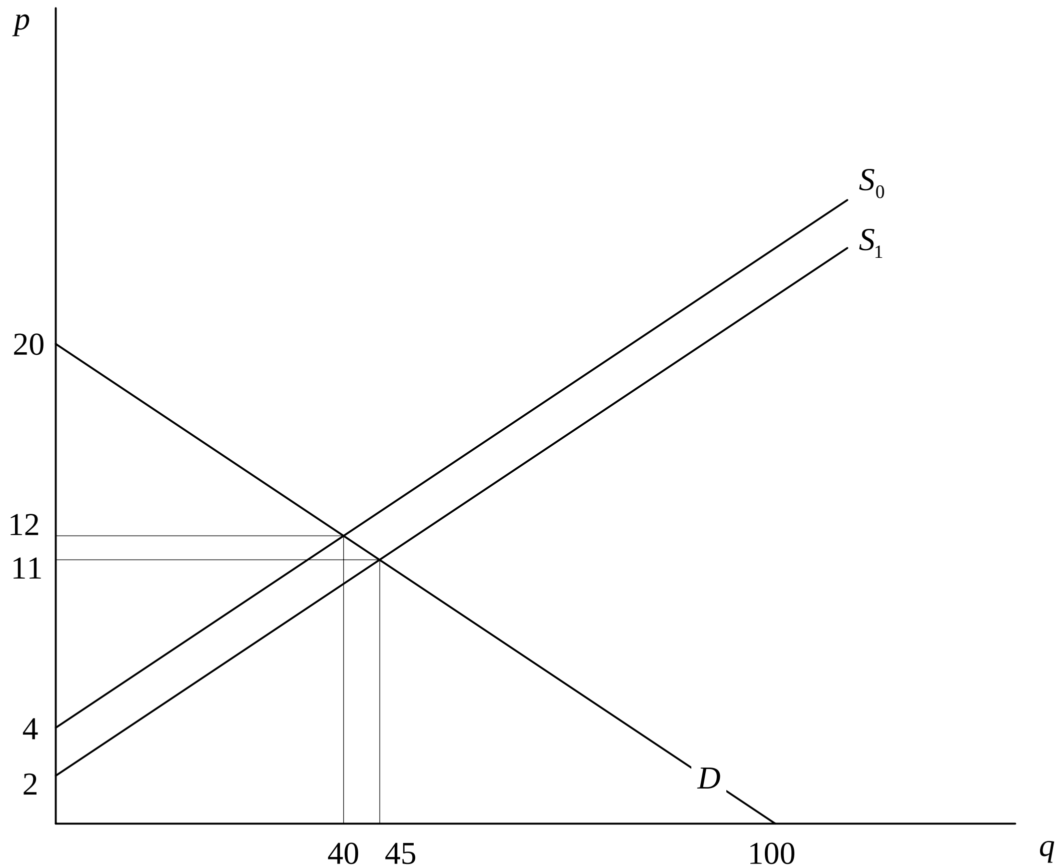

Figure R4-5

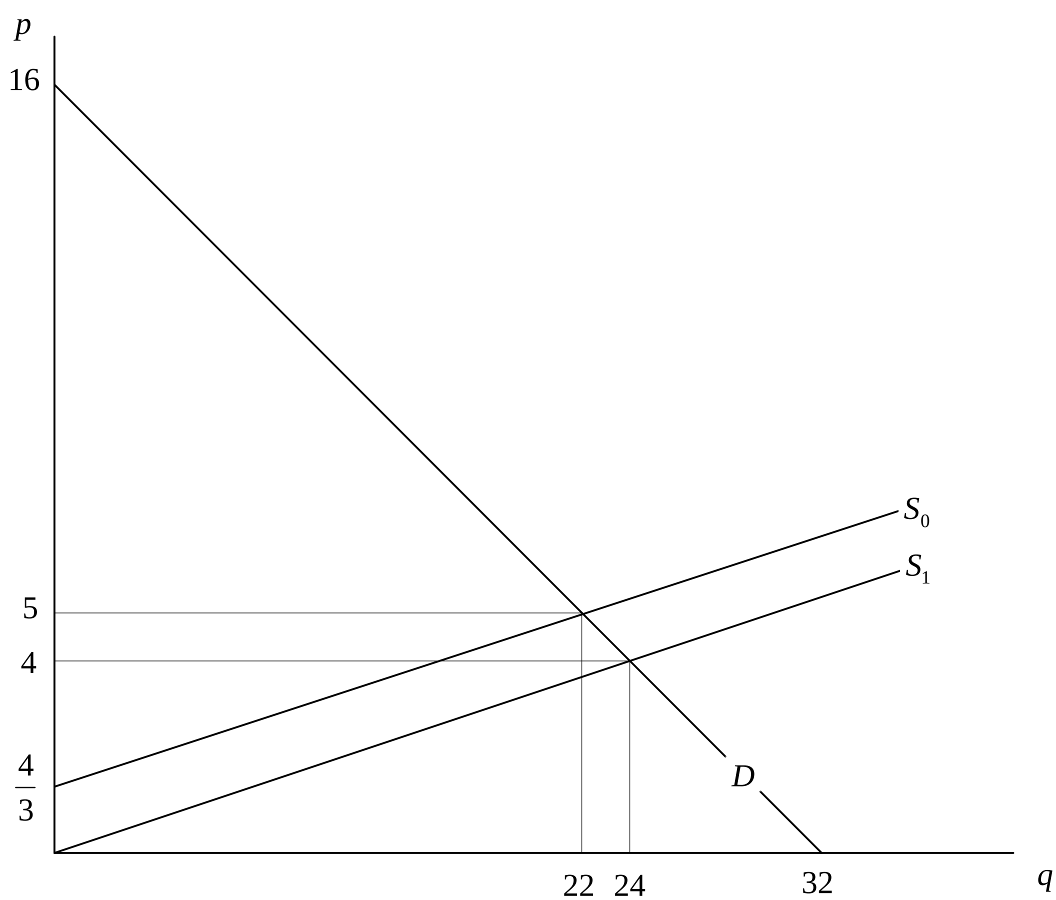

Figure R4-6

Figure R4-7

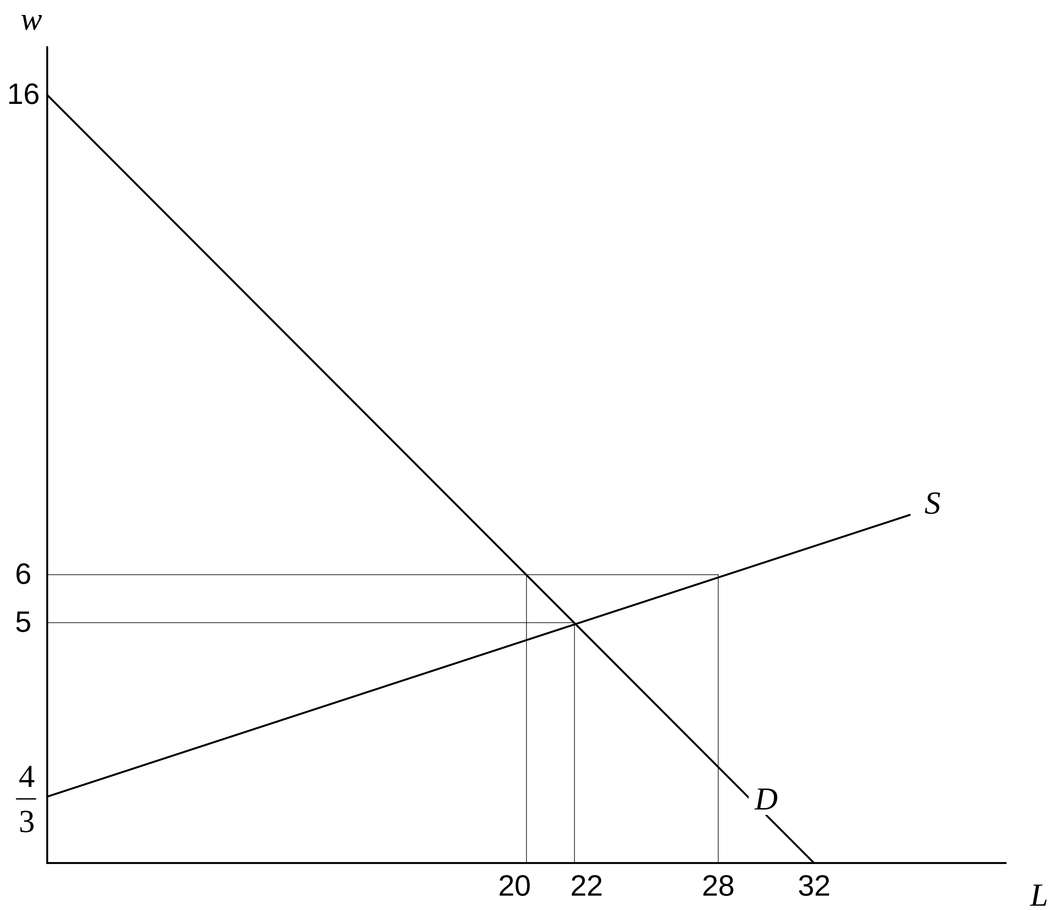

Figure 4R-8

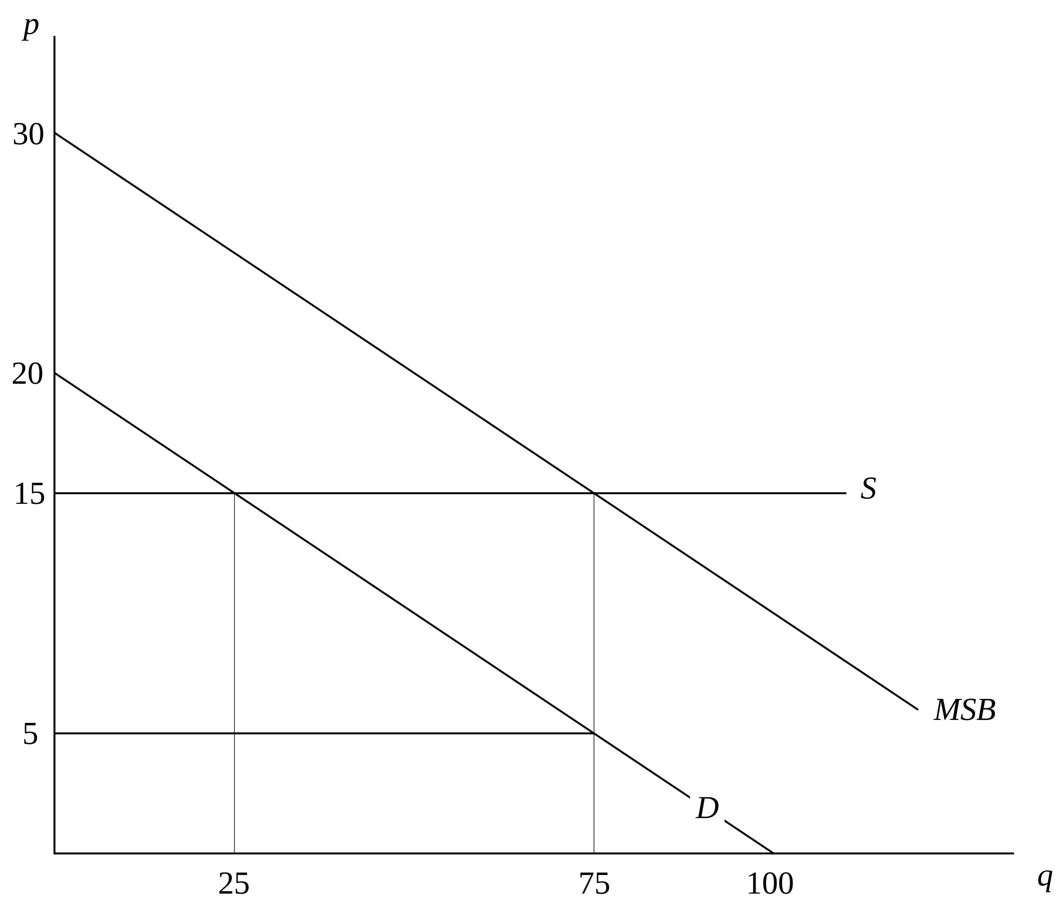

Figure R4-9

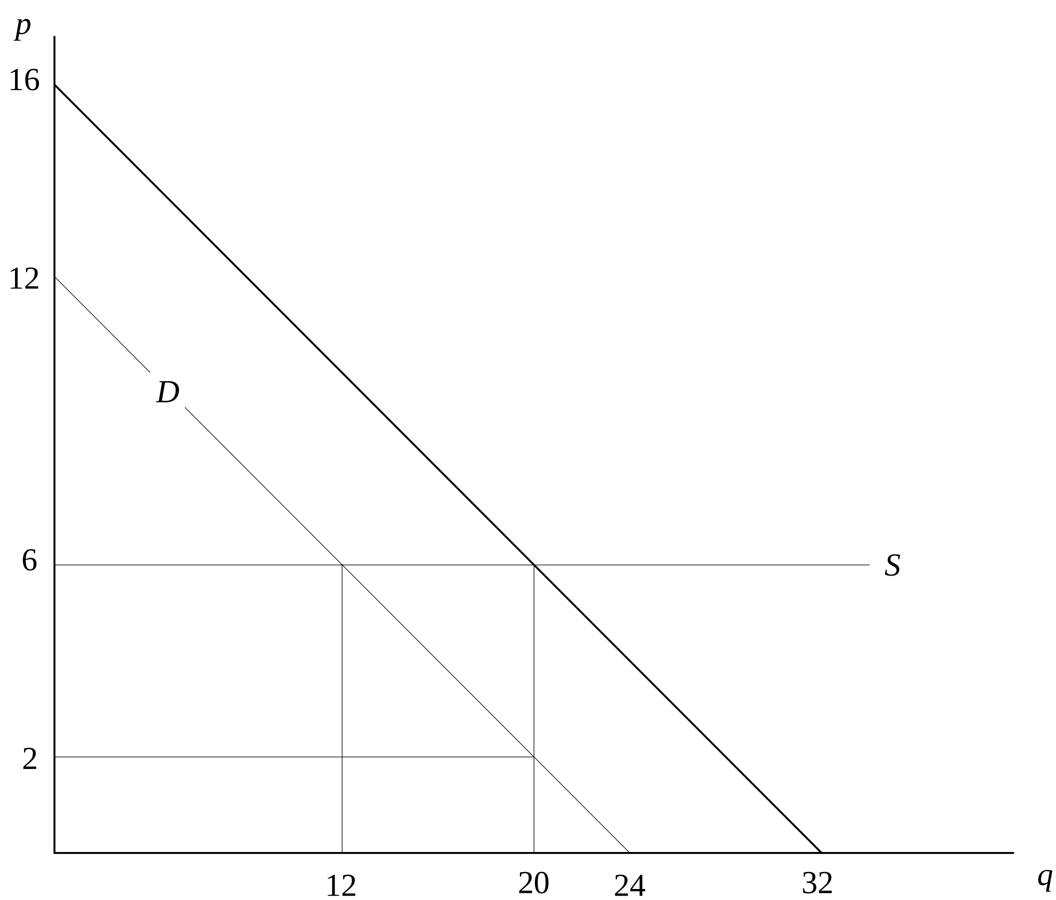

Figure R4-10

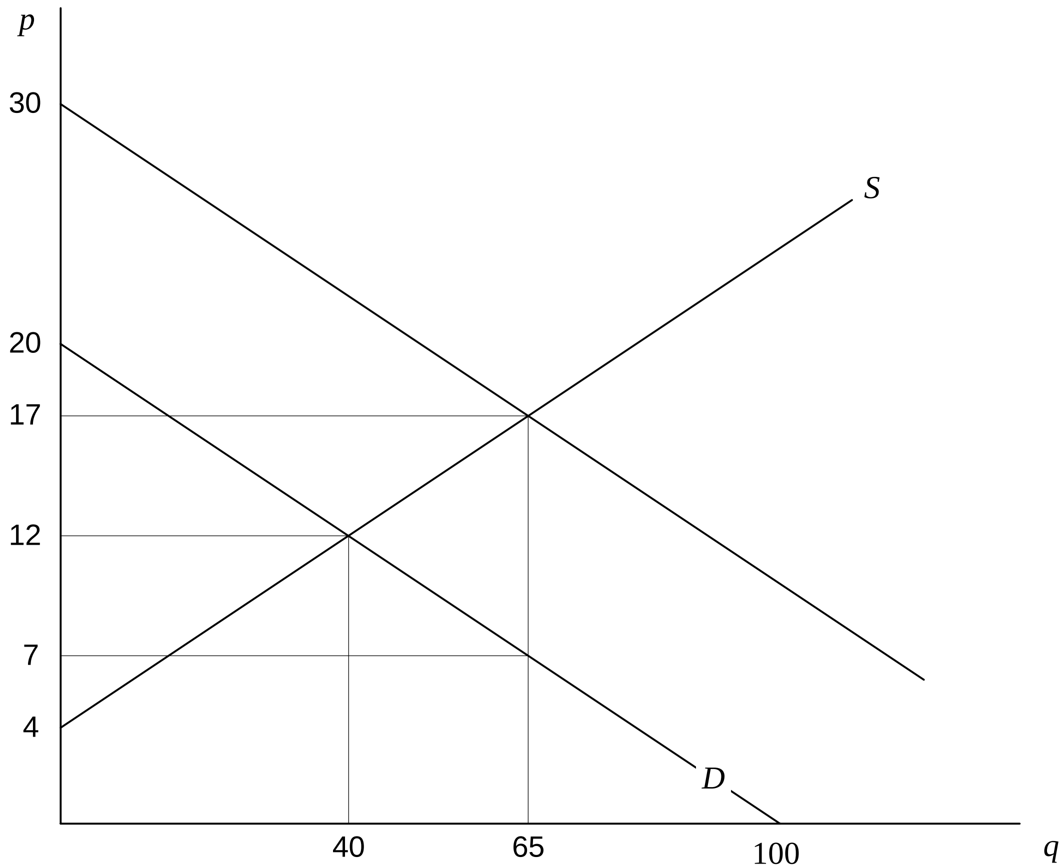

Figure R4-11

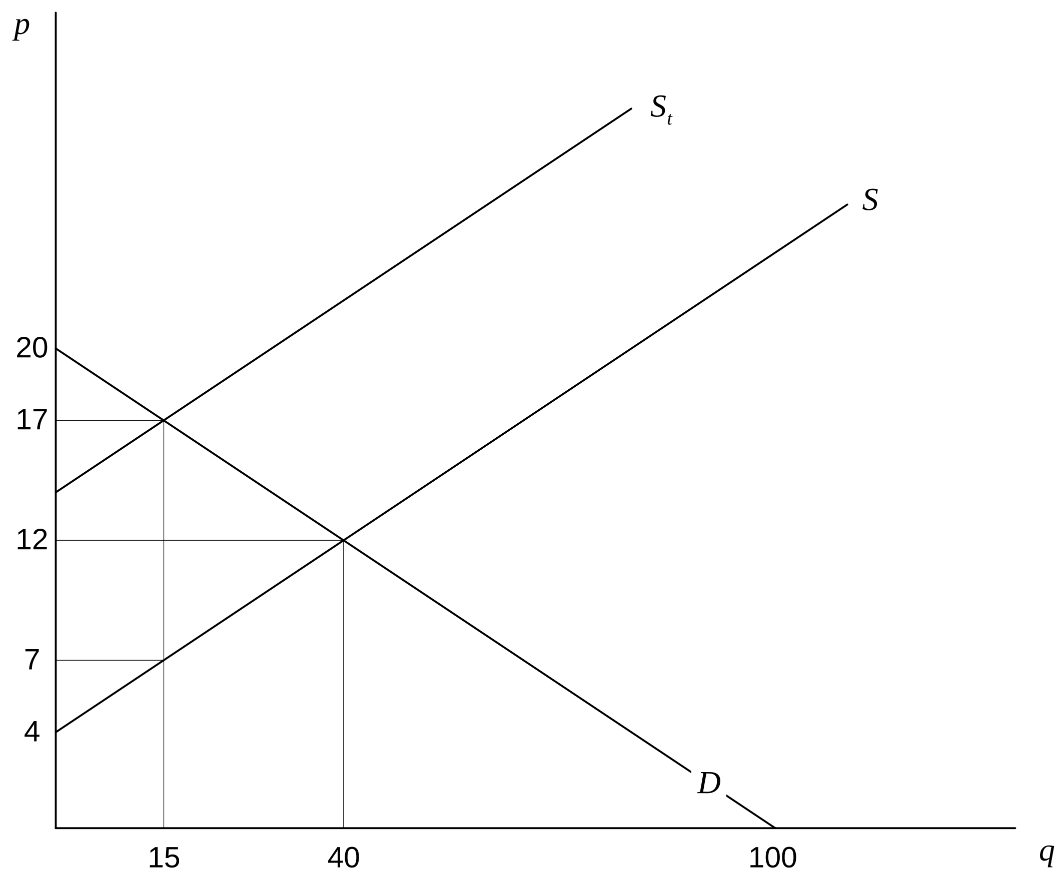

Figure R4-12

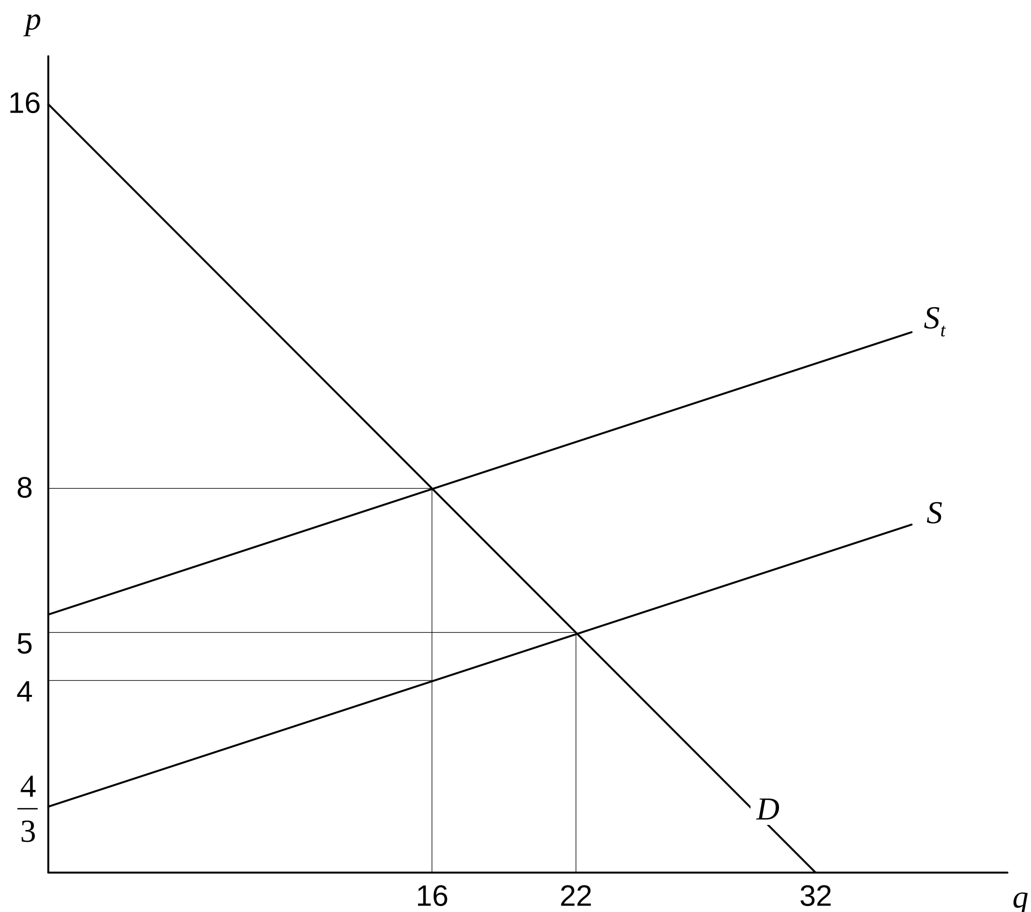

Figure R4-13

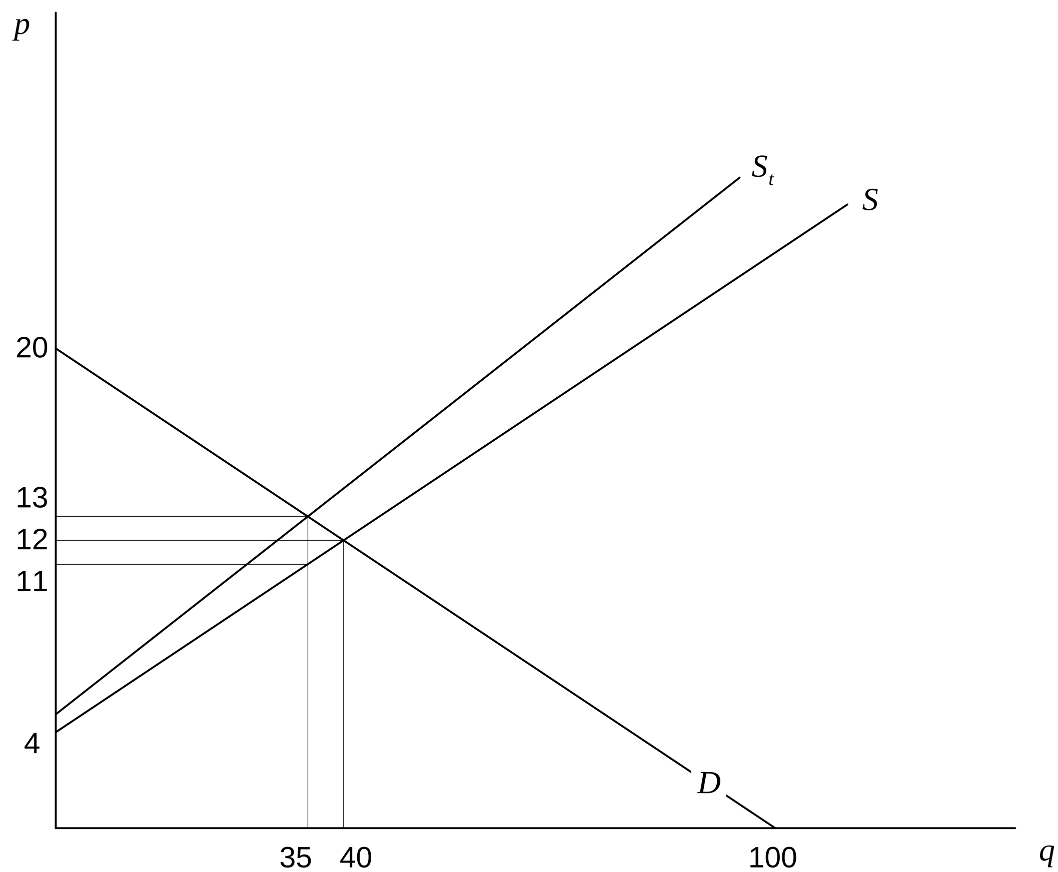

Figure R4-14

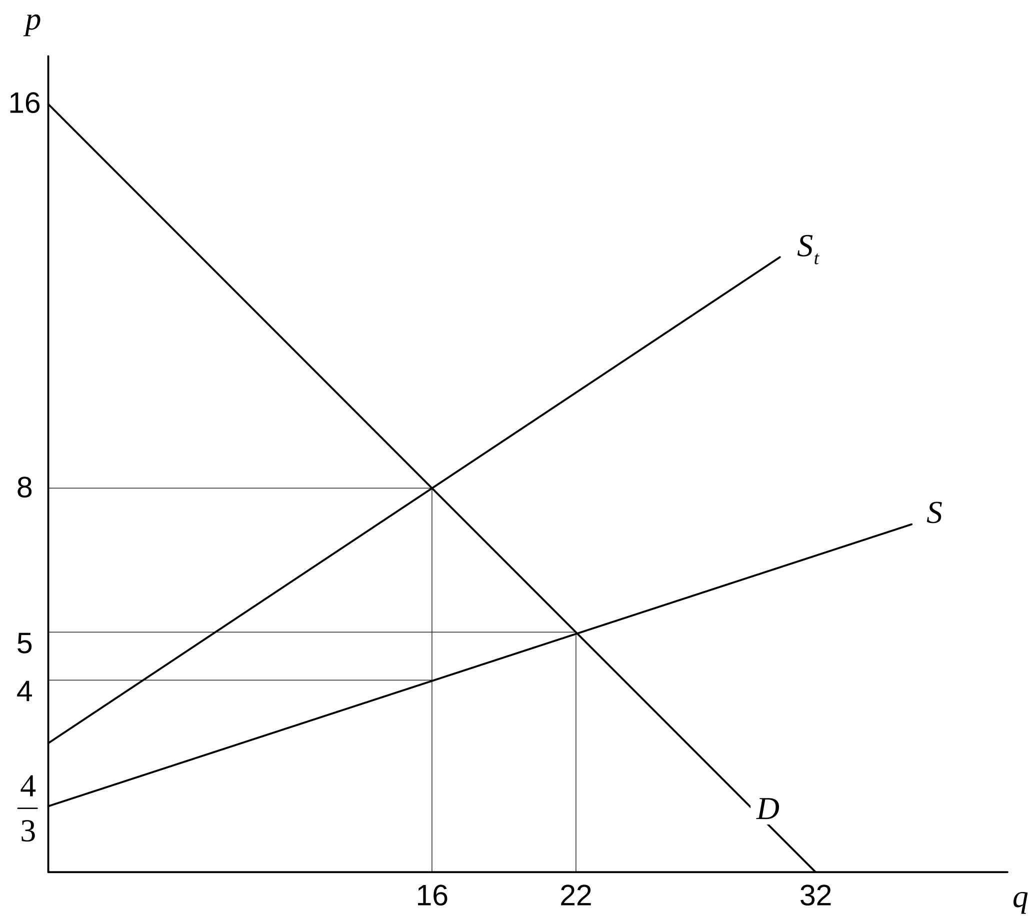

Figure R4-15

ANSWER KEY

1.	B	It augments the existing demand
2.	B	Via "crowding out"
3.	A	See Figure R4-A1 at the end of this answer key.
4.	D	Consumer surplus falls so *dCS*<0. See Figure R4-A2 at the end of this answer key.
5.	A	Producer surplus rises so *dPS*>0. See Figure R4-A3 at the end of this answer key.
6.	D	The outlay is positive. See Figure R4-A4 at the end of this answer key.
7.	C	Calculated as: outlay + (negative of *dCS*) – (*dPS*). See Figure R4-A5 at the end of this answer key.
8.	C	See Figure R4-A6 at the end of this answer key.
9.	A	See Figure R4-A7 at the end of this answer key.
10.	A	but the relationship between the answer to Q7 and the sum of the answers to Q8 and Q9 is not a coincidence.
11.	A	The difference between outlay and cost
12.	A	
13.	D	The reduction in consumer surplus is a consequence of the crowding out
14.	B	
15.	D	
16.	B	
17.	C	This is the negative of the financial outlay by government
18.	D	The cost is [outlay + (negative of *dCS*) – (*dPS*) = $210]
19.	B	
20.	C	
21.	B	Approximating the cost using the average of pre- and post-project prices is exact when supply and demand are both linear
22.	C	
23.	B	It augments the existing supply
24.	A	The reduction in price leads to the reduction in the amount privately

supplied but the price reduction per se does absorb production

25. B
26. C
27. B Consumer surplus rises so $dCS>0$
28. A Producer surplus falls so $dPS<0$
29. D
30. A The benefit is [financial receipt + dCS + dPS]
31. C
32. C
33. B
34. A
35. D
36. B
37. A
38. C
39. D The change in producer surplus is $-\$19$
40. C
41. B
42. B
43. C
44. B
45. B The supply curve is <u>shifted</u>
46. C More firms might enter the market but this does not cause costs to fall; causation runs in the other direction.
47. A The % reduction is <u>not</u> a constant; it depends on where production occurs on the supply curve.
48 D The change in <u>equilibrium</u> marginal cost is $1 per unit (same as the price change) because production rises, moving production up along the new supply curve.
49 C See Figure R4-A8 at the end of this answer key.
50. C The original PS is $160 (as measured under the original supply schedule).

