



**University  
of Victoria**

## UNIVERSITY OF VICTORIA

### FINAL EXAMINATION

**December 2012**

**CLASSICAL MECHANICS: I (PHYS 321A A01)**

**CRN: 12002**

**INSTRUCTOR: R. DE SOUSA**

**DURATION: 3 HOURS**

**TOTAL: 60**

**NAME:** Answer Key

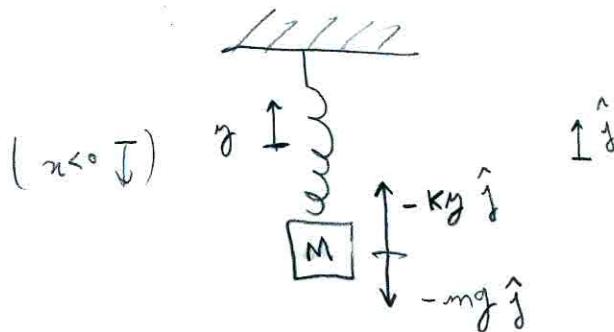
**STUDENT NUMBER:** V00 7

#### **INSTRUCTIONS:**

- This exam has a total of 8 problems. **You must choose 6 of these problems.** Each problem is worth 10 marks, giving a total of 60 marks for the exam. If you decide to solve more than 6 problems, only the first 6 will be graded. Please mark clearly the problems that you do not want to be graded.
- Write your answers into the space provided for each problem. Clearly explain your reasoning. If you need more space, please use the back of the page.
- This exam has a total of **9** pages including this cover page.
- Students must count the number of pages and report any discrepancy to the Invigilator.
- This examination must be answered on the question paper.
- Students are only allowed one formula sheet, handwritten on both sides of an 8.5x11 inch page, and one calculator: the *Sharp EL-510R*.
- Write your name and student number in the space provided at the top of this page.

**TURN OVER**

1. Consider a mass  $M$  suspended from a vertical spring of force constant  $k$ . Show that the force of gravity acting on  $M$  does not change the period of oscillation but only the center point of the oscillation.



Newton's 2nd law:  $m\ddot{y} = \sum F_i = -mg - ky$  writing N2 correctly ③

$$m\ddot{y} = -k(y + \frac{mg}{k})$$

$$\underbrace{m(y + \frac{mg}{k})}_{y' = y - y_{eq}} = -k(y + \frac{mg}{k}) \quad \text{Recognizing that } y_{eq} = -\frac{mg}{k} \quad ④$$

$y' = y - y_{eq}$  where  $y_{eq} = -\frac{mg}{k}$

$$\ddot{y}' + (\frac{k}{m})y' = 0 \Rightarrow y'(t) = A \cos(\sqrt{\frac{k}{m}}t + \delta) \quad \begin{matrix} \text{Arguing that } \sqrt{\frac{k}{m}} \\ \text{is freq., remains same} \end{matrix} \quad ③$$

Hence the force of gravity only shifts the equilibrium point to  $y_{eq} = -\frac{mg}{k}$ .  
The freq. of oscillation remains equal to  $\sqrt{\frac{k}{m}}$ .

2. (10) The force acting on a particle of mass  $m$  is given by  $F = k v x$ , where  $x$  is the particle's position,  $v$  is the particle's velocity, and  $k$  is a positive constant. The particle passes through the origin with speed  $v_0$  at  $t=0$ . Find  $x$  as a function of  $t$ .

Hint:  $\int \frac{dx}{1+x^2} = \arctan(x)$ .

$$m \frac{dN}{dt} = k N x \quad (2) \Rightarrow \frac{dN}{dt} = \left(\frac{k}{m}\right) x \frac{dx}{dt} \Rightarrow dN = \left(\frac{k}{m}\right) x dx \quad (2)$$

