Experimental observation of the Aharonov-Bohm effect

R.G. Chambers, Phys. Rev. Lett. **5**, 3 (1960)

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SHIFT OF AN ELECTRON INTERFERENCE PATTERN BY ENCLOSED MAGNETIC FLUX

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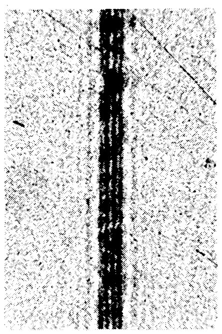
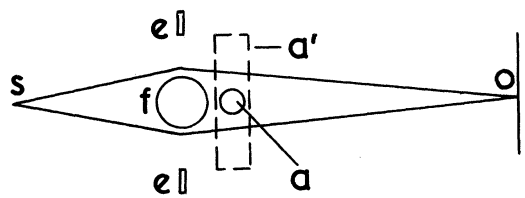
(Received May 27, 1960)





Interference Fringes *without* magnetic iron whisker

Tilted fringes with a tapering magnetic whisker at 'a'

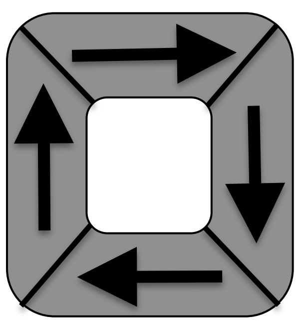
Direct imaging of the phase interference in the Aharonov-Bohm experiment with an holographic electron microscope

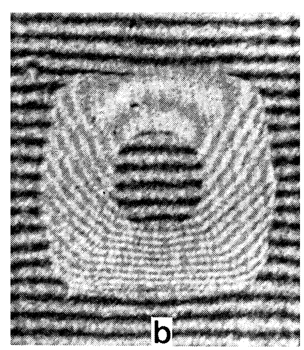
A. Tonomura et al, Phys. Rev. Lett. **48**, 1443 (1982)

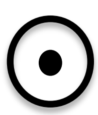
Observation of Aharonov-Bohm Effect by Electron Holography

Akira Tonomura, Tsuyoshi Matsuda, Ryo Suzuki, Akira Fukuhara, Nobuyuki Osakabe, Hiroshi Umezaki, Junji Endo, Kohsei Shinagawa, Yutaka Sugita, and Hideo Fujiwara
Central Research Laboratory, Hitachi Ltd., Kokubunji, Tokyo 185, Japan
(Received 16 February 1982)

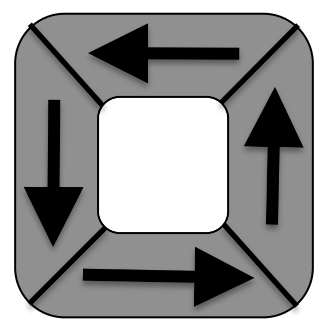
Interferogram of electron's phase for an electron beam going through a toroidal magnet (beam direction coming out of page). Observation of interference pattern in the toroid's shadow proves the A-B effect!