The new producer surplus is \$202.5 (as measured under the new supply schedule). See Figure R4-A9 at the end of this answer key.

51. D There are no implications for taxpayers from the changes in this market.

52. A See Figure R4-A10 at the end of this answer key.

53. B A reduction of \$2 per unit on each of 40 units. See Figure R4-A11 at the end of this answer key.

54. D Calculated as the difference in areas under the demand and new supply over the increment in quantity supplied. See Figure R4-A12 at the end of this answer key.

55. B

56. A

57. D

58. B

59. D

60. A

61. C

62. D The benefit is $\$30\frac{2}{3}$

63. A

64. D

65 B

66. A

67. A

68. B We know only that the 5 workers are drawn from somewhere in the 4 – 14 range.

69. B Calculated as $5\frac{(14+0)}{2}=35$

70. B Calculated as $5\left(14-\frac{14+0}{2}\right)=35$

71. B

72. B

73. D

74. C Calculated as $3\left(\frac{6+\frac{4}{3}}{2}\right)=11$

75. A Calculated as $3\left(6-\frac{6+\frac{4}{3}}{2}\right)=7$

76. B

77. A

78. D

79. B

80. D The external benefit is $500 (calculated as $10 x the increase in consumption)

81. C

82. B

83. C

84. B

85. D The optimal subsidy is $4 per unit

86. C

87. A

88. A

89. B

90. D The net social benefit is $16

91. C

92. B

93. A

94. C See Figure R4-A13 at the end of this answer key.

95. B See Figure R4-A14 at the end of this answer key.

96. B See Figure R4-A15 at the end of this answer key.

97. A See Figure R4-A16 at the end of this answer key.

98. C See Figure R4-A17 at the end of this answer key.

99. D See Figure R4-A18 at the end of this answer key.

100. C See Figure R4-A19 at the end of this answer key.

101. A See Figure R4-A20 at the end of this answer key.

102. A
103. C
104. C
105. B
106. D
107. B
108. B
109. A
110. A
111. B
112. C
113. A
114. C
115. B
116. B
117. C
118. A
119. B
120. A
121. A
122. A
123. B
124. C
125. A

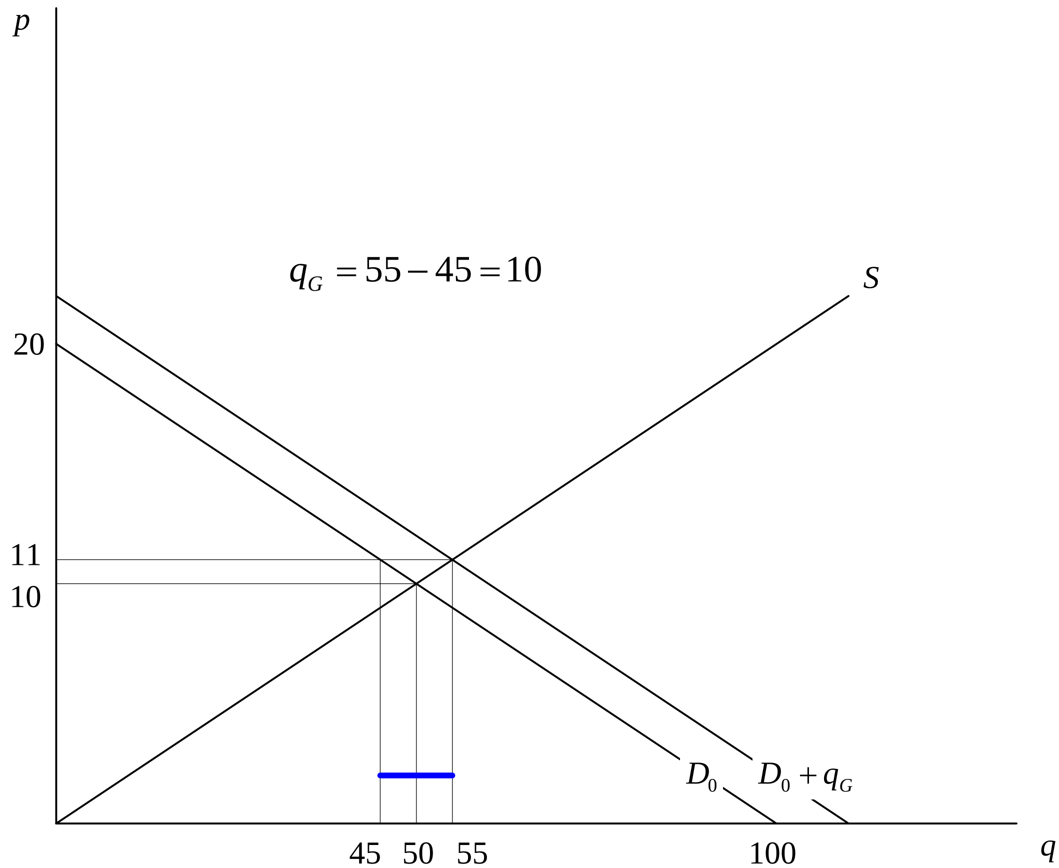

Figure R4-A1

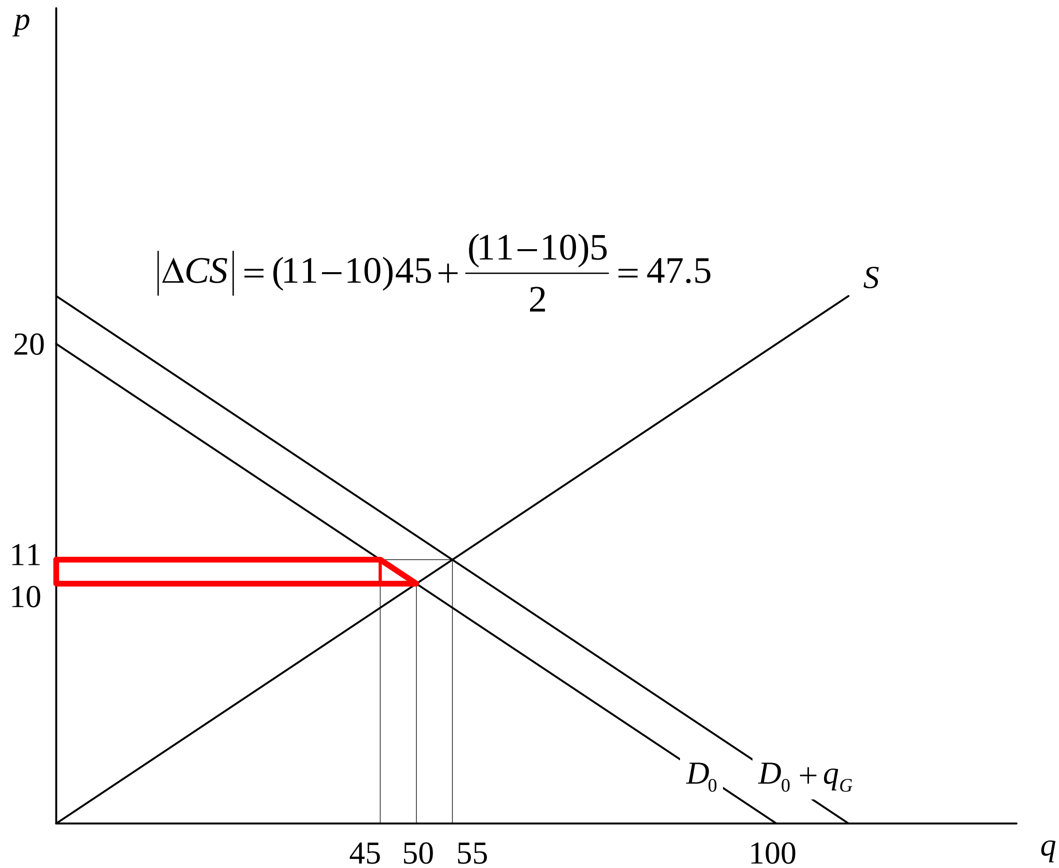
p
$|\Delta CS| = (11-10)45 + \frac{(11-10)5}{2} = 47.5$
S
20
11
10
D_0
$D_0 + q_G$
45
50
55
100
q

Figure R4-A2

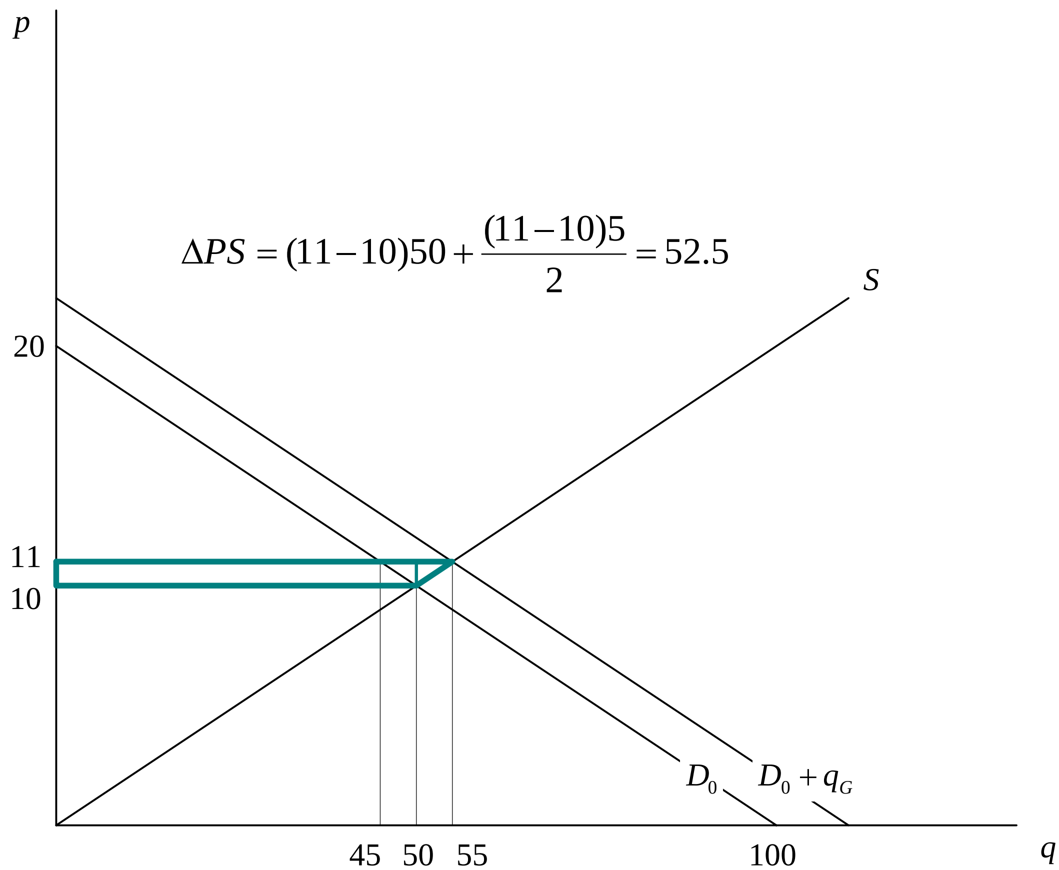
p
$\Delta PS = (11-10)50 + \frac{(11-10)5}{2} = 52.5$
S
20
11
10
D_0
$D_0 + q_G$
45
50
55
100
q

Figure R4-A3

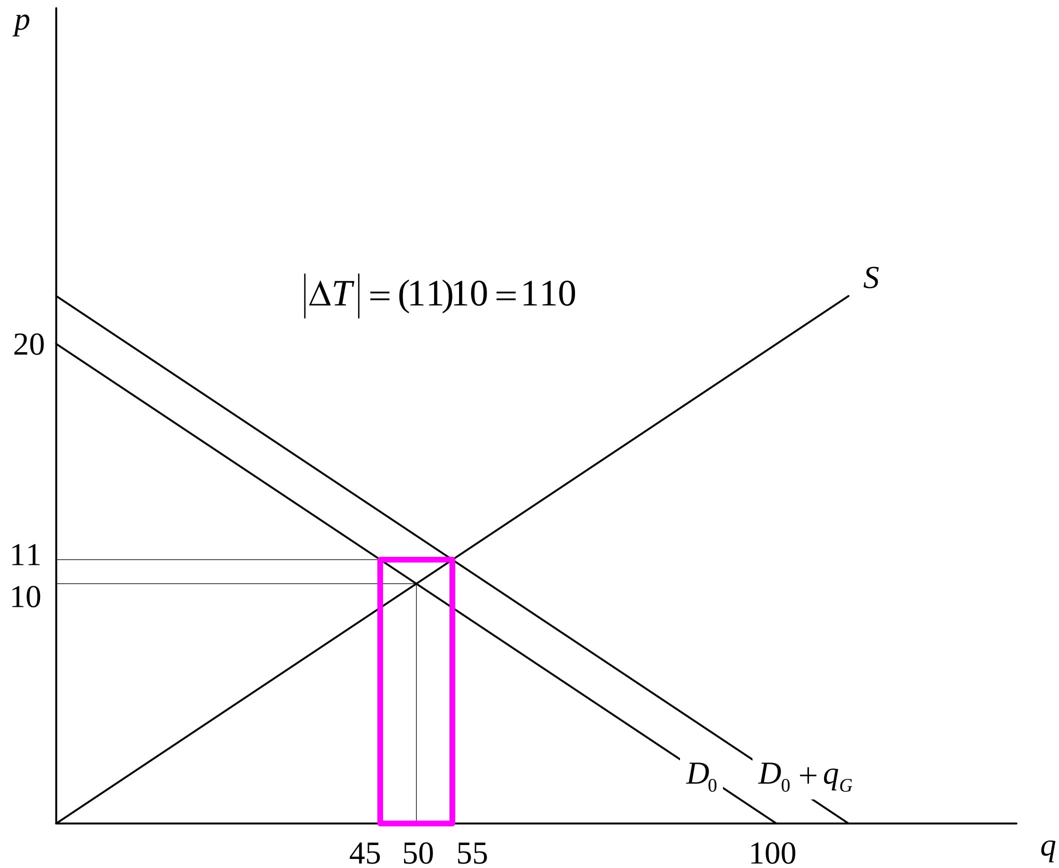

Figure R4-A4

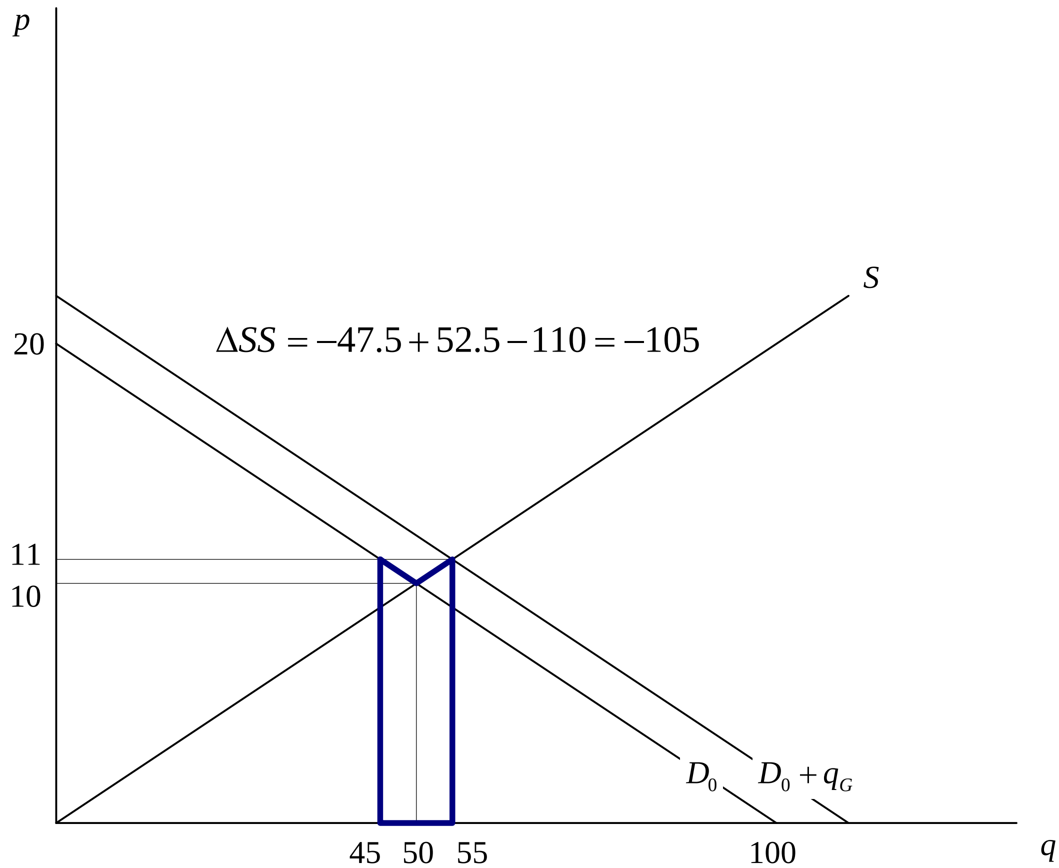

Figure R4-A5

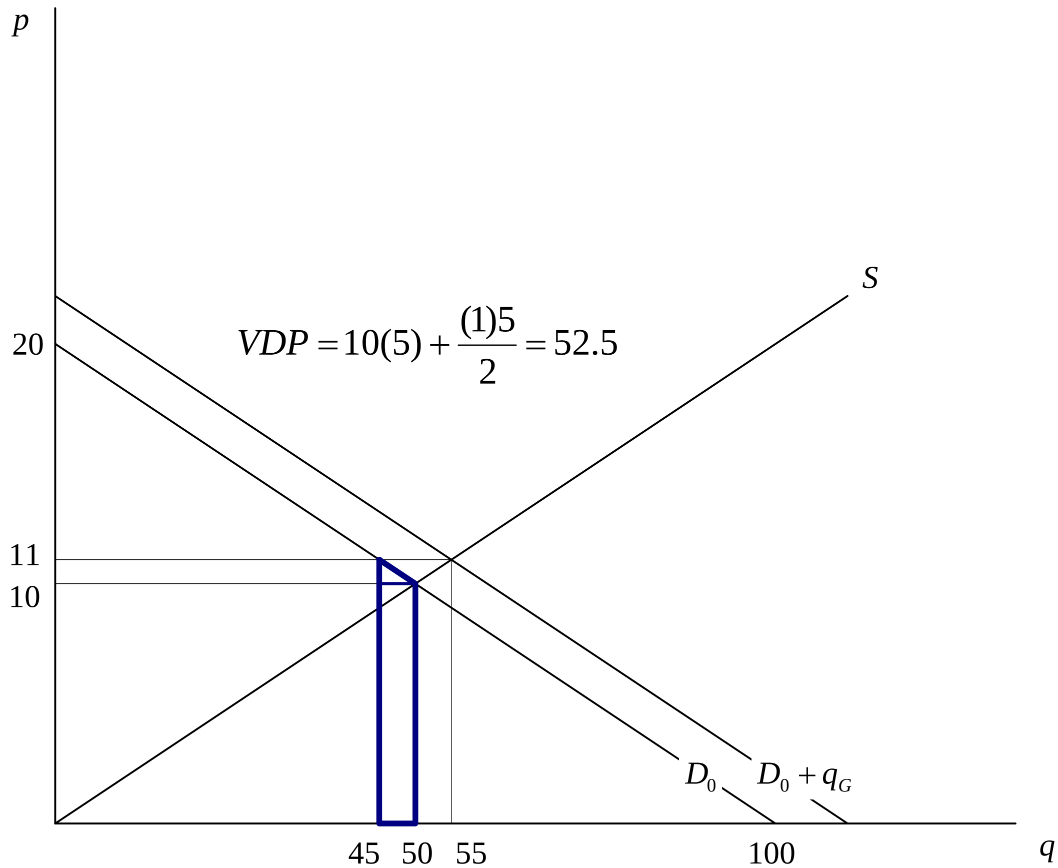

Figure R4-A6

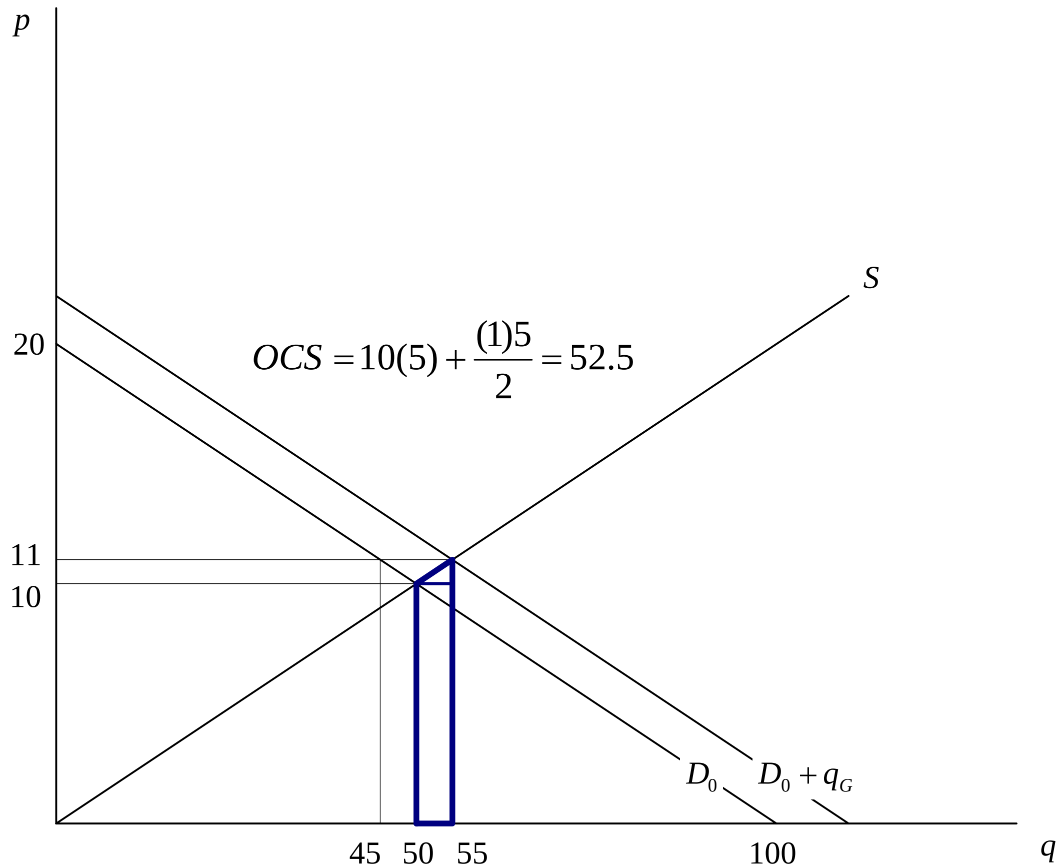

Figure R4-A7

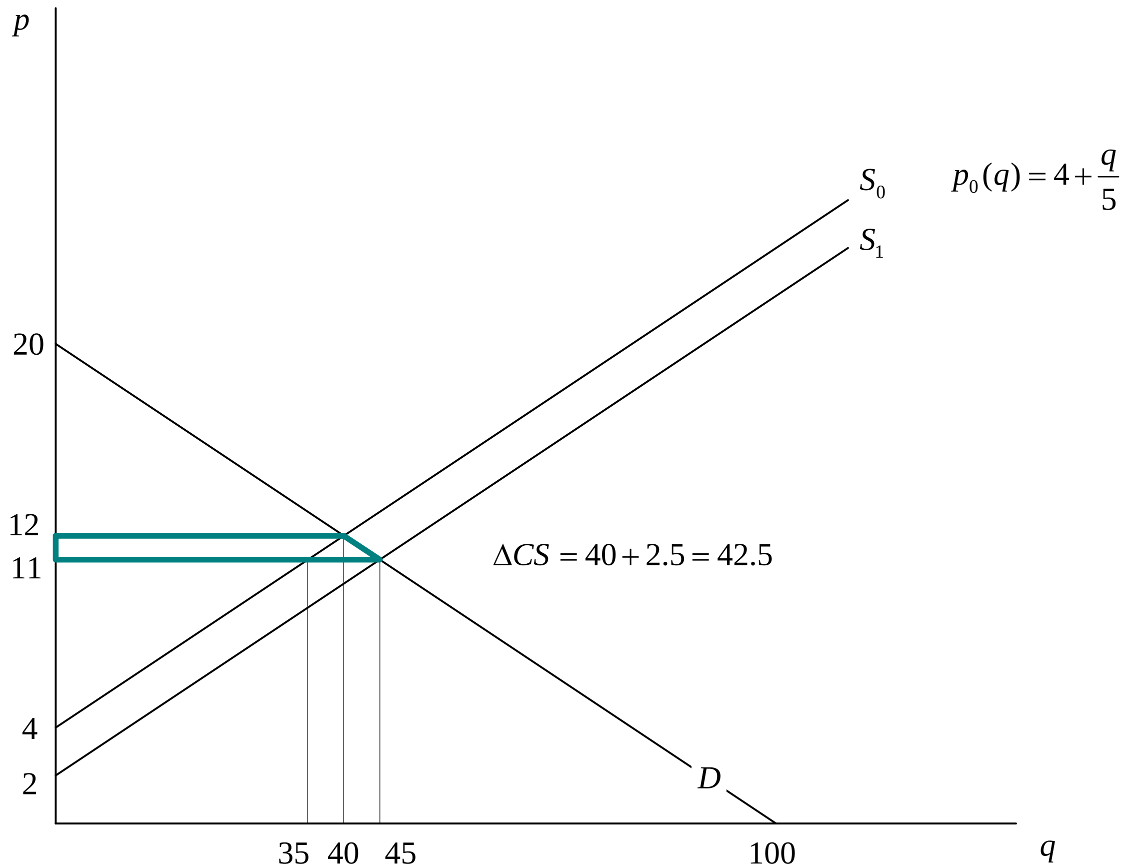

Figure R4-A8

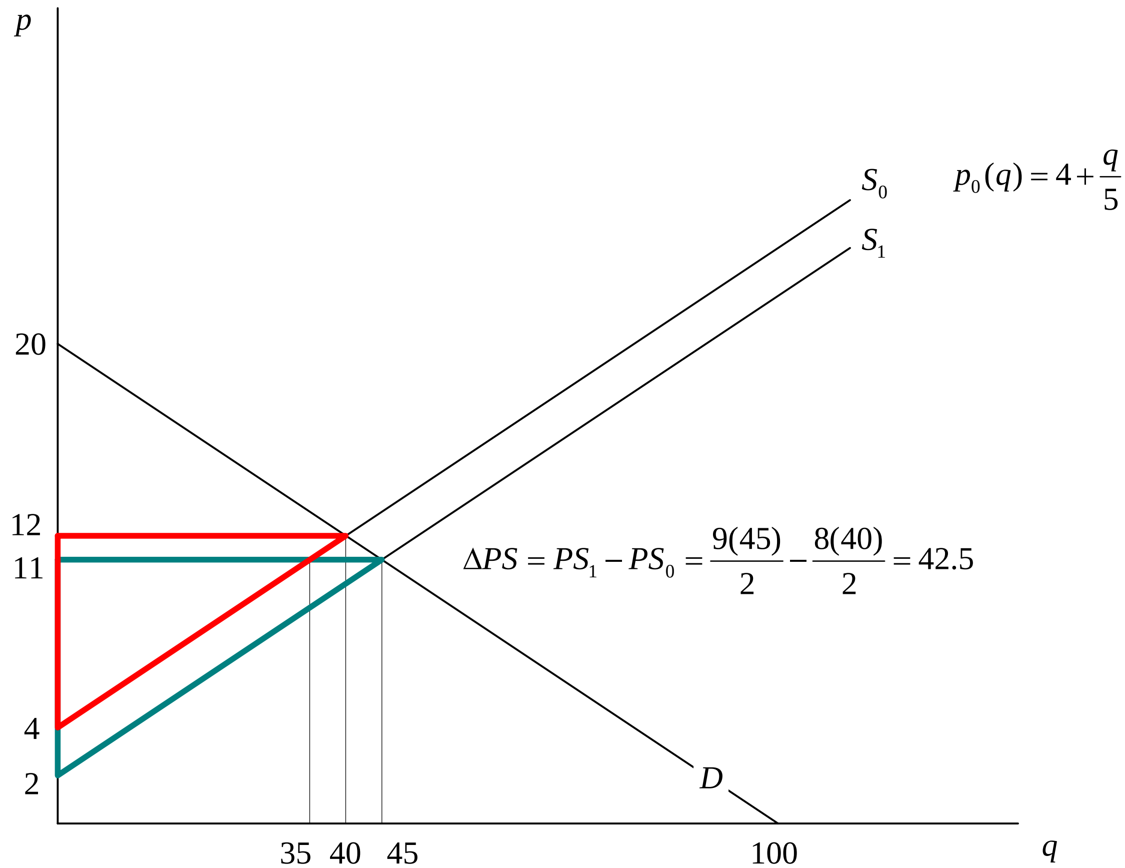
p
q
20
12
11
4
2
35
40
45
100
S_0
S_1
D
$p_0(q) = 4 + \frac{q}{5}$
$\Delta PS = PS_1 - PS_0 = \frac{9(45)}{2} - \frac{8(40)}{2} = 42.5$