Or we  $\frac{dN}{dt} = \frac{dN}{dx} \frac{dx}{dt}$

$$\Rightarrow \frac{dN}{dx} = \frac{k}{m} x \Rightarrow dN = \frac{k}{m} x dx$$

$$\Rightarrow \int_{N_0}^{N} dN = \frac{k}{m} \int_0^x x dx \Rightarrow (N - N_0) = \frac{k}{2m} x^2 \quad (3)$$

$$\frac{dx}{dt} = N_0 + \frac{k}{2m} x^2 \Rightarrow \int_0^x \frac{dx}{1 + \frac{k}{2mN_0} x^2} = N_0 \int_0^t dt$$

$$x' = \sqrt{\frac{k}{2mN_0}} x \Rightarrow dx = \sqrt{\frac{2mN_0}{k}} dx'$$

$$\int_0^{\sqrt{\frac{k}{2mN_0}} x} \frac{dx'}{1+(x')^2} = N_0 t \Rightarrow \sqrt{\frac{2mN_0}{k}} \left[ \arctan\left(\sqrt{\frac{k}{2mN_0}} x\right) - \arctan(0) \right] = N_0 t$$

$$\arctan\left(\sqrt{\frac{k}{2mN_0}} x\right) = \sqrt{\frac{N_0 k}{2m}} t$$

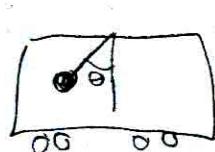
$$\Rightarrow x(t) = \sqrt{\frac{2mN_0}{k}} \tan\left(\sqrt{\frac{N_0 k}{2m}} t\right) \quad (3)$$

Didn't change variable appropriately  
in  $\int dx$ : (2)

3. A plumb line is held steady while being carried along in a moving train. The mass of the plumb bob is  $m$ , and the train is accelerating forward with constant acceleration  $g/10$ .

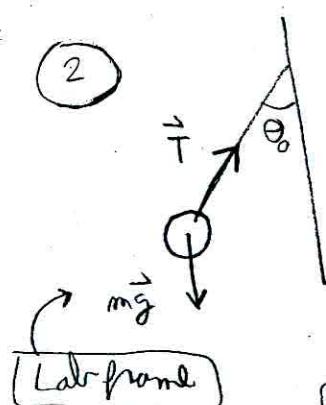
- Find the tension in the cord and the deflection from the local vertical (Ignore any effects of the earth's rotation).
- If the plumb line is not held steady but oscillates as a simple pendulum, find the period of oscillation for small amplitude.

a)



$$\vec{a} = \frac{\vec{g}}{10}$$

(2)



Lab frame

$$\begin{cases} T \cos(\theta_0) = mg \\ T \sin(\theta_0) = m \frac{g}{10} \end{cases}$$

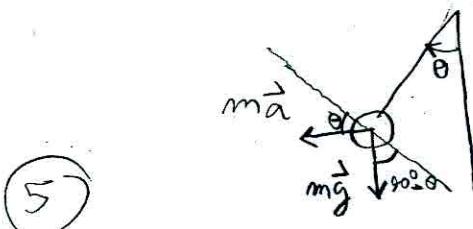
$$\Rightarrow \tan(\theta_0) = \frac{1}{10} \Rightarrow \theta_0 = \tan^{-1}\left(\frac{1}{10}\right) = 5.7^\circ$$

$$T^2 = (mg)^2 \left(1 + \frac{1}{10^2}\right)$$

$$T = mg \sqrt{1 + \frac{1}{10^2}} = 1.005 mg$$

(2)

b) Non inertial frame (inside train)



(5)

Torque

$$\tau = Ima \cos(\theta) - mg \sin(\theta)$$

$$= Img \left[ \frac{1}{10} \cos(\theta) - \sin(\theta) \right] = \frac{Img}{\omega(\theta_0)} \left[ \frac{\sin(\theta_0)}{\omega(\theta_0)} \cos\theta - \sin(\theta) \right]$$

$$\tau = -\frac{Img}{\omega(\theta_0)} \sin(\theta - \theta_0) \approx -\sqrt{1 + \left(\frac{1}{10}\right)^2} Img (\theta - \theta_0)$$

$$\frac{1}{\omega^2 \theta_0} = 1 + \tan^2(\theta_0) = 1 + \left(\frac{1}{10}\right)^2 \Rightarrow \omega(\theta_0) = \frac{1}{\sqrt{1 + (1/10)^2}}$$

$$T = 1.005 \pi \sqrt{\frac{l}{g}}$$

From Newton's 2nd law,  $\tau = I\alpha \Rightarrow ml^2 \ddot{\theta} = -\sqrt{1 + \left(\frac{1}{10}\right)^2} Img (\theta - \theta_0)$

