UNIVERSITY OF VICTORIA

MIDTERM EXAMINATION

October 3rd 2014

CLASSICAL MECHANICS: I (PHYS 321A A01)

INSTRUCTOR: R. DE SOUSA

DURATION: 45 MINUTES **TOTAL CREDIT:** 30

NAME:_____

STUDENT NUMBER: **V00**_____

INSTRUCTIONS:

- Write your answers into the space provided for each problem. Clearly explain your reasoning. If you need more space, please use the back of the page.
- This exam has a total of 6 pages including this cover page; there are 3 problems.
- Students must count the number of pages and report any discrepancy to the invigilator.
- This examination must be answered on the question paper.
- Students are only allowed one formula sheet, handwritten on both sides of an 8.5x11 inch page, and one calculator: the *Sharp EL-510R*.
- Write your name and student number in the space provided at the top of this page.

1) Potential energies and force fields.

a) (/5)The interaction between nucleons (protons and neutrons) may be represented by the Yukawa potential,

$$U(r) = -U_0 \left(\frac{r_0}{r}\right) \mathrm{e}^{-\left(\frac{r}{r_0}\right)}.$$

Here r_0 and U_0 are constants and $r = \sqrt{x^2 + y^2 + z^2}$.
Find the force field associated to *U(r)*.

Hint: $\frac{\partial r}{\partial x} = \frac{x}{r}$, $\frac{\partial r}{\partial y} = \frac{y}{r}$, $\frac{\partial r}{\partial z} = \frac{z}{r}$.

b) (/5) Consider the force field

$$\vec{F} = (x^2 - y^2)\,\hat{\mathbf{i}} - 2xy\,\hat{\mathbf{j}}$$

Is this force field conservative? Prove your answer mathematically. If your answer is yes, find the corresponding potential energy.

Hint: To find the potential energy, you will need to integrate over a particular path. Find a simple path to integrate.

2) Anisotropic harmonic oscillator (in two dimensions).

Consider the two dimensional harmonic oscillator with potential energy function,

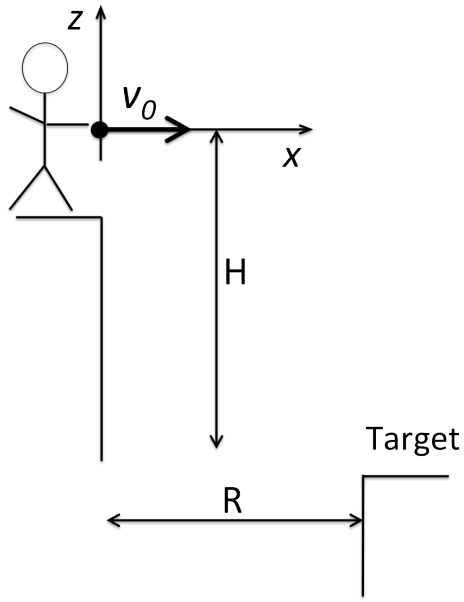
$$V(x,y) = \frac{1}{2} m\omega_1^2 \, x^2 + \frac{1}{2} m\omega_2^2 \, y^2$$

a) (/5) Write down the equations of motion, separate them into *x,y* coordinates and solve them for *x(t)* and *y(t)*.

b) (/5) Assume *m=1*, ω_1*=1*, ω_2*=2*, and initial conditions x_0*=0*, y_0*=1*, and $\dot{x}_0 = 1$, $\dot{y}_0 = 0$. Draw an orbit in the xy plane representing this motion.

3) Projectile motion with air resistance linear in v.

A person is at the edge of the abyss shown in the figure. He/she throws a rock with mass m and velocity $\vec{v}_0 = v_0\hat{i}$ (along the horizontal, see figure). Assume that the rock is subject to air resistance that is linear in v, $\vec{F}_{air} = -m\gamma\vec{v}$.



a) (/5) Write down the equation of motion for the projectile and find its position as a function of time assuming the origin of the coordinate system shown.

b) (/5) Assuming $H >> g/\gamma^2$, find the value of initial velocity v_0 required for the rock to hit the target shown in the picture. Draw the trajectory.