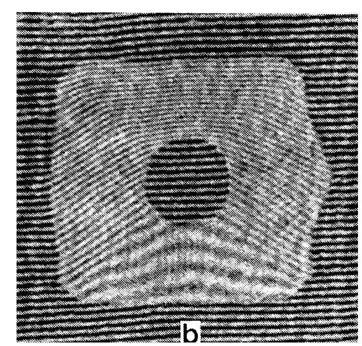






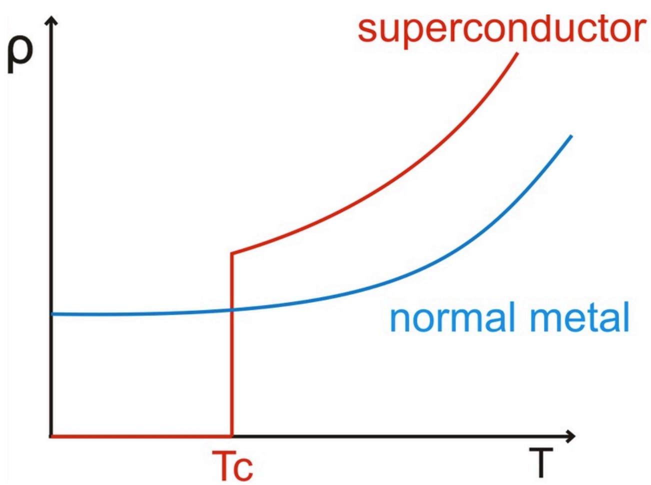
Electron beam direction

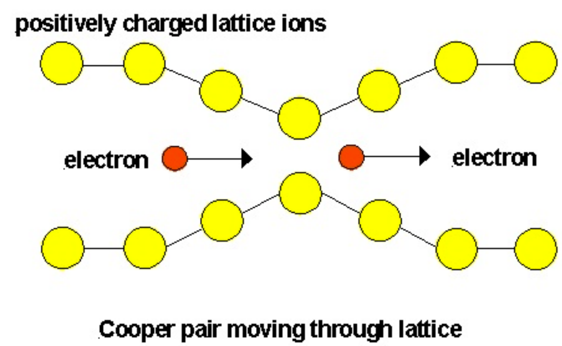




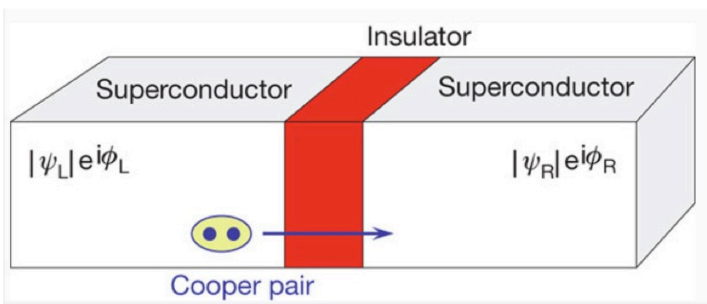
Same with magnet "flipped"

Aharonov-Bohm effect in devices: The SQUID





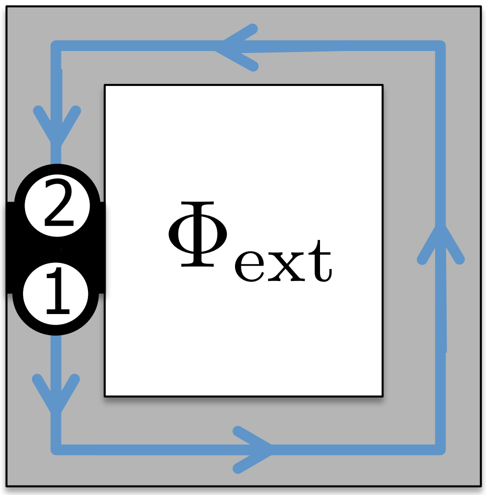
- In a superconductor, electrons act as a "macroscopic quantum state" $\psi = \mathrm{e}^{i\phi}$. A remarkable consequence is the Josephson effect.



Josephson junction

$$\begin{cases} I = I_c \sin\phi \\ \dfrac{d\phi}{dt} = -\dfrac{2e}{\hbar} V \end{cases}$$

The superconducting quantum interference device (SQUID)