$(\theta - \theta_0) + \sqrt{1 + \left(\frac{1}{10}\right)^2} \left(\frac{g}{l}\right) (\theta - \theta_0) = 0 \Rightarrow \omega_{oc} = \left(1 + \frac{1}{10^2}\right)^{1/4} \sqrt{\frac{g}{l}}$

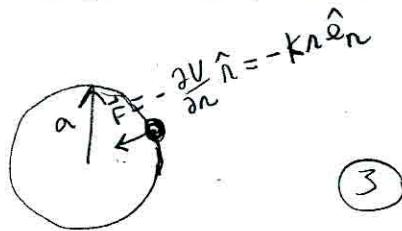
$T = \frac{2\pi}{\omega_{oc}} = 2\pi \sqrt{\frac{l}{g}} \left(1 + \frac{1}{10^2}\right)^{-1/2}$

TURN OVER ↑

4. A particle of mass  $m$  moves under the action of a harmonic oscillator force with potential energy  $\frac{1}{2}kr^2$ . Initially, it is moving in a circle of radius  $a$ .
- Find its orbital speed  $v$ .
  - The particle is then given a blow of impulse  $mv'$  in a direction perpendicular to its velocity. Sketch the effective potential  $V_{\text{eff}}(r)$  of the particle.
  - Use the conservation laws to determine the minimum and maximum distances from the origin during the subsequent motion.

$$a) \frac{m v^2}{a} = k a$$

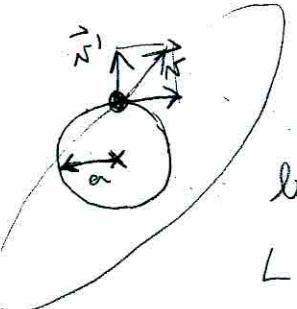
$$\Rightarrow v = \sqrt{\frac{k}{m}} a$$



(3)

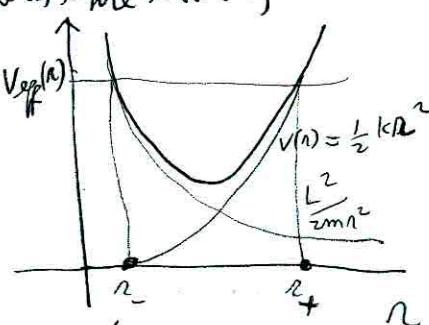
$$b) L = (\vec{r} \times \vec{p}) = m r |\hat{e}_r \times (r \dot{\theta} \hat{e}_\theta + i \hat{e}_r)| = m r^2 \dot{\theta}$$

$$E = \frac{\vec{p}^2}{2m} + V(r) = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 + V(r) = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{L^2}{2mr^2}}_{V_{\text{eff}}(r)} + V(r).$$



Angular momentum is the same as before the blow,  
because blow is along radius!

$$L = m \sqrt{\frac{k}{m}} a^2$$



(3)

c) After blow the energy is

$$E_{\text{after blow}} = \frac{1}{2} m ((v')^2 + v^2) + \frac{1}{2} K a^2 = \underbrace{\frac{L^2}{2mr^2} + \frac{1}{2} k r^2}_{V_{\text{eff}}(r)} + \frac{1}{2} m \dot{r}^2 \quad \text{at } r = r_+$$

$$\Rightarrow \frac{1}{2} k r^4 - \left( \frac{1}{2} m (r_+^2 + v^2) + \frac{1}{2} k a^2 \right) r^2 + \frac{1}{2} m \left( \frac{L^2}{2mr^2} + \frac{1}{2} k a^2 \right) = 0$$

TURN OVER



$$n^4 - \left[ \frac{m}{k} (n'^2 + n^2) + a^2 \right] n^2 + a^4 = 0$$

$$n_{\pm}^2 = \frac{1}{2} \left\{ \left[ \frac{m}{k} (n'^2 + n^2) + a^2 \right] \pm \sqrt{\left[ \frac{m}{k} (n'^2 + n^2) + a^2 \right]^2 - 4a^4} \right\}$$

