GEOPHYSICAL RESEARCH LETTERS, VOL. 33, L19310, doi:10.1029/2006GL027233, 2006

A note on pore fluid pressure ratios in the Coulomb wedge theory

Kelin Wang,[1,2] Jiangheng He,[1] and Yan Hu[2]

Received 14 June 2006; revised 22 August 2006; accepted 9 September 2006; published 11 October 2006.

[1] The widely used exact Coulomb-wedge stress solution presented by F. A. Dahlen contains an approximation in dealing with pore fluid pressure ratios within the wedge and in the basal fault. We discuss the theoretical and practical ramifications of the approximation and propose simple modifications to make the solution exact. We illustrate that errors caused by the approximation are negligible for taper angles less than about 10° but become significant for larger tapers. **Citation:** Wang, K., J. He, and Y. Hu (2006), A note on pore fluid pressure ratios in the Coulomb wedge theory, *Geophys. Res. Lett.*, *33*, L19310, doi:10.1029/2006GL027233.

1. Introduction

[2] In two seminal papers on the theory of the critically tapered Coulomb wedge [*Davis et al.*, 1983; *Dahlen*, 1984], the generalized Hubbert-Rubey pore fluid pressure ratio in the wedge is defined as follows

$$\lambda = \frac{P - \rho_w g D}{-\sigma_z - \rho_w g D} \tag{1}$$

where P is pore fluid pressure within the wedge, $\rho_w gD$ is the weight of the overlying water column, and σ_z is normal stress in the z direction and in many geological situations is nearly vertical (Figure 1). The fluid pressure ratio along the basal fault is similarly defined as

$$\lambda_b = \frac{P_b - \rho_w g D}{-\sigma_z - \rho_w g D} \tag{2}$$

where P_b is pore fluid pressure along the basal fault. The hydrological process in the fault zone may differ from that in the wedge and thus cause a sharp change in fluid pressure across the wedge base. By allowing λ and λ_b to be different, we use a pressure discontinuity to approximate what should be a sharp gradient. The strength of the wedge material can be represented by $\mu(1 - \lambda)$, where μ is the coefficient of internal friction, and the strength of the basal fault by $\mu_b(1 - \lambda_b)$, where μ_b is the coefficient of friction of the fault.

[3] Parameters λ and λ_b as defined above have been used in numerous applications of the Coulomb wedge theory to subduction zones and orogenic zones (see review by *Wang and Hu* [2006]). However, the definitions in *Davis et al.* [1983] and *Dahlen* [1984] are actually different, because of the different coordinate systems used and thus the different orientations of σ_z (Figure 1). In the following, we explain the theoretical and practical consequences of the difference and suggest ways of reconciliation.

2. An Approximation in Dahlen's Solution

[4] Because normal stress σ_n along the basal boundary must be continuous across the boundary, the assumed jump in pore fluid pressure causes the normal effective stress to be discontinuous, that is, $\overline{\sigma}_{n\_b} = \sigma_n + P_b$ in the fault zone but $\overline{\sigma}_n = \sigma_n + P$ just above the fault. The basal boundary condition is given in terms of fault friction $\tau_b = -\mu_b\overline{\sigma}_{n\_b}$, where τ_b is shear stress on the fault. To be able to derive a solution within the wedge, we must rewrite this condition in terms of $\overline{\sigma}_n$. *Dahlen* [1984] explains that if an effective coefficient of basal friction

$$\mu_b' = \frac{1 - \lambda_b}{1 - \lambda}\mu_b \tag{3}$$

is defined, we will have

$$\tau_b = -\mu_b \bar{\sigma}_{n\_b} = -\mu_b' \bar{\sigma}_n \tag{4}$$

This formula allows an exact solution to be derived using the coordinate system of Figure 1b for a uniform wedge in which cohesion is nil [*Dahlen*, 1984] or proportional to z [*Zhao et al.*, 1986]. Equations (3) and (4) imply that

$$\frac{1 - \lambda_b}{1 - \lambda} = \frac{\bar{\sigma}_{n\_b}}{\bar{\sigma}_n} \tag{5}$$