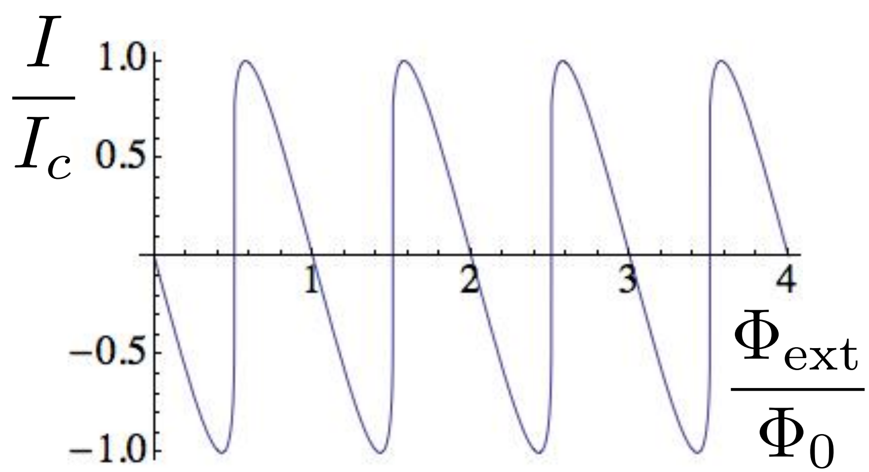
$$\psi = e^{i(\phi_2 - \phi_1 + 2\pi m)}$$

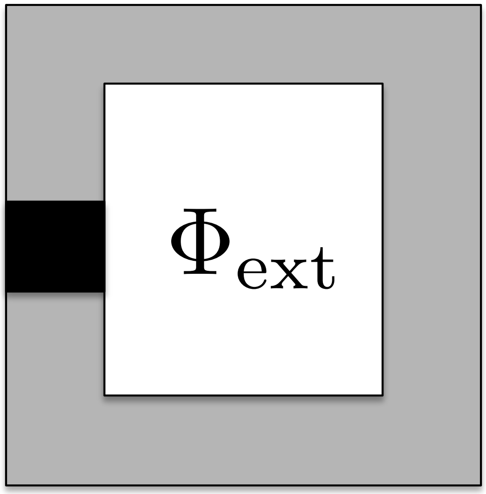
$$= e^{i\frac{q}{\hbar}\int_1^2 \mathbf{A}\cdot dx} = e^{i\frac{q}{\hbar}\Phi}$$

Hence we get $\phi_2 - \phi_1 = -2\pi\left(\frac{\Phi}{\Phi_0} + m\right)$, with flux quantum $\Phi_0 = -\frac{h}{q} = \frac{h}{|2e|} = 2\times 10^{-15}\mathrm{Wb}$.

$$\Phi = \Phi_{\rm ext} + LI$$

$$I = I_c \sin\phi = -I_c \sin\left(2\pi\frac{\Phi}{\Phi_0}\right)$$





SQUID as a qubit

- Potential energy of a JJ:

$$W = \int V\, dq = -E_J \cos\phi$$

- So we can think of ϕ as "position".

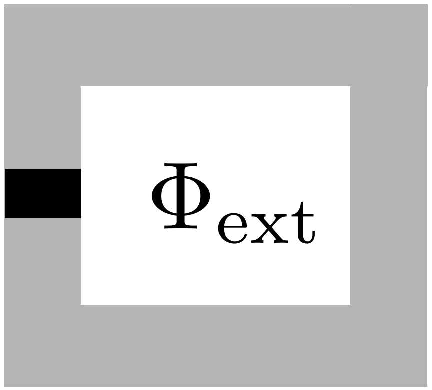
- Also, from $Q = CV = \left(\frac{C\hbar}{2e}\right)\frac{d\phi}{dt}$, think of charge as "momentum".