Use  $n = \sqrt{\frac{k}{m}} a$ :

$$\begin{aligned} n_{\pm}^2 &= \frac{1}{2} \left\{ \left[ 2a^2 + \frac{m}{k} n'^2 \right] \pm \sqrt{\underbrace{\left[ \frac{m}{k} n'^2 + 2a^2 \right]^2 - 4a^4}_{\left( \frac{m}{k} n' \right)^4 + 4a^2 \left( \frac{m}{k} n' \right)^2}} \right\} \\ n_{\pm}^2 &= \frac{1}{2} \left\{ \left[ 2a^2 + \frac{m}{k} n'^2 \right] \pm 2a n' \sqrt{\frac{m}{k}} \sqrt{1 + \frac{1}{4a^2} \left( \frac{m}{k} n' \right)^2} \right\} \end{aligned} \quad (4)$$

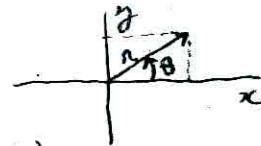
5. The orbit of a particle moving in a central force field is given by  $r = r_0 \cos(\theta)$ , where  $r_0$  is a constant.

a. Prove that this orbit is a circle. Find the origin and the radius of the circle.

b. Find the force field that created this orbit.

obtained a circle by plotting: ②

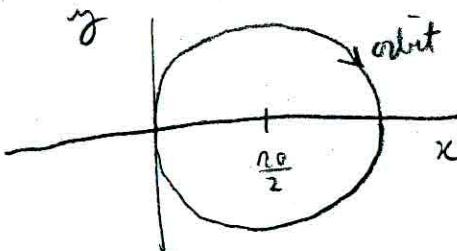
$$\textcircled{2} \quad \begin{cases} x = r_0 \cos(\theta) = r_0 \cos^2(\theta) \\ y = r_0 \sin(\theta) = r_0 \cos(\theta) \sin(\theta) \end{cases}$$



$$\Rightarrow \begin{cases} x = r_0 \frac{1 + \cos(2\theta)}{2} \\ y = \frac{1}{2} r_0 \sin(2\theta) \end{cases} \Rightarrow \begin{cases} \left(x - \frac{r_0}{2}\right) = \frac{r_0}{2} \cos(2\theta) \\ y = \frac{r_0}{2} \sin(2\theta) \end{cases}$$

$$\Rightarrow \boxed{\left(x - \frac{r_0}{2}\right)^2 + y^2 = \left(\frac{r_0}{2}\right)^2} \quad \textcircled{3}$$

Hence the orbit is a circle with radius  $\frac{r_0}{2}$  centered at  $\left(\frac{r_0}{2}, 0\right)$ :



Q) The orbit eqn is given by  $\frac{d^2 r}{d\theta^2} + \mu = -\frac{f(1/n)}{mr^2 \mu^2}$ , where  $n(\theta) = \frac{1}{r(\theta)}$

②

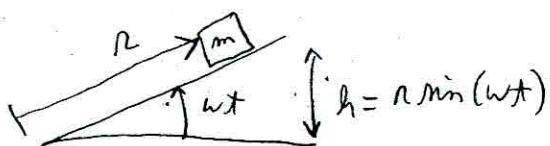
Hence:  $f(n) = -\frac{ml^2}{n^2} \left[ \frac{d}{d\theta^2} \left( \frac{1}{n} \right) + \frac{1}{n} \right]$ , plug  $r = r_0 \cos(\theta)$ :

$$\begin{aligned} \frac{d^2}{d\theta^2} \left( \frac{1}{n} \right) &= \frac{1}{n_0} \frac{d}{d\theta} \left[ \frac{-1}{\cos^2(\theta)} (\sin \theta) \right] = \frac{1}{n_0} \frac{\cos^3(\theta) + 2\cos(\theta)\sin^2(\theta)}{\sin^4(\theta)} = \frac{1}{n_0} \frac{\sin^2(\theta) + 2\sin^2(\theta)}{\cos^3(\theta)} \\ &= \frac{1}{n_0} \frac{2 - \sin^2(\theta)}{\cos^3(\theta)} = \frac{2n_0^2}{l^2} - \frac{1}{n_0} \end{aligned}$$

$$\Rightarrow f(n) = -\frac{ml^2}{n^2} \left[ + \frac{2n_0^2}{l^2} - \frac{1}{n_0} + \frac{1}{n_0} \right] = -\frac{2ml^2 n_0^2}{n^5} // \quad \textcircled{3}$$