Figure R4-A9

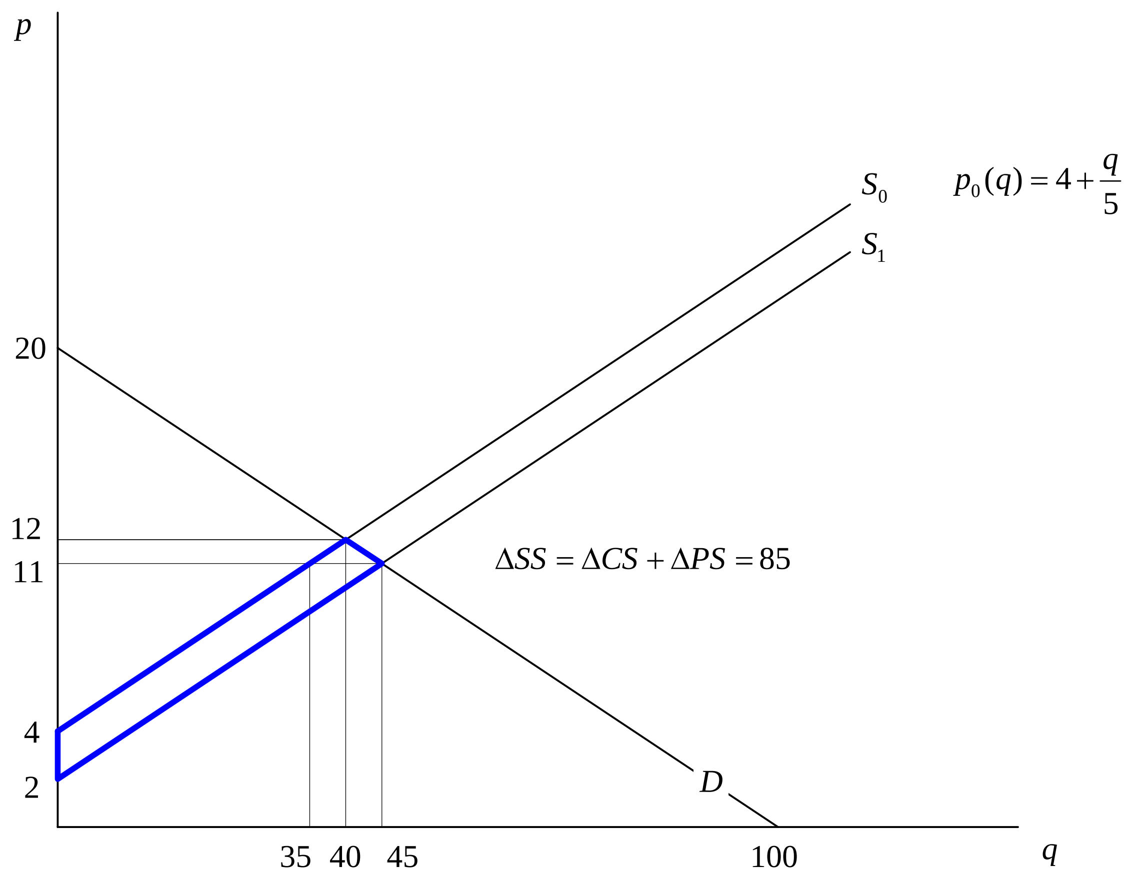

Figure R4-A10

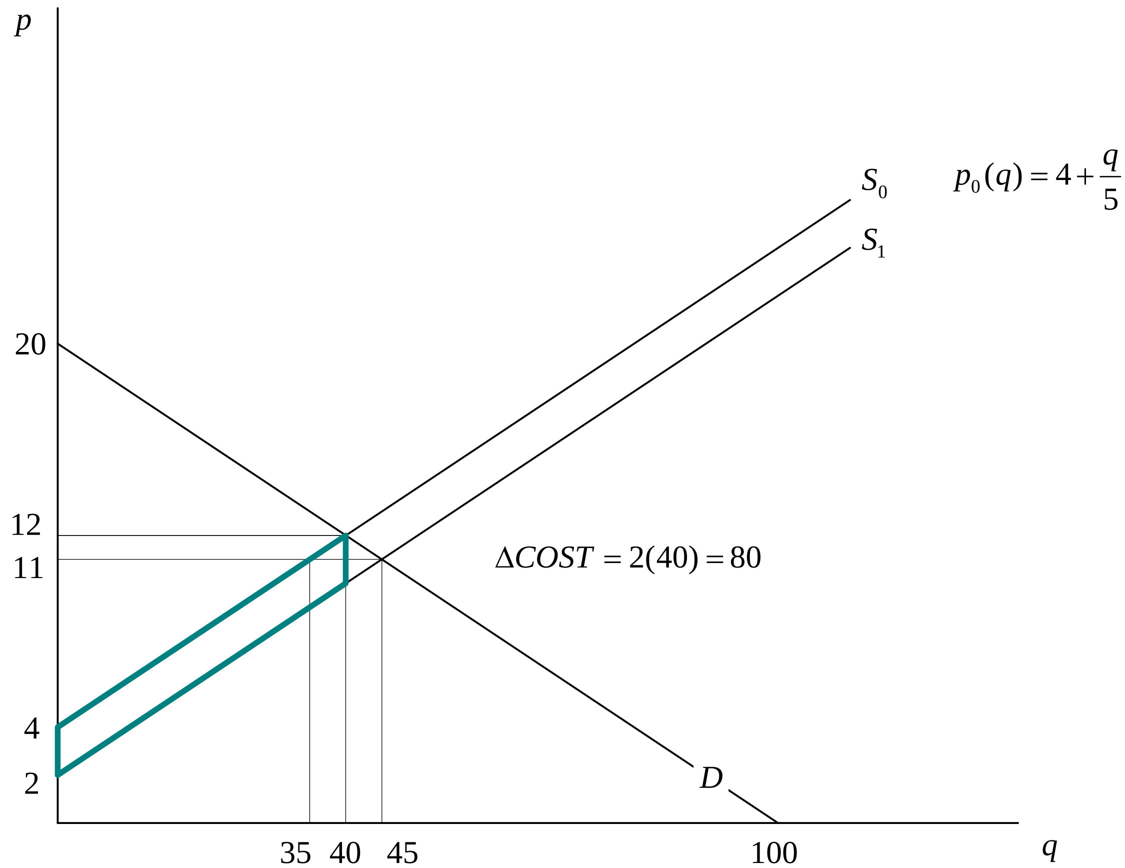

Figure R4-A11

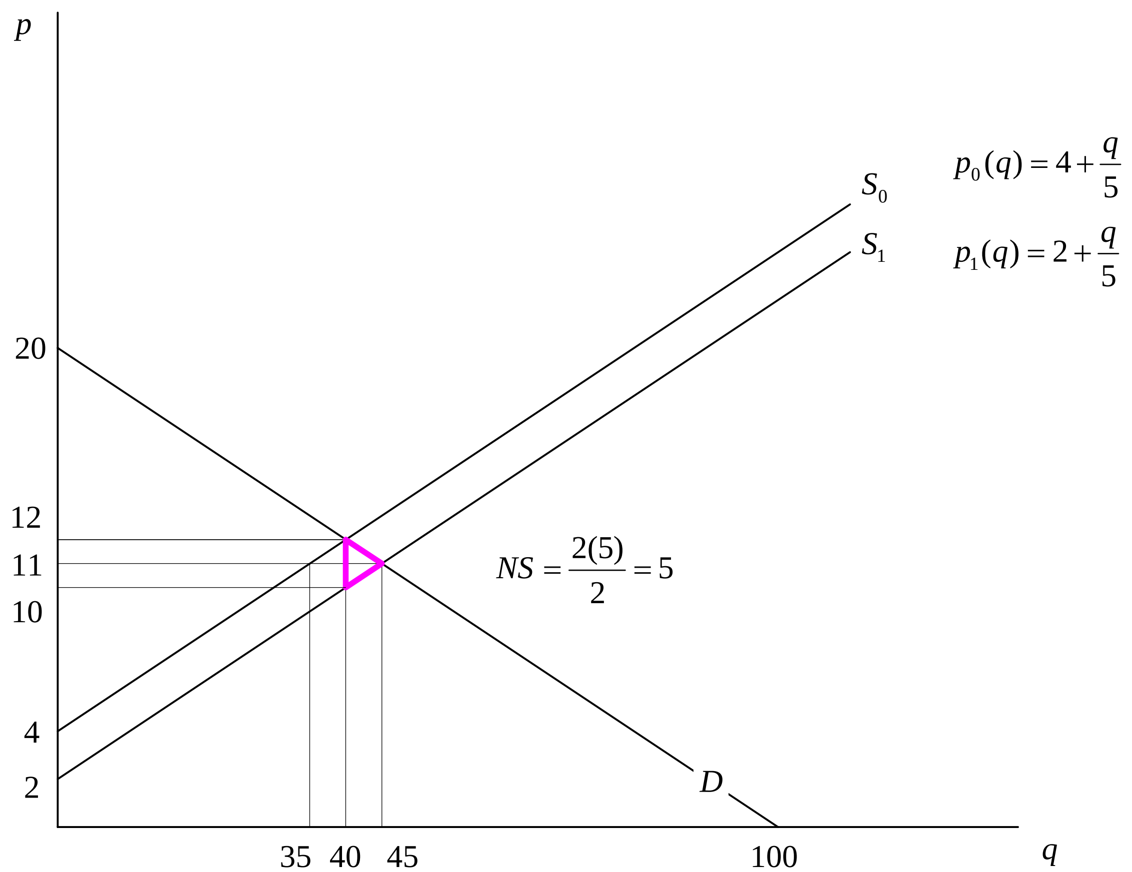

Figure R4-A12

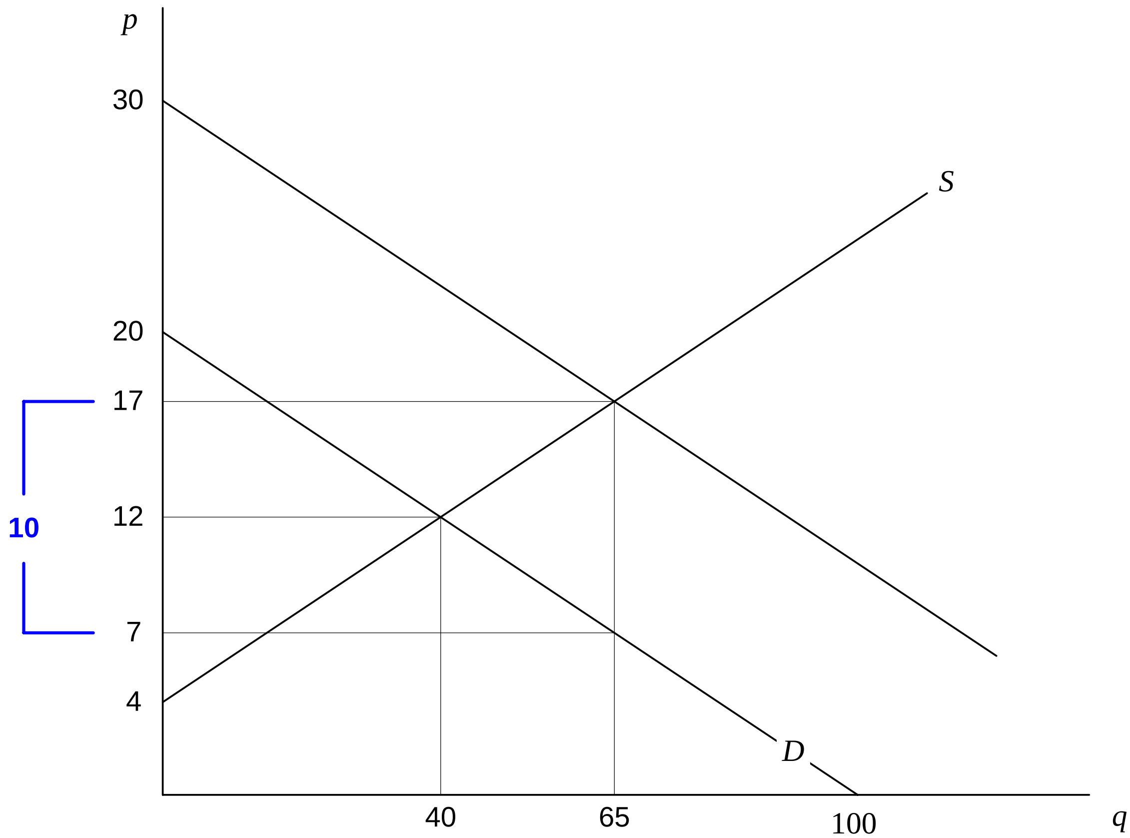

Figure R4-A13

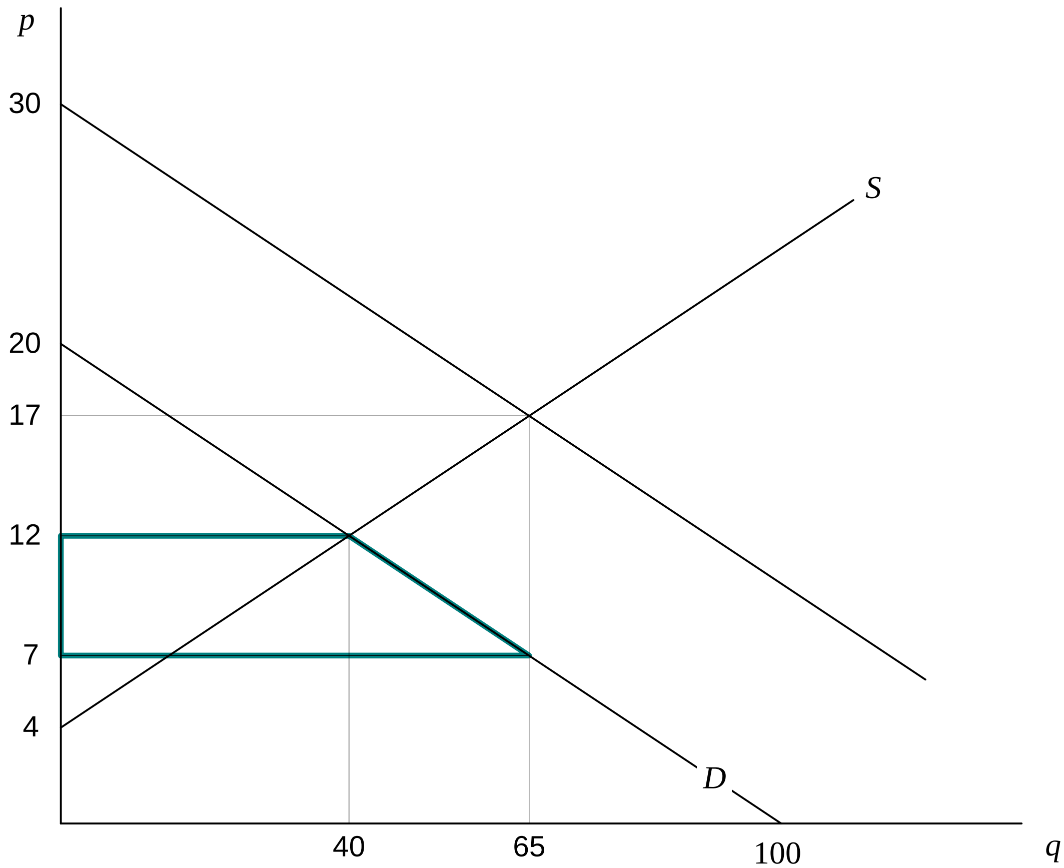

Figure R4-A14

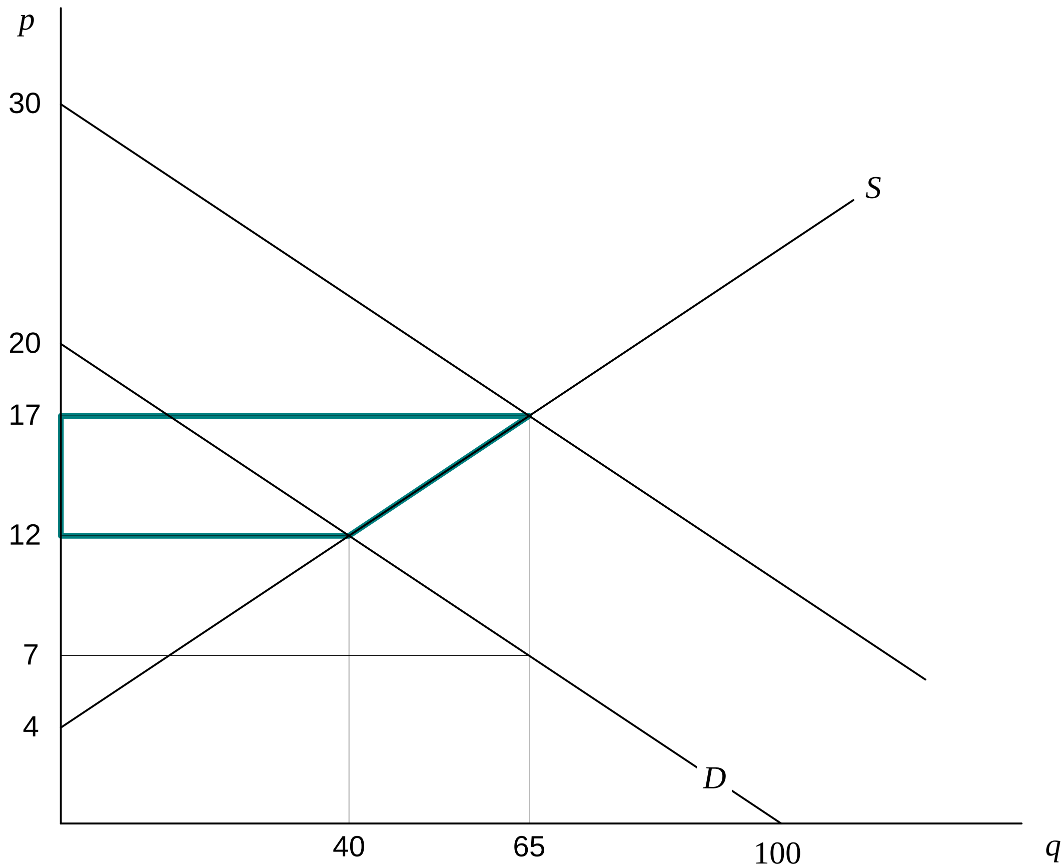

Figure R4-A15

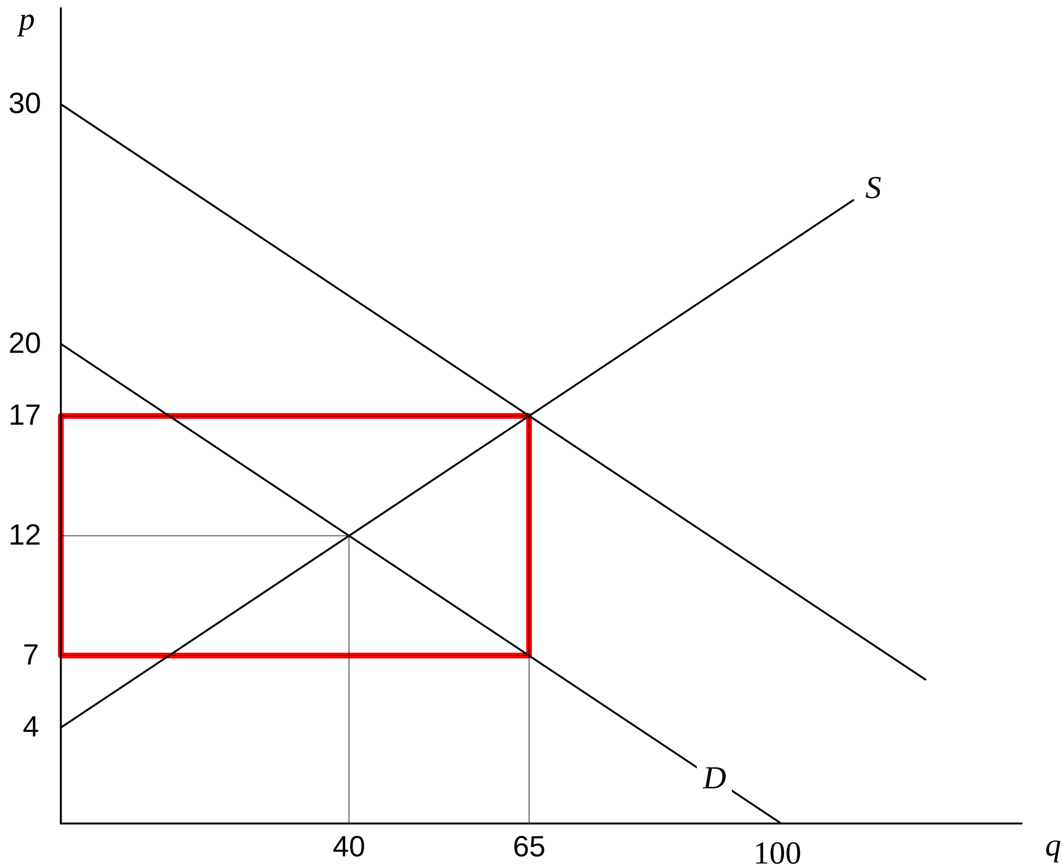

Figure R4-A16

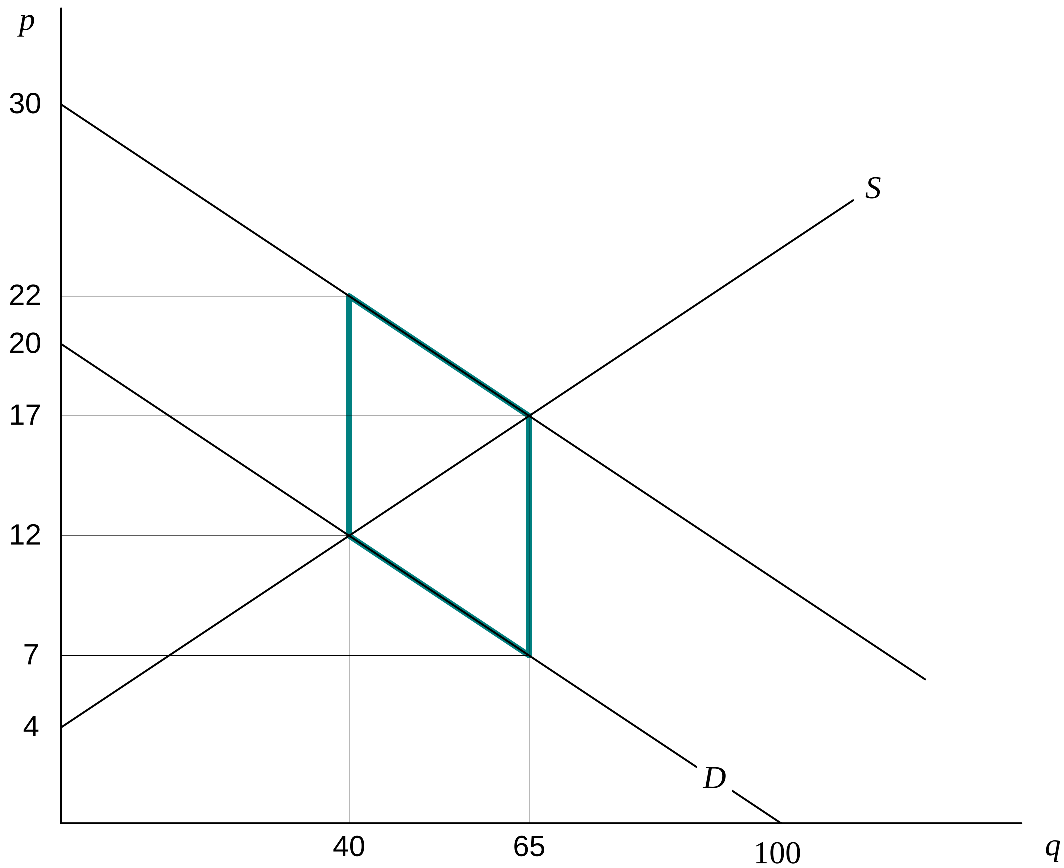

Figure R4-A17

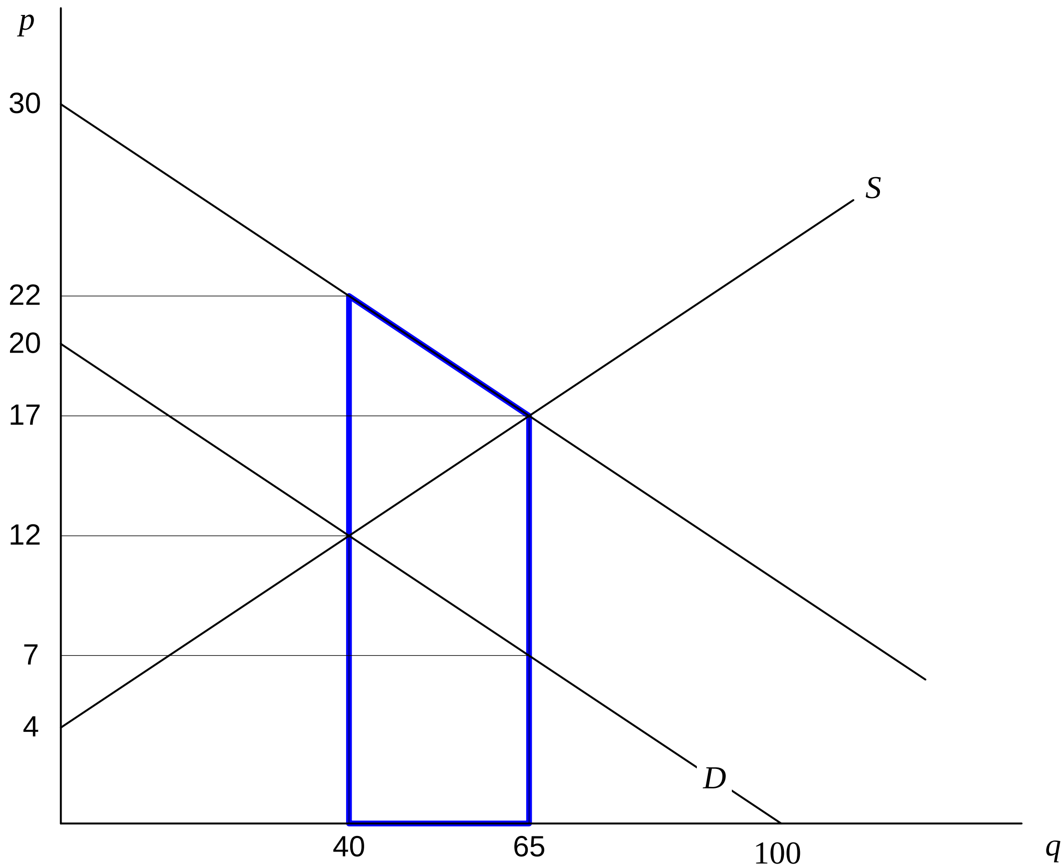

Figure R4-A18

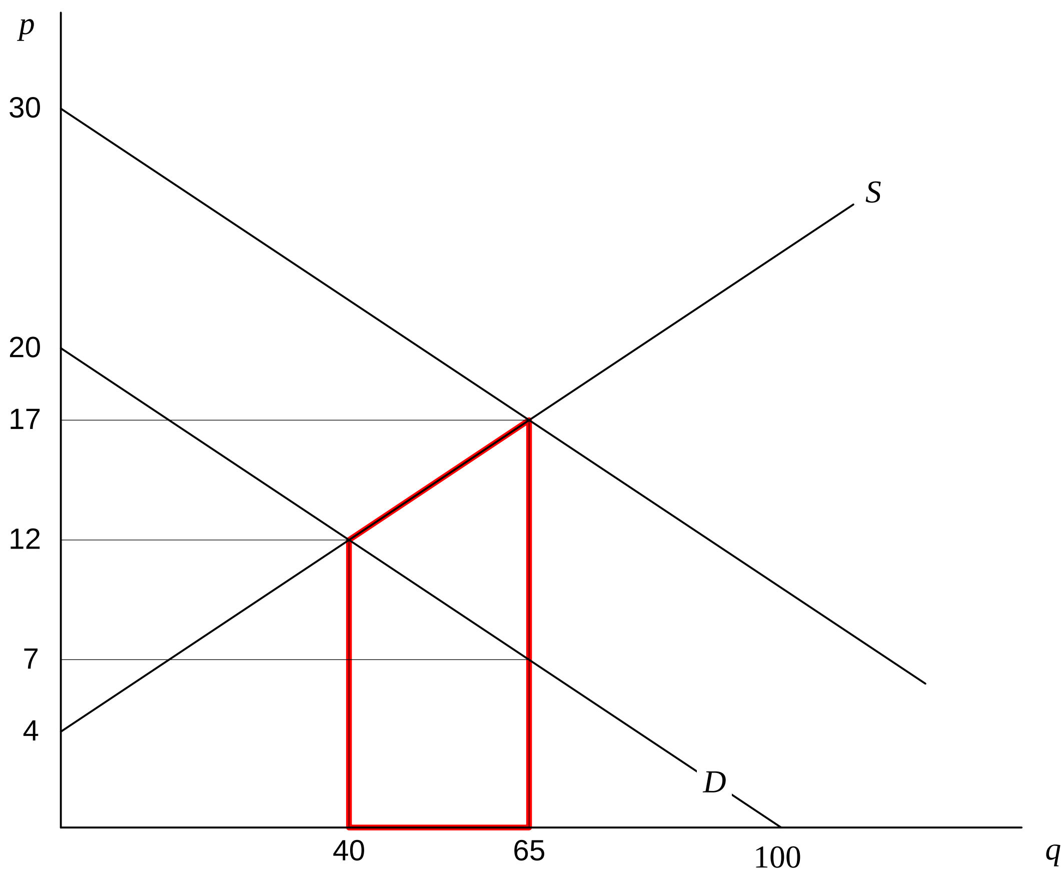

Figure R4-A19

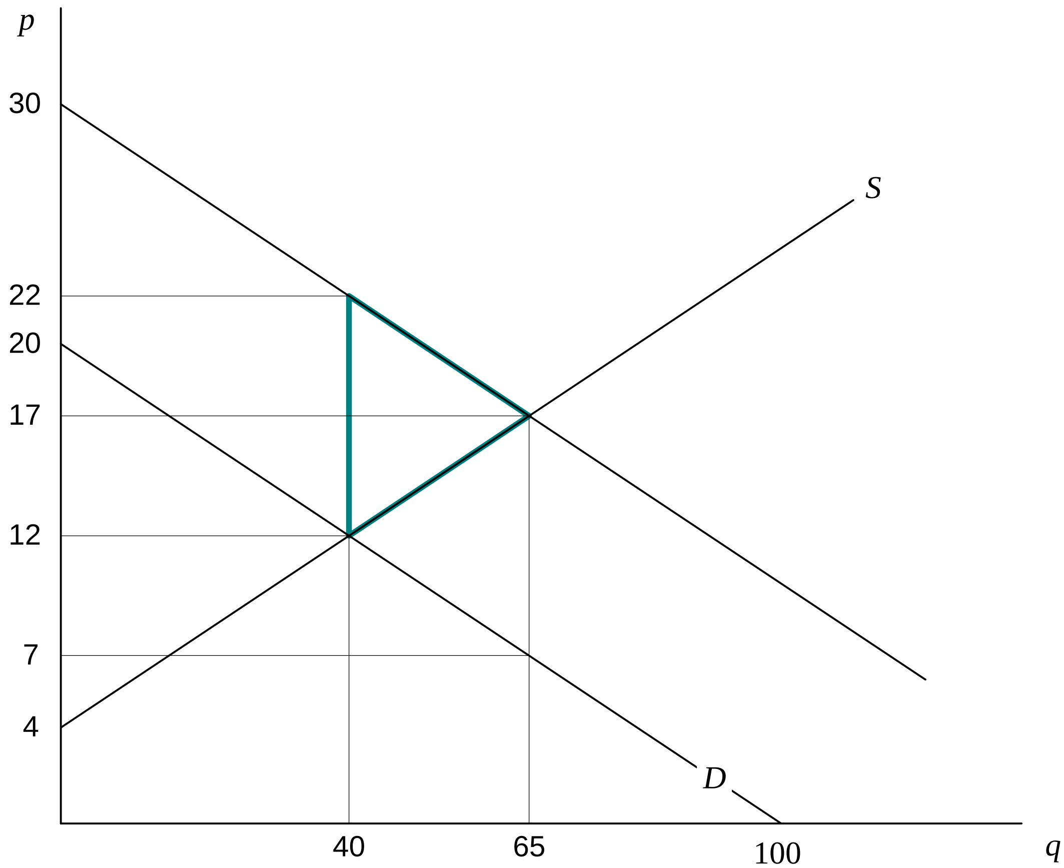

Figure R4-A20

5. NET PRESENT VALUE

OUTLINE

5.1 Introduction

5.2 A Simple Model of Savings and Investment

5.3 Calculating Net Present Value

5.4 Project Scale

5.5 Project Timing

5.6 Project Re-Appraisal

2

3

5.1 INTRODUCTION

- Most projects produce a stream of costs and benefits across time, and a dollar gained or lost today is not the same as a dollar gained or lost next year.
- In CBA future benefits and costs are discounted at the **public sector discount rate** (PSDR).

4

- Why discount the future?
  - marginal rate of time preference
  - investment rate of return

- We will soon see that together these determine the supply of savings and the demand for investment, and the equilibrium interest rate.
- We start with Figure 5-1.

5

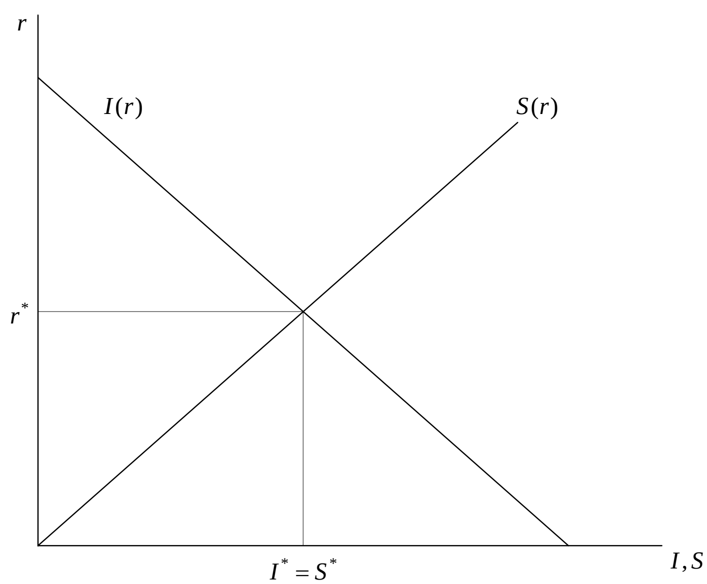

Figure 5-1

6

- In a simple undistorted capital market like the one depicted in Figure 5-1, there is a single interest rate, and in this economy the PSDR is simply equal to this interest rate.

- To understand why, we will explore the issue further in the context of a simple model of savings and investment.

7

5.2 A SIMPLE MODEL OF SAVINGS AND INVESTMENT

- Consider a hypothetical economy with a single good that can be consumed or invested.

- Imagine this good as being harvested grain.