Quantum theory: $\left[\hat{\phi}, \hat{Q}\right] = -2e\, i \Rightarrow \hat{Q} = 2ei\frac{d}{d\phi}$

$$\mathcal{H} = -E_J \cos\left(\frac{2\pi\Phi}{\Phi_0}\right) + \underbrace{\frac{\left(\Phi - \Phi_{\text{ext}}\right)^2}{2L}}_{\frac{1}{2}LI^2} - \underbrace{\frac{\hbar^2}{2C}\frac{d^2}{d\Phi^2}}_{\frac{1}{2}CV^2}$$

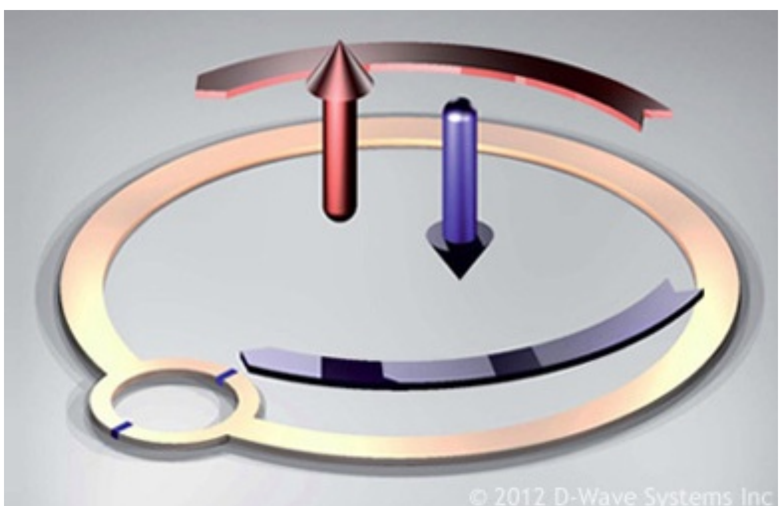
SQUID as a qubit: "Artificial spin"

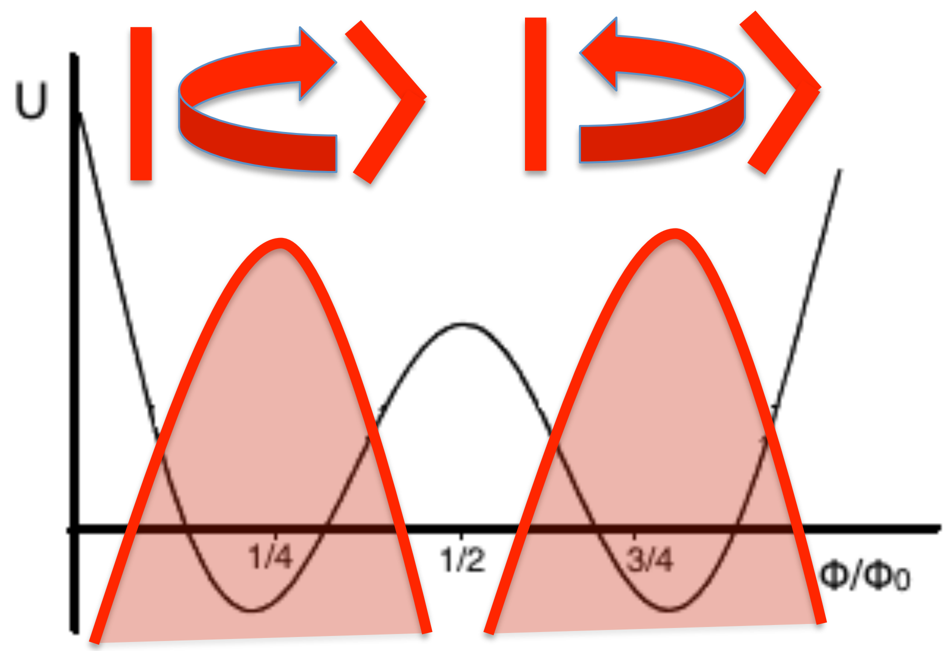
Makhlin, Schon, Shnirman, Rev. Mod. Phys. 2001



