Problem Set #2 Solutions

Coverage: Chapter 9 and 9A "Production" and Chapter 10 and 10A "Cost" and Chapter 11 "Perfect Competition." Many questions are from the Frank and Parker text.

Section 1

Coverage: Chapter 9 and 9A "Production"

Question 1:

a) $F(K,L) = 4K^{\frac{1}{2}}L^{\frac{1}{2}}$; $F(cK,cL) = 4(cK)^{\frac{1}{2}}(cL)^{\frac{1}{2}} = 4c^{\frac{1}{2}+\left(\frac{1}{2}\right)}K^{\frac{1}{2}}L^{\frac{1}{2}} = cF(K,L)$, so constant returns to scale. $MP_L = \frac{2K^{\frac{1}{2}}}{L^{\frac{1}{2}}}$ and $MP_k = \frac{2L^{\frac{1}{2}}}{K^{\frac{1}{2}}}$, both of which decline with respect to L and K respectively, so this production function satisfies the "law of diminishing returns" (LDR).

b) $F(cK,cL) = a(cK)^2 + b(cL)^2 = c^2(aK^2 + bL^2) = c^2F(K,L)$, so increasing returns to scale. $MP_L = 2bL, MP_K = 2aK$, both of which are increasing, so neither satisfies the LDR.

c) $F(K,L) = \min(aK,bL)$. $F(cK,cL) = \min(acK,bcL) = cmin(aK,bL) = cF(K,L)$ so constant returns to scale. $MP_L = b$ when $K > \frac{b}{a}L$ and 0 when $K < \frac{b}{a}L$. $MP_K = a$ when MP_K = a when $K < \frac{b}{a}L$, and 0 when $K > \frac{b}{a}L$. Here the marginal product of each variable input drops precipitously at $K = \frac{b}{a}L$ then remains constant at 0, so this production function satisfies the LDR with a vengeance!

d) $F(K,L) = 4K + 2L$; $F(cK,cL) = 4cK + 2cL = cF(K,L)$, so constant returns to scale. The marginal products of both inputs are constant, so this production function does not satisfy the LDR.

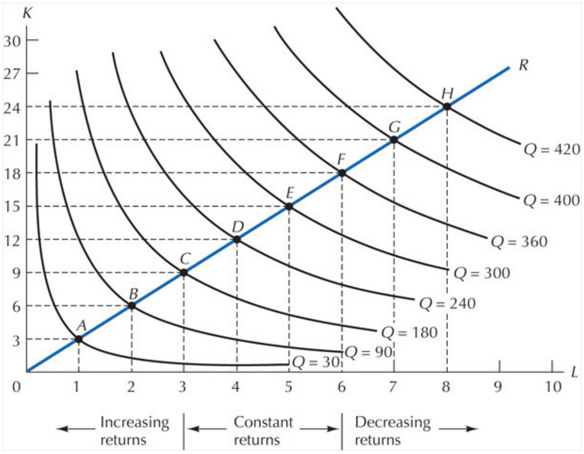
e) $F(K,L) = K^{0.5}L^{0.6}$; $F(K,L) = (cK)^{0.5}(cL)^{0.6} = c^{1.1}F(K,L)$, so increasing returns to scale. Both marginal products decline, so satisfies LDR.

f) $F(K,L) = K_1^{0.3}K_2^{0.3}L^{0.3}$; $F(cK,cL) = (cK_1)^{0.3}(cK_2)^{0.3}(cL)^{0.3} = c^{0.9}F(K,L)$, so decreasing returns to scale. All MPs decline, so satisfies LDR.

Question 2: For K = 27, we have $MP_L = 2\left(\frac{1}{3}\right)K_1^{\frac{1}{3}}L^{\left(\frac{1}{3}-1\right)} = \left(\frac{2}{3}\right)(27)^{\frac{1}{3}}L^{-\frac{2}{3}} = 2L^{-\frac{2}{3}}$, $Q = 6L^{\frac{1}{3}}$.