8

- An individual with current income y_0 makes an intertemporal consumption choice based on the market interest rate *r*.
- She has no future income and must allocate her current income between current consumption c_0, and future consumption c_1.
- See Figure 5-2.

9

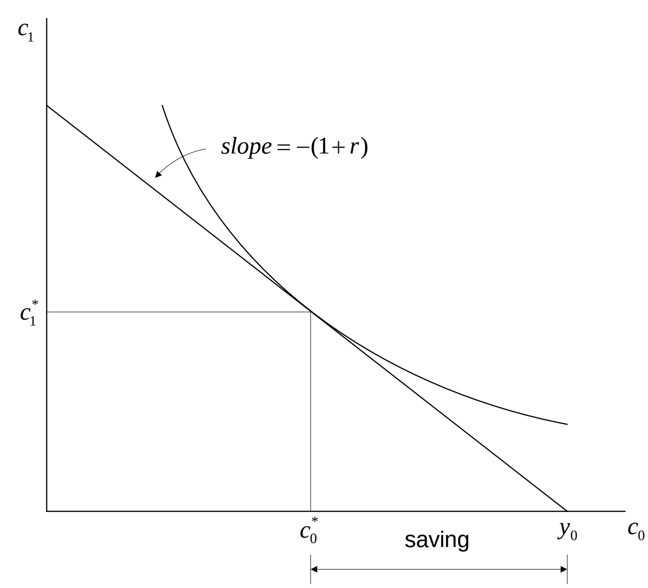

Figure 5-2

10

- The optimal choice is characterized by

$$MRS = (1 + r)$$

where MRS is the **marginal rate of substitution** between current consumption and future consumption.

11

- It turns out to be more convenient to express the MRS as

$$MRS = 1 + MRTP$$

where MRTP is the **marginal rate of time preference**.

12

- If the capital market is free of distortions then all individuals face the same interest rate *r*, so in equilibrium, for any two individuals *i* and *j*:

$$MRTP_i = r = MRTP_j \quad \forall i, j$$

13

- Under these distortion-free conditions, we can identify a single **social rate of time preference** (SRTP) for this economy:

$$SRTP = MRTP_i = r \quad \forall i$$

- This SRTP is called the **social discount rate** for this economy.

14

- Aggregate savings in this economy are invested by firms in activities that yield a future consumption stream.

- For example, in our grain economy, planting one seed today yields a return of harvested grain next year.

15

- The relationship between investment today and consumption in the future is described by the **intertemporal production possibility frontier** (IPPF) for this economy.

- See Figure 5-3, where the axes measure aggregate consumption, C_0 and C_1.

16

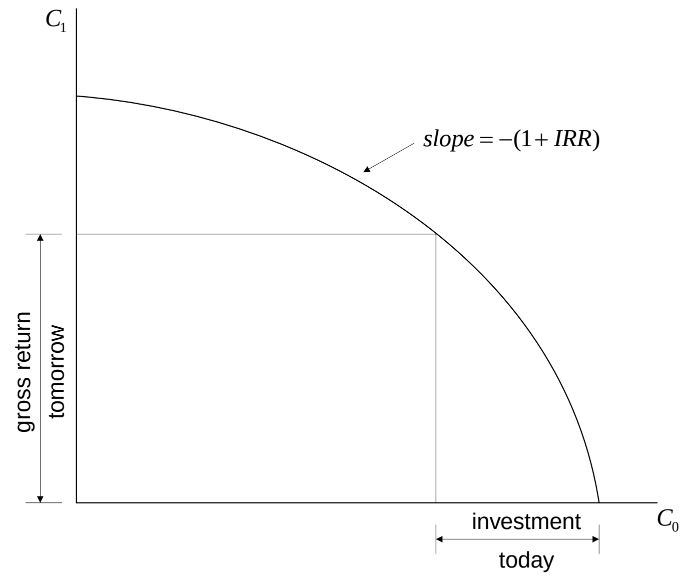

Figure 5-3

17

- The slope of the IPPF is the **intertemporal marginal rate of transformation** (IMRT).

- It turns out to be more convenient to express the IMRT as

$$IMRT = 1 + IRR$$

where IRR is the **investment rate of return**.

18

- Firms in this economy will invest up to the point where the IRR is just equal to the rate at which they can borrow:

$$IRR = r$$

19

- The market rate of interest adjusts to equate aggregate saving and aggregate investment.

- This equilibrium occurs where

$$IRR = r = SRTP$$

- See Figures 5-1 and 5-4.

20

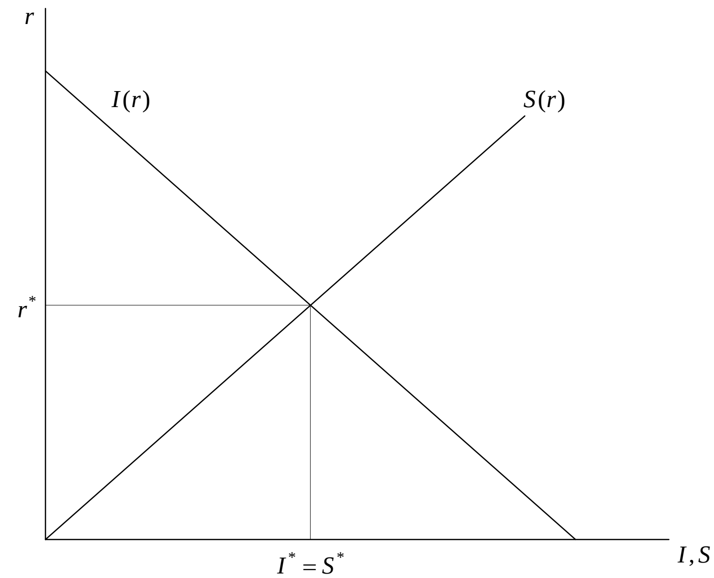

Figure 5-1 (repeat)

21

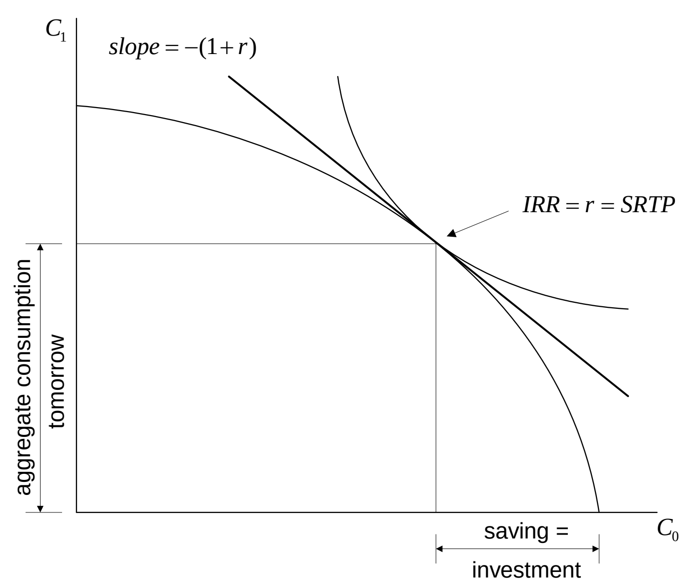

Figure 5-4

22

- The indifference curve in Figure 5-4 should not be interpreted as a "social indifference curve" (there is no such thing).

- It is an indifference curve for one individual and illustrates the maximum consumption profile that would be possible for that individual in this economy if no other individual consumed anything.

23

- In that sense, the indifference curve tangency depicted illustrates an extreme point on a continuum of Pareto-efficient divisions of aggregate consumption.

- That continuum is illustrated for a two-person economy as the Pareto-frontier (PF) in Figure 5-5.

24

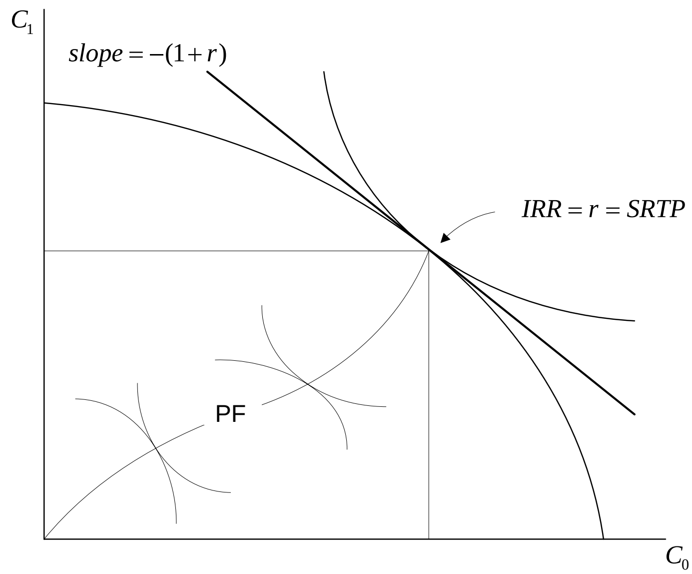

Figure 5-5

25

- In this very simple hypothetical economy, there is a unique rate at which everyone discounts the future, and so in this economy

$$PSDR = SRTP = IRR = r$$

26

- In reality, there are many sources of distortion in capital markets, including
  - asymmetry of information between borrowers and lenders
  - market power among intermediaries
  - taxes on the returns from saving and investment

27

- These distortions have two important implications for the characteristics of the equilibrium, and for determination of the PSDR.

28

- First, even if all agents face the same interest rate, the SRTP and the IRR are typically not equated in equilibrium in the presence of distortions.

29

- Second, all agents typically do not face the same interest rate, and this means that MRTPs are not equated across individuals.

- This in turn means that we cannot even define the SRTP.

30

- What does all this mean for the determination of the PSDR?

- Technically, it means that each dollar of cost and benefit of a project should have its own unique discount rate derived from first principles on the basis of which specific individuals incur those costs and receive those benefits.

31

- This individualized discounting approach is completely impractical.

- In practice, we discount all costs and benefits at the same PSDR, but we conduct that discounting for a wide range of rates to gauge the sensitivity of our results to the rate choice.

32

- A sensible rate range to report is 1% to 10% but most economists would probably cite a range of 2% to 4% as most relevant.

33

- It should be stressed that discounting is not a method for dealing with expected inflation.

- All future costs and benefits should be estimated in inflation-adjusted (constant dollar) terms before any discounting is applied.

34

5.3 CALCULATING NET PRESENT VALUE

- Net present value can be calculated in two equivalent ways.

Method 1: Present Value of Net Benefits

– calculate the net benefit in each year of the project as:

$$NB_t = B_t - C_t$$

35

– then sum the discounted net benefits over time:

$$NPV = \sum_{t=1}^{T} \frac{NB_t}{(1+r)^{t-1}}$$

- Note that by convention we do not discount the first period.

36

Example of Method 1

- Suppose anticipated costs and benefits are

	Year 1	Year 2	Year 3
benefits		100	383
costs	100	210	20
net benefits	-100	-110	363

Table 5-1

37

- Suppose PSDR = 10%. Then

$$NPV = -100 - \frac{110}{1.1} + \frac{363}{(1.1)^2} = 100$$

38

Method 2: Present Value of Benefits Minus Present Value of Costs

– calculate the present value of benefits (PVB):

$$PVB = \sum_{t=1}^{T} \frac{B_t}{(1+r)^{t-1}}$$

– calculate the present value of costs (PVC):

$$PVC = \sum_{t=1}^{T} \frac{C_t}{(1+r)^{t-1}}$$

39

– then calculate NPV as the difference:

$$NPV = PVB - PVC$$

40

Example of Method 2

- Suppose anticipated costs and benefits are

	Year 1	Year 2	Year 3
benefits		100	383
costs	100	210	20
net benefits	-100	-110	363

Table 5-1 (repeat)

41

- Suppose PSDR = 10%

$$PVB = 0 + \frac{100}{1.1} + \frac{383}{(1.1)^2} = 407.4$$

$$PVC = 100 + \frac{210}{1.1} + \frac{20}{(1.1)^2} = 307.4$$

$$NPV = 407.4 - 307.4 = 100$$

42

Present Value of a Perpetuity

$$\left[1+\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\ldots\ldots\ldots+\frac{1}{(1+r)^\infty}\right]=\frac{1+r}{r}$$

- Example: $100 per year received forever, discounted at 10%, is worth

$$\frac{100(1.1)}{0.1}=1100$$

43

- The simplest form of the NPV rule (with no consideration of distributional impacts):
  - adopt the project if NPV > 0
  - reject the project if NPV < 0

- Three important complications:
  - project scale
  - project timing
  - project reappraisal

44

5.4 PROJECT SCALE

- The scale of a project is often a choice variable.
- For example:
  - the number of lanes on a bridge
  - the generating capacity of a power plant
  - the number of beds in a hospital
  - the size of a sports arena

45

- The scale should be chosen to maximize the NPV of the project.
- Competing projects should be compared at their optimal scales (not necessarily at the same scale).
- Example:
  - a four lane bridge versus a two lane tunnel.

46

5.5 PROJECT TIMING

- The starting date for a project is also often a choice variable.
- Why might waiting be worthwhile? Example reasons:
  - expected costs may fall due to new technologies
  - population may increase, thereby changing benefits

47

- The starting date should be chosen to maximize NPV as viewed from today.
- Example:
  - suppose a project is scheduled to begin in a future year, say $y = 5$.
  - its present value viewed from today ($y = 1$) is

$$NPV_{y=1} = \frac{NPV_{y=5}}{(1+r)^4}$$

48

- More generally, suppose a project can begin in any future year $y = t$.

- Then t should be chosen to maximize:

$$NPV(t)_{y=1} = \frac{NPV(t)_{y=t}}{(1+r)^{t-1}}$$

49

- Competing projects should be compared at their optimal starting times (not necessarily at the same starting time).

- Among competing projects, the project with the highest NPV (discounted back to today) is the best project.

50

5.6 PROJECT RE-APPRAISAL

- Suppose a project begins, and then conditions change.
- Example:
  – a mining project begins and then the price of the ore falls.
- Do we proceed with the project or do we abandon it?

51

- Re-appraisal rule:
  – continue the project if and only if NPV > 0 from now on, regardless of any <u>sunk</u> costs already incurred.

- **Sunk costs** are costs that cannot be recovered if the project is terminated and are therefore irrelevant to whether or not the project should proceed.

52

- Suppose anticipated costs and benefits, as viewed in year 1, are

	Year 1	Year 2	Year 3
benefits		100	383
costs	100	210	20
net benefits	-100	-110	363

Table 5-1 (repeat)

53

- Suppose r = 10%. Then

$$NPV_{y=1} = -100 - \frac{110}{1.1} + \frac{363}{(1.1)^2} = 100$$

- Thus, we would commence the project.

54

- Now suppose that in Year 2 there is a change in conditions that affects the anticipated future net benefit stream:

	Year 1	Year 2	Year 3
benefits		100	**152**
costs	100	210	20
net benefit	-100	-110	**132**

Table 5-2

55

- As viewed from Year 1:

$$NPV_{y=1} = -100 - \frac{110}{1.1} + \frac{132}{(1.1)^2} = -90.91$$

- Thus, had we anticipated the change, the project would not have proceeded.

56

- However, the only relevant question is
  - **do we continue?**

- Whether or not we proceed depends on whether or not the $100m cost already incurred in Year 1 is sunk.

57

- Suppose the entire first-year amount is sunk. Then the **continuation payoff** is

$$NPV_{y=2} = -110 + \frac{132}{1.1} = 10$$

- Since NPV > 0, we should proceed with the project.

58

- Now suppose instead that \$15m of that initial cost can be recovered in year 2 if the project is terminated.
- Then the continuation payoff is

$$NPV_{y=2} = -15 - 110 + \frac{132}{1.1} = -5$$

- Key point: the \$15m that could be recovered by terminating the project is treated as an opportunity cost of continuing.

59

- Since NPV < 0, the project should be terminated, despite the “wasted” (non-recoverable) investment of \$85m.

60

5.7 THE BENEFIT-COST RATIO

- Many CBAs present results in terms of a **benefit-cost ratio** (BCR) instead of, or in addition to, the NPV.
- The BCR is:

$$BCR = \frac{PVB}{PVC}$$

61

- The simple BCR decision rule is:
  - – accept the project if BCR > 1
  - – reject the project if BCR < 1

62

- This rule is related to the NPV rule:

$$BCR = \frac{PVB}{PVC} = \frac{NPV + PVC}{PVC} = 1 + \frac{NPV}{PVC}$$

- Thus, if BCR > 1 then NPV > 0.

63

- However, the BCR rule has two major shortcomings:

1. the BCR depends on the arbitrary labeling of costs and benefits

2. the BCR can give the wrong ranking of competing projects

64

1. Arbitrary Labeling

- Relabel a benefit *b* as a negative cost.
- The NPV is invariant to this change:

$$(PVB - b) - (PVC + (-b)) = PVB - PVC$$

- The BCR is <u>not</u> invariant to this change:

$$\frac{PVB - b}{PVC + (-b)} \neq \frac{PVB}{PVC}$$

65

2. Project Ranking

- Consider two projects A and B with the following payoffs:

	PVB	PVC
project A	100	50
project B	800	600

Table 5-3

66

- Project B is the better project:

$$NPV_B = 200 > NPV_A = 50$$

- But by the BCR rule:

$$BCR_B = 1.33 < BCR_A = 2$$

- The BCR rule gives the wrong ranking.

67

5.8 DISCOUNTING AND FUTURE GENERATIONS

- It is often claimed that discounting unfairly penalizes future generations by putting less weight on future costs and benefits.
- Examples:
  - discounting the long-term costs of nuclear power
  - discounting the long-term benefits of greenhouse gas emission reductions

68

- This argument against discounting is flawed:
  - discounting does <u>not</u> implicitly assign less importance to future generations

- Consider an example:
  - suppose a cost of $100m will be incurred in 50 years time to decommission a nuclear power plant built today

69

- discounted at 5%, the present value of that future cost is only $8.72m and so it has a relatively small impact on the project NPV.

- if instead that cost was incurred today it would receive its full $100m weight in the NPV.

- does this mean that this cost is effectively receiving less weight because it is incurred by a future generation?

70

- Not at all.
- If we set aside \$8.72m today and invest it at 5% (the rate at which we discounted the cost) then we will have exactly \$100m available to fully compensate those who incur the cost in 50 years time.

71

- Of course, if the \$8.72m is <u>not</u> set aside and invested then there will be nothing available to compensate the future generation.
- However, a decision not to set aside those resources is a political choice not to compensate, no different from a choice made not to compensate losers today; it is not a fault with the logic of discounting.

72

- Discounting only harms future generations if the discount rate chosen is higher than the investment rate of return in the economy.

- For example, if we discount at 5% but can only invest at 4% then $8.72m set aside today will yield only $62m in 50 years time, and so the future generation cannot be fully compensated for the $100m cost.

73

- It is worth noting that a decision not to compensate future generations can affect the discount rate itself because the supply of savings is then lower than it otherwise would be.

- That lower savings level has a positive effect on the equilibrium interest rate by shifting the savings schedule (Figure 5-6).

74

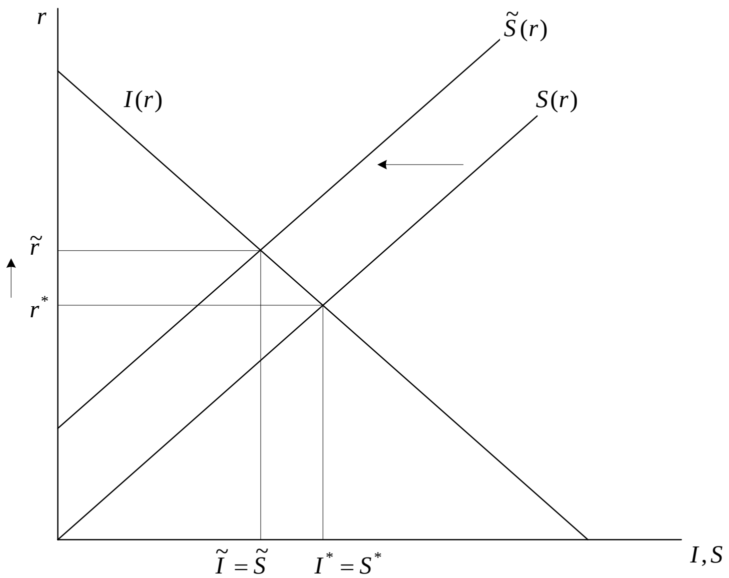

Figure 5-6

75

- We might expect that effect to be relatively small in the context of global capital markets where trillions of dollars are invested annually.

- However, in the case of climate change – where future costs could be extremely large – the effect could be significant.

76

- In that regard, a decision to not save now to compensate future generations for future climate costs could be doubly harmful to those future generations:
  - a direct harm via the lack of compensation itself
  - an indirect harm via discounting future costs more heavily than we otherwise would, and thereby taking less action to reduce emissions now as a consequence.

END

77

APPENDIX A5

DISTORTED CAPITAL MARKETS AND THE DISCOUNT RATE

Suppose that interest paid on household savings is taxed at rate *t*. Then households earn an effective (after-tax) return of $r(1-t)$, and base their saving decision on that rate.

Suppose also that the return on investment by firms is taxed at rate τ. Then firms must earn a return of $r/(1-\tau)$ to make borrowing (from households) at interest rate *r* worthwhile.

The equilibrium in this economy with taxes is illustrated in Figure A5-1. Saving decisions are characterized by $MRS = r(1-t)$ and investment decisions are characterized by $IRR = r/(1-\tau)$. Note that this equilibrium is inefficient: there is too little saving and investment. In this distorted equilibrium: $SRTP < r < IRR$.

What is the appropriate PSDR in this case? We must calculate a shadow price. An investment of $1 in a public project will be drawn partly from displaced private investment (with an opportunity cost of IRR) and partly from displaced current consumption (with an opportunity cost of MRTP). Let α denote the fraction drawn from displaced private investment; then $(1-\alpha)$ is the fraction drawn from displaced current consumption. We can then use the **Harberger weighted average rule** to construct a shadow price:

(A5-1) $$PSDR = \alpha(IRR) + (1-\alpha)SRTP$$

where $IRR = r/(1-\tau)$ and $SRTP = r(1-t)$, and *r* is the risk-free real interest rate (usually measured as the real rate on long-term government bonds). What is α in practice? In an open economy like Canada it is probably close to zero.

The Harberger rule is theoretically correct (even in this hypothetical economy) only if the returns on private and public investment are consumed. If a fraction of the investment returns are reinvested then we should take account of the multiplied opportunity cost of the displaced private investment, and the multiplied effect of the return on public investment.

A further complication is that not all individual face the same tax rate (due to progressive tax systems). This means that MRTPs are not even equated across individuals, and so the SRTP cannot even be defined.

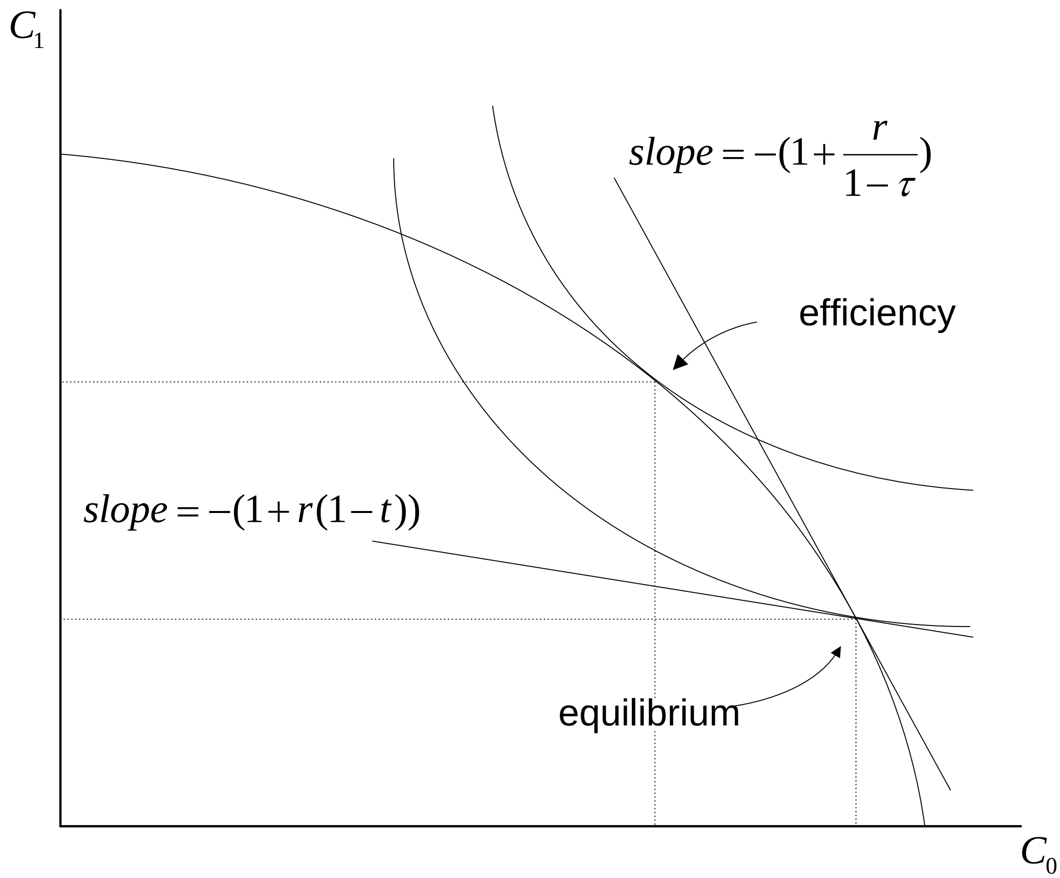

Figure A5-1

TOPIC 5 REVIEW QUESTIONS

Questions 1 – 4 relate to the following information. Consider a project with a profile of costs and benefits as illustrated in **Table R5-1**. Assume a discount rate of 2%.

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
Benefits	0	0	500	500	500
Costs	1000	50	50	50	50
Net Benefits	-1000	-50	450	450	450

Table R5-1

1. The present value of benefits (PVB) is approximately

A. 1414

B. 1515

C. 1616

D. 1717

2. The present value of costs is (PVC) is approximately

A. 1630

B. 1451

C. 1190

D. 973

3. The benefit-cost ratio for this project is

A. approximately 1.23

B. approximately 1.09

C. invariant to the labeling of costs and benefits.

D. None of the above.

4. "The profile of costs and benefits for this project is such that its net present value (NPV) falls as the discount rate rises".

A. True.

B. False.

Questions 5 – 7 relate to the following information.

Consider a project with a profile of costs and benefits as illustrated in **Table R5-2**. Assume a discount rate of 3%.

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
Benefits	0	1000	1000	1000	0
Costs	1000	0	0	0	2100
Net Benefits	-1000	1000	1000	1000	-2100

Table R5-2

5. The present value of benefits (PVB) is approximately

A. 2324

B. 2829

C. 3132

D. 3334

6. The present value of costs (PVC) is approximately

A. 2688

B. 2866

C. 2992

D. 3113

7. The net present value (NPV) for this project is

A. approximately **–** 37

B. approximately + 37

C. invariant to the labeling of costs and benefits.

D. Both A and C.

8. Consider a project with a profile of costs and benefits as illustrated in **Table R5-3**. Note that benefits and costs continue in perpetuity.

	Yr 1	Yr 2	Yr 3 (+)
Benefits	0	0	500
Costs	1000	50	50
Net Benefits	-1000	-50	450

Table R5-3

If the discount rate is 5%, the net present value of the project is

A. 7524

B. 8256

C. 8764

D. 9056

9. Consider a project with a profile of costs and benefits as illustrated in **Table R5-4**. Note that benefits and costs continue in perpetuity.

	Yr 1	Yr 2	Yr 3	Yr 4 (+)
Benefits	0	200	200	500
Costs	1000	50	50	50
Net Benefits	-1000	150	150	450

Table R5-4

If the discount rate is 4%, the net present value of the project is

A. 8728

B. 9684

C. 10864

D. 11674

Questions 10 to 16 relate to the following information. A bridge project has two competing designs under consideration. Design **A** has lower constructions costs than design **B**, but higher ongoing maintenance costs. The profiles of costs and benefits are summarized in **Tables R5-5A** and **R5-5B** for designs **A** and **B** respectively.

BRIDGE A	**Yr 1**	**Yr 2**	**Yr 3 (+)**
Benefits	0	0	190
Costs	1000	500	40
Net Benefits	-1000	-500	150

Table R5-5A

BRIDGE B	**Yr 1**	**Yr 2**	**Yr 3 (+)**
Benefits	0	0	200
Costs	2000	1000	15
Net Benefits	-2000	-1000	185

Table R5-5B

10. With a discount rate of 2%, the benefit-cost ratios (BCRs) for Bridge **A** and Bridge **B** respectively are

A. 2.09 and 1.87

B. 2.64 and 2.49

C. 1.87 and 2.09

D. 2.70 and 2.64

11. Based on a comparison of BCRs using a discount rate of 2%, the best bridge design appears to be

A. Bridge **A**

B. Bridge **B**

12. With a discount rate of 2%, the net present values (NPVs) for Bridge **A** and Bridge **B** respectively are

A. 6088 and 5863

B. 3369 and 3016

C. 5863 and 6088

D. 5863 and 3369

13. Based on a comparison of NPVs using a discount rate of 2%, the best bridge design is

A. Bridge **A**

B. Bridge **B**

14. “The benefit-cost ratio is an evil concept”.

A. True.

B. False.

15. With a discount rate of 3%, the net present values (NPVs) for Bridge **A** and Bridge **B** respectively are

A. 6088 and 5863

B. 3369 and 3016

C. 5863 and 6088

D. 5863 and 3369

16. Based on a comparison of NPVs using a discount rate of 3%, the best bridge design is

A. Bridge **A**

B. Bridge **B**

Questions 17 to 26 relate to the following information. A tunnel has been proposed as an alternative to the bridge project described in Qs 10 – 16. There are two competing designs for the tunnel. Tunnel design **1** has one lane in each direction, while tunnel design **2** has two lanes in each direction. The profiles of costs and benefits are summarized in **Tables R5-6-1** and R5-**6-2** for designs **1** and **2** respectively.

TUNNEL 1	**Yr 1**	**Yr 2**	**Yr 3 (+)**
Benefits	0	0	160
Costs	1050	350	15
Net Benefits	-1050	-350	145

Table R5-6-1

TUNNEL 2	**Yr 1**	**Yr 2**	**Yr 3 (+)**
Benefits	0	0	180
Costs	1550	450	15
Net Benefits	-1550	-450	165

Table R5-6-2

17. With a discount rate of 2%, the benefit-cost ratios (BCRs) for Tunnel **1** and Tunnel **2** respectively are

A. 2.19 and 2.87

B. 3.68 and 3.24

C. 2.87 and 2.22

D. 3.84 and 3.96

18. Based on a comparison of BCRs using a discount rate of 2%, the best tunnel design appears to be

A. Tunnel **1**

B. Tunnel **2**

19. With a discount rate of 2%, the net present values (NPVs) for Tunnel **1** and Tunnel **2** respectively are

A. 6088 and 5863

B. 5819 and 5789

C. 5874 and 6112

D. 5715 and 6097

20. Based on a comparison of NPVs using a discount rate of 2%, the best tunnel design is

A. Tunnel **1**

B. Tunnel **2**

21. "The benefit-cost ratio is a very evil concept".

A. True.

B. False.

22. With a discount rate of 3%, the net present values (NPVs) for Tunnel **1** and Tunnel **2** respectively are

A. 3303 and 3353

B. 3417 and 3168

C. 4874 and 3996

D. 3876 and 4012

23. Based on a comparison of NPVs using a discount rate of 3%, the best tunnel design is

A. Tunnel **1**

B. Tunnel **2**

We now want to choose between the bridge and the tunnel.

24. If the discount rate is 2%, then the best project is

A. Bridge **A**

B. Bridge **B**

C. Tunnel **1**

D. Tunnel **2**

25. If the discount rate is 3%, then the best project is

A. Bridge **A**

B. Bridge **B**

C. Tunnel **1**

D. Tunnel **2**

26. Based on a comparison of BCRs using a discount rate of 3%, the best project appears to be

A. Bridge **A**

B. Bridge **B**

C. Tunnel **1**

D. Tunnel **2**

27. In general, a comparison of competing projects should be based NPV calculated for the optimal design for each project.