However, if we write P and P_b in terms of λ and λ_b using (1) and (2), respectively, and substitute the results into the expressions of $\overline{\sigma}_n$ and $\overline{\sigma}_{n\_b}$ in (5), we will find that, for the general situation of $\lambda \neq \lambda_b$, (5) does not hold unless $\sigma_z = \sigma_n$. The condition $\sigma_z = \sigma_n$ is naturally met in the coordinate system used by *Davis et al.* [1983] (Figure 1a), who derived an approximate solution based on the assumption of a very small wedge taper $\theta = \alpha + \beta$, where α and β are the surface slope angle and basal dip, respectively. Obviously, to allow $\sigma_z \approx \sigma_n$ in *Dahlen*'s [1984] coordinate system (Figure 1b), θ must also be small. For this reason, Dahlen's solution is also a small-taper approximation except for $\lambda = \lambda_b$.

3. Reconciliation

[5] There are two ways to modify Dahlen's solution to make it exact. The first way is to replace σ_z with σ_n in (1) and (2) and use the resultant λ and λ_b for (3) to define μ'_b. In other words, we use *Davis et al.*'s [1983] definition in *Dahlen*'s [1984] coordinate system (Figure 1b). This makes expressions of all the parameters somewhat cumbersome, because σ_n, at angle θ with z, depends on σ_x, σ_z, and shear stress τ_{xz}. After some derivation, we find that the only

[1] Pacific Geoscience Centre, Geological Survey of Canada, Sidney, British Columbia, Canada.
[2] School of Earth and Ocean Sciences, University of Victoria, Victoria, British Columbia, Canada.

0094-8276/06/2006GL027233$05.00

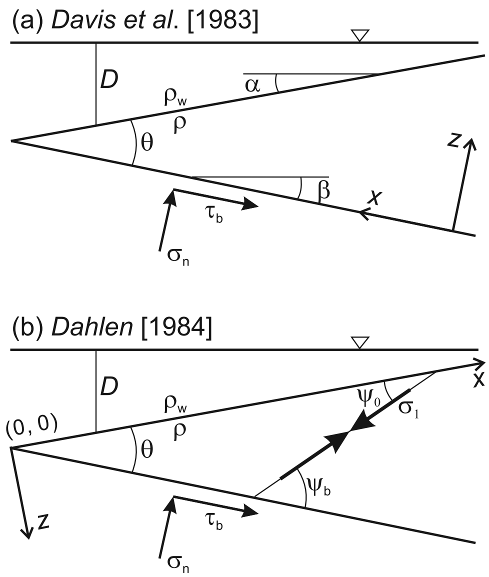

Figure 1. Schematic illustration of coordinate systems used in the Coulomb wedge theory (x, z), maximum compressive stress σ_1, angles α, β, θ, ψ_o, and ψ_b, water depth D, and basal tractions. ρ and ρ_w are densities of the wedge material and overlying water, respectively.

modification we need to make to other parts of Dahlen's solution is to replace $(1 - \lambda)$ with

$$\frac{1-\lambda}{1+\lambda(J-1)} \tag{6a}$$

where

$$J = \frac{\overline{\sigma}_n}{\overline{\sigma}_z} = \frac{\csc\varphi - \cos 2\psi_b}{\csc\varphi - \cos 2\psi_o} \tag{6b}$$

where effective stress $\overline{\sigma}_z = \sigma_z + P$, $\varphi = \tan^{-1}\mu$, and ψ_o and ψ_b are the angles between maximum compressive stress σ_1 and the upper and basal surfaces (Figure 1b), respectively. Because $\psi_b - \psi_o = \theta$, the small-taper approximation $\theta \approx 0$ leads to $J \approx 1$ making *Dahlen*'s [1984] solution exact .