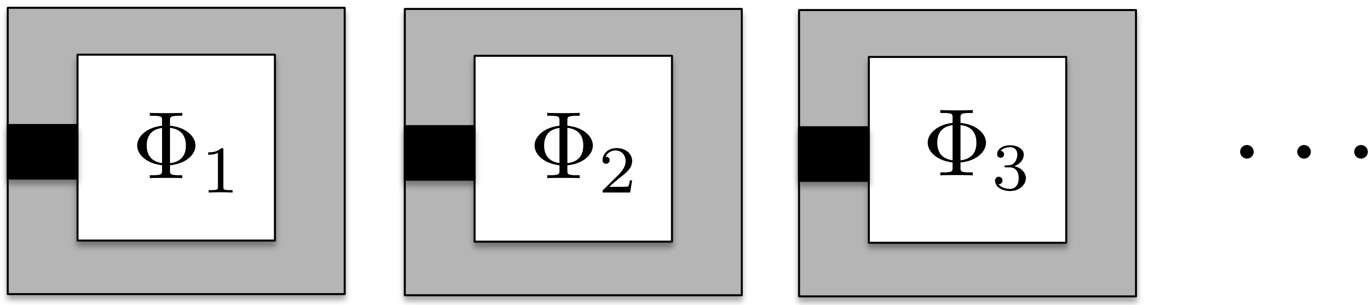
rf-SQUID

$$\mathcal{H} = -E_J \cos\left(\frac{2\pi\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_{ext})^2}{2L} - \frac{\hbar^2}{2C}\frac{d^2}{d\Phi^2}$$





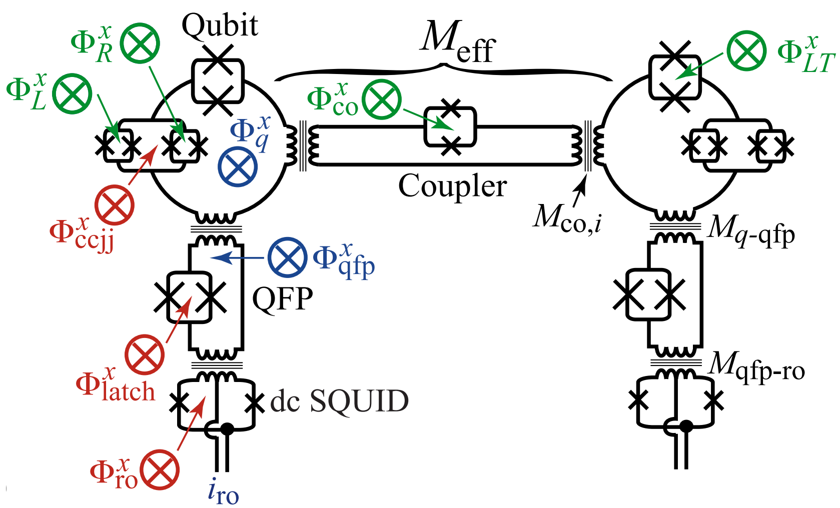
The SQUID advantage: Inductive coupling

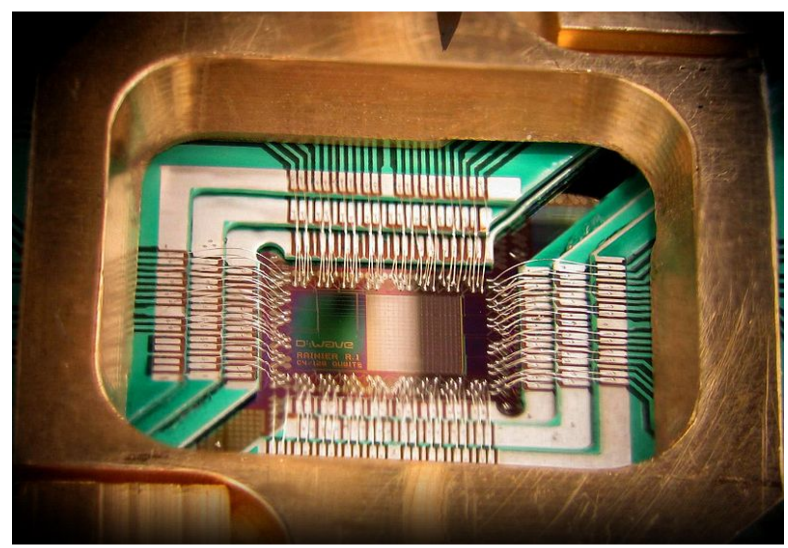


$$\mathcal{H} = M_{12}\Phi_1\Phi_2 + M_{23}\Phi_2\Phi_3 + M_{34}\Phi_3\Phi_4 + \cdots$$

