An incremental analysis of the value of expanding a wilderness area

Jonathan Buttle *Natural Resources Canada – Canadian Forestry Service*
Daniel Rondeau *Department of Economics, University of Victoria*

Abstract. Forsyth (2000) concludes from an option value analysis that when considered together, Killarney Provincial Park and two adjacent parcels of land should be preserved in an expanded park. With a simple discrete difference approach, we assess the incremental value of the adjacent land and decompose the benefits of park expansion. We identify a change in the value of the option to log the current park as a new component of value not previously reported. Applying the method to Killarney suggests that logging may be the most efficient use of the adjacent land. We also provide corrections for some of Forsyth's numerical results. JEL Classification: D81, Q20.

La valeur à la marge de l'agrandissement d'une aire protégée. Forsyth (2000) conclut d'une analyse de valeur d'option que le parc provincial Killarney et deux parcelles de forêt adjacentes devraient ensemble être préservées dans leur état naturel. Ici, les auteurs développent une méthodologie simple permettant de déterminer la valeur des terres adjacentes indépendamment du parc, et de décomposer numériquement la valeur du projet d'agrandissement. Un changement dans la valeur d'option de la ressource forestière est identifié comme nouvelle composante des coûts et bénéfices du projet d'expansion. Une application de la méthode à la situation de Killarney indique qu'il pourrait être avantageux d'exploiter les ressources forestières des terres adjacentes plutôt que de les ajouter au parc. Certains des résultats numériques obtenus par Forsyth sont corrigés.

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1. Introduction

In an article published in this journal Forsyth (2000) applies option value theory to assess the desirability of protecting Ontario's Killarney Provincial Park and two adjacent land areas from logging. The adjacent land areas were part of a proposed expansion of the park (which has now taken place) that would increase its size from 48,500 hectares to 76,866 hectares. In doing so, the park's visitor capacity would increase from 72,345 to an estimated 85,000 visitor-days per year (Forsyth 2000).

Treating the current park and adjacent lands as a single entity, Forsyth studied the effect of postulating that the stochastic process generating the flow of benefits from the park is characterized by white noise around a logistic growth process. With this departure from previous literature (Conrad 1997, 2000), she concluded from her analysis that conservation in the form of a park was a better use of the area than logging. Yet, whether or not *adding* adjacent lands to the existing park is economically desirable and how to tackle this issue in an option value framework remain unanswered questions.

In this paper we introduce a simple method to numerically identify the incremental value of expanding a park and to decompose the value of the decision to expand into its component parts. The method yields an estimate of incremental conservation benefits and provides a basis for optimally allocating the land between competing uses. It also identifies a change in the value of the option to log the existing park that would result from expanding the park. This component of value has not previously been identified in the literature.

While our focus is on methodology, we illustrate the approach for the Killarney area using a rough estimate of the value of timber on the land proposed for expansion. The results suggest that retaining the existing park and logging the land proposed for expansion may be most efficient. This is in contrast with the conclusion one reaches when the entire area is considered as a single entity. Having discovered a small programming error in Forsyth's numerical algorithm, we also provide corrections for some of her results.

In the remainder of the paper, we briefly review Forsyth's option value model, introduce a finite difference method to assess and decompose the incremental value of expanding the park, and illustrate the method by applying it to the Killarney situation. Final remarks conclude.

2. The option value model

To set the proper context for our analysis, it is useful first to outline Forsyth's model. Consider a forest with a known timber value \$$N$ (if logged) or generating a rate of amenity benefits $A(t)$ if it is maintained in its natural state at time t. The change over time in the rate of amenity benefits follows a stochastic process around a logistic growth function such that

$$dA(t) = rA(t)\left(1 - \frac{A(t)}{Amax}\right) dt + A(t)\sigma\, dz, \tag{1}$$

where $dz = \varepsilon_t \sqrt{dt}$, $E(dz) = 0$ and Var $(dz) = dt$. As $A(t)$ approaches $Amax$ (the maximum rate of benefits that the park can generate) the deterministic component of the growth rate goes to zero and the amenity flow is attracted to $Amax$. Here, as in Forsyth, $A(t)$ and $Amax$ are taken to be directly proportional to the number of visitors at (t) and to the maximum carrying capacity of the area, respectively; r is the intrinsic growth rate of the deterministic component of the law of motion.