TURN OVER

6. A particle slides on a smooth inclined plane whose inclination angle  $\theta$  is increasing at a constant rate  $\omega$ . If  $\theta = 0$  at time  $t = 0$ , and the particle starts from rest, find the subsequent motion of the particle.



$$\begin{aligned} r(t=0) &= r_0, \dot{r}(t=0) = 0 \\ \theta(t=0) &= 0, \theta(t) = \omega t \end{aligned} \quad (2)$$

$$V = mg h = mg R \sin(\omega t)$$

$$r^2 = (\dot{r})^2 + (R\omega)^2$$

$$L = \frac{1}{2}m[\dot{r}^2 + (R\omega)^2] - mg \sin(\omega t) R \quad (2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = \frac{\partial L}{\partial r}$$

$$\cancel{m\ddot{r}} = m\omega^2 r - mg \sin(\omega t)$$

$$\dot{r} - \omega^2 r = -g \sin(\omega t) \quad (2)$$

$$\text{put } y = Ae^{i\omega t}; \text{ solution will be } r = \text{Im}(y)$$

$$-2\omega^2 Ae^{i\omega t} = -g e^{i\omega t} \Rightarrow A = \frac{g}{2\omega^2}$$

$$r = \text{Im}\left(\frac{g}{2\omega^2} e^{i\omega t}\right) = \frac{g}{2\omega^2} \sin(\omega t) \quad (\text{particular solution}) \quad (2)$$

Solved N2  
without centrifugal  
 $\Rightarrow 5/10$

The general solution is

$$r(t) = \frac{g}{2\omega^2} \sin(\omega t) + A'e^{-\omega t} + B'e^{\omega t}$$

$$\dot{r}(t) = \frac{g}{2\omega} \cos(\omega t) - \omega A'e^{-\omega t} + \omega B'e^{\omega t}$$

$$(t=0) = \frac{g}{2\omega} + \omega(B' - A') = 0$$

$$(t=0) = A' + B' = r_0$$

$$\frac{g}{2\omega} + \omega(r_0 - 2A') = 0 \Rightarrow A' = \left(\frac{g}{4\omega^2} + \frac{r_0}{2}\right), B' = \left(-\frac{g}{4\omega^2} + \frac{r_0}{\omega}\right)$$

$$\Rightarrow r(t) = \frac{g}{2\omega^2} \sin(\omega t) + r_0 \cosh(\omega t) - \frac{g}{2\omega^2} \sinh(\omega t)$$

$$r(t) = r_0 \cosh(\omega t) + \frac{g}{2\omega^2} [\sin(\omega t) - \sinh(\omega t)]$$

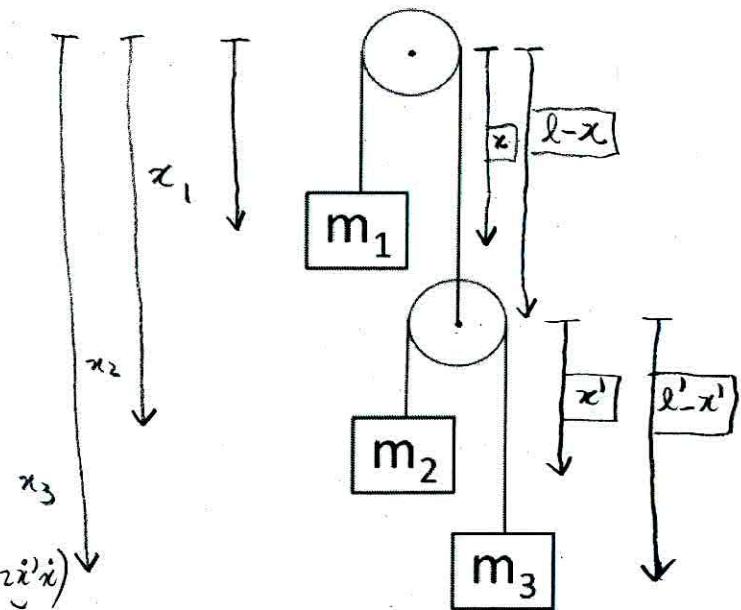
(2)

TURN OVER

7. Consider the Atwood machine shown in the figure. Find the acceleration of each of the three masses when they are subject to gravity (assume the pulleys are massless and the cables are non-elastic).

