Recursive Definitions

Every recursive definition has two parts: a Basis and a Recursion. In the Basis part you define what you want for a few small cases (possibly only one). In the Recursion part you describe how to build the rest of what you want from what you already have.

One classic example is the Fibonacci Numbers $f_0, f_1, f_2, \ldots$ which are defined recursively by

Basis: $f_0 = 1$, and $f_1 = 1$.

Recursion: $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$.

Sets are an object that are sometimes defined recursively. For example, the following defines the set $\{3, 4\} \cup \{6, 7, 8, \ldots\}$

Basis: $3 \in S$, and $4 \in S$.

Recursion: If $n \in S$, then $n + 3 \in S$ and $n + 4 \in S$.

To prove that S is the set described above, use strong induction.