Given N, the value of the optimally managed land is captured by the Bellman equation:

$$\delta V(A) = \max\left[\delta N, A + \frac{1}{dt} E[dV(A)]\right], \tag{2}$$

where δ is the discount rate and E is the expectation operator. The instantaneous return from the optimally used forest asset $[\delta V(A)]$ is given by the largest of the instantaneous return on timber revenue, δN, or the sum of the instantaneous rate of amenity benefits, $A(t)$, and the expected rate of change in the value of the forest asset.

If the land is preserved, the Bellman equation for the value of conservation is thus

$$\delta V(A) = A + \frac{1}{dt} E[dV(A)]. \tag{3}$$

Using Ito's lemma, equation (3) becomes

$$\delta V(A) = A + V_t + rA\left(1 - \frac{A}{Amax}\right) \cdot V_A + \frac{\sigma^2}{2} A^2 \cdot V_{AA}. \tag{4}$$

In what follows, we numerically estimate (4) to extract the value of conservation function using the solution algorithm programmed in Matlab by Forsyth. For a given set of parameter values, the algorithm returns the expected value of the land (using equations (4) and then (2)) for a tight grid of possible values of $A(0)$ between 0 and $Amax$. When logging is optimal for lower initial amenity benefits but conservation is preferred over a higher range of observed rate of benefits, the maximum value of $A(0)$ for which logging is optimal becomes the stopping rule. The optimal policy is then to log the area if $A(t)$ ever takes a value below this trigger point.

The questions tackled in this paper revolve around the implications of increasing $Amax$ (through an expansion of the park) on the value and efficient use of the existing and expanded park areas, and how to extract incremental values for the proposed expansion land from this information.

3. Incremental value of park expansion

3.1. Total and net value

With the logistic growth assumption, it is not possible to directly estimate the value of the land proposed for expansion. In addition to increasing expected benefits over time, enlarging the park initially increases the deterministic rate of growth in amenity benefits and decreases the expected amount of time necessary for the flow of benefits to reach the maximum capacity of the original park. As a result, the value of the adjacent land is tied in non-linear ways not only to the observed rate of amenity benefits, but also to the carrying capacity of the original park.

Conceptually, however, the incremental benefits associated with the decision to expand the park can be expressed simply as the difference between the value of the expanded park (V_{ep}) and the value of original park (V_{op}). This can be written as

$$V_{ex}^{c} = V_{ep} - V_{op}, \tag{5}$$

where the argument of $V(A)$ has been suppressed to simplify the notation. Equation (5) simply states that the benefits of conserving the extension land (V_{ex}^{c}) is given by the increment in total value (in the continuing region) gained by expanding the park.

The optimal value of the expansion at time t is then obtained by comparing the benefits of conservation with the amount of logging rents forgone on the expansion land (N_{ex}). Equation (6) embodies the criterion necessary to determine whether the incremental unit of land should be preserved or logged.

$$V_{ex} = \max\left[N_{ex}, V_{ex}^{c}\right]. \tag{6}$$

This methodology can be applied to units of increment as small as the limitations of the numerical algorithm will allow. It is therefore conceptually possible to carry out a truly marginal analysis to determine the optimal park size. The mechanics of such an analysis simply calls for successive applications of the algorithm and incremental analysis (equations (1) to (6)) to an increasingly large protected area. This would require detailed knowledge of the functional relationship between park size, visitor carrying capacity, as well as precise information on the distribution of timber rents for various park sizes and locations. Such information is not available for the Killarney area. The numerical illustration below is thus limited to assessing the value of the proposed park expansion as a discrete increment.

3.2. Decomposition of incremental benefits

In option value theory, the total value of the continuation alternative (conservation) typically has two components: the expected value of the contributions made by the stochastic variable (amenity flow) and the value of the option to terminate this activity in favour of an irreversible course of action (logging). In the incremental analysis considered here, however, the decision to expand the park has a third component: a reduction in the value of the option to log the original park area.

This is due to the fact that increasing the park's size increases the upper bound on recreational benefits and therefore the expected value of the amenity flow. This reduces the probability that $A(t)$ will ever fall to a level sufficiently low to warrant logging the original park. It follows that the value of the option to log the original park area in the future decreases.