Question 3: Given a Cobb-Douglas production function of the form $Q = F(K,L) = mK^aL^b$, $F(cK,cL) = m(cK)^a(cL)^b = c^{a+b}mK^aL^b = c^{a+b}F(K,L)$. Since c^{a+b} is fixed for all levels of output, we will have increasing returns if a+b > 1, and decreasing returns if a+b < 1. But we can't have *both* with the *same* production function.

A function that has IRS initially and DRS later on is not hard to draw (Fig 9-12):



Finding a formula is not so easy as everybody will have discovered.

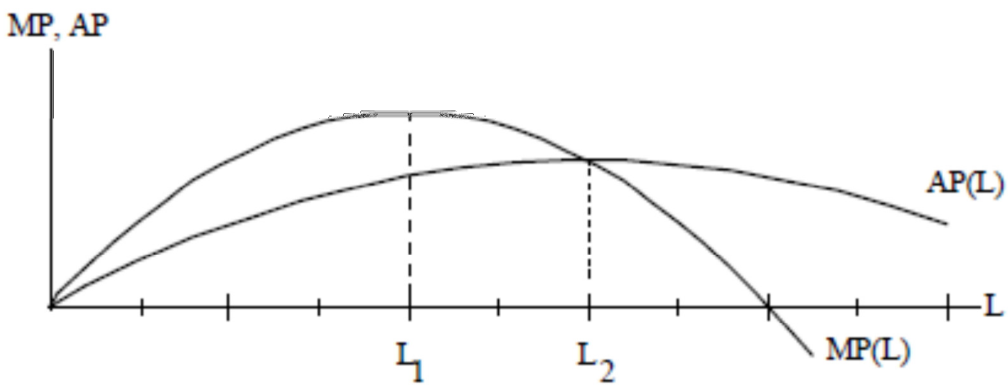
Question 4: Given K=6 and L=5 and that $Q = \min(2K, 3L)$, then $Q = \min(12,15) = 12$.
$MP_K = F(7,5) - F(6,5) = 14 - 12 = 2$ and $MP_L = F(6,6) - F(6,5) = 12 - 12 = 0$.

Another way to see this is that close to the point (K,L)=(6,5) we must have Q=min(2K,3L)=2K. With Q=2K, we clearly have that $MP_K = 2$ and $MP_L = 0$.

If (K,L)=(15,10) then $MP_K = 0$ and $MP_L = 0$.

Question 5: They are equivalent mathematically, but the measure for production is a *cardinal* number, whereas the index for utility is of only *ordinal* (“greater than” or “less than”) significance.

Question 6: False. As shown in the diagram, MP is decreasing from L1 onward, whereas AP does not begin to decline until L2. As long as MP > AP, AP will be increasing.



Question 7: Under constant returns to scale, output will be 2, under decreasing returns to scale the output will be < 2, and under increasing returns to scale the output will be > 2 units of output.

Section 2

Coverage: Chapter 10 and 10A "Cost"

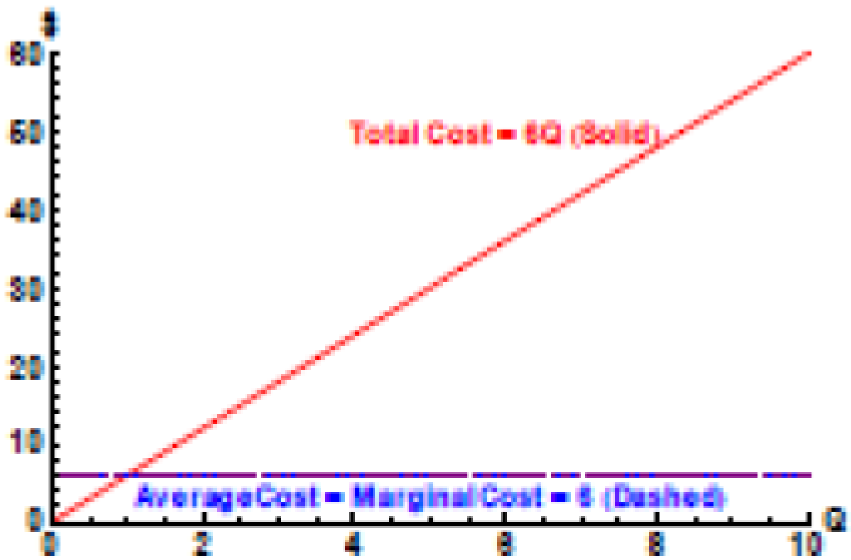
Question 8:

Recall that here is how you find the cost function:

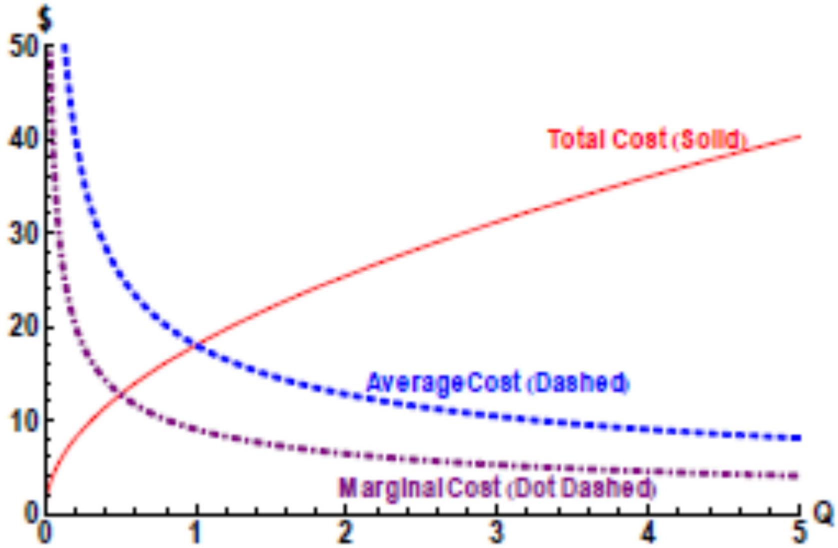
(i) First solve the cost minimization problem (but leave Q in as an unknown). This will give you the cost-minimizing bundle of inputs K and L.

(ii) Then compute the costs associated with the cost-minimizing combination of K and L. (These costs will then obviously also be a function of Q!)

a. For a perfect substitutes production function you can solve the cost minimization problem graphically. The graphical analysis will show that in this case we have K*=Q and L*=0. An alternative way to see that you should have positive K and L=0 is to compare MP_k/p_k and MP_l/p_l . We have $MP_k/p_k = 1/6 > MP_l/p_l = 1/10$, so a dollar invested in capital is always more productive! Hence we have C(Q)=6K*+10L*=6Q. Average and marginal costs will be equal at AC(Q)=6Q/Q=6 and MC(Q)=C'(Q)=6.



b. Here the two conditions that will yield the optimum bundle are 9L/81=9K/9, or L=9K & 9KL=Q. Substituting yields $K^* = \frac{1}{9}\sqrt{Q}$ and $L^* = \sqrt{Q}$. From this optimum bundle we learn that $C(Q) = \frac{81}{9}\sqrt{Q} + 9\sqrt{Q} = 18\sqrt{Q}$, so that AC(Q)=$\frac{18}{\sqrt{Q}}$ and MC(Q)=$\frac{9}{\sqrt{Q}}$.



Question 9:

a) The firm minimizes costs when it distributes production across the two processes so that marginal cost is the same in each. If Q1 denotes production in the first process and Q2 is production in the second process, we have Q1 + Q2 = 8 and 0.4Q1 = 2 + 0.2Q2, which yields Q1 = 6, Q2 = 2. The common value of marginal cost will be $2.40 per unit of output.

b) Note that for output levels less than 5, it is always cheapest to produce all units with process 1. Hence $Q = Q_1 = 4$ units, and $MC = MC_1 =$ \$1.60/unit of output.