More complicated circuit allows each M_{ij} to be"programmable":

D-Wave's chip (128 and 512 qubits)





D-Wave's approach to QC

- Their chip realizes a programmable Ising model

$$\mathcal{H}_0 = -\sum_{i,j} J_{ij} s_{iz} s_{jz} - \sum_i h_i s_{iz}$$

With each SQUID qubit an artificial Ising spin $s_{iz}=0, 1$.

- Finding set of $\{s_{iz}\}$ that minimizes energy is a NP-hard problem – no polynomial time algorithm exists (i.e. current algorithms take t ~ exp(size) to find answer).

- Commercial value of solving this problem: All NP-hard problems map on each other – particularly, the family of travelling salesman optimization problems - really important in business (i.e. optimal way of loading trucks, etc).

Quantum annealing: Using quantum mechanics to find the ground state

$$\mathcal{H} = -\sum_{i,j} J_{ij} s_{iz} s_{jz} - \sum_{i} h_i s_{iz} - \Gamma \sum_{i} s_{ix}$$

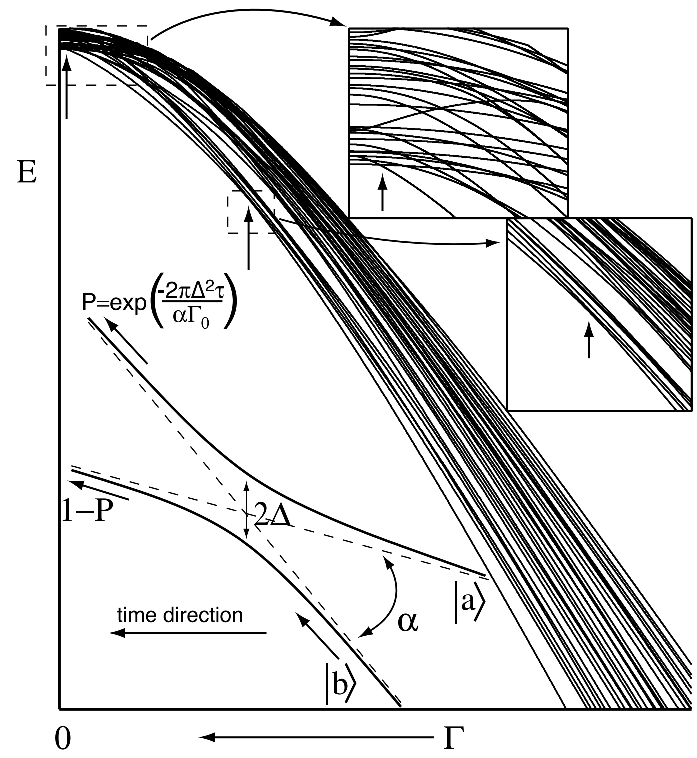
- Start with $\Gamma \gg J_{ij}, h_i$, with qubits reset to known ground state:

$$|\psi(0)\rangle = \prod_{i=1}^{N} (|0\rangle + |1\rangle)$$

- Decrease Γ *slowly* to zero

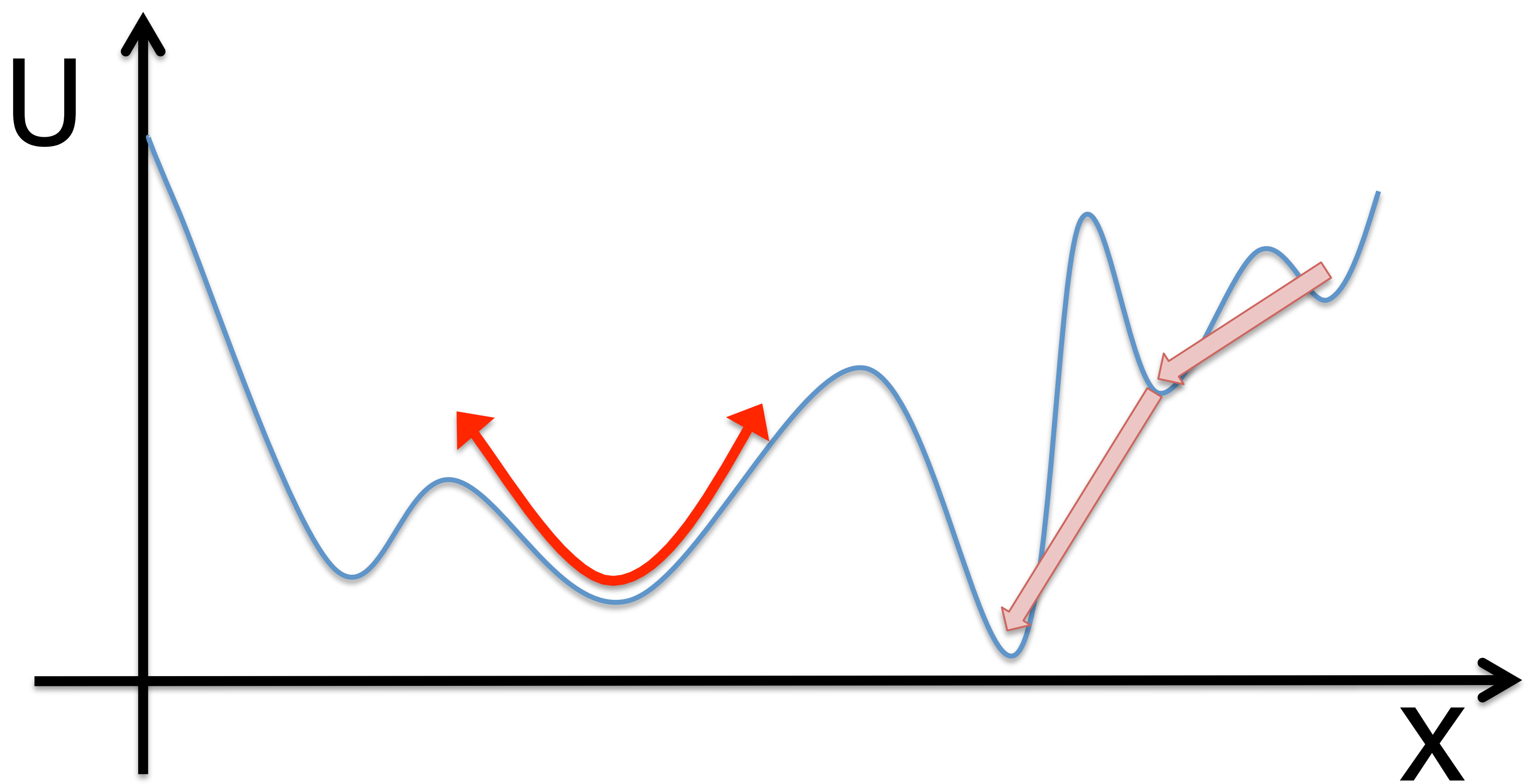
If $\tau \gg \hbar/\text{gap}$,

system will end up in ground state!

