University of Victoria

Practice Midterm examination

October 3rd, 2014

QUANTUM MECHANICS II (PHYS 423)

Professor: R. de Sousa

Duration: 45 minutes – Total credit: 30

NAME:__________________________________

STUDENT NUMBER: V00__________________________________

INSTRUCTIONS:

- Write your answers into the space provided for each problem. Clearly explain your reasoning. If you need more space, please use the back of the page.
- This exam has a total of 5 pages including this cover page; there are 3 problems.
- Students must count the number of pages and report any discrepancy to the invigilator.
- This examination must be answered on the question paper.
- Students are only allowed one formula sheet, handwritten on both sides of an 8.5x11 inch page, and one calculator: the Sharp EL-510R.
- Write your name and student number in the space provided at the top of this page.

1. **Effect of an electric field in the energy levels of the harmonic oscillator.**– A particle in one dimension is confined by an harmonic potential,

$$V(x) = \frac{1}{2} m\omega_0^2 x^2. \tag{1}$$

The particle has charge q and is subject to an applied external electric field E.

(a) **(/5)** Calculate the energy levels as a function of E exactly.

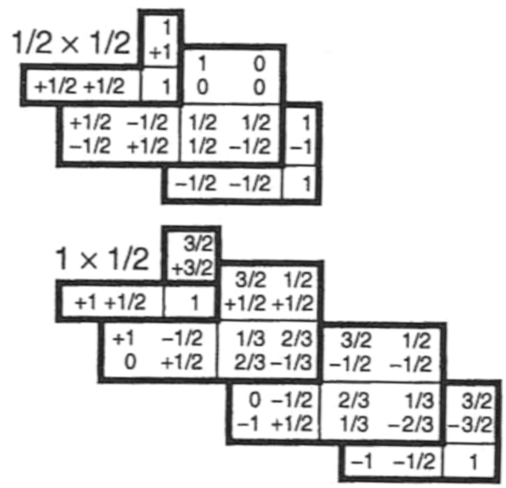
(b) **(/5)** Do the same calculation using perturbation theory to 1st and 2nd orders in E.

(Problem 1, extra space)

2. **Fine structure of the hydrogen atom.**– Consider the $n = 2$ manifold of the hydrogen atom within 1st order perturbation theory on the relativistic effects.

   (a) **(/5)** Compute the energy levels and write down all energy eigenstates in terms of the unperturbed wavefunctions $\psi^{(0)}_{nlm}(\boldsymbol{r})$ and the spin eigenstates $|\uparrow\rangle, |\downarrow\rangle$. For this calculation, ignore nuclear spin and the hyperfine interaction.

   (b) **(/5)** What happens to the $n = 2, l = 0, j = 1/2$ manifold when the hyperfine interaction is included? Calculate the energy splittings and energy eigenstates by using degenerate perturbation theory in this manifold. Write down your energy eigenstates showing the electron coordinate, spin, and nuclear spin states explicitly.
   Hint: You might want to use the Clebsh-Gordan table shown below.



3. **Rotating quantum sphere with intrinsic spin 1.**– Consider a quantum sphere that can rotate with orbital angular momentum, and in addition has intrinsic spin equal to 1. The sphere is described by the Hamiltonian

$$\mathcal{H} = \frac{(\boldsymbol{J})^2}{2I}, \tag{2}$$

where $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{M}$ is the total angular momentum of the system, with $\boldsymbol{L}$ the orbital angular momentum operator, $\boldsymbol{M}$ the spin-1 intrinsic angular momentum operator, and $I > 0$ the moment of inertia.

(a) **(/5)** Find the energy levels of the system and their degeneracies.

(b) **(/5)** What is the orbital angular momentum l of the ground state? Is it degenerate? Write down the ground state wavefunction(s).
Hint: You might want to use the Clebsh-Gordan table shown below.

1×1

	2 +2
+1 +1	1

	2 +1	1 +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

	2 0	1 0	0 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

	2 −1	1 −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

	2 −2
−1 −1	1