Question 10: This is not an easy question, because you need to organise the information methodically. But it is given that for Q=5 we have K*=2 and L*=3. From this optimum bundle at Q=5 and the info on the expansion path we can also infer that for this production function, as long as the input price ratio (PL/PK) equals 1/2, the optimal input bundle will always have 2 units of capital for every 3 units of labour. In other words the equation for the output expansion path is given by K = 2L/3. At the isocost line of \$70, we must therefore have $P_k K + P_L L = 2\left(\frac{2L}{3}\right) + L = 70$, or L*=30. This then also shows that K*=20. Now let's use the fact that we know this is a CRS technology. At K*=2 and L*=3 we have Q=5. Therefore at 10 times that scale, i.e. K*=20 and L*=30 we have ten time the output: Q=50. Clearly the average cost is 70/50 =\$1.40/unit.

Question 11:

a. $\frac{MP_L}{MP_K} = \frac{\left(\frac{1}{3}\right)L^{-\frac{2}{3}}K^{\frac{2}{3}}}{\frac{2}{3}L^{\frac{1}{3}}K^{-\frac{1}{3}}} = \frac{K}{2L} = \frac{w_L}{w_K} = 1 \rightarrow K = 2L\ .\ Also\!: Q = L^{\frac{1}{3}}(2L)^{\frac{2}{3}} = 2^{\frac{2}{3}}L \rightarrow L^* = \frac{q^{\frac{3}{2}}}{2}, K^* =$ $q^{\frac{3}{2}}$, therefore the cost function is: $C(q) = 2\left(\left(\frac{q^{\frac{3}{2}}}{2}\right)\right) + 2\left(q^{\frac{3}{2}}\right) = 3q^{\frac{3}{2}}$

b. $\frac{MP_L}{w_L} = \frac{1}{2}, \frac{MP_K}{w_K} = \frac{2}{6} = \frac{1}{3} \rightarrow use\ only\ labour! \rightarrow L^* = q, K^* = 0 \rightarrow C(q) = 2q$

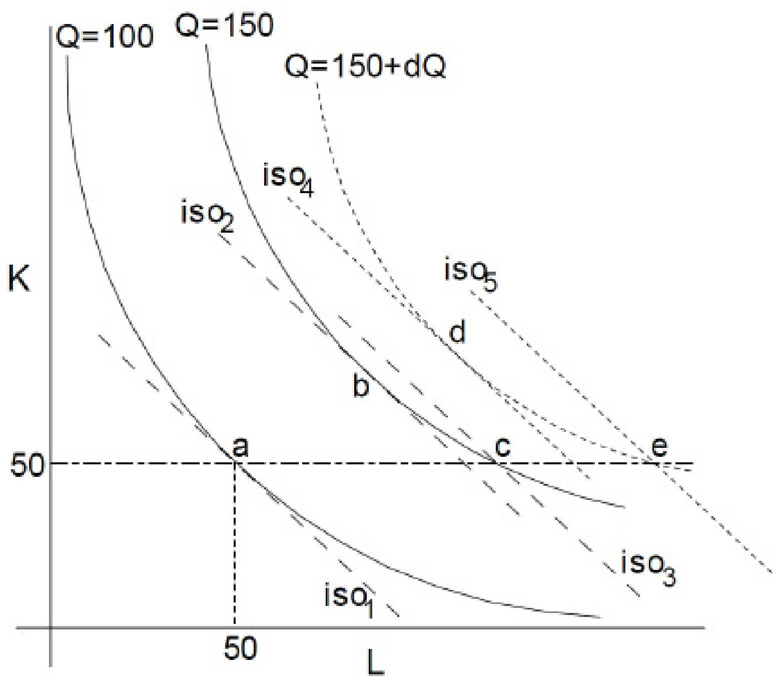
c. In optimum, $2L = K = q$. So for q=90 we have $L^* = 45, K^* = 90. C(q) = 3\left(\frac{q}{2}\right) + 6q \rightarrow \frac{3q+12q}{2} \rightarrow C(q) = \frac{15q}{2}$