A. True.

B. False.

Questions 28 – 30 relate to the following information. Consider a project with a profile of costs and benefits as illustrated in **Table R5-7**.

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
Benefits	0	0	700	700	700
Costs	1000	50	50	50	50
Net Benefits	-1000	-50	650	650	650

Table R5-7

If this project is delayed by two years, the first-year cost is expected to fall by 5% to $950 (due to an anticipated technological breakthrough). All other costs and benefits are expected to remain the same.

28. Using a discount rate of 2%,

A. the project should be delayed because the net-benefit of doing so (as viewed from today) is $17

B. the project should be delayed because the net-benefit of doing so (as viewed from today) is $9

C. the project should not be delayed because the net-benefit of doing so (as viewed from today) is negative $17

D. the project should not be delayed because the net-benefit of doing so (as viewed from today) is negative $9

29. Using a discount rate of 4%,

A. the project should be delayed because the net-benefit of doing so (as viewed from today) is $8

B. the project should be delayed because the net-benefit of doing so (as viewed from today) is $4

C. the project should not be delayed because the net-benefit of doing so (as viewed from today) is negative $5

D. the project should not be delayed because the net-benefit of doing so (as viewed from today) is negative $9

30. Using a discount rate of 4%, by how much would the first-period cost have to fall to make a two-year delay just worthwhile?

A. approximately $42

B. approximately $45

C. approximately $56

D. approximately $61

Questions 31 – 33 relate to the following information. Consider a project with a profile of costs and benefits as illustrated in **Table R5-8**.

	Yr 1	Yr 2	Yr 3	Yr 4
Benefits	0	700	700	700
Costs	1000	0	0	0
Net Benefits	-1000	700	700	700

Table R5-8

If this project is delayed by three years, the benefits in the last three years of the project are expected to rise by 5% to $735 (due to an anticipated technological breakthrough). The first-year cost is expected to remain the same.

31. Using a discount rate of 3%,

A. the project should be delayed because the net-benefit of doing so (as viewed from today) is $7

B. the project should be delayed because the net-benefit of doing so (as viewed from today) is $9

C. the project should not be delayed because the net-benefit of doing so (as viewed from today) is negative $7

D. the project should not be delayed because the net-benefit of doing so (as viewed from today) is negative $9

32. Using a discount rate of 4%, by how much would the benefit in years 2 through 4 have to rise to make a three-year delay just worthwhile?

A. approximately $42

B. approximately $47

C. approximately $51

D. approximately $53

Questions 33 – 36 relate to the following information. Consider a project with a profile of costs and benefits as illustrated in **Table R5-9**.

	Yr 1	Yr 2	Yr 3	Yr 4
Benefits	0	0	500	500
Costs	200	500	50	50
Net Benefits	-200	-500	450	450

Table R5-9

Suppose this project begins, and the first-year cost of $200 is incurred. At the beginning of the second year, a change in the global price of oil leads to a revision of future costs. In particular, costs in years 2 through 4 are now expected to be 50% higher. Assume a discount rate of 3%.

33. Had the information on oil prices been available before the project began, what would have been the NPV of the project?

A. minus $213

B. minus $139

C. $121

D. $152

34. Is the calculation from Q33 relevant to whether or not the project should continue?

A. Yes.

B. No.

35. Suppose the entire $200 incurred in year 1 is sunk. What is the continuation payoff for the project?

A. minus $27

B. minus $4

C. $63

D. $81

36. Suppose instead that an amount $R can be recovered from the first-year cost if the project is terminated at the beginning of year 2. For what values of R should the project be terminated?

A. greater than $4

B. less than $4

C. greater than $63

D. less than $63

Questions 37 – 40 relate to the following information. Consider a project with a profile of costs and benefits as illustrated in **Table R5-10**.

	Yr 1	Yr 2	Yr 3	Yr 4
Benefits	0	0	600	600
Costs	500	600	0	0
Net Benefits	-500	-600	600	600

Table R5-10

Suppose this project begins, and the first-year cost of $500 is incurred. At the beginning of the second year, a technological change reduces the value of this project. In particular, the benefits in years 3 and 4 are now expected to be 50% lower. Assume a discount rate of 3%.

37. Had the information on the technological change been available before the project began, what would have been the NPV of the project?

A. $210

B. $139

C. minus $321

D. minus $525

38. Was the decision to start the project an *ex post* mistake?

A. Yes.

B. No.

39. Suppose the entire $500 incurred in year 1 is sunk. What is the continuation payoff for the project?

A. minus $26

B. minus $14

C. $23

D. $61

40. Suppose instead that $200 can be recovered from the first-year cost if the project is terminated at the beginning of year 2. What is the continuation payoff for the project?

A. minus $177

B. minus $184

C. minus $226

D. minus $214

ANSWER KEY

1. A
2. C
3. D
4. A In general, the relationship between the NPV and the PSDR is complicated. However, we can make two general statements. First, if net-benefit is initially negative <u>and</u> rises monotonically over time <u>and</u> eventually becomes positive, then the NPV falls as the PSDR rises. Second, if net-benefit is initially positive <u>and</u> falls monotonically over time <u>and</u> eventually becomes negative, then the NPV rises as the PSDR rises.
5. B
6. B
7. D
8. A

$$NPV = -1000 - \frac{50}{1.05} + \frac{P}{(1.05)^2}$$

where P is the value the perpetuity of 450 that begins in year 3. Using the formula from slide 43,

$$P = 450\left(\frac{1.05}{0.05}\right) = 9450$$

So we have

$$NPV = -1000 - \frac{50}{1.05} + \frac{9450}{(1.05)^2} = 7523.81$$

9. B

$$NPV = -1000 + \frac{150}{1.04} + \frac{150}{(1.04)^2} + \frac{P}{(1.04)^3}$$

where P is the value of the perpetuity of 450 that begins in year 4. Using the formula from slide 43,

$$P = 450\left(\frac{1.04}{0.04}\right) = 11700$$

So we have

$$NPV = -1000 + \frac{150}{1.04} + \frac{150}{(1.04)^2} + \frac{11700}{(1.04)^3} = 9684.16$$

10. D

For Bridge A, the present value of benefits is

$$PVB = 0 + \frac{0}{1.02} + \frac{P_B}{(1.02)^2}$$

where P_B is the value the perpetuity of 190 that begins in year 3. Using the formula from slide 43,

$$P_B = 190\left(\frac{1.02}{0.02}\right) = 9690$$

So we have

$$PVB = 0 + \frac{0}{1.02} + \frac{9690}{(1.02)^2} = 9314$$

The present value costs is

$$PVC = 1000 + \frac{500}{1.02} + \frac{P_C}{(1.02)^2}$$

where P_C is the value the perpetuity of 40 that begins in year 3. Using the formula from slide 43,

$$P_C = 40\left(\frac{1.02}{0.02}\right) = 2040$$

So we have

$$PVC = 1000 + \frac{500}{1.02} + \frac{2040}{(1.02)^2} = 3451$$

So the BCR of Bridge A is

$$BCR = \frac{PVB}{PVC} = \frac{9314}{3451} = 2.70$$

The BCR for Bridge B is calculated using the same method.

11. A
12. C
13. B
14. A
15. B
16. A
17. B
18. A
19. D
20. B
21. A
22. A
23. B
24. D Listed in order of the answer options, the NPVs are 5863, 6088, 5715, **6097**
25. A Listed in order of the answer options, the NPVs are **3369**, 3016, 3303, 3353
26. C Listed in order of the answer options, the BCRs are 2.21, 1.87, **2.76**, 2.36. Thus, the BCR comparison would lead us not only to pick the tunnel when we should pick the bridge, it would lead us to pick the worst of the two tunnel designs.
27. A
28. A If the project begins now, the NPV is $789. If the project is delayed by two years, the NPV as viewed from that start date is $839. Discounting that value back two years yields a NPV of $806. Thus, the net-benefit of waiting is $17.
29. C If the project begins now, the NPV is $686. If the project is delayed by two years, the NPV as viewed from that start date is $736. Discounting that value back two years yields a NPV of $681. Thus, the net-benefit of

waiting is negative \$5.

30. C If the project begins now, the NPV is \$686. If the project is delayed two years, and the first-period cost thereby falls to $\$1000 - x$, then the NPV as viewed from today is

$$NPV(x) = \frac{686 + x}{(1.04)^2}$$

Setting $NPV(x) = 686$ and solving for *x*, yields approximately $x = 56$.

31. A If the project begins now, the NPV is \$980. If the project is delayed by three years, the NPV as viewed from that start date is \$1079. Discounting that value back two years yields a NPV of \$987. Thus, the net-benefit of waiting is \$7.

32. A If the project begins now, the NPV is \$943. If the project is delayed three years, and the benefits thereby rise to $\$700 + x$ in years 2 through 4, then the NPV as viewed from today is

$$NPV(x) = \frac{-1000 + (700 + x)\left(\frac{1}{1.04} + \frac{1}{(1.04)^2}\ \frac{1}{(1.04)^3}\right)}{(1.04)^3}$$

Setting $NPV(x) = 943$ and solving for *x*, yields approximately $x = 42$.

33. B

34. B

35. C

36. C

37. D

38. A

39. A

40. C

6. DEALING WITH UNCERTAINTY

OUTLINE

2

6.1 INTRODUCTION

- Most projects have uncertain costs and benefits either because the future is uncertain or because we simply do not have full knowledge of key parameters.

- That uncertainty can often be reduced through further research but some uncertainty will always remain.

3

- How do we evaluate projects under uncertainty?

- Two related but separate issues arise here:
  - the measurement of WTP and WTA for an individual under uncertainty
  - dealing with aggregate uncertainty in the assessment of the project

4

- The measurement of WTP and WTA under uncertainty is theoretically challenging, and the results of that theory are difficult to apply in practice. (See Appendix A6-1 for a brief introduction).

- Instead we will focus on dealing with uncertainty in the assessment of the project.

5

- A variety of ways are used for dealing with uncertainty, including use of a so-called “risk-adjusted discount rate”, but many of these have dubious theoretical foundations.

- We will focus on a **Bayesian approach**.

6

- There are four key components to a **Bayesian approach** to decision-making under uncertainty:
  - – specification of prior beliefs
  - – sensitivity testing
  - – belief-updating
  - – simulation

- We will describe each of these in turn.

7

6.2 SPECIFICATION OF PRIOR BELIEFS

- For each uncertain parameter a_i we specify a set of beliefs, described by a probability density function, $f_i(a_i)$.

- The density function assigned to any given parameter is specific to that parameter.

8

- For example, our beliefs about parameter a_1 may be represented by a uniform distribution over some interval [c,d], while our beliefs about parameter a_2 may be represented by a log-normal distribution.

9

- The **mean** of the distribution specified for a given parameter can be thought of as our "best guess value" for that parameter.

- The **variance** of the distribution reflects our level of confidence in that best guess.

10

- It is more standard to measure our level of confidence by the **precision** of the density function, which is the reciprocal of variance.
- Thus, infinitely precise beliefs correspond to perfect certainty that our best guess is correct.

11

- One of the most flexible distributions for the specification of beliefs is the **Beta distribution**.
- Three sample Beta distributions are depicted in Figure 6-1.

12

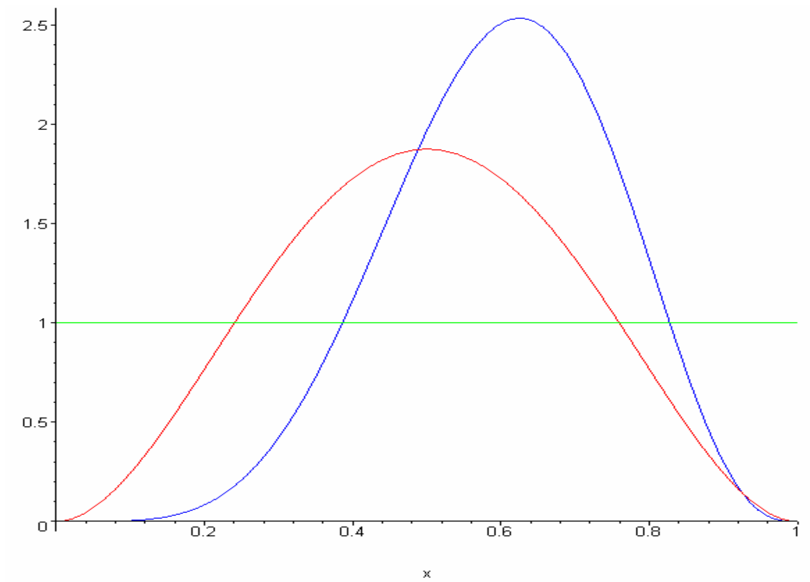

Figure 6-1

13

- Note from Figure 6-1 that the uniform distribution (in green) is a special case of the Beta distribution.
- The standard Beta distribution is defined on the interval [0,1] but it can be scaled to cover any finite interval, so there are very few instances where it cannot be shaped to fit beliefs about a given parameter.

14

6.3 SENSITIVITY TESTING

- For each uncertain parameter, we conduct sensitivity testing with respect to the NPV, and the distributional impacts, holding all other parameters at their mean values.
- These sensitivity results should be reported as **elasticities**.

15

- The results of sensitivity testing tell us something about which parameters are most important in terms of their impact on the viability of the project.
- This in turn tells us where further research can be directed most usefully.

16

- From a Bayesian perspective, the value of information arises from its role in improving our decision-making.
- For example, we do not want to waste time and effort acquiring more information about a parameter that does not matter much in terms of our decision-making.

17

- In the CBA context, the more sensitive is NPV to variation in a parameter, the more valuable is information about that parameter.
- This should guide any further research we undertake to obtain better information.

18

6.4 BELIEF-UPDATING

- We use the results of any additional research to refine our beliefs about the uncertain parameters.
- This process of refinement is called **belief-updating**.

19

- The revised beliefs are called **posterior beliefs** (as distinct from the prior beliefs with which we started).

20

6.5 SIMULATION

- For each parameter, we generate *n* random values based on the posterior distribution specified for that parameter.
- This provides us with *n* possible scenarios.
- We then calculate NPV for each scenario.

21

- We then create a **histogram** based on the simulation results, which tells us the relative likelihood of possible NPVs.
- We can then calculate the **expected NPV** (the mean of the NPV distribution), and the **variance** of the NPV distribution.

22

- The variance can be used to construct a measure of **project risk**, which is often measured by the ratio

$$\rho \equiv \frac{\textbf{standard deviation}}{\textbf{mean}}$$

23

APPENDIX A6-1

WILLINGNESS-TO-PAY UNDER UNCERTAINTY

Suppose we are assessing a project whose impacts are uncertain to the agents affected by the project. How do we measure WTP (or WTA) in that situation?

Suppose the payoff from a project depends on the state of the world. Suppose further that there are two possible states, s_1 and s_2, with associated probabilities π_1 and π_2 respectively (where $\pi_1 + \pi_2 = 1$). Let $u_i(w, \delta)$ denote utility in state *i* as a function of private wealth *w*, and whether or not the project proceeds: $\delta = 1$ if the project proceeds and $\delta = 0$ otherwise.

A6-1.1 THE WTP LOCUS

Let x_1 and x_2 be state-contingent payments if the project proceeds. Define the **willingness-to-pay locus** as $\{x_1, x_2\}$ such that

(A6.1) $$\pi_1 u_1(w - x_1, 1) + \pi_2 u_2(w - x_2, 1) = \pi_1 u_1(w, 0) + \pi_2 u_2(w, 0) \equiv \bar{u}$$

That is, $\{x_1, x_2\}$ is the set of state-contingent payments that would leave the agent just indifferent (in expected utility terms) between having the project and making the payments, and not having the project and paying nothing. We can think of the WTP locus as an indifference curve in (x_1, x_2) space reflecting a given level of expected utility. It is illustrated in Figure A6-1. The strict concavity of the illustrated WTP locus reflects an underlying assumption of risk aversion.

Any $\{x_1, x_2\}$ pair on the WTP locus is a legitimate measure of WTP under uncertainty. There are three "natural" measures that are used most commonly:

- expected surplus
- option price
- the fair bet

We will describe each of these in turn.

A6-1.2 EXPECTED SURPLUS

Define σ_i such that

(A6.2) $$u_i(w-\sigma_i,1)=u_i(w,0) \qquad \text{for } i=1,2$$

That is, σ_i is the maximum WTP for the project in state *i*; if an agent knew that state *i* was going to occur for certain then she would be willing to pay σ_i for the project. (For example, if an agent knew that a flood was going to occur next year then she would be willing to pay an amount σ_{flood} for a flood mitigation project today. If she knew that no flood would occur then she would be willing to pay only $\sigma_{noflood} < \sigma_{flood}$).

Note that if the agent did not have to pay for the project then σ_i would be the private surplus she would derive from the project in state *i*. The pair $\{\sigma_1,\sigma_2\}$ is marked as the point *S* on the WTP locus in Figure A6-1. (It could lie above or below the 45° line).

Expected surplus is defined as

(A6.3) $$ES=\pi_1\sigma_1+\pi_2\sigma_2$$

ES is represented graphically as a line with slope $-\pi_1/\pi_2$ passing through *S*; it is labeled *ES* in Figure A6-1.

A6-1.3 OPTION PRICE

Option price is defined as *OP* such that

(A6.4) $$\pi_1 u_1(w-OP,1)+\pi_2 u_2(w-OP,1)=\bar{u}$$

That is, option price is the maximum state-independent amount that an agent would be willing to pay for the project.

OP may be greater than or less than *ES*; they are equal only if $\sigma_1=\sigma_2$ or if the agent is risk-neutral. (As illustrated in figure A6-1, $OP > ES$). The difference between *OP* and *ES* is called **option value** (*OV*):

(A6.5) $$OV \equiv OP-ES$$

Inspection of Figure A6-1 reveals that *OV* is increasing in π_1 when $\sigma_1 < \sigma_2$, and increasing in π_2 when $\sigma_2 < \sigma_1$. (Note that a higher value of π_1 means a steeper line in Figure A6-1). That is, for a risk averse agent, *OV* tends to be positive when private surplus is smallest in the state that is most likely.[1]

A6-1.4 THE FAIR BET

The **fair bet** is the payment pair $\{x_1, x_2\}$ that maximizes the expected payment subject to maintaining indifference. That is,

(A6.6) $$\{x_1^*, x_2^*\} = \arg\max_{x_1, x_2} \; \pi_1 x_1 + \pi_2 x_2$$

$$s.t. \quad \pi_1 u_1(w - x_1, 1) + \pi_2 u_2(w - x_2, 1) = \bar{u}$$

The fair bet is represented by the point *F* on the WTP locus in Figure A6-1.

A6-1.5 THE RELATIONSHIP BETWEEN OP AND ES

If the agent agrees to a state contingent contract $\{\sigma_1, \sigma_2\}$ in payment for the project then she eliminates all *risk* (her *ex post* net surplus is zero in both states). Conversely, if she agrees to pay *OP* then she faces some risk whenever $\sigma_1 \neq \sigma_2$. For example, suppose $\sigma_1 < \sigma_2$. Then her *ex post* net surplus ($\sigma_i - OP$) will be positive in state 2 and negative in state 1. The risk associated with that uncertain prospect (as measured by its risk premium) is *decreasing* in the likelihood of state 1. That is, the *OP* contract becomes less risky when the likelihood of state 1 increases. Thus, the *OP* amount must increase relative to *ES* to ensure that the two contracts remain equivalent in risk-adjusted terms (in the sense that a risk averse-agent is indifferent between them).

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[1] See Section A6-1.5 following.

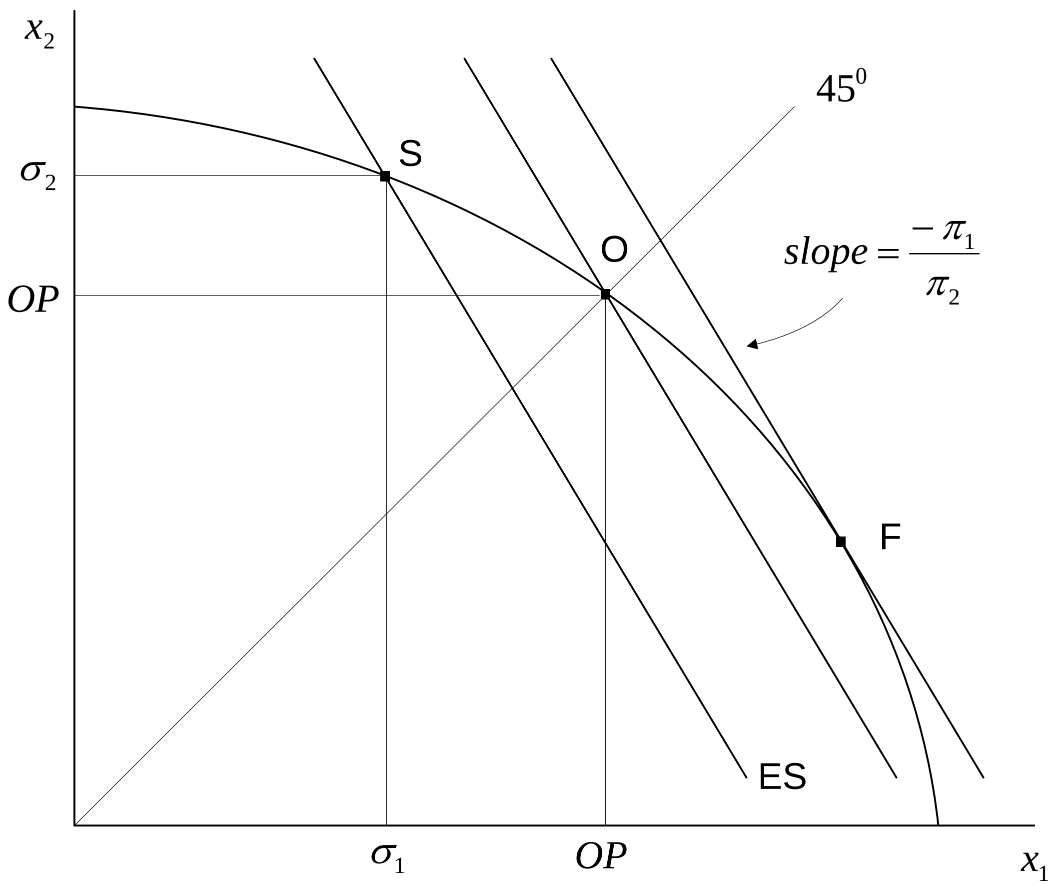


Figure A6-1

TOPIC 6 REVIEW QUESTIONS

1. Which of the following is not a component of a Bayesian approach to decision-making under uncertainty?

A. Specification of prior beliefs.

B. Sensitivity testing.

C. Belief-updating.

D. Application of a risk-adjusted discount rate.

2. When assigning a probability density to describe beliefs, the same density should be used for all uncertain parameters, with appropriate adjustments to the mean and variance.

A. True.

B. False.

3. The precision of a density function is the reciprocal of its standard deviation, and reflects the confidence of the beliefs represented by that density.

A. True.

B. False.

4. The Beta distribution is too inflexible for use as a representation of beliefs because

A. it is defined on the interval [0,1].

B. it is a symmetric distribution.

C. it has a fixed variance.

D. None of above.

5. From a Bayesian perspective, the value of information arises from

A. its role in raising the NPV of the project.

B. its role in updating prior beliefs.

C. its role in improving our decision-making.

D. All of the above.

6. Suppose we undertake sensitivity-testing with respect to two parameters a_1 and a_2, and find that $|\varepsilon_1| > |\varepsilon_2|$, where

$$\varepsilon_i = \frac{\%\Delta NPV}{\%\Delta a_i}$$

is the elasticity of NPV with respect to a_i. Based on this result,

A. we should devote more effort to acquiring additional information about parameter a1
B. we should devote more effort to acquiring additional information about parameter a2
C. we should devote equal effort to acquiring additional information about parameters a1 and a2
D. our efforts toward acquiring more information will also depend on the relative cost of those efforts, and so we cannot make a judgement based on the elasticity values alone.

7. Posterior beliefs are constructed on the basis of information obtained from observing the actual performance of a project once it is begun.

A. True.
B. False.

Questions 8 and 9 relate to Table 1. The table reports the results of a simulation. There are four uncertain parameters (a_1 – a_4). Five random values were generated for each parameter based on a specified distribution for each. (In practice, we would generate 10,000 values rather than five). These generated values are grouped into five scenarios (S1 – S5), and NPV is reported for each scenario.

	S1	**S2**	**S3**	**S4**	**S5**
a1	0.382	0.101	0.596	0.899	0.204
a2	1.304	3.493	3.325	3.759	2.571
a3	1	0	1	1	0
a4	-1.577	0.176	1.033	0.598	-0.036
NPV	**-1.336**	**11.755**	**3.413**	**8.058**	**7.165**

Since each scenario was created using the same set of distributions, we can calculate the expected NPV as the arithmetic average of the scenario-specific values. Similarly, we can calculate the variance (and standard deviation) among the scenario-specific values. For the latter, we use the unbiased standard deviation because we are looking only at sample values (not the population), and this is calculated as

$$\sigma = \left(\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \right)^{\frac{1}{2}}$$

where $\bar{x}$ is the sample mean.

8. The expected NPV for this project is

A. 2.452

B. 5.811

C. 6.343

D. 7.213

9. The ρ value for this project is

A. 0.856

B. 0.682

C. 0.934

D. 0.397

ANSWER KEY

1. D
2. B
3. B
4. D
5. C
6. D

If the effort-costs are the same for both, then A would be the correct response.

7. B
8. B
9. A

7. VALUATION OF NON-MARKET GOODS: AN INTRODUCTION

OUTLINE

* Advanced Topic

7.1 INTRODUCTION

- Many policies and projects involve inputs and outputs for which there are no formal markets.

- Important examples include public goods (such as environmental quality), health and safety, and risks to human life.

3

- Measuring the value of non-market goods is still a rough science, and there is currently much research underway to improve it.

- Nonetheless, these methods can provide useful ways of putting “ball park” values on important costs and benefits that might otherwise be ignored simply because they are hard to measure.

4

- Most of the research on non-market valuation has focused on the valuation of environmental amenities.

- We will make that our focus here.

5

- There are two broad classes of non-market valuation methods:
  – revealed-preference methods
  – stated-preference methods

6

- **Revealed-preference methods** use observable market data on related goods to make inferences about the value of non-market values.

- **Stated-preference methods** use surveys to elicit from respondents their valuation of non-market values.

7

- We will describe both methods here but first we may want to say something more about the values that are being measured.

8

7.2 THE VALUE OF ENVIRONMENTAL AMENITIES*

- Let the vector *q* denote environmental quality.

- For example, *q* could reflect a flow of emissions, whether or not a dam is built on a river, the amount of protected land, whether or not a species goes instinct, etc.

* Advanced Material

- We assume that individuals make private decisions to maximize utility given *q*, and given market prices *p* for other goods.

- We will assume any change in *q* is not so large as to cause market prices to change.

- Thus, we will take *p* as fixed and omit it from our notation.

- We can construct an **expenditure function** $e(q^0, u^0)$ that specifies the minimum level of expenditure needed to achieve u^0 given q^0.

11

- We can then measure the value of a change in environmental quality, from q^0 to q^1, using either the **equivalent variation** (EV) or the **compensating variation** (CV).

12

- The **CV** is the (negative of the*) amount of money that would have to be given to the individual after the change in *q* to enable her to attain the same level of utility she enjoyed at q^0.

*By convention, *CV*>0 for an improvement; hence the "negative of".

13

- Formally,

$$CV = e(q^0, u^0) - e(q^1, u^0)$$

where q^1 is the post-change level of environmental quality.

14

- The **EV** is the (negative of the*) the amount of money that would have to be taken away from the individual, in the absence of the change in *q*, to leave her with the same level of utility she would have derived at q^1.

*By convention, *EV*>0 for an improvement; hence the "negative of".

15

- Formally,

$$EV = e(q^0, u^1) - e(q^1, u^1)$$

where u^1 is the post-change level of utility.

16

- The EV and CV relate to WTP and WTA.
- If the change $q^0 \rightarrow q^1$ is an improvement in environmental quality for an individual then
  - – CV measures her WTP for the change
  - – EV measures her WTA to forego the change

17

- If the change $q^0 \rightarrow q^1$ is a deterioration in environmental quality for an individual then
  - – EV measures her WTP to avoid the change
  - – CV measures her WTA to compensate for the change

18

7.3 USE VALUE AND PASSIVE-USE VALUE

- It can sometimes be conceptually useful to decompose the value of an environmental amenity into separate components:
  - use value
  - passive-use value

19

- **Use value** is the value to the agent from sensuous interaction with the environmental amenity (via hiking, sight seeing, wildlife viewing, etc).

- This sensuous interaction can include vicarious interaction (for example, via watching wildlife films).

20

- **Passive-use value** comprises three parts:
  - existence value
  - bequest value
  - option value

21

- **Existence value** is the value gained purely from the existence of the environmental amenity at a particular quality.

- For example, an individual may derive utility from knowing that there are whales swimming in the oceans even if they never have any sensuous interaction with them.

22

- **Option value** is the value an individual places on preserving the option to make use of an environmental amenity in the future.

23

- **Bequest value** is the value an individual places on preserving an environmental amenity for future generations.

- Bequest value arises out of intergenerational altruism.

24

7.4 REVEALED-PREFERENCE METHODS

- Revealed-preference methods use observable market data on related goods to make inferences about non-market values.

- They are “revealed-preference” methods in the sense that an individual’s valuation of a non-market good is revealed through her behaviour with respect to market goods.

25

- We will describe three such methods:
  - averting behaviour method
  - hedonic price method
  - travel cost method

26

7.4-1 AVERTING BEHAVIOUR METHOD

- The value of a small reduction in environmental quality can in principle be measured by the amount an individual is willing to spend on some defensive action ("averting behaviour") to prevent it.
- Use of the averting behavior method is limited to cases where market-based defensive actions are actually available.

27

- A further limitation is that an individual makes a defensive expenditure based on his own beliefs about the efficacy of the defensive action.

- As analysts, we can only observe the actual efficacy of the defensive action (based on statistical evidence).

28

- If an individual believes that the defensive action is more effective than statistical tests suggests, then we will over-estimate his WTP for the protection that the defensive action actually provides.

- Many people believe in, and pay for, various remedies whose efficacy has no scientific support.

29

7.4-2 HEDONIC PRICE METHOD

- The market price for a good (or the market wage for labor services) can be decomposed into **hedonic prices** of the characteristics of the good (or job).
- Example:
  - a house price reflects attributes of that house, incl. environmental attributes such as proximity to a park or dump, noise levels, views, etc.

30

An **hedonic price function** is estimated by a cross-section regression of house prices against a vector of measured attributes, including environmental ones.

$$p_i = \beta_0 + \sum_{j=1}^{K} \beta_j a_{ij}$$

31

- The estimated coefficients on the environmental attributes can then be used to measure the value of marginal changes in those attributes.

32

- A similar approach can be taken to the **hedonic decomposition of wages** across different job types.
- The particular attribute of interest in that case is often the risk of death on the job.

33

- The wage premium demanded for higher risk can be used as a way to measure WTA for increased risk of death more generally.
- This approach has sometimes been used to estimate the value of a statistical life.

34

7.4-3 TRAVEL COST METHOD

- The travel cost method measures the value of an environmental amenity (such as a recreation site) by drawing inferences from expenditures that are made in order to “consume” the good (including the cost of traveling to the site).

35

FOUR MAIN STEPS IN A TRAVEL COST STUDY

- The main steps in a travel cost study:
  - on-site survey
  - estimation of travel cost
  - estimation of the trip generating function
  - surplus calculation

36

Step 1: On-Site Survey

- Data is collected from site users on
  - point of origin
  - time taken for the trip
  - expenditures *en route*
  - cost of any equipment used
  - site-entry fee paid
  - socioeconomic variables (incl. income, age, family size, education, gender).

37

Step 2: Travel Cost Estimation

- Travel cost for user *i* is estimated from the information gathered from the survey.

- "Travel cost" actually includes <u>all</u> costs incurred to use the site (some of which may not be related to travel *per se*).

38

Step 3: Trip Generating Function

- Users are typically grouped into zones of origin based on where they began their trip.