[6] The second way to make *Dahlen*'s [1984] solution exact is to comply with his definition of λ using (1) but argue that λ_b can be defined differently from (2) such that (5) holds. This is the approach used by *Wang and Hu* [2006], although they did not give an explicit expression for λ_b. It can be readily verified that, if (1) is used to define λ, in order for (5) to hold we must have

$$\lambda_b = \frac{P_b - \rho_w gD + \lambda H}{-\sigma_z - \rho_w gD + H} \tag{7a}$$

where

$$H = \frac{\sigma_z - \sigma_n}{1-\lambda} \tag{7b}$$

For very small θ, $\sigma_z \approx \sigma_n$, and (7a) reduces to (2). This second approach is preferred over the first one, because, with λ_b redefined using (7), there is no need to revise previously published results that were derived in the coordinate system shown in Figure 1b.

[7] Figure 2a shows a comparison between λ_b defined using (7) or (2), as functions of wedge taper, for subareal noncohesive wedges. There are two branches of λ_b for either definition. The upper branch (dashed line) represents extensionally critical states, and the lower branch (solid line) represents compressively critical states. They encompass the region of stable wedges for which a stress solution is given by *Wang and Hu* [2006]. The stable region based on (7) is shown in white. To derive the results in Figure 2a, we first obtain stresses for each θ value using *Dahlen*'s [1984] solution with a prescribed λ_b value assumed to be given by (7), and we then calculate λ_b as defined by (2)

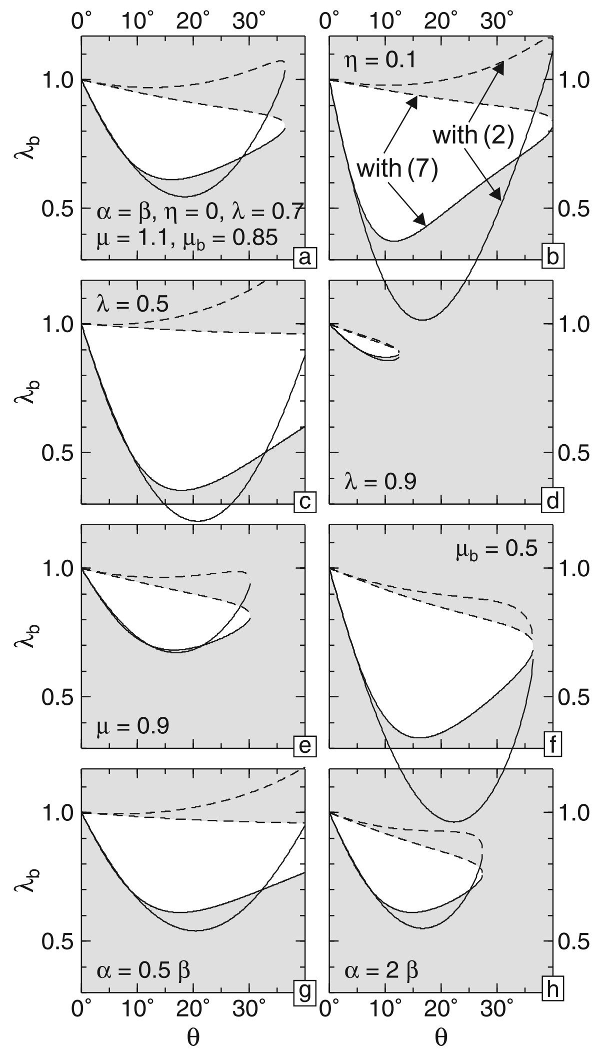

Figure 2. (a–h) Examples to illustrate how the difference between λ_b defined using (7) and (2) changes with taper angle θ. The stable region encompassed by λ_b values defined using (7) is show in white, with the unstable region shaded. Dashed lines represent extensionally critical states, and solid lines represent compressively critical states. The wedge is assumed to be subareal ($\rho_w = 0$) with density ρ = 2500 kg/m^3. Every case in Figures 2b through 2h is a variation from that of Figure 2a by changing only one parameter or condition as indicated.

using the P_b and σ_z values given by the exact stress solution. All other cases in Figure 2 are variations of the reference case of Figure 2a.

