UVic Mathematics Competition
September 24, 2019

University of Victoria

- No calculators, books or notes are allowed.
- Write solutions in the booklets provided. Clearly separate rough work from solutions.
- All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit.
- Partial credit will be given only for substantial progress toward a solution.
- Questions are of equal value.

Duration: 2 hours

Question 1. The polynomial $f(x) = x^3 - 3x$ has a local maximum at $P = (-1, 2)$ and a local minimum at $Q = (1, -2)$. From the graph of $f(x)$, create a new graph by first deleting the portion between P and Q, and then translating the two remaining pieces by $\overrightarrow{PO}$ and $\overrightarrow{QO}$, so that they join together at the origin. Is the resulting graph that of a polynomial?

Question 2. Find an infinite sequence of sets $A_1, A_2, A_3, \ldots$ such that $|A_n| = n$ for any positive integer n and $|A_m \setminus A_n| = 1$ for any positive integers $m < n$.
(Here, $|A|$ denotes the cardinality of set A and $A \setminus B$ is the set of elements in A but not B.)

Question 3. Is it possible for a rectangle R to contain a rectangle R' (with the edges of R and R' not necessarily parallel) so that the perimeter of R' exceeds that of R?

Question 4. Suppose n points are placed randomly on a circle. Find the probability that the the convex polygon determined by the given points does not include the centre of the circle in its interior.