$$\begin{cases} x_1 = x \\ x_2 = (l - x) + x' \\ x_3 = (l - x) + l' - x' \end{cases} \quad (2)$$

$$\begin{aligned} T &= \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_3\dot{x}_3^2 \\ &= \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}' - \dot{x})^2 + \frac{1}{2}m_3(\dot{x} + \dot{x}')^2 \\ &= \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}'^2 + \dot{x}^2 - 2\dot{x}'\dot{x}) + \frac{1}{2}m_3(\dot{x}^2 + \dot{x}'^2 + 2\dot{x}'\dot{x}) \end{aligned}$$



$$\Gamma = \frac{1}{2}(m_1 + m_2 + m_3)\dot{x}^2 + \frac{1}{2}(m_2 + m_3)\dot{x}'^2 + (m_3 - m_2)\dot{x}'\dot{x} \quad (2)$$

$$\begin{aligned} V &= -m_1g x_1 - m_2g x_2 - m_3g x_3 \\ &= -m_1g x - m_2g(l - x + x') - m_3g(l + l' - x - x') \end{aligned}$$

$$= -(m_1 - m_2 - m_3)g x - (m_2 - m_3)g x' + \text{const.} \quad (2)$$

$$\begin{aligned} T - V &= \frac{1}{2}(m_1 + m_2 + m_3)\dot{x}^2 + \frac{1}{2}(m_2 + m_3)\dot{x}'^2 + (m_3 - m_2)\dot{x}'\dot{x} \\ &\quad + (m_1 - m_2 - m_3)g x + (m_2 - m_3)g x' + \text{const} \end{aligned}$$

Eqns of motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \Rightarrow (m_1 + m_2 + m_3)\ddot{x} + (m_3 - m_2)\ddot{x}' = (m_1 - m_2 - m_3)g \Big| \div (m_3 - m_2) \quad (2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}'}\right) = \frac{\partial L}{\partial x'} \Rightarrow (m_3 - m_2)\ddot{x}' + (m_2 + m_3)\ddot{x} = (m_2 - m_3)g \Big| \div (m_2 + m_3) \quad (2)$$

$$\begin{cases} \frac{m_1 + m_2 + m_3}{m_3 - m_2} \ddot{x} + \ddot{x}' = \frac{m_1 - m_2 - m_3}{m_3 - m_2} g \\ \frac{m_3 - m_2}{m_2 + m_3} \ddot{x}' + \ddot{x} = \frac{m_2 - m_3}{m_2 + m_3} g \end{cases}$$

PS: Give 10/10 for people who got the signs of motion right.

→ TURN OVER

Subtract eqns:

$$\frac{(m_1+m_2+m_3)(m_3+m_2) - (m_3-m_2)^2}{(m_2+m_3)(m_3-m_2)} \ddot{x} = \frac{(m_1-m_2-m_3)(m_2+m_3) - (m_2-m_3)(m_3-m_2)}{(m_2+m_3)(m_3-m_2)} g$$

$$\ddot{x} = \frac{[m_2(m_1-m_2-m_3-m_3+m_2) + m_3(m_1-m_2-m_3+m_2-m_3)]g}{[m_2(m_1+m_2+m_3-m_2+m_3) + m_3(m_1+m_2+m_3-m_3)]}$$

$$\ddot{x} = \frac{m_2(m_1-2m_3) + m_3(m_1-2m_2)}{m_2(m_1+3m_3) + m_3(m_1+m_2)} g \quad \ddot{x}_1 = \left[ 1 - \frac{8m_2m_3}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g$$

$$\text{Use } \ddot{x}' = \frac{(m_2-m_3)(g + \ddot{x})}{(m_2+m_3)} = \left( \frac{m_2-m_3}{m_2+m_3} \right) \left[ 2 - \frac{8m_2m_3}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g$$

$$\ddot{x}' = \left( \frac{m_2-m_3}{m_2+m_3} \right) \left[ \frac{2m_2(m_1+3m_3) + 2m_3(m_1+m_2) - 8m_2m_3}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g = \left( \frac{m_2-m_3}{m_2+m_3} \right) \left[ \frac{2(m_2+m_3)m_1}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g$$