- A demand schedule for trips (sometimes called a "trip generating function") is then estimated by a cross-section regression across zones of origin.

39

- Dependent variable:
  - number of visitors from a zone

- Independent variables:
  - average travel cost for users in that zone
  - average socioeconomic variables for users in that zone
  - population in the zone

40

- The estimated function can be interpreted as a demand curve for visits, plotting the number of visits against travel cost, which is now interpreted as the effective price of a visit; see Figure 7-1.

- All other variables are fixed at their mean values along that demand curve.

41

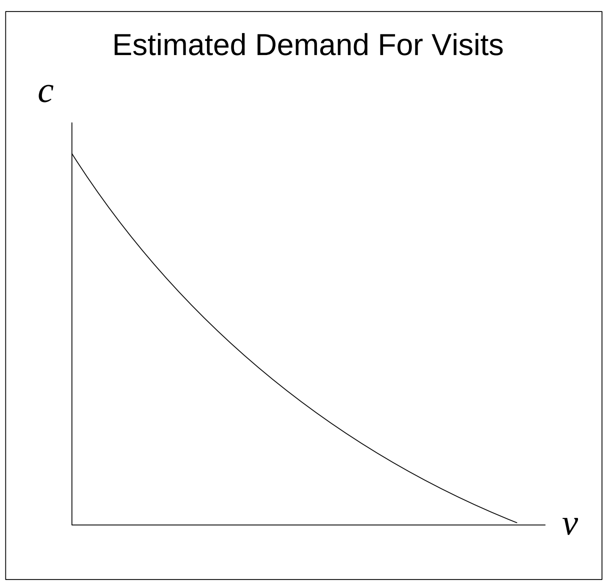


Figure 7-1

42

Step 4: Surplus Calculation

- The area under the estimated demand curve is the total value (as measured by WTP) that users place on the site.

43

- The sensitivity of the site-value to small changes in other variables (such as income) can be measured by simulating shifts in the estimated demand curve and then calculating the associated change in surplus.

44

SOME COMPLICATIONS AND LIMITATIONS WITH THE TRAVEL COST METHOD

- Multi-purpose trips
  - Visitors to a site may visit that site in the course of a larger trip that includes visits to a number of other sites.
  - The best solution here is to identify these visitors in the survey and ask a question about the importance of the studied site in their overall trip.

45

- The value of time
  - The appropriate measure of the time-cost of travel is its opportunity cost but this can be hard to measure.
  - If time spent on a trip would otherwise have been spent at work, then the appropriate time cost is the value of foregone earnings.

46

– However, time spent on a trip may otherwise have been spent in an alternative leisure activity, possibly worth more than foregone earnings.

– The time-cost problem is further complicated by the fact that the trip itself may have a positive utility value.

47

– A possible solution is to ask visitors whether or not they “enjoyed the trip”, and use a zero time cost for those respondents who did, and a fraction of their wage for those who did not.

48

- Alternative sites
  - The demand for a market commodity depends not only on the price of that commodity but also on the prices of substitute and complementary commodities.
  - The same should be true of recreation sites.
  - Ideally, the cost of visiting close substitute sites should be taken into account in deriving a demand for visits to any particular site.

49

- Ideally, one should estimate simultaneously a system of demand equations for a set of alternative sites.
- More generally, the system of demands for recreation sites should ideally be "nested" in a system of demands for leisure activity generally.

50

7.5 STATED-PREFERENCE METHODS

- Revealed-preference valuation methods are limited in applicability, and in particular, they are limited to measuring use values.
- **Stated-preference methods** provide an alternative that can be applied in a wider range of circumstances, and can be used to measure passive-use values.

51

- Stated-preference methods use surveys to elicit from respondents their valuation of non-market goods.
- They are "stated-preference" methods in the sense that they measure what individuals state to be their preferences (in contrast to revealed preference methods).

52

- There are a wide-range of stated-preference methods, but we will focus on a particular method called **referendum-format contingent valuation**.

53

7-5-1 REFERENDUM-FORMAT CONTINGENT VALUATION

- Referendum-format contingent valuation is a special "all-or-nothing" case of a stated choice method in which the respondent is asked to choose between a defined environmental project for a given price, and no project at all.

54

KEY ELEMENTS OF THE SURVEY

- The key elements of the survey are:
  - a description of the good to be valued
  - a method of hypothetical payment (sometimes called the "payment vehicle")
  - the payment question
  - a set of questions to obtain data on socioeconomic characteristics

55

THE REFERENDUM QUESTION

- The distinguishing feature of the referendum format is the payment question:
  - the respondent is asked to answer "yes" or "no" as to whether she is willing to pay an amount $*b* for the project
  - the $*b* amount (the "bid price") is specified in the survey she is given

56

BID GROUPS

- The survey of *N* individuals is split into *n* groups of equal size, called "bid groups".
- Each group is given a different bid price.
- The respondent is asked whether or not she is willing to pay the bid price in her survey.
- A typical survey would split the sample into around six bid groups.

57

RESPONSE HISTOGRAM

- The percentage of "yes" responses from each group can be plotted as a histogram.
- For example:
  – suppose the bid prices are 100, 200, 300, 400, 500, 600
  – plotting the % of "yes" responses against these bid prices will yield a histogram like that in Figure 7-2.

58

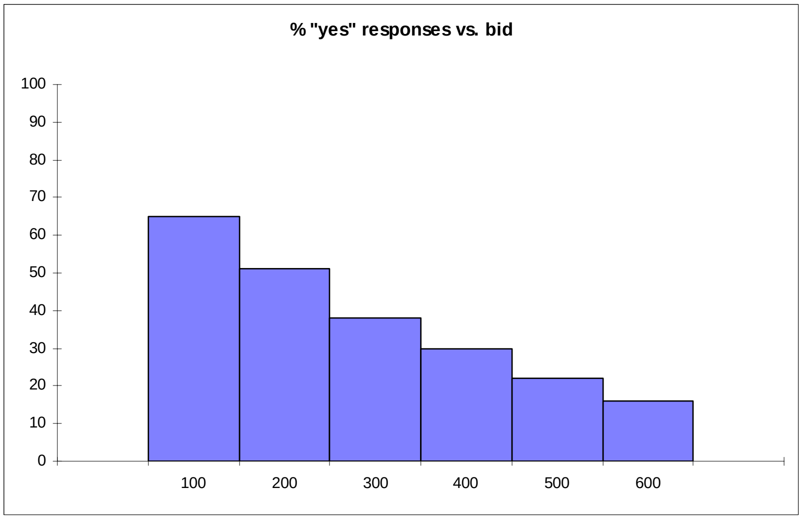


Figure 7-2

59

THE INVERSE CUMULATIVE DENSITY

- We can interpret this histogram as a segment of the **inverse cumulative density** of WTP among the survey respondents.

60

- Typically we plot a cumulative density *F*(*y*) as an increasing function, rising from 0 to 1 (or to 100%).

- For example, Figure 7-3 plots the cumulative density for a standard normal distribution.

61

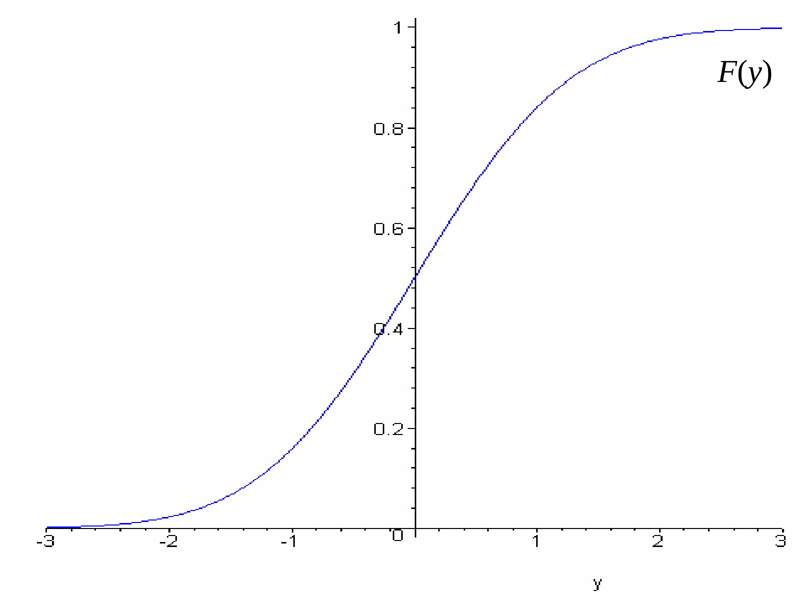


Figure 7-3

62

- An inverse cumulative density (ICD) conveys exactly the same information as a regular cumulative density but it plots

$$1-F(y)$$

and it is a decreasing function, from 1 to 0.

63

- Figure 7.4 plots the ICD for the normal distribution (in red) alongside the regular cumulative density from Figure 7.3.

64

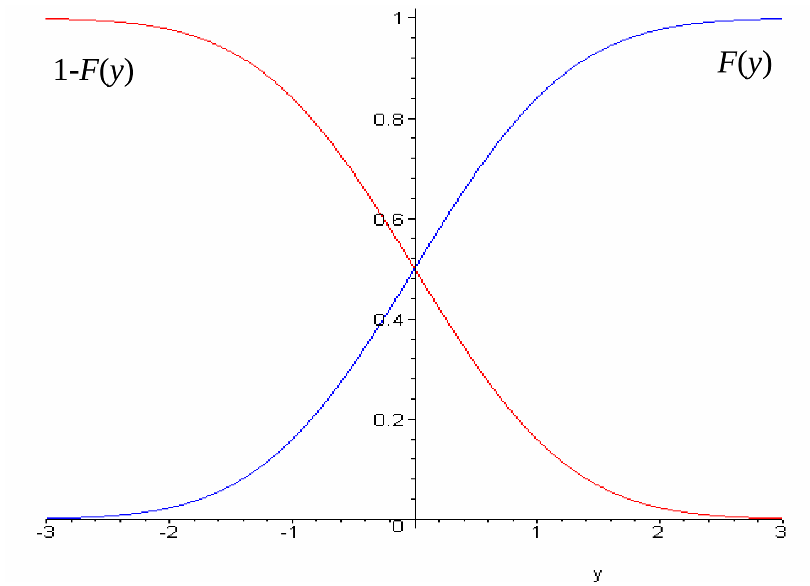


Figure 7-4

65

- If we fit an ICD to our histogram data it will look something like that in Figure 7-5.

66

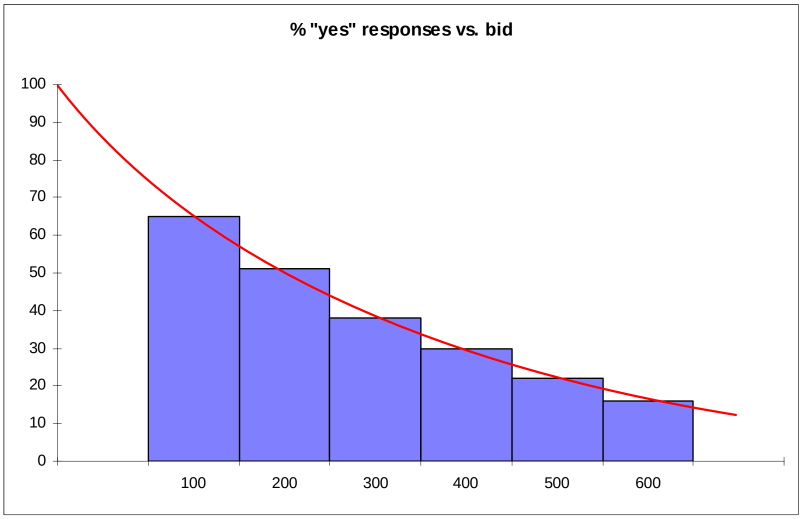


Figure 7-5

67

- The interpretation of Figure 7-5 is that among the respondents, approximately:
  - – 16% have a WTP of at least $600
  - – 22% have a WTP of at least $500
  - – 30% have a WTP of at least $400
  - – 38% have a WTP of at least $300
  - – 51% have a WTP of at least $200
  - – 65% have a WTP of at least $100
  - – 100% have a WTP of at least $0

68

ESTIMATING WTP

- Our goal is to use the survey-response data to estimate the ICD, and from this we can then derive
  - an estimate of the median WTP for the sample; and
  - an estimate of the underlying density function, from which we can then calculate the mean WTP for the sample

69

- To get a good estimate of the ICD we need to control for the factors other than bid price that drive the bid response.
- For example, two respondents with the same preferences may respond differently to the same bid price because one respondent is rich while the other respondent is poor.
- This is why we need to collect data on socioeconomic characteristics.

70

- We estimate the likelihood of a “yes” response as a function of the bid price <u>and</u> the characteristics of the respondent.

- We do this using a **logit regression**.

71

- The logit regression is a particular type of estimation method designed for **limited-dependent variable problems**, where the dependent variable is not a continuous variable (as assumed in an OLS regression) but instead takes on a limited number of possible values.

- In the referendum problem: yes or no.

72

THE LOGIT REGRESSION MODEL

- In the logit model, an index of behavior (in our case, "yes" or "no") is specified to be a linear function of a set of *K* explanatory variables *x* (one of which is the bid price):

$$I = \sum_{i=1}^{K} \beta_i x_i$$

73

- In general, we cannot hope to explain "yes" or "no" perfectly (due to unobserved determinants), so instead we attempt to explain the probability of a "yes" response.

- In our case we are attempting to model the probability of a "yes" response as a function of the bid price and the socioeconomic characteristics of the respondent.

74

- This probabilistic approach to modeling behavior derives from the so-called "random utility model".

- The idea is that factors unobservable to the analyst influence a respondent's response, and so the analyst treats those factors as a random error from her perspective.

75

- The choice of functional form assumed for the probability function distinguishes different classes of limited-dependent variable methods (logit vs. probit).

76

- In the case of the logit model, the functional form is the **logistic function**:

$$P = F(I) = \frac{1}{1 + \exp(-\sum_{i=1}^{K} \beta_i x_i)}$$

77

- The explanatory variables (the vector *x*) include the bid price faced by the agent and her socioeconomic characteristics.

- The estimated coefficients are then used to derive the estimated logistic function:
  - it tells us the likelihood of a "yes" response among respondents as a function of the bid price faced and socioeconomic characteristics.

78

THE ESTIMATED ICD

- The estimated ICD is then given by

$$1 - F = \frac{1}{1 + \exp(\sum_{i=1}^{K} \hat{\beta}_i x_i)}$$

79

- To plot our estimated ICD (like the one depicted in Figure 7-5), we set the value of all explanatory variables other than the bid price at their mean values for the sample, and then plot 1 – *F* against the bid price.

80

- That is, we plot the following estimated function against *b*:

$$1-F = \frac{1}{1+\exp(\hat{\beta}_{bid}\, b + \sum_{i=1}^{K-1} \hat{\beta}_i \bar{x}_i)}$$

- This plot will look something like the one in Figure 7-6, which reflects the shape of the inverse logistic function.

81

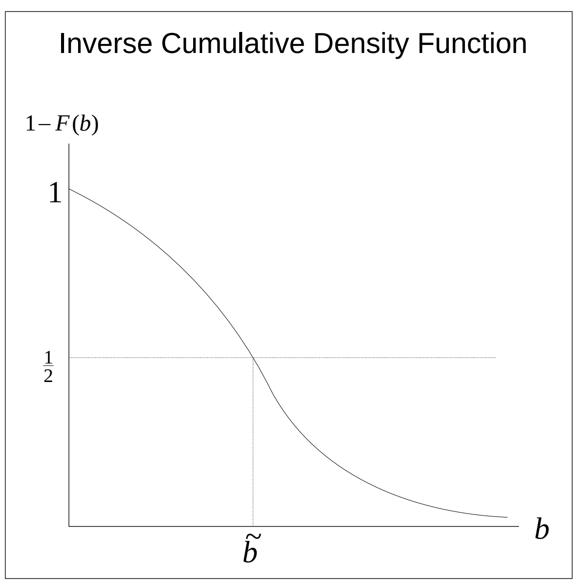


Figure 7-6

82

- We can then calculate the median WTP as the solution to

$$F(b) = \tfrac{1}{2}$$

- To find the mean WTP we first need to extract the density function from the estimated cumulative density function, and then take the expectation in the usual way.

83

EXTRAPOLATION TO THE POPULATION

- The final step is to extrapolate the sample results to the population.

- Ideally we would like to substitute population-average values for the socioeconomic variables in the estimated ICD.

84

- This would yield an estimated population ICD from which we could calculate mean and median WTP values for the population.

- This is often not possible in practice because we do not always have population data on all the explanatory variables.

85

- Where the required population data is not available the only feasible approach is to calculate a mean WTP from the sample and then simply multiply this value by the number of households in the referent population.

86

7.5-2 ISSUES IN SURVEY DESIGN

- Surveys should always be designed in consultation with survey-design experts.
- A draft survey should always be tested with a number of “focus groups”.

87

- An important function of focus groups in referendum contingent valuation studies is the determination of bid prices.
- The statistical efficiency of our valuation estimates relies on achieving a mix of “yes” and “no” responses in each bid group.

88

- If bid prices are set too high or too low then the results will show too high a proportion of “no” or “yes” responses, and the data will not be very informative.

89

- It is important to conduct any survey under conditions that ensure the attentiveness of the respondents.
- Postal surveys are inexpensive but unreliable; phone surveys are better.
- The best administration method is to use a “central facility” but this is costly.

90

- It is important to impress upon respondents the need for realism in their responses.
- It is standard to remind respondents of their income constraints and household budget commitments when soliciting responses.
- Respondents should be asked to review their responses.

91

- Evidence indicates that respondents do not treat very short surveys seriously but become fatigued if surveys are too long.

92

7.5-3 LIMITATIONS AND COMPLICATIONS

- Stated preference methods have a number of limitations and potential problems that have often made their use controversial.
- Among the most important of these are
  - strategic bias
  - artificiality
  - information bias

93

STRATEGIC BIAS

- Survey respondents may have an incentive to misrepresent their WTP depending on how they believe any actual payment will be tied to their stated valuation.
- If respondents believe that they may actually be required to pay an amount equal to their stated valuation, then they may have an incentive to understate their valuation.

94

- Conversely, if they believe that they will not actually have to pay anything then they may have an incentive to overstate their valuation.

95

ARTIFICIALITY

- Critics of these methods claim that "if one asks a hypothetical question then one will get a hypothetical answer".
- That is, because respondents in a survey do not actually have to pay, they have little incentive to put much thought into their responses.

96

- Evidence of the artificiality problem is alleged to show up in **embedding effects**.
- "Embedding" a candidate environmental project in a basket of projects should not significantly affect the WTP for that project.
- Experiments often suggest otherwise.

97

- The artificiality problem can be alleviated by setting up a well-controlled survey environment, in which the respondent is reminded of her income and other prices.
- However, the problem cannot be eliminated entirely.

98

INFORMATION BIAS

- Survey respondents often have very little prior knowledge of the amenity to be valued.

- For example, would you be willing to pay $20 per year to protect the habitat of quokas?

99

100

- How much information about the amenity should be provided to survey respondents?
- The information provided should, in principle, be the same as the respondent would have acquired had the amenity been offered on a real market.
- This will generally be less than “full information”, because information acquisition is costly.

101

- Moreover, by informing survey respondents, we make them “non-representative” of the population because those individuals who have not participated in the survey do not receive the same information.
- This is a difficult theoretical issue yet to be fully resolved.

END

102

TOPIC 7 REVIEW QUESTIONS

1. Which of the following is not a component of passive-use value?

A. Existence value.

B. Information value.

C. Bequest value.

D. Option value.

2. Suppose an individual believes that she might one day visit the Great Barrier Reef off the coast of north-east Australia, and she contributes $100 every year to a conservation charity that works to protect the reef. Eventually she does visit the reef, and the next year she stops her annual charitable contribution. This behaviour suggests that her earlier contributions were probably motivated by

A. Existence value.

B. Use value.

C. Option value.

D. Information value.

3. Revealed-preference methods

A. use observable market data on related goods to make inferences about non-market values.

B. use non-market data to estimate non-market values.

C. are not well-suited to measuring use values.

D. Both A and C.

4. The "averting behaviour method" is based on the principle that if a well-informed individual is willing to pay at least $*x* to defend himself against potential damage then the cost to him of the undefended damage must be at least $*x*.

A. True.

B. False.

5. Suppose an individual spends $120 per year on dandelion tea because she believes it will reduce by 50% the likelihood that she will become ill from drinking the local water. A scientific study into the issue found that the risk reduction is only about 2%. An averting behavior study uses the dandelion tea expenditure and the scientifically-based risk-reduction estimate to measure the value this person puts on healthy drinking water. The study is likely to

A. under-estimate this value.

B. over-estimate this value.

Questions 6 – 8 relate to the following information

Suppose an hedonic price study is conducted using house-price data in a given area, and the estimated relationship is as follows:

$$p = 400000 + 100s + 50000b + 1000000\left(\frac{\ln(k)}{1+\ln(k)}\right)$$

where *p* is the house price, *s* is the floor area of the house (measured in square meters), *b* is the number of bedrooms, and *k* is the number of kilometers between the house of the local landfill. The average house in the region has a floor area of $200m^2$ and three bedrooms.

6. Approximately how much more valuable is an average house located 3km from the landfill relative to an average house located only 2km from the landfill?

A. $20,000

B. $76,000

C. $114,000

D. $186,000

7. At what distance from the landfill would the presence of the landfill make the average house worthless?

A. 0.696 km

B. 1.352 km

C. 2 km

D. 0 km

8. What would be the price of an average house in the area if the landfill could be magically removed? (Hint: what would be equivalent to removing the landfill in terms of putting distance between the house and the landfill?)

A. $2m

B. $1.57m

C. $870,000

D. $600,000

9. The "travel cost method" is based on the principle that the amount of money an individual is willing to spend to visit and utilize a recreation site reflects the value she places on using that site.

A. True.

B. False.

10. The demand for site visits estimated using the travel cost method plots visits against

A. the market price paid to enter the site.

B. the cost of traveling to the site.

C. a travel cost variable calculated using all costs incurred to use the site (including but not limited to travel costs).

D. None of the above.

11. The availability of alternative recreation sites and alternative leisure activities means that the demand for site visits estimated for any one site

A. will systematically over-estimate that demand.

B. would shift in response to changes in the prices of those alternatives.

C. should ideally be nested in a system of demands for leisure activity generally.

D. Both B and C.

12. Referendum-format contingent valuation (RFCV) is a particular type of

A. revealed-preference method.

B. stated-preference method.

13. In a RFCV, all survey respondents face the same bid price

A. to ensure consistency across responses.

B. to limit the scope for strategic responses.

C. Both A and B.

D. None of the above.

14. The response histogram generated from a RFCV survey displays the

A. percentage of “yes” responses for each bid price.

B. the relative likelihood that each bid group responded “yes” to the bid price for that survey.

C. the split between the number of “yes” responses and the number of “no” responses for each bid group.

D. None of the above.

15. The response histogram generated from a RFCV survey is a rough estimate of the inverse cumulative density (ICD) for the distribution of “yes” responses across bid groups.

A. True.

B. False.

16. The “random-utility model” that underlies the use of the logit regression in a RFCV

A. captures the idea that individuals do not always act rationally.

B. reflects the fact that unobservable factors appear to introduce randomness into behaviour from the perspective of the analyst.

C. models the utility function for an individual as a logistic function.

D. Both B and C.

17. The following is an ICD estimated using a logit regression model:

$$1 - F = \frac{1}{1 + \exp(\hat{\beta}_{bid} b + \sum_{i=1}^{K-1} \hat{\beta}_i \bar{x}_i)}$$

In this equation, $\bar{x}_i$

A. is the i^{th} explanatory variable evaluated at its sample mean.

B. is included in the regression to account for the fact that this explanatory variable differs across survey respondents.

C. is a shift parameter when $1 - F$ is plotted against the bid price b.

D. All of the above.

18. In a RFCV, "strategic bias" refers to the potential for estimated valuations to be biased because

A. survey respondents do not actually have to pay, and so they have little incentive to put much thought into their responses.

B. survey respondents often have very little prior knowledge of the amenity to be valued.

C. survey respondents may have an incentive to misrepresent their WTP depending on how they believe any actual payment will be tied to their stated valuation.

D. the analyst may deliberately choose bid values to influence the estimation results.

ANSWER KEY

1. B
2. C
3. A
4. A
5. B
6. C
7. A
8. B

Take the limit of p as $k \to \infty$

9. A
10. C
11. D
12. B
13. D
14. A
15. A
16. B
17. D
18. C

8. AN ILLUSTRATIVE EXAMPLE

8.1 THE PROPOSED PROJECT

- The government is considering constructing a bridge over a river.
- The river can currently be crossed only by ferry boat, operated by a private firm.

2

- The ferry firm currently charges $40 per vehicle-crossing, though its operating costs are only $35 per crossing.
- There are currently 30,000 crossings per year.
- The ferry service will close down once the bridge is completed and the operator will sell the ferry for scrap for $100,000.

3

- Construction of the bridge will take 2 years, with costs estimated to be $11m in year 1 and $11m in year 2.
- These costs will be funded out of general tax revenue. The marginal cost of funds from is estimated to be 1.2.
- The bridge is expected to last forever.

4

- Yearly maintenance costs for the bridge are estimated to be $10 per crossing (based on estimated annual resurfacing requirements). This cost will be financed by a toll of $10 per crossing.

- The estimated number of yearly crossings at this $10 price is 90,000.

5

- The demand curve for crossings is assumed to be linear, as illustrated in Figure 8.1.

6

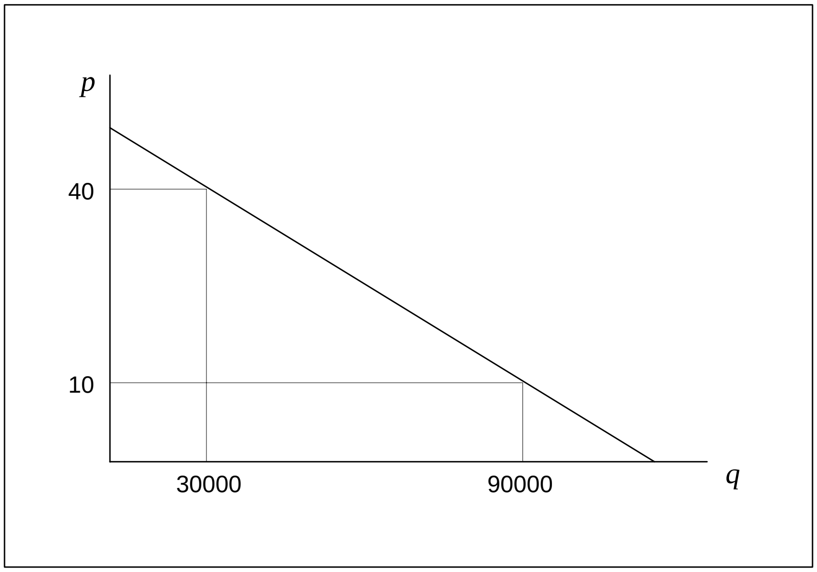


Figure 8-1

7

8.2 COST-BENEFIT ANALYSIS

1. Referent Group

- No information is provided, so we assume that all impacted parties have standing.

2. Select the Portfolio of Projects

- We have information on only one project, so consideration is confined to that one.

8

3. Catalogue Potential Impacts

- increase in the number of crossings
- cost savings on ferry crossings eliminated
- scrapping of ferry
- construction and maintenance costs
- government finances and the cost of funds

9

4. Quantitative Impacts

- These are provided in the project information so we will not repeat them here.

10

5. Monetize all Impacts

(a) Increase in the Number of Crossings

- There are 60,000 additional crossings per year.
- The value of these crossings is indicated as the shaded area in Figure 8.2.

11

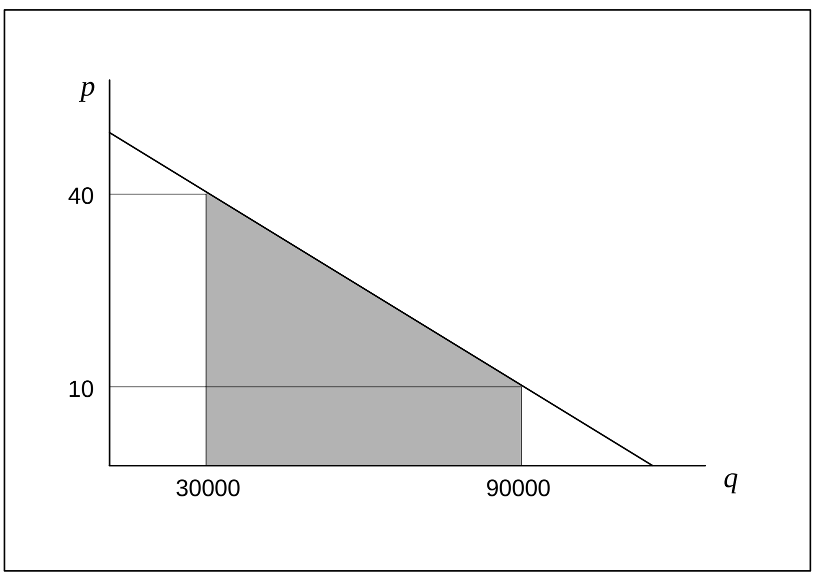


Figure 8-2

12

- The value of these additional crossings is (10 x 60,000) + (30 x 60,000)/2 = **$1.5m**
- This is a benefit of the project. It accrues in year 3 and every year thereafter.

13

(b) Savings on Ferry Crossings Eliminated

- The price of a crossing by ferry is $40, but the cost of that crossing is only $35.
- Thus, the cost savings on ferry crossings eliminated is:

  35 x 30,000 = **$1.05m**
- This is a benefit of the project. It accrues in year 3 and every year thereafter.

14

(c) Scrap Value of the Ferry

- Scrap value is **$100,000**.
- This is a benefit of the project, accruing once, in year 3.

15

(d) Construction and Maintenance Costs

- Construction costs are **$11m** in each of years 1 and 2.
- Maintenance costs are **$900,000** per year in year 3 and every year thereafter.
- These are costs of the project.

16

(e) Government Finances and Cost of Funds

- The impact on government finances is summarized in Table 8-1, where the "+" in the last column indicates that this column repeats for all future years.

17

$m	year 1	year 2	year 3 (+)
outlays			
construction	11	11	
maintenance			0.9
receipts			
tolls			0.9
net outlay	**11**	**11**	**0**
cost of funds	**2.2**	**2.2**	**0**

Table 8-1

18

- The cost of funds (in years 1 and 2) is
  0.2 x $11m = **$2.2m**

19

Summary of Benefits and Costs

$m	year 1	year 2	year 3	year 4(+)
benefits				
new crossings			1.50	1.50
cost savings			1.05	1.05
ferry scrap value			0.10	
total benefits			2.65	2.55
costs				
construction	11.00	11.00		
maintenance			0.90	0.90
cost of funds	2.20	2.20		
total costs	13.20	13.20	0.90	0.90
net benefits	**-13.20**	**-13.20**	**1.75**	**1.65**

Table 8-2

20

6. Calculate the NPV

- Method 1

$$NPV = -13.2 - \frac{13.2}{(1+r)} + \frac{1.75}{(1+r)^2} + \frac{\left[\frac{1.65(1+r)}{r}\right]}{(1+r)^3}$$

21

- Method 2

$$PVB = \frac{2.65}{(1+r)^2} + \frac{\left[\frac{2.55(1+r)}{r}\right]}{(1+r)^3}$$

$$PVC = 13.2 + \frac{13.2}{(1+r)} + \frac{\left[\frac{0.9(1+r)}{r}\right]}{(1+r)^2}$$

$$NPV = PVB - PVC$$

22

Net Present Value

r	1	2	3	4	5	6	7	8	9	10
NPV	137.20	54.84	27.48	13.86	5.75	0.38	-3.42	-6.24	-8.41	-10.12

Table 8-3

23

Net Present Value

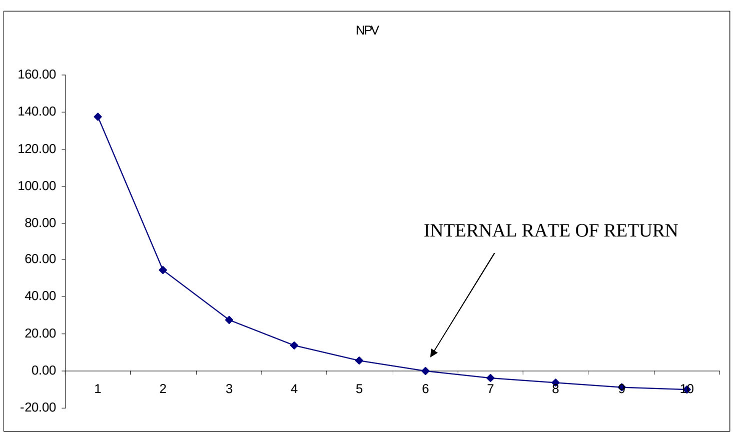


Figure 8-3

24

7. Identify the Distribution of Costs and Benefits

(a) Bridge Users

- Gain in consumer surplus (shaded areas in Figure 8-4):

  (30 x 30000) + (30 x 60000)/2 = **$1.8m**

- These gains accrue in year 3 and each year thereafter.

