

UVic Mathematics Competition

September 26, 2017



University
of Victoria

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- No calculators, books or notes are allowed.
 - Write solutions in the booklets provided. Clearly separate rough work from solutions.
 - All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit.
 - Partial credit will be given only for substantial progress toward a solution.
 - Questions are of equal value.
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Duration: 2 hours

- Question 1.** Given are n fair coins, where the j th coin has value '1' printed on one side and ' $2j - 1$ ' on the other, $j = 1, \dots, n$. All n coins are flipped, and the product of the face-up values is recorded as a score. Determine the expected score, that is, the average score over all possible flips.
- Question 2.** Consider a row of 2017 switches, whose initial states are: off, on, off, on, \dots , off. A legal move consists of changing the state of each of two consecutive switches. Using only a sequence of legal moves, is it possible to arrive at the situation in which the middle switch is on and all others are off?
- Question 3.** A circular arc divides the interior of a circle with radius 1 into two regions of equal areas. Prove that the length of the arc is greater than 2.
- Question 4.** Let $p(x)$ be a polynomial with real coefficients, and suppose $p(n) \geq 0$ for each integer n . Show that $p(x)$ is a sum of polynomials of the form $a(x)a(x+1)$.