Set Operations and the Laws of Set Theory

- The *union* of sets A and B is the set $A \cup B = \{x : x \in A \vee x \in B\}$.
- The *intersection* of sets A and B is the set $A \cap B = \{x : x \in A \wedge x \in B\}$.

- The *set difference of A and B* is the set $A \setminus B = \{x : x \in A \wedge x \notin B\}$.
  Alternate notation: $A - B$.
- The *symmetric difference of A and B* is $A \oplus B = (A \setminus B) \cup (B \setminus A)$.
  Note: $A \oplus B = \{x : (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$.

The *universe*, $\mathcal{U}$, is the collection of all objects that can occur as elements of the sets under consideration.

- The *complement of A* is $A^c = \mathcal{U} \setminus A = \{x : x \notin A\}$.

For each Law of Logic, there is a corresponding Law of Set Theory.

- Commutative: $A \cup B = B \cup A, \quad A \cap B = B \cap A$.
- Associative: $A \cup (B \cup C) = (A \cup B) \cup C, \quad A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C), \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  and also on the right: $(B \cap C) \cup A = (B \cup A) \cap (C \cup A), \quad (B \cup C) \cap A = (B \cap A) \cup (C \cap A)$
- Double Complement: $(A^c)^c = A$
- DeMorgan's Laws: $(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c$
- Identity: $\emptyset \cup A = A, \quad \mathcal{U} \cap A = A$
- Idempotence: $A \cup A = A, \quad A \cap A = A$
- Dominance: $A \cup \mathcal{U} = \mathcal{U}, \quad A \cap \emptyset = \emptyset$

Arguments that prove logical equivalences can be directly translated into arguments that prove set equalities.

Set equalities of note:

- $A \setminus B = A \cap B^c$
- $A \oplus B = (A \cup B) \setminus (A \cap B)$