25

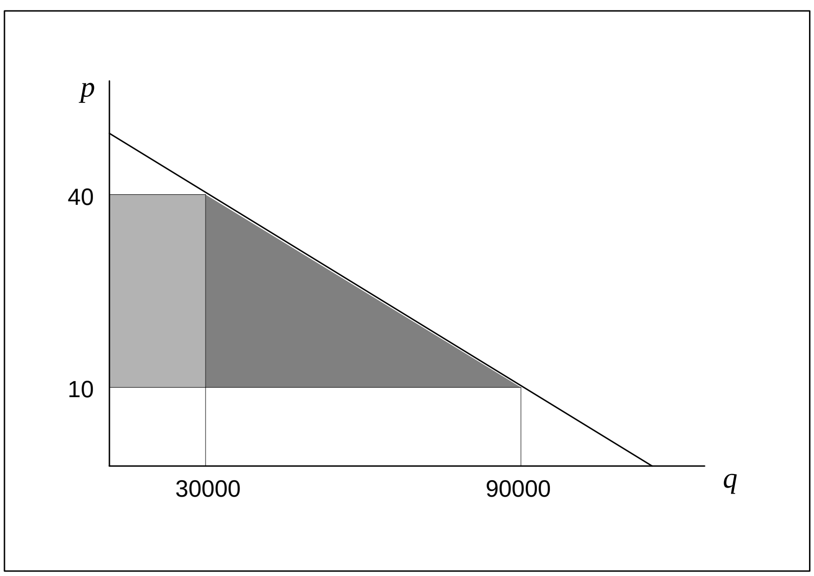


Figure 8-4

26

- Present value of gain to bridge users:

$$G_B = \frac{\left(1.8\left(\frac{1+r}{r}\right)\right)}{(1+r)^2}$$

27

(b) Ferry Operator

- Loss of producer surplus:

(40 – 35) x 30000 = $150,000

- This loss occurs in year 3 and each year thereafter.

28

- This loss is partly offset by the one-time scrap value of the ferry in year 3.

- Thus, the net loss to the ferry operator (in present value terms) is

$$L_F = \frac{\left(0.15\left(\frac{1+r}{r}\right) - 0.1\right)}{(1+r)^2}$$

29

(c) Taxpayers

- Construction costs plus the associated cost of funds, in years 1 and 2.

- Present value of this loss:

$$L_T = 13.2 + \frac{13.2}{1+r}$$

30

Summary of Distributional Impacts

$m (present value)	at 2%	at 5%	at 7%
winners			
bridge users	88.23	34.29	24.03
losers			
ferry operator	7.25	2.77	1.92
taxpayers	26.14	25.77	25.54
aggregate impact	**54.84**	**5.75**	**-3.43**

Table 8-4

31

- Note that the aggregate net impact at any given discount rate is necessarily equal to the NPV of the project at that discount rate, as calculated in Step 6 above; see Table 8-3.
- This comparison provides a useful check that all calculations are correct.

32

Net Present Value

r	1	2	3	4	5	6	7	8	9	10
NPV	137.20	**54.84**	27.48	13.86	**5.75**	0.38	**-3.42**	-6.24	-8.41	-10.12

Table 8-3 (repeat)

33

8.3 THE OPTIMAL TOLL

- The results from Table 8-4 tell us that bridge users are the big beneficiaries of the project, and that taxpayers are the big losers.
- The policy-maker may decide that this distribution of wins and losses is too unbalanced, and that a higher toll should be set to help fund the construction costs.

34

- Ordinarily we expect social surplus to be maximized when price is set equal to marginal cost, which in this case means setting the toll equal to the marginal maintenance cost, and this underlies the logic of a $10 toll.
- Thus, any deviation from that policy in pursuit of a distributional goal might be expected to reduce the overall social surplus of the project.

35

- However, in this project there is a significant cost of funds associated with taxpayer-financing of construction costs, and we need to account for this when thinking about an optimal toll.
- In particular, there is a benefit from reducing that cost of funds by raising revenue from the toll beyond the amount that will cover the cost of maintenance.

36

- Thus, in this setting where MCF > 1, the toll that maximizes social surplus (the “optimal toll”) will be higher than $10.

- It is important to stress that setting the toll at its optimal value (higher than $10) is <u>not</u> driven by distributional concerns directly, but it may help to address those concerns at the same time as raising social surplus.

37

- To determine the optimal toll, we first need to make the toll a variable, and express NPV in terms of the toll.

- The toll can then be chosen to maximize NPV.

38

- There are three components of the NPV calculation that vary with the toll:
  - the value of new crossings
  - maintenance costs
  - revenue raised (and hence the cost of funds)

39

- To derive these three components as functions of the toll, we begin by deriving the equation for the demand for crossings.
- We can then determine the number of crossings as a function of the toll.
- See Figures 8-5 and 8-6.

40

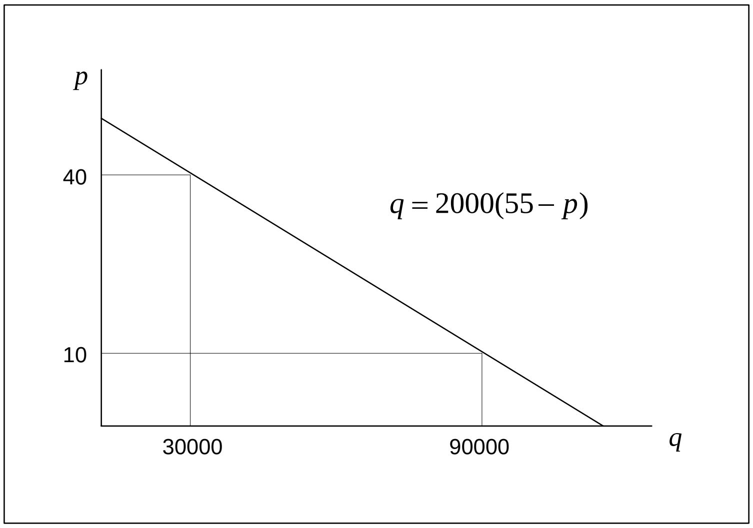


Figure 8-5 41

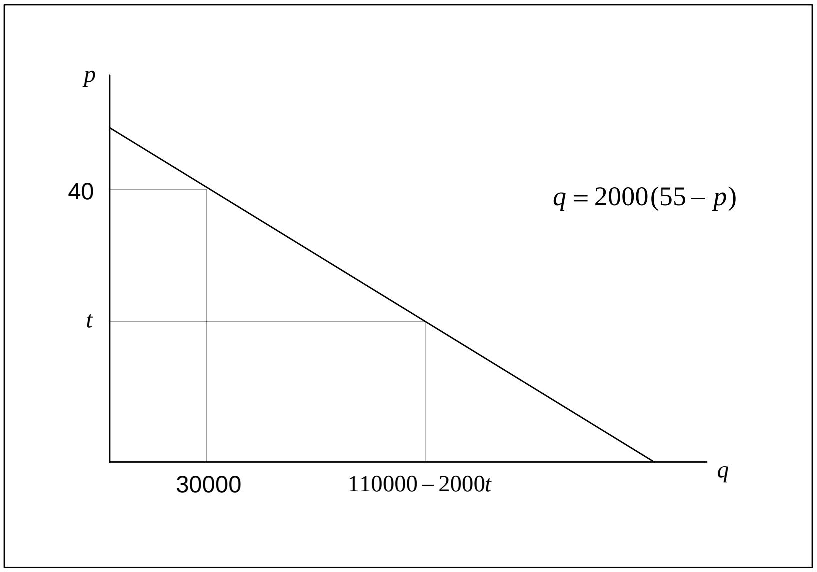


Figure 8-6 42

- The value of the additional crossings can now be calculated as the shaded area in Figure 8-7.

43

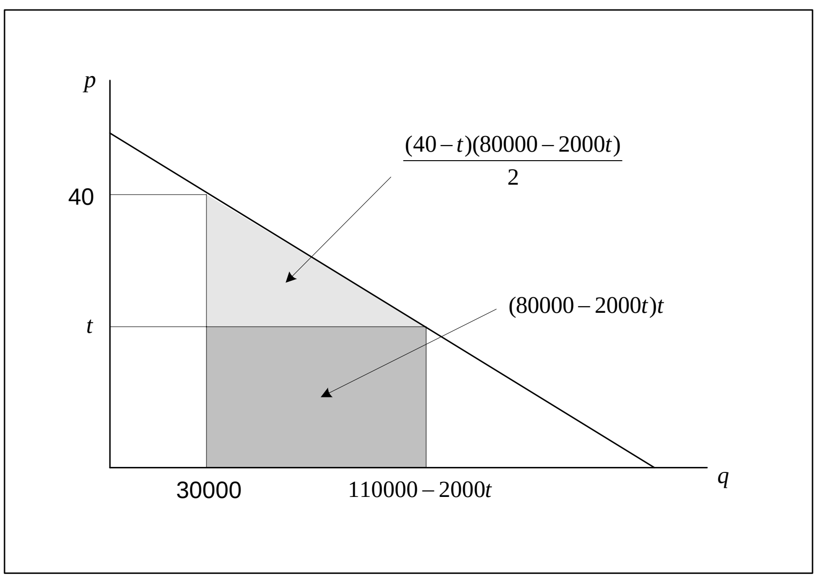


Figure 8-7

44

- Value of new crossings:

$$V = (80000 - 2000t)\left(t + \frac{40 - t}{2}\right)$$

$$= 1600000 - 1000t^2$$

$$= \left(1.6 - \frac{t^2}{1000}\right) million$$

45

- Similarly, we can use the endogenized number of crossings (as a function of the toll) to calculate maintenance costs and toll revenue, as per the following calculations.

46

- Maintenance costs:

$$M = 10q(t)$$

$$= 10(110000 - 2000t)$$

$$= \left(1.1 - \frac{2t}{100}\right) million$$

47

- Toll revenue:

$$T = tq(t)$$

$$= t(110000 - 2000t)$$

$$= \left(0.11t - \frac{2t^2}{1000}\right) million$$

48

- We can now replace the hard numbers in our Excel Tables with the new formulas for these three components.
- See Tables 8.1R and 8.2R.

$m	year 1	year 2	year 3 (+)
outlays			
construction	11	11	
maintenance			**0.9**
receipts			
tolls			**0.9**
net outlay	11	11	0
cost of funds	**2.2**	**2.2**	**0**

maintenance ← $\left(1.1-\frac{2t}{100}\right)$

tolls ← $\left(0.11t-\frac{2t^2}{1000}\right)$

cost of funds ← $T(t)(MCF-1)$

Table 8-1 R

$$\left(1.6 - \frac{t^2}{1000}\right)$$

$m	year 1	year 2	year 3	year 4(+)
benefits				
new crossings			**1.50**	**1.50**
cost savings			1.05	1.05
ferry scrap value			0.10	
total benefits			2.65	2.55
costs				
construction	11.00	11.00		
maintenance			**0.90**	**0.90**
cost of funds	2.20	2.20	**0**	**0**
total costs	13.20	13.20	0.90	0.90
net benefits	**-13.20**	**-13.20**	**1.75**	**1.65**

Figure 8-2 R

51

- We can now experiment with different values of t to determine the toll that maximizes NPV.
- See Figure 8-8 and Table 8-5.

52

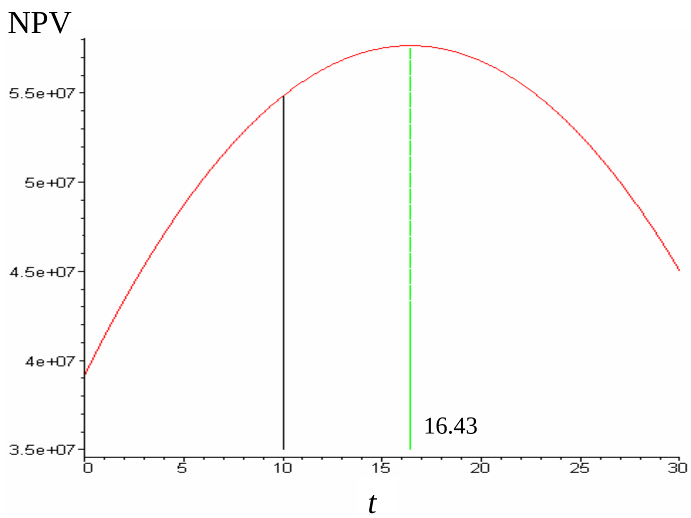


Figure 8-8 53

Experiment		
	Policy 1	**Variant**
Toll	10	**16.43**
MCF	1.2	1.2
Surplus Changes (2%)		
NPV	54.84	57.67
Winners		
Bridge Users	88.24	61.89
Losers		
Ferry Operator	7.26	7.26
Taxpayers	26.14	1.83

Table 8-5 54

- While Excel can be used for analyzing simple questions like the optimal toll it is not well-suited to handling more complicated models where a large number of values are specified as variables.

55

- For more complicated models it is better to use a package like
  - Maple
  - Mathematica
  - MatLab

  or to code it directly using a language like Python.
- Appendix A8 presents the **Maple** code for the bridge project and the optimal toll.

56

8.4 DEALING WITH UNCERTAINTY

- Suppose now we are uncertain about the demand for crossings.

57

- We will continue to assume that demand is linear but we do not know either its slope or its intercept.

- We know only one point on the demand:

  (40, 30000)

  and there are an infinite number of possible demands running through that point; see Figure 8-9.

58

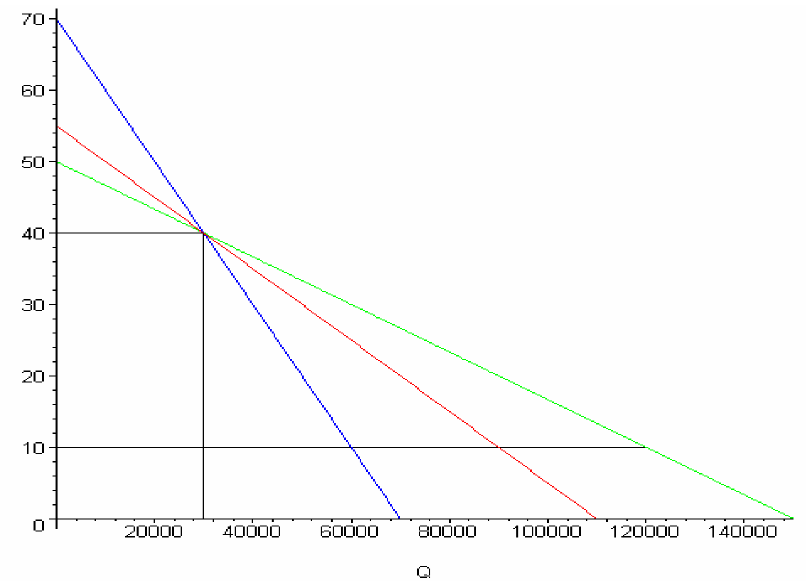


Figure 8-9

59

- Suppose the demand is

$$p = a - bq$$

- We know that if *p* = 40 then *q* = 30000, so we know the relationship between *a* and *b*:

$$b = \frac{a - 40}{30000}$$

60

- This means that we can express our uncertainty about demand in terms of beliefs about a single parameter, *a*.

61

SPECIFICATION OF BELIEFS

- In our initial assessment we simply assumed that *a* = 55.

- Suppose we now specify our beliefs as a uniform distribution on [50,60].

- Note that the mean of this distribution is 55.

62

BUILDING A MODEL

- To conduct sensitivity testing and simulations, we now need to construct all of our calculations in terms of this uncertain parameter *a*.

- This can be done in Excel, in much the same way that in Topic 8.3 we expressed values in terms of the variable "toll".

63

- However, it is generally better to use a package or programming language designed for analytical modeling.

- The results reported in the slides that follow were produced using **Maple**, and the associated code is attached as Appendix A8.2.

64

SENSITIVITY TESTING

- How sensitive is the NPV within the specified range of *a*?
- To illustrate, assume *t* = 10 and *r* = 0.02.
- The result is reported in Figure 8-10.

65

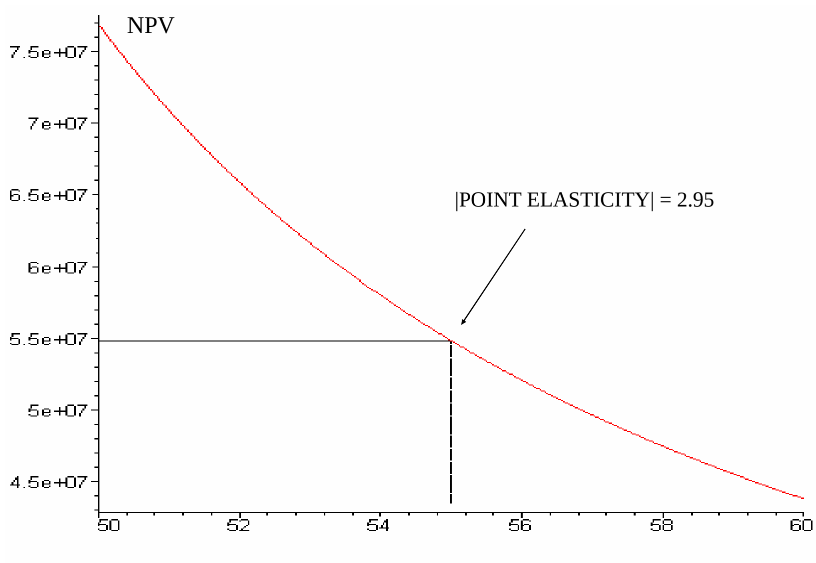


Figure 8-10

66

- Note from Figure 8-10 that NPV is not linear in *a*; it is **strictly convex** in *a*.
- This will be important for some of the results that follow.

67

SIMULATION

- Now let us draw *n* = 10,000 values from a uniform distribution on [50,60] and calculate the NPV for each of those draws.
- The results can be plotted as a histogram, reported in Figure 8-11.

68

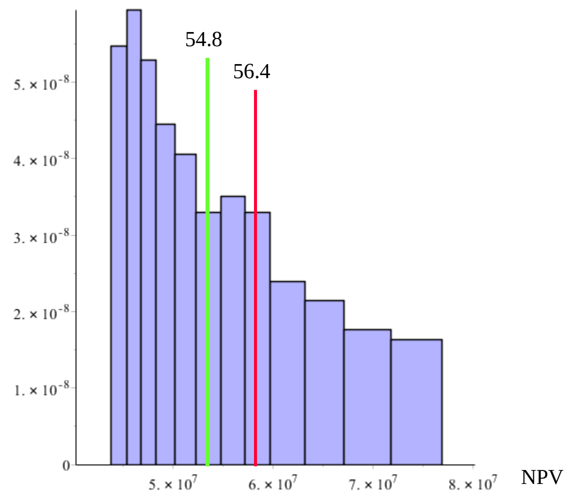


Figure 8-11

69

- The expected NPV is calculated as the mean of the 10,000 values generated.

- This mean is highlighted on the histogram in red: $56.4m

- In comparison, the NPV when *a* = 55 is highlighted in green: $54.8m

70

- Thus, in this example, the expected NPV is greater than the NPV evaluated at the expected value of *a*.

- Why?
  - NPV is a strictly convex function of *a*.

71

- Note too that the distribution of NPV in our histogram is **positively skewed** even though the uniform distribution on *a* is symmetric (not skewed).

- This skewness is also due to the fact that NPV is a strictly convex function of *a*.

72

THE OPTIMAL TOLL REVISITED

- What is the optimal toll when *a* is uncertain?

- We now need to specify our model in terms of the uncertain parameter *a* <u>and</u> the variable *t*, and calculate the expected NPV as a function of *t*.

73

- The Maple code in Appendix 8-2 builds-in the flexibility to choose the toll.

- Figure 8-12 illustrates the relationship between expected NPV and the toll, as generated by that model.

74

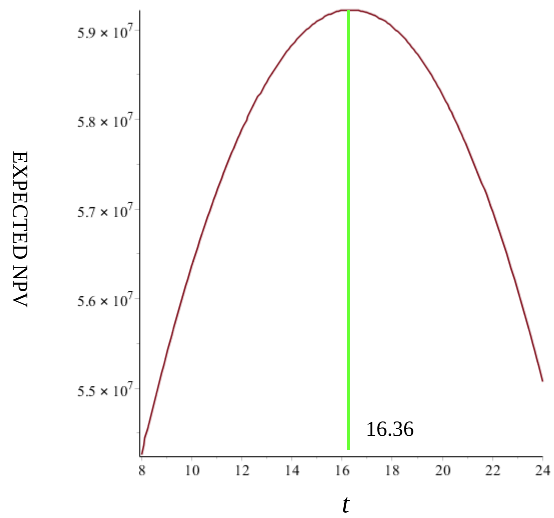


Figure 8-12 75

- The optimal toll is $16.36.
- In comparison, recall from our initial assessment that when *a* is known to be 55, the optimal toll is $16.43.
- Thus, the uncertainty has little impact on our choice of toll in this example.

END 76

APPENDIX A8-1

MAPLE CODE FOR THE ILLUSTRATIVE EXAMPLE

```
> restart:
> with(plots):
Warning, the name changecoords has been redefined
```

NUMBER OF CROSSINGS

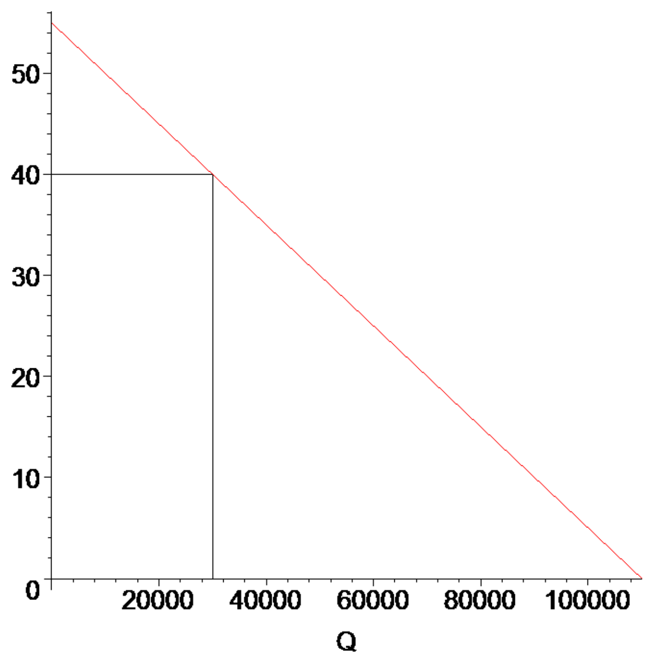
Assumed Linear Demand

```
> P:=a-b*Q;
```

$$P := a - b\,Q$$

Example

```
> g1:=plot(subs(a=55,b=1/2000,P),Q=0..110000):
> g2:=plot(40,Q=0..30000,color=black):
> g3:=implicitplot(Q=30000,Q=0..30000,p=0..40,color=black):
> display(g1,g2,g3);
```



Inverse demand

```
> q:=solve(P-p,Q);
```

$$q := \frac{a - p}{b}$$

We know only that q=30000 at p=40. This implies a relationship between a and b:

```
> b:=solve(subs(p=40,q)=30000,b);
```

$$b := \frac{a}{30000} - \frac{1}{750}$$

GOVERNMENT FINANCES

Toll revenue

```
> TR:=simplify(q*p);
```

$$TR := \frac{30000\,(a-p)\,p}{a-40}$$

Aside: revenue-maximizing toll:

```
> solve(diff(TR,p),p);
```

$$\frac{a}{2}$$

Maintenance Costs

```
> MC:=simplify(m*q);
```

$$MC := \frac{30000\,m\,(a-p)}{a-40}$$

Construction Costs

```
> CC;
```

$$CC$$

Net Outlays

```
> NO[1]:=CC;
```

$$NO_1 := CC$$

```
> NO[2]:=CC;
```

$$NO_2 := CC$$

```
> NO[3]:=simplify(MC-TR);
```

$$NO_3 := \frac{30000\,(a-p)\,(m-p)}{a-40}$$

```
> COF[1]:=(MCF-1)*NO[1];
```

$$COF_1 := (MCF-1)\,CC$$

```
> COF[2]:=(MCF-1)*NO[2];
```

$$COF_2 := (MCF-1)\,CC$$

```
> COF[3]:=(MCF-1)*NO[3];
```

$$COF_3 := \frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}$$

BENEFITS

New Crossings

```
> NCB:=simplify(int(P,Q=F..q));
```

$$NCB := \frac{900000000\, a^2 - 900000000\, p^2 + F^2\, a^2 - 80\, F^2\, a + 1600\, F^2 - 60000\, F\, a^2 + 2400000\, F\, a}{60000\,(a - 40)}$$

Cost Savings on Ferry Crossings Eliminated

```
> CSF:=F*fc;
```

$$CSF := F\, fc$$

where

```
> F:=30000;
```

$$F := 30000$$

Ferry Scrap

```
> FSV;
```

$$FSV$$

COSTS

Construction Costs

```
> CC;
```

$$CC$$

```
> MC;
```

$$\frac{30000\, m\,(a - p)}{a - 40}$$

```
> COF;
```

$$COF$$

NET BENEFITS

```
> NB[1]:=-CC-COF[1];
```

$$NB_1 := -CC - (MCF - 1)\, CC$$

```
> NB[2]:=-CC-COF[2];
```

$$NB_2 := -CC - (MCF - 1)\, CC$$

```
> NB[3]:=NCB+CSF+FSV-MC-COF[3];
```

$$NB_3 := \frac{-900000000\, p^2 + 1440000000000}{60000\,(a - 40)} + 30000\, fc + FSV - \frac{30000\, m\,(a - p)}{a - 40} - \frac{30000\,(MCF - 1)\,(a - p)\,(m - p)}{a - 40}$$

```
> NB[4]:=NCB+CSF-MC-COF[3];
```

$$NB_4 := \frac{-900000000\,p^2 + 1440000000000}{60000\,(a-40)} + 30000\,fc - \frac{30000\,m\,(a-p)}{a-40} - \frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}$$

NET PRESENT VALUE

```
> NPV:=NB[1]+NB[2]/(1+r)+NB[3]/(1+r)^2+NB[4]*(1+r)/r/(1+r)^3;
```

$$NPV := -CC - (MCF-1)\,CC + \frac{-CC-(MCF-1)\,CC}{1+r} + \left(\frac{-900000000\,p^2 + 1440000000000}{60000\,(a-40)} + 30000\,fc + FSV - \frac{30000\,m\,(a-p)}{a-40} - \frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}\right) \Big/ (1+r)^2 + \left(\frac{-900000000\,p^2 + 1440000000000}{60000\,(a-40)} + 30000\,fc - \frac{30000\,m\,(a-p)}{a-40} - \frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}\right) \Big/ ((1+r)^2\,r)$$

DISTRIBUTION OF COSTS AND BENEFITS

<u>Bridge Users</u>

Change in CS (year 3+):

```
> dCS:=int(q,p=p..40);
```

$$dCS := -\frac{1600-p^2}{2\left(\frac{a}{30000}-\frac{1}{750}\right)} + \frac{a\,(40-p)}{\frac{a}{30000}-\frac{1}{750}}$$

Present value:

```
> PVdCS:=dCS*(1+r)/r/(1+r)^2;
```

$$PVdCS := \frac{-\frac{1600-p^2}{2\left(\frac{a}{30000}-\frac{1}{750}\right)} + \frac{a\,(40-p)}{\frac{a}{30000}-\frac{1}{750}}}{(1+r)\,r}$$

Ferry Operator

Change in PS (year 3+):

```
> dPS:=(40-fc)*F;
```

$$dPS := 1200000 - 30000\, fc$$

Present value, offset by FSV:

```
> PVdPS:=dPS*(1+r)/r/(1+r)^2-FSV/(1+r)^2;
```

$$PVdPS := \frac{1200000 - 30000\, fc}{(1+r)\, r} - \frac{FSV}{(1+r)^2}$$

Taxpayers

Net Outlays and COF:

```
> dT[1]:=CC+COF[1];
```

$$dT_1 := CC + (MCF - 1)\, CC$$

```
> dT[2]:=CC+COF[2];
```

$$dT_2 := CC + (MCF - 1)\, CC$$

```
> dT[3]:=-TR+MC+COF[3];
```

$$dT_3 := -\frac{30000\,(a-p)\,p}{a-40} + \frac{30000\, m\,(a-p)}{a-40} + \frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}$$

Present value:

```
> PVdT:=dT[1]+dT[2]/(1+r)+dT[3]*(1+r)/r/(1+r)^2;
```

$$PVdT := CC + (MCF-1)\, CC + \frac{CC + (MCF-1)\, CC}{1+r} + \frac{-\frac{30000\,(a-p)\,p}{a-40} + \frac{30000\, m\,(a-p)}{a-40} + \frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}}{(1+r)\, r}$$

Change in Social Surplus

```
> PVdSS:=PVdCS-PVdPS-PVdT;
```

$$PVdSS := \frac{-\frac{1600 - p^2}{2\left(\frac{a}{30000} - \frac{1}{750}\right)} + \frac{a\,(40-p)}{\frac{a}{30000} - \frac{1}{750}}}{(1+r)\, r} - \frac{1200000 - 30000\, fc}{(1+r)\, r} + \frac{FSV}{(1+r)^2} - CC - (MCF-1)\, CC - \frac{CC + (MCF-1)\, CC}{1+r} - \frac{-\frac{30000\,(a-p)\,p}{a-40} + \frac{30000\, m\,(a-p)}{a-40} + \frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}}{(1+r)\, r}$$

CONSISTENCY CHECK

```
> check:=simplify(NPV-PVdSS);
```

$$check := 0$$

OPTIMAL TOLL

```
> FOC:=diff(NPV,p):
> pstar:=solve(FOC,p);
```

$$pstar := \frac{MCF\ a - a + MCF\ m}{2\ MCF - 1}$$

Scenario 1 Parameters

```
> a:=55;
```

$$a := 55$$

```
> m:=10;
```

$$m := 10$$

```
> CC:=11*1000000;
```

$$CC := 11000000$$

```
> fc:=35;
```

$$fc := 35$$

```
> FSV:=100000;
```

$$FSV := 100000$$

```
> MCF:=1.2;
```

$$MCF := 1.2$$

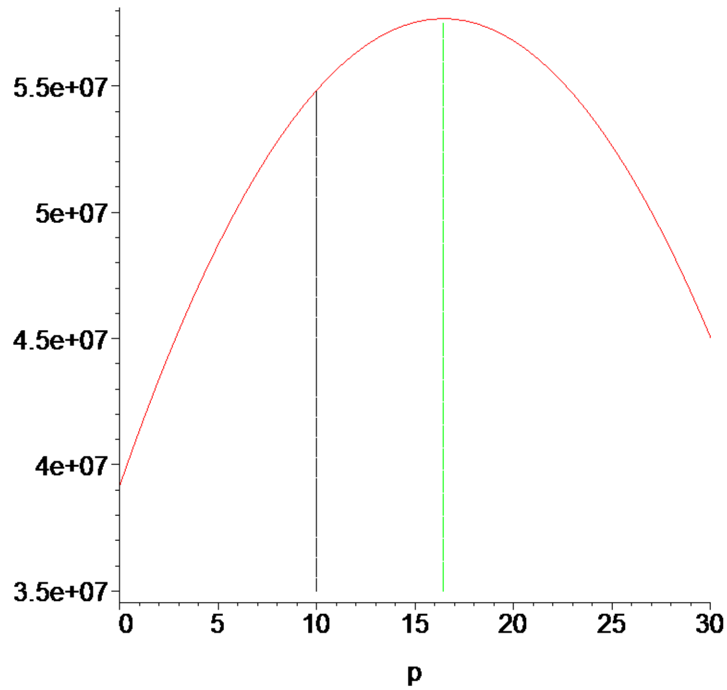
Optimal Toll

```
> pstar;
```

$$16.42857143$$

NPV Plotted Against Toll (with r=2%)

```
> g1:=plot(subs(r=0.02,NPV),p=0..30):
> g2:=implicitplot(p=m,p=0..30,npv=3.5e+07..5.484e+07,color=black):
> g3:=implicitplot(p=16.43,p=0..30,npv=3.5e+07..5.75e+07,color=green
):
> display(g1,g2,g3);
```



