Lecture 1: Newtonian Mechanics in 1-d

Newton's Laws

First: A body will continue in its state of rest, or by uniform motion in a straight line, unless it is compelled to change that state by forces imposed on it. (It took 2000 years for people to realize this!)

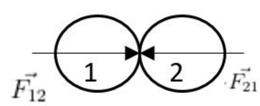
$\rightarrow$ *Inertial Frame of Reference*: Frame of reference in which the first law is valid. Rules out accelerated frames of reference.

Second: The change of motion is proportional to the motive force imposed and is made in the direction of the line in which that force is imposed

$$\implies \vec{F} = m\frac{d^2\vec{x}}{dt^2}$$

Third: To every action there is always imposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary points.

$$\vec{F_{12}} = -\vec{F_{21}}$$



Later we will show that conservation of momentum $\implies$ the third law! (Note that the symbol $\implies$ represents the logical meaning "implies").

A Few Words

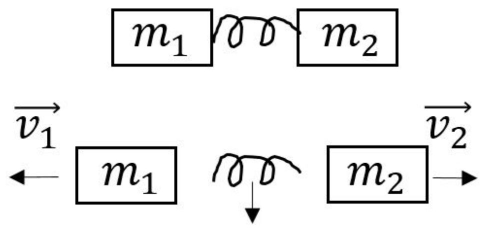
-These laws are mathematical prescriptions that allow us to predict accurately the motion of particles as a function of time, given a knowledge of their current state (initial position and velocity). These laws describe **how** things work, but they do not tell us **why**.

-As you know, these laws have to be modified for objects moving with relativistic velocities compared to the speed of light (*Theory of Relativity*)

-They also have to be modified completely for objects with small dimensions and small energies (*Quantum Mechanics*)

-Another formulation of Newton's laws, the so called *Lagrangian Mechanics* is extremely convenient and fundamental: it forms the laws of several other theories in physics. Our goal is to introduce Lagrangian Mechanics towards the end of the course.

Consider the following mass spring experiment:



We define the ratio of masses to be $\frac{m_2}{m_1} = \frac{|\vec{v_1}|}{|\vec{v_2}|}$ and let m_1 be the standard of mass.

This new statement which we have proposed (conservation of linear momentum) is consistent with Newton's laws. In other words, if we assume that conservation of linear momentum holds, we do not contradict any of Newton's laws. In fact, we get back the third law!

(Note that our derivation below DOES NOT show that the third law implies conservation of momentum- but rather, the opposite).

Our definition is equivalent to the statement $\Delta(m_1\vec{v_1}) = -\Delta(m_2\vec{v_2})$ since the initial velocity is zero in both cases and v_2 and v_1 are opposite in direction. Dividing by Δt and taking the limit as $\Delta t \to 0$ yields

$$\boxed{\frac{d}{dt}(m_1\vec{v_1}) = -\frac{d}{dt}(m_2\vec{v_2})}$$

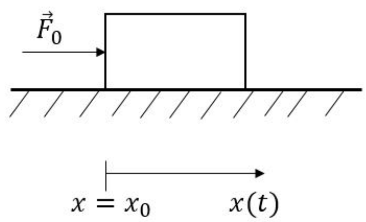
Now define linear momentum $\vec{p_i} = m_i\vec{v_i} \implies \frac{d}{dt}(\vec{p_1} + \vec{p_2}) = 0$ and thus clearly total linear momentum is conserved since we must have $\vec{p_1} + \vec{p_2} =$ constant.

Combining this with Newton's second law $\vec{F}_{acting} = \frac{d}{dt}\vec{p_i}$, we get Newton's third law:

$$\boxed{\vec{F}_{actingon1} = -\vec{F}_{actingon2}}$$

Therefore, our definition of momentum implies Newton's third law. You can also prove that N3 $\implies$ conservation of momentum for simple mechanical system.

Motion in 1-d Under a Constant Force



In this example, we know that

$$\begin{cases} m\ddot{x} = F_0, \quad \text{want to find} \ \ \dot{x}(t), x(t) \\ \dot{x}(t=0) = v_0 \\ x(t=0) = x_0 \end{cases}$$

Finding Velocity:

$$m\frac{dv}{dt} = F_0 \quad \Longrightarrow \quad \int_{v_0}^{v(t)} dv = \frac{F_0}{m}\int_{t_0}^{t} dt \quad \Longrightarrow \quad v(t) - v_0 = \frac{F_0}{m}(t - t_0)$$

$$\text{OR} \quad \boxed{v = \frac{dx}{dt} = v_0 + a(t - t_0)}$$

In addition, we have that

$$\int_{x_0}^{x(t)} dx = \int_{t_0}^{t} [v_0 + a(t - t_0)]dt \quad \Longrightarrow \quad x(t) - x_0 = v(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

$$\int_{t_0}^{t} dt(t - t_0) \; = \; \frac{t^2}{2} - t_0 t\Big|_{t_0}^{t} \; = \; \frac{1}{2}(t - t_0)^2$$

$$\boxed{x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2}$$

We can eliminate $(t - t_0)$ to obtain

$$\boxed{v^2 - v_0^2 = 2a(x - x_0)}$$