Question 12:

a. $ATC = \frac{TC}{Q} = 3Q^{-\frac{1}{2}} + 15 + \frac{30}{Q}$
b. $VC = 3Q^{\frac{1}{2}} + 15Q$
c. $AVC = 3Q^{-\frac{1}{2}} + 15$
d. $MC = \frac{3}{2}Q^{-\frac{1}{2}} + 15$
e. $FC = 30$
f. $AFC = \frac{30}{Q}$

Question 13

We did this question in the labs. Consider the figure below: The isoquant lines represent a constant level of output and the isocost lines represent a constant cost. Isoquants closer to the origin represent lower levels of output. Isocost lines closer to the origin represent lower costs. Note that the lowest possible cost associated with the production of Q=150 units is represented by iso2. However, if capital is fixed at 50 units, the lowest cost you can produce 150 units of output with is the cost associated with iso3, which is further away from the origin than iso3. Short run costs (were one factor is fixed) are always associated with a higher or equal total cost than in the long run (were all factors are variable).



Section 3

Coverage: Chapter 11 “Perfect Competition

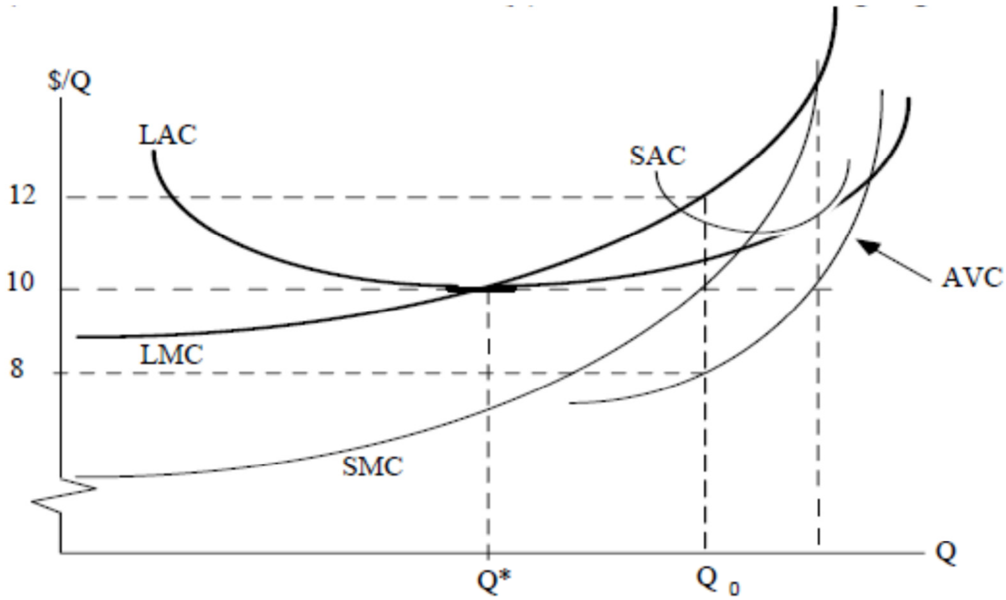
<u>Question 14:</u> Economic profit includes all costs, including the *opportunity* cost of all resources used by the firm, whether or not they involve outside payments. Accounting profit looks only those costs that entail cash outlays. The firm should consider economic profit only.

<u>Question 15:</u> Price = TR/Q, so this firm's demand curve is given by P = 3 – 0.02Q. Since its price is a *decreasing* function of its output, it would not be acting as a price taker, like a perfectly competitive firm.

<u>Question 16:</u> The effect of such a tax is to produce a parallel upward movement in each firm's long-run average cost curve. The output level for which the minimum value of LAC occurs will thus be the same as before, which means that firms in long-run equilibrium will each have the same amount of output as before. Thus the statement is true. The long-run market equilibrium price will be equal to the minimum LAC + the amount of the per-unit tax, and buyers will pay 100% of the tax.