$$\ddot{x}' = \frac{2(m_2-m_3)m_1}{m_2(m_1+3m_3) + m_3(m_1+m_2)} g$$

$$\ddot{x}_2 = (-\ddot{x} + \ddot{x}') = \left[ -1 + \frac{8m_2m_3 + 2(m_2-m_3)m_1}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g = \left[ \frac{m_2(m_1+3m_3) - m_3(m_1+m_2) + 2m_1(m_2-m_3)}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g$$

$$\ddot{x}_2 = \frac{m_2(-m_1-3m_3-m_3+2m_1) + m_3(-m_1-2m_1)}{m_2(m_1+3m_3) + m_3(m_1+m_2)} g = \frac{m_2(m_1+4m_3) - 3m_3m_1}{m_2(m_1+3m_3) + m_3(m_1+m_2)} g$$

$$\ddot{x}_3 = -(\ddot{x} + \ddot{x}') = -\left[ 1 + \frac{2m_1(m_2-m_3) - 8m_2m_3}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g = -\left[ 1 + \frac{2m_2(m_1-4m_3) - 2m_3m_1}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g$$

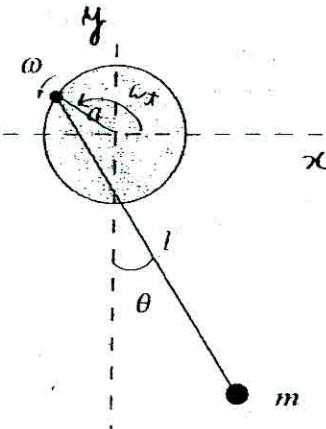
$$i_3 = -\left[ \frac{m_2(3m_1-5m_3) + m_3(m_2-m_1)}{m_2(m_1+3m_3) + m_3(m_1+m_2)} \right] g$$

Checked limits  $m_1 \rightarrow 0, m_1 \rightarrow \infty$ .

- 8) A simple pendulum of length  $l$  and mass  $m$  is suspended from a point on the circumference of a thin massless disc of radius  $a$  that rotates with a constant angular velocity  $\omega$  about its central axis as shown in the figure. Find the equation of motion of the mass  $m$ .

Coordinates of the mass w.r.t origin:

$$\begin{cases} x = a \cos(\omega t) + l \sin(\theta) \\ y = +a \sin(\omega t) - l \cos(\theta) \end{cases} \quad (2)$$



$$\begin{cases} \dot{x} = -a\omega \sin(\omega t) + l\dot{\theta} \cos(\theta) \\ \dot{y} = +a\omega \cos(\omega t) + l\dot{\theta} \sin(\theta) \end{cases}$$

$$= \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) = \frac{1}{2} m \left\{ a^2 \omega^2 \sin^2(\omega t) - 2l\dot{\theta} a\omega \sin(\omega t) \cos(\theta) + l^2 \dot{\theta}^2 \cos^2(\theta) \right. \\ \left. + a^2 \omega^2 \cos^2(\omega t) + 2l\dot{\theta} a\omega \sin(\theta) \cos(\omega t) + l^2 \dot{\theta}^2 \sin^2(\theta) \right\}$$

$$\Gamma = \frac{1}{2} m \left\{ a^2 \omega^2 + 2l\dot{\theta} a\omega \sin(\theta - \omega t) + l^2 \dot{\theta}^2 \right\} \quad (2)$$

$$I = mg y \quad (2)$$

$$f(\theta, \dot{\theta}) = \frac{1}{2} m \left\{ a^2 \omega^2 + 2l\dot{\theta} a\omega \sin(\theta - \omega t) \dot{\theta} + l^2 \dot{\theta}^2 \right\} - mg [a \sin(\omega t) - l \cos(\theta)]$$

$$\frac{\partial f}{\partial \dot{\theta}} = ml\omega \sin(\theta - \omega t) + ml^2 \ddot{\theta}$$

$$\frac{\partial f}{\partial \theta} = -mg l \sin(\theta) + ml\omega \cos(\theta - \omega t) \dot{\theta}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial \dot{\theta}} \Rightarrow ml^2 \ddot{\theta} + ml\omega \cos(\theta - \omega t) (\dot{\theta} - \omega) = -mg l \sin(\theta) + ml\omega \dot{\theta} \cancel{\cos(\theta - \omega t)}$$

$$\ddot{\theta} + \frac{g}{l} \sin(\theta) - \frac{a}{l} \omega^2 \cos(\theta - \omega t) = 0 \quad (2)$$

TURN OVER