UVic Mathematics Competition
September 25, 2018

University of Victoria

- No calculators, books or notes are allowed.
- Write solutions in the booklets provided. Clearly separate rough work from solutions.
- All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit.
- Partial credit will be given only for substantial progress toward a solution.
- Questions are of equal value.

Duration: 2 hours

Question 1. Find, with proof, which positive integers N are a sum of two or more consecutive positive integers.

Question 2. A cat is chasing a mouse. They are represented by points in the plane, starting 10 units apart. The cat runs at 2 units per second, always running directly toward the mouse; the mouse runs at speed 3 units per second, always running at an angle 60° left of the line pointing directly away from the cat as shown in the figure. Show that the cat will catch the mouse, and determine whether the mouse makes a finite number of turns around the cat before capture.

Question 3. Consider a 100×30 grid of (square) boxes, with 100 boxes in each row. Each box is to be filled with a number 1, 2 or 3 in such a way that each pair of horizontally or vertically neighbouring squares receive different numbers. Initially, the bottom row is filled with the numbers $1, 2, 3, 1, 2, 3, \ldots, 1, 2, 3, 1$, in this order, and the top row is filled with the numbers $3, 2, 1, 3, 2, 1, \ldots, 3, 2, 1, 3$, in this order. Is it possible to fill in the remainder of the grid according to the rules?

Question 4. Does there exist a pentagon in the plane with all vertices at integer coordinates and all five side lengths equal?