5.5e+07
5e+07
4.5e+07
4e+07
3.5e+07
0
5
10
15
20
25
30
p

APPENDIX A8-2

MAPLE CODE FOR THE ILLUSTRATIVE EXAMPLE: DEALING WITH UNCERTAINTY

```
> restart:
> with(plots):
Warning, the name changecoords has been redefined
> with(stats):
```

PART 1: THE BASIC MODEL

NUMBER OF CROSSINGS

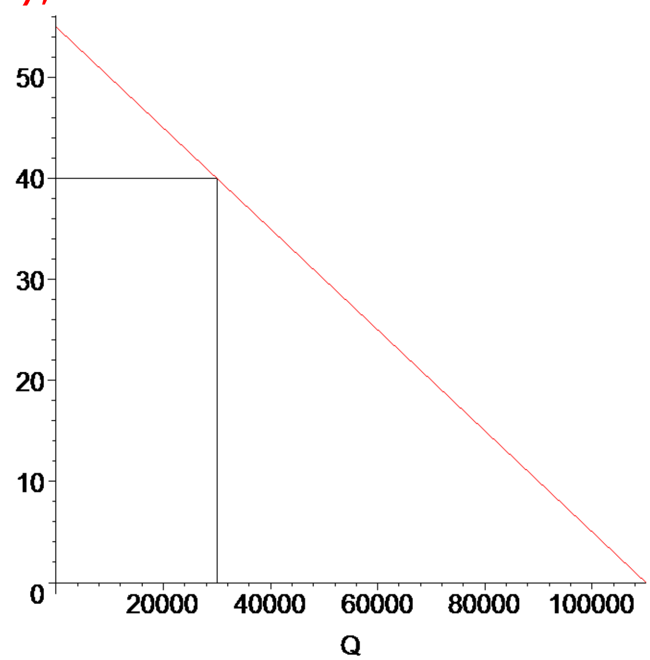
Assumed Linear Demand

```
> P:=a-b*Q;
```

$$P := a - b\,Q$$

Example

```
> g1:=plot(subs(a=55,b=1/2000,P),Q=0..110000):
> g2:=plot(40,Q=0..30000,color=black):
> g3:=implicitplot(Q=30000,Q=0..30000,p=0..40,color=black):
> display(g1,g2,g3);
```



Inverse demand

```
> q:=solve(P-p,Q);
```

$$q := \frac{a-p}{b}$$

We know only that q=30000 at p=40. This implies a relationship between a and b:

```
> b:=solve(subs(p=40,q)=30000,b);
```

$$b := \frac{a}{30000} - \frac{1}{750}$$

GOVERNMENT FINANCES

Toll revenue

```
> TR:=simplify(q*p);
```

$$TR := \frac{30000\,(a-p)\,p}{a-40}$$

Aside: revenue-maximizing toll:

```
> solve(diff(TR,p),p);
```

$$\frac{a}{2}$$

Maintenance Costs

```
> MC:=simplify(m*q);
```

$$MC := \frac{30000\,m\,(a-p)}{a-40}$$

Construction Costs

```
> CC;
```

$$CC$$

Net Outlays

```
> NO[1]:=CC;
```

$$NO_1 := CC$$

```
> NO[2]:=CC;
```

$$NO_2 := CC$$

```
> NO[3]:=simplify(MC-TR);
```

$$NO_3 := \frac{30000\,(a-p)\,(m-p)}{a-40}$$

```
> COF[1]:=(MCF-1)*NO[1];
```

$$COF_1 := (MCF-1)\,CC$$

```
> COF[2]:=(MCF-1)*NO[2];
```

$$COF_2 := (MCF - 1)\ CC$$

```
> COF[3]:=(MCF-1)*NO[3];
```

$$COF_3 := \frac{30000\ (MCF - 1)\ (a - p)\ (m - p)}{a - 40}$$

BENEFITS

New Crossings

```
> NCB:=simplify(int(P,Q=F..q));
```

$$NCB := \frac{900000000\ a^2 - 900000000\ p^2 + F^2\ a^2 - 80\ F^2\ a + 1600\ F^2 - 60000\ F\ a^2 + 2400000\ F\ a}{60000\ (a - 40)}$$

Cost Savings on Ferry Crossings Eliminated

```
> CSF:=F*fc;
```

$$CSF := F\ fc$$

where

```
> F:=30000;
```

$$F := 30000$$

Ferry Scrap

```
> FSV;
```

$$FSV$$

COSTS

Construction Costs

```
> CC;
```

$$CC$$

```
> MC;
```

$$\frac{30000\ m\ (a - p)}{a - 40}$$

```
> COF;
```

$$COF$$

NET BENEFITS

```
> NB[1]:=-CC-COF[1];
```

$$NB_1 := -CC - (MCF - 1)\ CC$$

```
> NB[2]:=-CC-COF[2];
```

$$NB_2 := -CC - (MCF - 1)\, CC$$

```
> NB[3]:=NCB+CSF+FSV-MC-COF[3];
```

$$NB_3 := \frac{-900000000\, p^2 + 1440000000000}{60000\, (a - 40)} + 30000\, fc + FSV - \frac{30000\, m\, (a - p)}{a - 40} - \frac{30000\, (MCF - 1)\, (a - p)\, (m - p)}{a - 40}$$

```
> NB[4]:=NCB+CSF-MC-COF[3];
```

$$NB_4 := \frac{-900000000\, p^2 + 1440000000000}{60000\, (a - 40)} + 30000\, fc - \frac{30000\, m\, (a - p)}{a - 40} - \frac{30000\, (MCF - 1)\, (a - p)\, (m - p)}{a - 40}$$

NET PRESENT VALUE

```
> NPV:=NB[1]+NB[2]/(1+r)+NB[3]/(1+r)^2+NB[4]*(1+r)/r/(1+r)^3;
```

$$NPV := -CC - (MCF - 1)\, CC + \frac{-CC - (MCF - 1)\, CC}{1 + r} + \left(\frac{-900000000\, p^2 + 1440000000000}{60000\, (a - 40)} + 30000\, fc + FSV - \frac{30000\, m\, (a - p)}{a - 40} - \frac{30000\, (MCF - 1)\, (a - p)\, (m - p)}{a - 40} \right) \Big/ (1 + r)^2 + \left(\frac{-900000000\, p^2 + 1440000000000}{60000\, (a - 40)} + 30000\, fc - \frac{30000\, m\, (a - p)}{a - 40} - \frac{30000\, (MCF - 1)\, (a - p)\, (m - p)}{a - 40} \right) \Big/ ((1 + r)^2\, r)$$

DISTRIBUTION OF COSTS AND BENEFITS

<u>Bridge Users</u>

Change in CS (year 3+):

```
> dCS:=int(q,p=p..40);
```

$$dCS := -\frac{1600 - p^2}{2 \left(\frac{a}{30000} - \frac{1}{750} \right)} + \frac{a\, (40 - p)}{\frac{a}{30000} - \frac{1}{750}}$$

Present value:

```
> PVdCS:=dCS*(1+r)/r/(1+r)^2;
```

$$PVdCS := \frac{-\frac{1600 - p^2}{2\left(\frac{a}{30000} - \frac{1}{750}\right)} + \frac{a\,(40 - p)}{\frac{a}{30000} - \frac{1}{750}}}{(1 + r)\, r}$$

Ferry Operator

Change in PS (year 3+):

```
> dPS:=(40-fc)*F;
```

$$dPS := 1200000 - 30000\, fc$$

Present value, offset by FSV:

```
> PVdPS:=dPS*(1+r)/r/(1+r)^2-FSV/(1+r)^2;
```

$$PVdPS := \frac{1200000 - 30000\, fc}{(1 + r)\, r} - \frac{FSV}{(1 + r)^2}$$

Taxpayers

Net Outlays and COF:

```
> dT[1]:=CC+COF[1];
```

$$dT_1 := CC + (MCF - 1)\, CC$$

```
> dT[2]:=CC+COF[2];
```

$$dT_2 := CC + (MCF - 1)\, CC$$

```
> dT[3]:=-TR+MC+COF[3];
```

$$dT_3 := -\frac{30000\,(a - p)\, p}{a - 40} + \frac{30000\, m\,(a - p)}{a - 40} + \frac{30000\,(MCF - 1)\,(a - p)\,(m - p)}{a - 40}$$

Present value:

```
> PVdT:=dT[1]+dT[2]/(1+r)+dT[3]*(1+r)/r/(1+r)^2;
```

$$PVdT := CC + (MCF - 1)\, CC + \frac{CC + (MCF - 1)\, CC}{1 + r} + \frac{-\frac{30000\,(a - p)\, p}{a - 40} + \frac{30000\, m\,(a - p)}{a - 40} + \frac{30000\,(MCF - 1)\,(a - p)\,(m - p)}{a - 40}}{(1 + r)\, r}$$

Change in Social Surplus

```
> PVdSS:=PVdCS-PVdPS-PVdT;
```

$$PVdSS := \frac{-\frac{1600-p^2}{2\left(\frac{a}{30000}-\frac{1}{750}\right)}+\frac{a\,(40-p)}{\frac{a}{30000}-\frac{1}{750}}}{(1+r)\,r}-\frac{1200000-30000\,fc}{(1+r)\,r}+\frac{FSV}{(1+r)^2}-CC$$
$$-(MCF-1)\,CC-\frac{CC+(MCF-1)\,CC}{1+r}$$
$$-\frac{-\frac{30000\,(a-p)\,p}{a-40}+\frac{30000\,m\,(a-p)}{a-40}+\frac{30000\,(MCF-1)\,(a-p)\,(m-p)}{a-40}}{(1+r)\,r}$$

CONSISTENCY CHECK

```
> check:=simplify(NPV-PVdSS);
```

$$check := 0$$

PART 2: SENSTIVITY TESTING AND SIMULATION

Scenario 1 Parameters

```
> m:=10;
```

$$m := 10$$

```
> CC:=11*1000000;
```

$$CC := 11000000$$

```
> fc:=35;
```

$$fc := 35$$

```
> FSV:=100000;
```

$$FSV := 100000$$

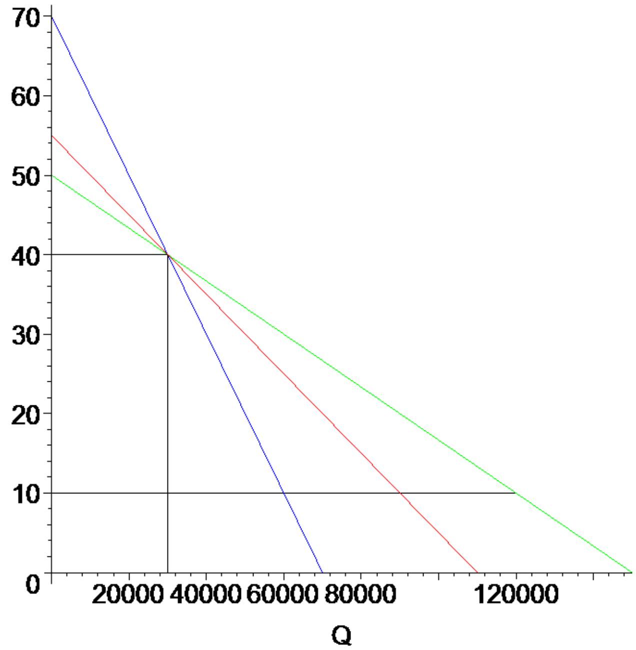
```
> MCF:=1.2;
```

$$MCF := 1.2$$

Sample Possible Demands

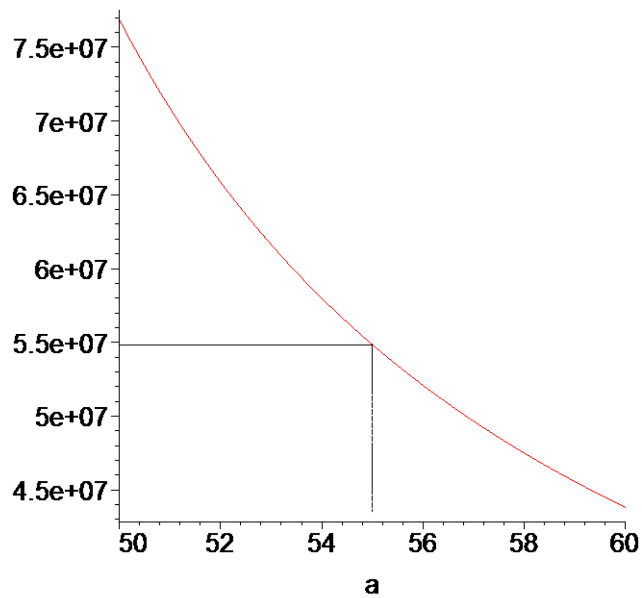
```
> g4:=plot(subs(a=70,P),Q=0..70000,color=blue):
> g5:=plot(subs(a=50,P),Q=0..150000,color=green):
> g6:=plot(10,Q=0..120000,color=black):
```

```
> display(g1,g2,g3,g4,g5,g6);
```



<u>Senstivity Testing</u>

```
> g1:=plot(subs(p=10,r=0.02,NPV),a=50..60):
> g2:=implicitplot(a=55,a=50..60,npv=43500000..54800000,color=black)
  :
> g3:=plot(54800000,a=50..55,color=black):
> display(g1,g2,g3);
```




```
> epsilon:=diff(NPV,a)*a/NPV:
> subs(a=55,r=0.02,p=10,epsilon);
```

-2.949903177

Simulation

Suppose a is uniform on [50,60]

Set the number of draws (set here to be very low so that the file runs quickly)

```
> n:=500:
```

Generate random values:

```
> A:=[random[uniform[50,60]](n,'default','inverse')]:
```

Construct an index vector of length n:

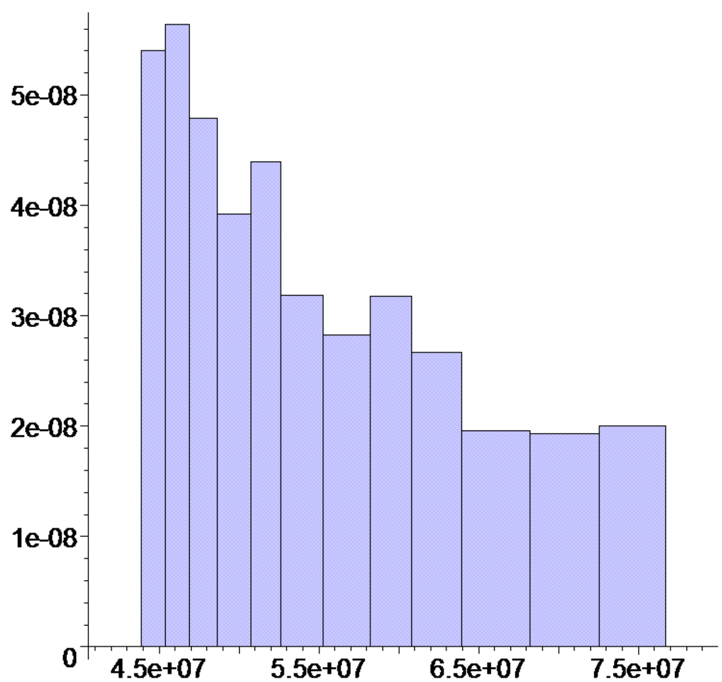
```
> L:=[seq(i,i=1..n)]:
```

Generate the NPV values:

```
> sim:=proc(i);subs(a=A[i],NPV);end:
> R:=map(sim,L):
```

Policy 1

```
> r:=0.02:
> p:=10:
> statplots[histogram[1]](R);
```



Mean and Variance

```
> mean:=sum(R[i],i=1..n)/n;
```

$$mean := 0.5689902452\ 10^{8}$$

```
> sd:=(sum((R[i]-mean)^2,i=1..n)/n)^(1/2);
```

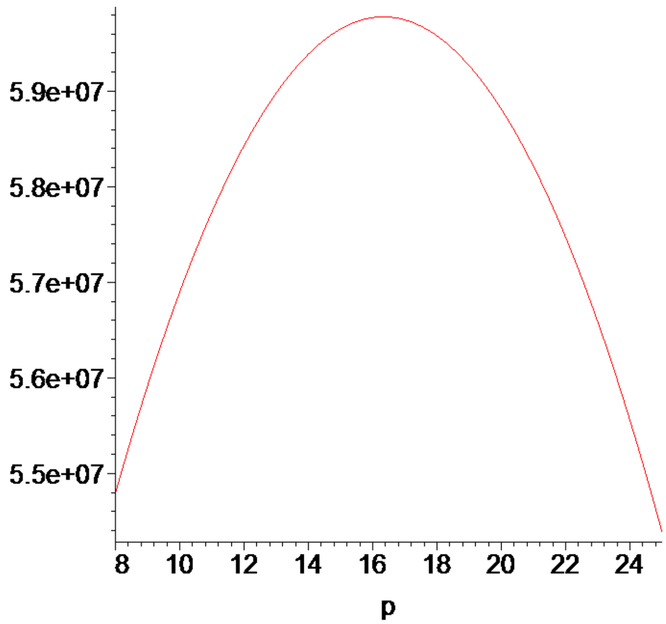
$$sd := 0.9374599836\ 10^7$$

Policy 2: Optimal Toll

```
> p:='p':
> mean:=sum(R[i],i=1..n)/n:
```

Plot against p:

```
> plot(mean,p=8..25);
```



Optimal Toll:

```
> pstar:=solve(diff(mean,p),p);
```

$$pstar := 16.33290115$$

TOPIC 8 REVIEW EXERCISE

This review involves the cost-benefit analysis of a proposed road construction project.

The project will upgrade a section highway between two cities. The highway currently carries two main types of traffic: cars and freight trucks. The current average driving time is 2 hours for cars and 2 ½ hours for trucks. The upgrade is expected to reduce those times to 1 ½ hours and 1 ¾ hours respectively.

There are currently 2m car trips per year and 1m truck trips per year. The reduction in driving times is expected to cause an increase in the number of trips to 2.2m cars per year and 1.5m trucks per year.

The estimated cost of travel time is $6 per hour for cars and $16 per hour for trucks.

The demand curves for car and truck trips are thought to be approximately linear.

The upgrade is expected to improve safety, with a corresponding reduction in the number of deaths from 2 per million vehicle trips to 1.5 per million vehicle trips. The estimated value of a life is $1m.

Construction will take place in year 1 and cost $250m. These costs are to funded out of general taxation revenue. The marginal of funds is estimated to be 1.2.

The upgrade will be complete at the end of year 1 and is expected to last forever.

<u>Exercise</u>

Conduct a cost-benefit analysis of this project. In particular,

- show that the NPV of this project (at a discount rate of 2%) is $787.5m
- identify the distributional impacts on car drivers, truck drivers and taxpayers

ANSWER GUIDE

1. REFERENT GROUP

No information is provided. Assume that all impacted parties have standing.

2. SELECT THE PORTFOLIO OF PROJECTS

We have information on only one project; consideration is confined to that one.

3. CATALOGUE POTENTIAL IMPACTS

- travel time savings for cars and trucks
- additional trips for cars and trucks
- reduced rate of traffic deaths
- construction costs
- government finances (the cost of funds)

4. QUANTITATIVE IMPACTS

(a) Travel Time Savings for Cars and Trucks

Cars save ½ hour per trip

Trucks save ¾ hour per trip.

(b) Additional Trips for Cars and Trucks

0.2m additional car trips

0.5m additional truck trips.

(c) Reduced Traffic Deaths

Current number of deaths per year: 2 x 3 = 6.

Post-project: 1.5 x 3.7 = 5.55.

Thus, the project will save 0.45 lives per year.

5. MONETIZE ALL IMPACTS

(a) Travel Time Savings

(i) Cars

Refer to figure A1. The current travel time price of a trip is 2 x \$6 = \$12. The post-upgrade price is 1.5 x \$6 = \$9. The travel time savings are equal to shaded area A:

\$3 x 2m = \$6m

This is a benefit of the project. It accrues in year 2 and every year thereafter.

(ii) Trucks

Refer to figure A2. The current travel time price of a trip is 2.5 x \$16 = \$40. The post-upgrade price is 1.75 x \$16 = \$28. The travel time savings are equal to shaded area A:

\$12 x 1m = \$12m

This is a benefit of the project. It accrues in year 2 and every year thereafter.

(b) Additional Trips

(i) Cars

Refer to figure A1. The net value of the additional trips (the value of the trips less the time cost required to make them) is the CS surplus indicated by area B:

(\$3 x 0.2m)/2 = \$0.3m

This is a benefit of the project. It accrues in year 2 and every year thereafter.

(ii) Trucks

Refer to figure A2. The net value of the additional trips (the value of the trips less the time cost required to make them) is the CS surplus indicated by area B:

(\$12 x 0.5m)/2 = \$3m

This is a benefit of the project. It accrues in year 2 and every year thereafter.

(c) Reduced Traffic Deaths

There are 0.45 lives saved @ $1m per life: $0.45m. This is a benefit of the project. It accrues in year 2 and every year thereafter.

(d) Construction Costs

$250m in year 1. This is a cost of the project.

(e) Government Finances and the Cost of Funds

The only impact on government finances is $250m in year 1. The associated cost of funds is

0.2 x $250m = $50m

This is a cost of the project. It occurs in year 1.

6. CALCULATE THE NPV

Summary table of cost and benefits:

$m	year 1	year 2 (+)
benefits		
time savings for cars		6.00
time savings for trucks		12.00
new car trips		0.30
new truck trips		3.00
lives saved		0.45
total benefits		21.75
costs		
construction	250.00	
cost of funds	50.00	
total costs	300.00	
net benefits	**-300.00**	**21.75**

Net Present Value

$$NPV = -300 + \frac{21.75}{r}$$

The NPV is positive at all reasonable discount rates:

r = 2%: *NPV* = $787.5m

r = 5%: *NPV* = $135m

r = 7%: *NPV* = $10.71m

7. THE DISTRIBUTION OF COSTS AND BENEFITS

(a) Car Drivers

Consumer surplus gain to existing drivers (shaded area A in figure A1): $6m.

Consumer surplus gain to new car drivers (shaded area B in figure A1): $0.3m

A proportionate fraction of lives saved:

pre-project car driver deaths: 2/3 x 6 = 4

post-project car driver deaths: (2.2/3.7) x 5.55 = 3.3

value of reduced deaths: 0.7 x $1m = $0.7m

Total benefits: $7m. These gains accrue in year 3 and each year thereafter.

(b) Truck Drivers

Consumer surplus gain to existing drivers (shaded area A in figure A2): $12m.

Consumer surplus gain to new truck drivers (shaded area B in figure A2): $3m

A proportionate fraction of lives saved:

pre-project truck driver deaths: 1/3 x 6 = 2

post-project truck driver deaths: (1.5/3.7) x 5.55 = 2.25

value of *increased* deaths: 0.25 x $1m = $0.25m

Total benefits: $14.75m. These gains accrue in year 3 and each year thereafter.

(c) Taxpayers

Construction costs plus associated cost of funds: $300m in year 1.

Summary table of distributional impacts:

$m (present value)	at 2%	at 5%	at 7%
winners			
car drivers	350	140	100
truck drivers	737.5	295	210.71
losers			
taxpayers	300	300	300
aggregate impact	**787.5**	**135**	**10.71**

Note that the aggregate net impact is necessarily equal to NPV calculated in step 6 above.

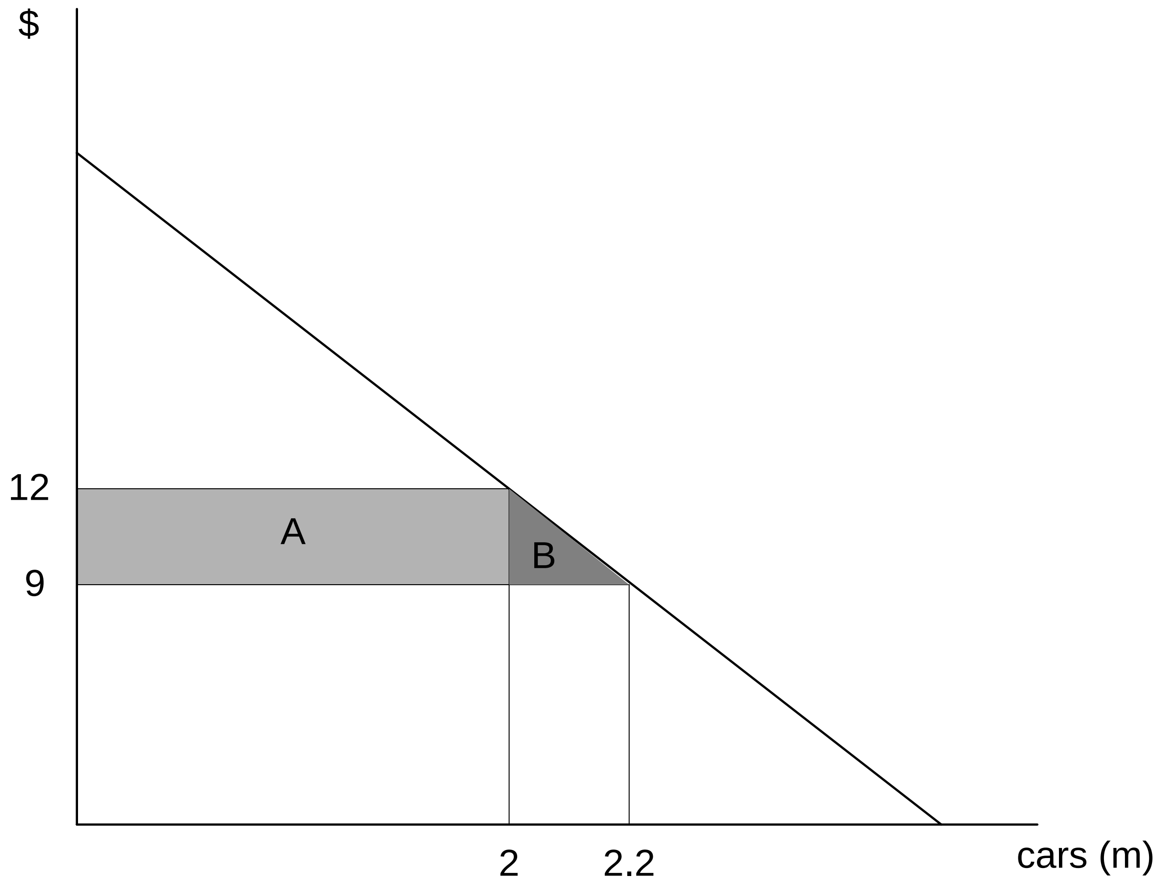


Figure A1

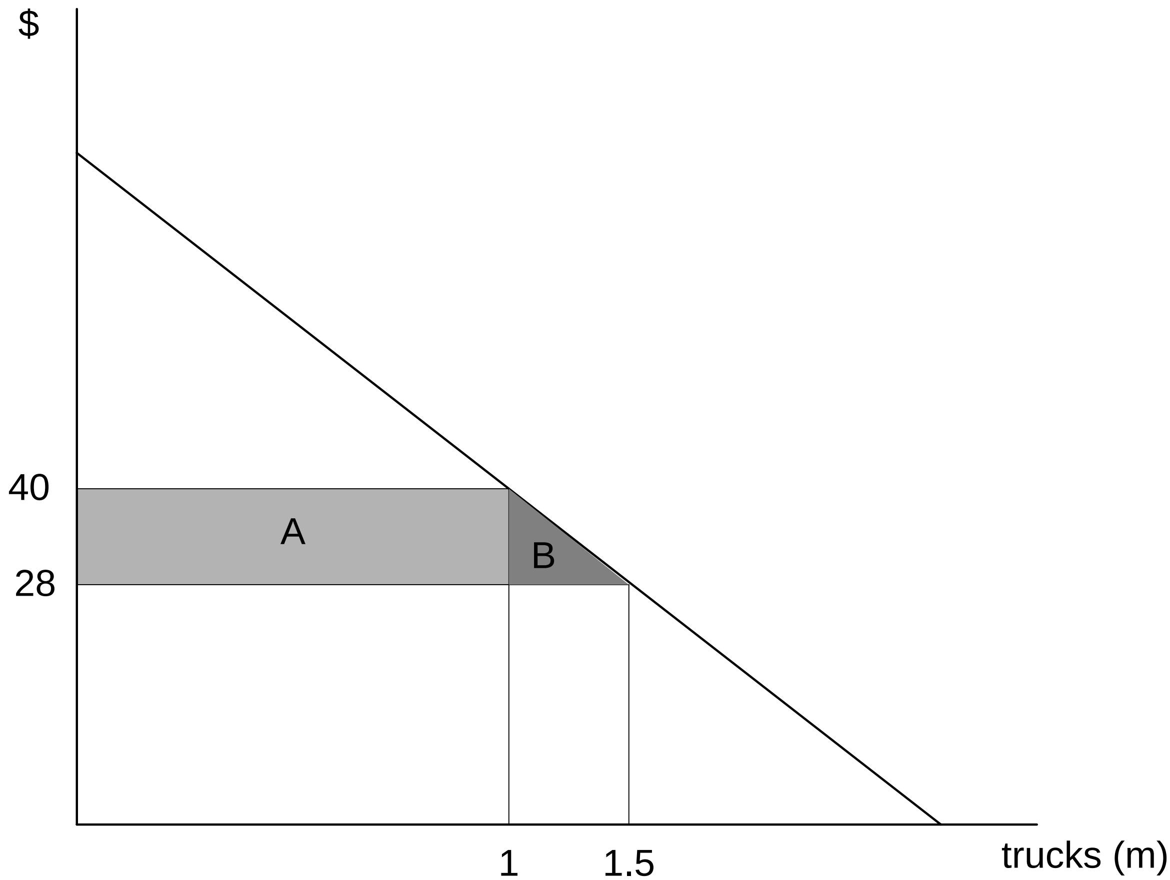


Figure A2

APPENDIX: A NOTE ON USING ELASTICITIES

Note that we can also calculate the surplus measures using estimated elasticities.

The estimated demand curve for car trips (in millions):

$$q_C = 2.8 - \frac{p_C}{15}$$

The implied point elasticity of demand at the current price is:

$$\varepsilon_0 = \frac{\Delta q}{\Delta p}\frac{p_0}{q_0} = \frac{-1}{15}.\frac{12}{2} = -0.4$$

Using the formula from Topic 4 Appendix

$$\Delta CS = -q_0 \Delta p + \frac{|\Delta p| \varepsilon_0 q_0 \Delta p}{2p_0}$$

Thus, we have

$$\Delta CS_C = -2(-3) + \frac{|(-3)|(-0.4)2(-3)}{2(12)} = 6 + 0.3$$

The estimated demand curve for truck trips (in millions):

$$q_T = 2.67 - \frac{p_T}{24}$$

The implied point elasticity of demand at the current price is:

$$\varepsilon_0 = \frac{\Delta q}{\Delta p}\frac{p_0}{q_0} = \frac{-1}{24}.\frac{40}{1} = -1.67$$

Using the formula from Topic 4 Appendix:

$$\Delta CS = -q_0 \Delta p + \frac{|\Delta p| \varepsilon_0 q_0 \Delta p}{2p_0}$$

Thus, we have

$$\Delta CS_T = -1(-12) + \frac{|(-12)|(-1.67)1(-12)}{2(40)} = 12 + 3$$