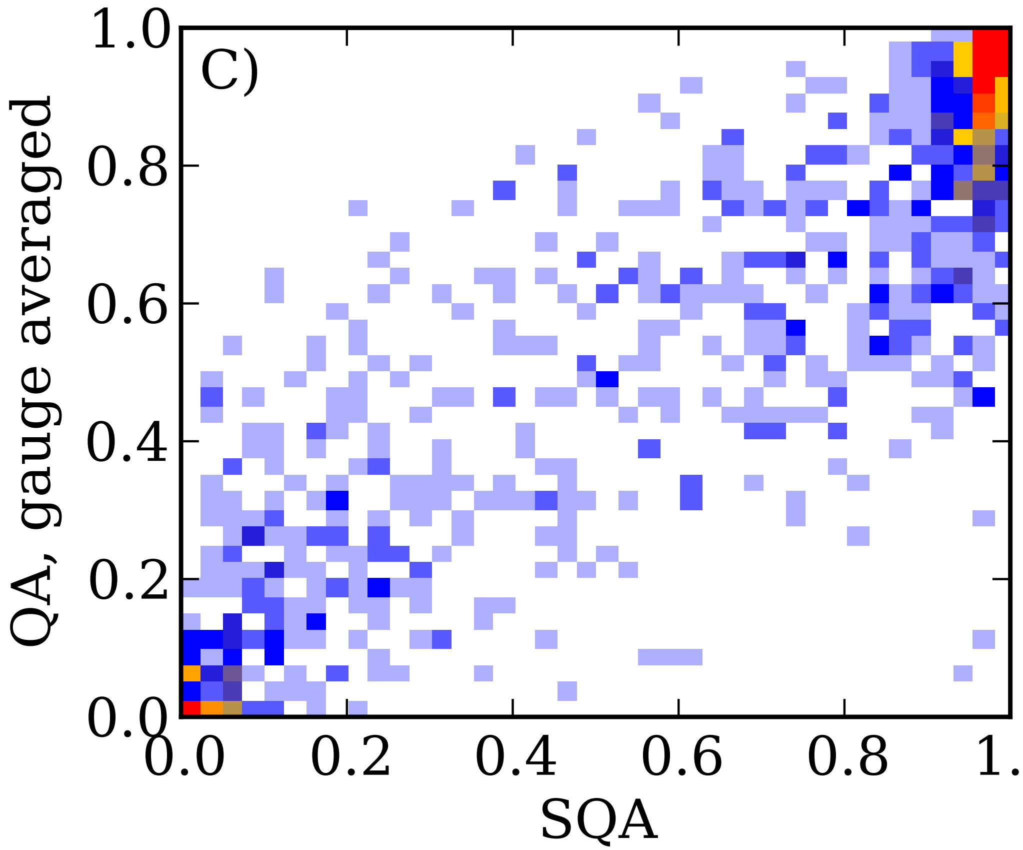


Quantum annealing versus classical annealing?

- Classical annealing: Start with $T=\infty$, reduce T slowly to zero
- Quantum annealing: Take a short cut by tunneling



Does it work? Comparison of D-Wave's 128 qubit chip with a *simulation* of quantum annealing on a classical computer



S. Boixo et al, arXiv:1304.4595 [quant-ph]

Why is quantum computing powerful?

- The qubit can exist in a superposition of its two states

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Consider N qubits

$$|\psi\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle)(\alpha_2|0\rangle + \beta_2|1\rangle)\cdots(\alpha_N|0\rangle + \beta_N|1\rangle)$$

$$= \underbrace{(\alpha_1\alpha_2\cdots\alpha_N)|00\cdots0\rangle + (\alpha_1\beta_2\cdots\alpha_N)|010\cdots0\rangle + \cdots}_{2^N \text{ states!}}$$

Processing →

$$a_1|\text{output } 1\rangle + a_2|\text{output } 2\rangle + \cdots a_{2^N}|\text{output } 2^N\rangle$$

- "Quantum parallelism": 2^N inputs processed simultaneously. *However*, when we read out the answer only one output survives.

Some problems such as factorization, searching disordered databases, etc can be solved much faster in QC