To see this, consider an area with possible timber values of 0, N, or N', where N and N' will be taken to represent the forgone stumpage rents in the original and expanded (including the original) parks, respectively, and 0 will be the useful reference point of an area hypothetically without stumpage value. Similarly, define the maximum benefit flow capacity of the protected area as $Amax$ and $Amax'$, taken to represent the maximum benefit flow capacity of the original and expanded parks, respectively.

With this notation, note that for a given $A(0)$, $V_{op} \equiv V^{N}_{Amax}$ and $V_{ep} \equiv V^{N'}_{Amax'}$. We can then expand the value of continued conservation (equation 5), into its components:

$$V^{c}_{ex} = \left[V^{0}_{Amax'} - V^{0}_{Amax}\right] + \left[\left(V^{N}_{Amax'} - V^{N}_{Amax}\right) - \left(V^{0}_{Amax'} - V^{0}_{Amax}\right)\right] + \left[V^{N'}_{Amax'} - V^{N}_{Amax'}\right]. \quad (7)$$

The term in the first square brackets isolates the increment in amenity value associated with the park's expansion and the increase in carrying capacity from $Amax$ to $Amax'$. Since the timber value is set to zero for both value functions in this term, no option values contaminate this measure. The second square brackets contain the incremental change in the value of the option to log the original park area. It is made up of two components. $\left(V^{N}_{Amax'} - V^{N}_{Amax}\right)$ is the change in value that would take place if the park's carrying capacity was increased without forgoing any additional stumpage value. By subtracting the net change in amenity benefits resulting from the expansion $\left(V^{0}_{Amax'} - V^{0}_{Amax}\right)$, one obtains the reduction in the value of the option to log the original area that follows from increasing the park's carrying capacity. Finally, the last term measures the value of the option to log the extension land only. This is best understood as the increment in the total value of the expanded park that occurs when stumpage value is increased from N to N', keeping visitor carrying capacity constant. Recalling that $V_{op} \equiv V^{N}_{Amax}$ and $V_{ep} \equiv V^{N'}_{Amax'}$ it is easily verified that simplifying equation (7) yields equation (5).

It is therefore possible within the option value framework both to estimate the incremental value of conservation and to measure the magnitude of each of its component parts, providing a basis for optimal land use decisions at the margin. We now illustrate the methodology with an application to Killarney Provincial Park.

4. An illustration of the methodology: the Killarney expansion

4.1. Total and incremental values

Table 1 summarizes the parameter values used to estimate the value function for the original and expanded park and later to compute the value of expanding the park.[1] It is assumed that the \$26 million in stumpage value of the entire area is homogeneously distributed across the land. Given the size of the original park and extension land, it is therefore assumed that \$16.4 million in timber are in the original park and an additional \$9.6 million are on the land proposed for expansion.[2]

Figure 1 provides a graphical representation of the total value functions for the original park, V_{op}, and expanded park, V_{ep}, as well as the application of the incremental analysis.[3] The flat segments of V_{op} and V_{ep} correspond to initial levels of amenity values insufficient to warrant continued conservation of the respective areas. Over those respective ranges, logging is optimal and yields \$16.4 million in the case of the original park or \$26 million when the larger area is considered.

To the right, the increasing portions of the value functions correspond to optimal continuation regions. For those ranges of initial amenity values, it is optimal to maintain the forest in its natural state and make it accessible to visitors. Where the two regions of a value function join defines the critical level of amenity value and the optimal stopping rule. If the benefit flow from continued conservation falls below this critical value, logging becomes optimal.

For the original park, the trigger value is \$119,240. At a mean WTP value of \$25 per visitor, this translates into 4,769 visitors, well below the 1997 visitation level (67,281). On this basis, the original park is clearly worth preserving. For

1 Parameter values for the expanded park are taken directly from Forsyth (2000). The values for r and σ were obtained from her analysis of data on visitors to the existing park. We therefore assume that they are also the relevant values for the original park. The 1997 visitor level (67,281) is reported to represent 93% of the original park's carrying capacity, yielding $K = 72{,}345$. Assuming that the WTP is unchanged at \$25 gives an estimate of \$1.8million for makes *Amax*.