<u>Question 17:</u> Consumer surplus in a competitive industry is the area between the price line and the *market* demand curve, not the individual *firm's* demand curve. Since the market demand curve is downward sloping, there will in general be positive consumer surplus. Indeed, compared to other market structures, perfect competition creates the maximum consumer surplus. Thus the statement is false.

<u>Question 18:</u> Since $P = SMC > AVC$, we know that the firm should continue at its current level of output (call it Q0) in the short run. Is the firm making economic profits? Since $LMC = 12 > \min LAC = 10$, we know that the firm is producing to the *right* of its long run cost-minimizing level, that LAC is rising, and therefore that $10 < LAC(Q_0) < 12$. Since $SMC \neq LMC$, $LAC(Q_0) < SAC(Q_0)$, and hence $P = MC < SAC$. Therefore the firm is incurring losses in the short run, and in the long run it should shift to a smaller size of plant (in fact, the size that minimizes LAC at Q^*), as shown in the following diagram.



Question 19:

a. $LAC = \frac{TC}{Q} = 4Q + 100 + \frac{100}{Q}$. The minimum point on LAC is found either by graphing the LAC curve or by taking the first derivative and setting it equal to zero: $\frac{\partial LAC}{\partial Q} = 4 - \frac{100}{Q_2} = 0$, which yields Q = 5 units. In the long run, P = LAC = $140/unit.

b. If demand is Q = 1000 – P, then at P = 140, we get Q = 860 units. So in long run equilibrium, there will be 860/5 = 172 firms.

c. Now $LAC = \frac{TC-36}{Q} = 4Q + 100 + \frac{640}{Q}$. Again, the minimum point on LAC is found either by graphing the LAC curve or by taking the first derivative and setting it equal to zero. $\frac{\partial LAC}{\partial Q} = 4 - \frac{64}{Q_2} = 0$, which yields Q = 4 units. In the long run, P = LAC =$132/unit. At this price, Q = 868 units, and the number of firms rises to 868/4 = 217.

Question 20:

(a) $LAC_i = 504 - 36Q_i + Q_i^2$, and $LMC_i = 504 - 72Q_i + 3Q_i^2$.

(b) Minimum LAC_i is found where $\frac{\partial LAC_i}{\partial Q_i} = -36 + 2Q_i = 0$ where *Qi* = 18. Minimum LMC_i is found where $\frac{\partial LMC_i}{\partial Q_i} = -72 + 6Q_i = 0$, or where *Qi* = 12. LMC_i reaches its minimum to the left of the point at which LAC_i reaches its minimum.

(c) Minimum LAC_i (which occurs at Q_i = 18 rings) = $504 - 36(18) + (18)^2$ =$180/ring, which determines the price of rings in long-run equilibrium. At this price, with demand given by *P* = 270 - .01*Q*, then long-run equilibrium *Q*= 9000 rings, and the equilibrium number of firms is 9000/18 = 500 firms.

(d) If market demand becomes $P = 243 - 0.01Q,$ then long-run equilibrium Q = 6300 rings, and the equilibrium number of firms is 6300/18 = 350 firms. The transition path from the initial equilibrium to the new equilibrium is as follows: when demand falls, there is an excess supply of rings at the original equilibrium price, and so the market price falls, and firms contract output along their short-run MC curves. At the lower market price, however, firms are experiencing economic losses, which will induce the exit of some firms, leading to a leftward shift in supply and a resultant return of the market price to its long-run equilibrium level, at the lower equilibrium level of output of 6300 rings.

Question 21:

a. If a firm would innovate while others are not it would become temporarily the leader in the industry. It could reap profits for as long as it takes the other firms in the industry to adjust their technologies to match the innovation. If the innovating firm could even patent the technology or otherwise restrict access to it, then there is an even bigger bonus for the innovator. In this case the structure of the industry would probably even change during the patent life (or until the innovation becomes obsolete).
b. The zero profit condition reflects zero economic profit. But accounting profits could well be structurally positive. Thus even without an innovation happening expenses on R&D could well be sustainable even in the long run.