[8] Figure 2b is for cohesive wedges in which cohesion S_0 is proportional to z in the coordinate system of Figure 1b. The cohesion function of *Zhao et al.* [1986] can be written as

$$S_0 = \eta(1 - \lambda)\mu\rho gz\cos\alpha \quad (8)$$

where η is a constant [*Wang and Hu*, 2006]. If $\eta = 0$, as in all other cases of Figure 2, the solution of *Zhao et al.* [1986] reduces to that of *Dahlen* [1984]. Even with $\eta = 0.1$, a value smaller than typical values assumed by *Zhao et al.* [1986], Figure 2b indicates significant difference between the two λ_b definitions. A larger η value will cause an even greater difference.

[9] Figure 2 shows that for θ less than about 10°, there is little difference between λ_b defined using (7) or (2). The difference increases for larger tapers and is greater if the wedge material is stronger due to either a nonzero cohesion (Figure 2b), a lower λ (Figure 2c), or a greater μ (Figure 2a vs. Figure 2e). The difference is also greater if the basal fault is weaker (Figure 2f). The two (solid) lines of compressively critical states cross at $\lambda_b = \lambda$, that is, 0.5 for Figure 2c and 0.7 for the other cases except Figure 2d in which the stable region is too small to reach this value. The crossing simply reflects the fact that the two definitions of λ_b are identical if $\lambda_b = \lambda$.

[10] When applying the stress solutions of *Dahlen* [1984], *Zhao et al.* [1986], and *Wang and Hu* [2006], we only need to consider $\mu_b(1 - \lambda_b)$ and do not need to specify μ_b and λ_b separately. Only when we fix μ_b and discuss λ_b independently, does the definition of λ_b become an issue. For example, one may obtain a value of λ_b, denoted λ^*_b, using one of these solutions assuming known wedge geometry and properties and μ_b, and wish to determine P_b from λ^*_b. In this case, the difference between P_b correctly derived using (7) and incorrectly using (2) is

$$\frac{\lambda_b^* - \lambda}{1 - \lambda}(\sigma_z - \sigma_n) \quad (9)$$

Examples in Figure 2 show that for small tapers, the issue of λ_b definition is largely a theoretical one and makes little practical difference. Errors caused by using the approximate definition of (2) may be significant for larger tapers, especially if the wedge material is strong relative to the basal fault.

References

Dahlen, F. A. (1984), Noncohesive critical Coulomb wedges: An exact solution, *J. Geophys. Res.*, *89*, 10,125–10,133.

Davis, D. M., J. Suppe, and F. A. Dahlen (1983), Mechanics of fold-and-thrust belts and accretionary wedges, *J. Geophys. Res.*, *88*, 1153–1172.

Wang, K., and Y. Hu (2006), Accretionary prisms in subduction earthquake cycles: The theory of dynamic Coulomb wedge, *J. Geophys. Res.*, *111*, B06410, doi:10.1029/2005JB004094.

Zhao, W. L., D. M. Davis, F. A. Dahlen, and J. Suppe (1986), Origin of convex accretionary wedges: Evidence from Barbados, *J. Geophys. Res.*, *91*, 10,246–10,258.

J. He and K. Wang, Pacific Geoscience Centre, Geological Survey of Canada, 9860 West Saanich Road, Sidney, BC, Canada V8L 4B2. (kwang@nrcan.gc.ca)

Y. Hu, School of Earth and Ocean Sciences, University of Victoria, Victoria, BC, Canada V8P 5C2.