2 The \$26 million timber value is already a rough estimate (Forsyth 2000, 421). The homogeneous distribution is the simplest assumption that can be made in the absence of more precise information. Thus, while this is sufficient to illustrate the workings of the incremental analysis, better estimates of total stumpage values and timber distribution would be required to support policy making.

3 Forsyth's program contained a minor programming error. A parameter first defined to represent the visitor-carrying capacity of the park was later redefined as a loop counter, thus unintentionally resetting the value of the parameter before using it again to represent carrying capacity. Correcting the error results in a critical trigger value roughly twice as large as values originally reported. See table 2, below, for details and corrections. From a policy perspective, the differences are inconsequential. The suggestion, based on the incremental analysis, that the expansion may not be warranted does not depend at all on the correction of this error.

TABLE 1
Parameter values

Parameter	Symbol	Original park	Expanded park
Volatility	σ	0.26	0.26
Intrinsic growth rate	R	0.26	0.26
Discount rate	δ	6%	6%
Visitation capacity (camper nights)	K, K'	72,345	85,000
Willingness to pay ($ per camper night)	WTP	$25	$25
Amenity capacity (K*WTP), (K'*WTP)	*Amax*, *Amax′*	$1.8M	$2.125M
Stumpage value ($)	N, N'	$16.4M	$26M

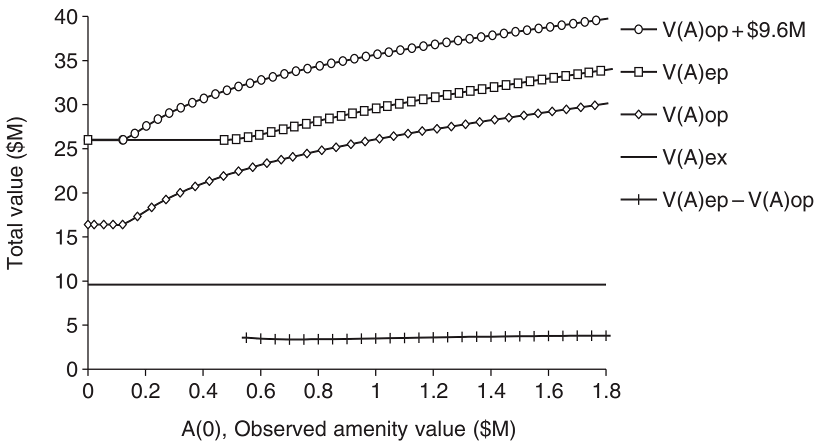

FIGURE 1 Total and incremental value functions

the expanded park, the trigger value is estimated at $474,100, or 19,200 visitors annually. This is well below the number of visitors counted every year since 1970. One may therefore conclude that the entire area is worth preserving, but this may be a misleading conclusion.

Figure 1 also plots (a) the total incremental benefits of protecting the extension land, $V_{ep}(A) - V_{op}(A)$, over their common continuation region; (b) a curve giving $V_{op}(A) + \$9.6$ million (the sum of the value of the original park plus $9.6 million in rents from logging the extension lands); and (c) the optimal value of the expansion land ($V_{ex}(A)$) as defined in equation (6).

A succinct analysis reveals that the incremental benefits of conservation $[V_{ep} - V_{op}]$ are everywhere lower than timber values ($9.6 million). Alternatively, comparing $V_{op} + \$9.6$ million (the total benefits from conserving the original park area and logging the land proposed for expansion) with the value of the expanded park V_{ep} for any initial flow of amenity value conveys the

TABLE 2
Optimal stopping rule

	Case 1	Case 2	Case 3	Case 4
r	0.26	0.26	0.02	0.02
σ	0.26	0.5	0.26	0.5
A* ($000) Expanded park	$474	$394	$808	$536
A* ($000) Original park	$119	$141	$467	$288
A* ($000) Extension	None	None	None	None

same conclusion that incremental conservation benefits are insufficient to justify the proposed expansion.[4]

Table 2 provides the corrected trigger values for the expanded park (corresponding to Forsyth's table 1), as well as the trigger values for the existing park and the expansion lands for the various values of r and σ considered by Forsyth. The absence of trigger value for the extension land simply means that there are no initial amenity flows for which park expansion is optimal.[5]

The conclusion that the entire area should be preserved on the one hand but that the extension land should be logged on the other is explained by the fact that the construction of V_{ep} compares only the benefits of either preserving or logging the entire area, without due consideration to incremental changes in costs and benefits. In the Killarney example with the roughly estimated distribution of timber values, the benefits of preserving the original park are sufficiently large to warrant retaining the entire area only if the alternative is to log the entire area. Retaining the original park and logging the extension is a valid alternative, but it is ignored when the option value analysis is focused on the expanded park.

