# UVic Mathematics Competition
## October 1, 2024

University of Victoria

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- No calculators, books or notes are allowed.
- Write solutions in the booklets provided. Clearly separate rough work from solutions.
- All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit.
- Partial credit will be given only for substantial progress toward a solution.
- Questions are of equal value.

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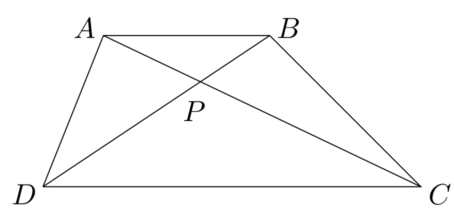
**Duration: 2 hours**

**Question 1.** The trapezoid $ABCD$ has diagonals intersecting at $P$, as shown. Suppose the areas of both $\triangle ABC$ and $\triangle CPD$ are equal to 1. Determine the area of the trapezoid.




**Question 2.** Let $a_0 = 1$ and define $a_{n+1} = 2^{a_n}$ for each $n \geq 0$. Similarly let $b_0 = 1$ and define $b_{n+1} = 2024^{b_n}$ for each $n \geq 0$. Show there exists $k \in \mathbb{N}$ such that $b_n \leq a_{n+k}$ for each $n \in \mathbb{N}$.

**Question 3.** Any three points in space that are not on the same line determine a unique plane passing through them. Show that there is an arrangement of 24 points in space, no three on the same line, which determine exactly 2000 distinct planes.

**Question 4.** A card passing game is played by $n \geq 3$ players seated around a circular table. Initially, the number of cards held by each player is a (possibly different) multiple of three. In each round, the players each simultaneously pass one-third of their cards to the player on their right and one-third of their cards to the player on their left. Then, each player discards either $0, 1$ or $2$ cards so that they again hold a multiple of three many cards.

Show that, after a finite number of rounds, all players will have the same number of cards.