4.2. Value decomposition

We decomposed the benefits of conservation function [V_{ex}^c] for three values of the net benefits per camper nights (WTP = 25, 40, and 75). It is important to keep in mind that for WTP = 25 and 40, it is never optimal to expand the park, since incremental conservation benefits never exceed the timber rents of $9.6 million assumed for the extension lands. We carried out those decompositions after observing that for WTP = 75 (for which conservation would be optimal), the two option value components are orders of magnitudes smaller than expected amenity benefits whenever $A(0) > 0$.

4 It is possible that the expansion lands also generate non-user benefits. However, given the absence of unique environmental features on this land and considering the fact that nearly 10% of the Province of Ontario is already protected, it is difficult to conceive that an additional 28,000 hectares provides significant additional non-use benefits.

5 With current assumptions on all other parameter values, we find that the extension would be justified if WTP > $58.

This brings us to the first noteworthy point about the decomposition of conservation benefits. For the level of uncertainty observed in the Killarney case, it appears that little is gained from modelling the policy decision in an option value framework. The two option components of value are rapidly dwarfed by expected amenity benefits when WTP = 25 and 40, and are negligible when the WTP is large enough to warrant conservation. For WTP = 25, the expected incremental amenity benefits exceed 95% of total incremental benefits at approximately $A(t) = \$1.0$ million (or 40,000 visitors). For WTP = 40, the 95% mark is passed at $A(t)$ slightly less than \$0.15 million (3,750 visitors). For WTP = 75, recreational benefits already represent 99.98% of total incremental benefits at $A(t) = \$0.05$ million (667 visitors).

In the case of the proposed expansion of Killarney Provincial Park, one could arguably make a policy recommendation simply by comparing the expected amenity benefits with the cost of park expansion and the forgone timber rents without serious consequences. This finding begs the broader question of whether the degree of uncertainty observed in economic applications of option value theory is sufficient to warrant the additional modelling and computing difficulties inherent in this methodology.

The second point of interest is the magnitude and behaviour of the change in the value of the option to log the existing park. For $A(0) = 0$, no change is observed, since the value of the option to log the existing park is in both cases identically equal to the value of the timber on that land. For $A(0) > 0$, the value of the option to log the existing park decreases. With the expansion, expected amenity benefits increase, reducing the probability that this option will ever be exercised. The change in the value of the option is greatest for $A(t)$ arbitrarily close to zero. For WTP = 25, the largest measurement (at $A(0) = \$0.05$ million) is a loss of \$1.7 million. This measure falls to \$12,000 when WTP is increased to 40 and to \$135 when WTP = 75. In each case, the change in option value rapidly decreases to zero as $A(t)$ increases.

5. Conclusions

It is one of the fundamental principles of economic analysis that efficient resource allocation decisions require a consideration of marginal benefits and costs. In this paper, we have shown how option value theory can be applied to assess incremental benefits and how the components of the incremental value function can be measured. While a precise analysis of the desirability of expanding Killarney Provincial Park would require better estimates of parameter values than those available, the results that one may wish to protect the entire area on the one hand but log the extension on the other clearly illustrate the importance of properly casting the decision problem in its marginal context.

The analysis also points out that applying option value theory in an incremental fashion can give rise to a change in the option value of infra-marginal units. In the Killarney case, the non-linear relationship between the carrying capacity of the area and the rate of change in the deterministic growth of the amenity value created a decrease in the value of the option to log the original park. Such a component of value, though it turned out to be of small magnitude for Killarney, is a theoretically valid cost component associated with incremental decisions that should be accounted for in the decision making process.

With effects on option value components orders of magnitude smaller than expected recreational values, ignoring them in the case at hand would seem to cause little prejudice to the quest for optimality and would allow policy analysts to reach valid policy decisions with greater ease. Whether or not the levels of uncertainty typically dealt with in economic applications of option value theory are large enough to warrant the added modelling complexity is a question that seems worthy of future investigation.

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