Lecture 3

Keynesian Models

In this lecture, we will analyze Keynesian models. To do this, we will first develop IS-LM and AD-AS models. These models are widely used to analyze macroeconomic issues and policies. We will see that both models are equivalent. They are based on same assumptions about economic behavior and price adjustments, and they give same answers when used to analyze the effects of various shocks. Using these models, we will analyze the Keynesian view of the effects of monetary and real shocks on the macro-economic aggregates.

A. The IS-LM Model

1. The LM Curve

Let us begin with the LM curve. The LM curve gives a relationship between the nominal interest rate (i) and real output/income/expenditure (Y), such that the money market is in equilibrium *i.e.*, the demand for money is equal to the supply of money. In the derivation of the LM curve, it is generally assumed that money and other financial assets are perfect substitutes.

We can divide financial assets in two categories: monetary assets (M) and non-monetary assets (NM). The demand for monetary and non-monetary assets is part of **portfolio-allocation decision**. Agents decide what part of their wealth they should keep in the form of monetary assets and what part as non-monetary assets. This decision is based on the relative desirability of these assets. The most attractive characteristic of money is that it is a medium of exchange and the most liquid asset. By liquidity, we mean that money can be exchanged for goods, services, and other assets quickly and without transaction costs. Non-monetary assets, on the other hand, are relatively illiquid, but they provide some return, which is proxied by the nominal rate of interest, i. Any agent allocates his wealth among these two assets such that he is indifferent between these two types of assets at margin.

The equilibrium in the asset market requires that the demand for financial assets should be equal to the supply of financial assets.

$$M^d + NM^d = M + NM. \tag{1.1}$$

If we define the excess demand for an asset (E^A) as a difference between its demand and supply, then the equilibrium in the asset market requires

$$M^A + NM^A = 0. \tag{1.2}$$

Thus if the money market is in equilibrium, *i.e.*, $M^A = 0$, it must be the case that the market of non-monetary assets is also in equilibrium, $NM^A = 0$. This is the application of the **Walras Law**, which says that if there are n markets and $n-1$ markets are in equilibrium, then n^{th} market is also in equilibrium. Thus, more broadly, the LM curve represents the combination of the nominal interest rate, i, and real output, Y, such that the financial market is in equilibrium.

In the derivation of the LM curve, money supply, M, is taken as given. The demand for money is expressed as a function of price level, P, interest rate, i, and real output Y.

$$M^d = PL(i, Y),\ L_i < 0,\ L_Y > 0 \tag{1.3}$$

where $L(i, y)$ is the demand function for **real money balance**, $\frac{M^d}{P}$, or the liquidity preference. By assumption, the demand for money, M^d, increases one for one with the price level, P. It is also an increasing function of real income Y. Idea is that the demand for money increases with these two variables as the number and the size of transactions become larger (the transaction motive for holding money).

The demand for money declines as the nominal rate of interest rises, i, since it represents the opportunity cost of holding money. Idea is that an agent by holding money is forgoing the opportunity to hold other financial assets, which yield return i. A higher i implies a higher opportunity cost, and thus a lower demand for money, M^d.

The equilibrium in the money market requires that the demand for money, M^d, be equal to the supply of money, M, and thus

$$\frac{M}{P} = L(i, Y). \tag{1.4}$$

Using (1.4), we can derive the LM curve. Given the properties of liquidity preference, one can easily see that the equilibrium in the money market requires a positive relationship between interest rate, i, and real output, Y, i.e., the LM curve is upward sloping in the (Y, i) space. A higher Y implies a higher $L(i, Y)$, other thing remaining the same. Then in order to achieve equilibrium, i should be higher, which pushes down $L(i, y)$.

In order to derive the slope of the LM curve formally, differentiate both sides of (1.4) by Y.

$$0 = L_i \frac{di}{dY} + L_Y. \tag{1.5}$$

Rearranging (1.5), we have

$$\frac{di}{dY} = -\frac{L_Y}{L_i} > 0 \tag{1.6}$$

as $L_i < 0$ and $L_y > 0$. The intuition is as follows: A higher output, Y, leads to a greater demand for money. For a given supply of money and price level, in order to acquire more money agents in the economy try to sell their non-monetary assets, which reduces the demand for these assets and thus their prices. The decline in the prices of non-monetary assets (given their pay-offs) is reflected in the increase in the rate of return of these assets, which is proxied by the interest rate, i.

(1.6) shows that a higher L_Y and a lower L_i make the LM curve steeper. In other words, a relatively small change in income, Y, leads to a larger change in the interest rate, i. Intuitively, with a higher, L_Y, a given increase in income leads to relatively large increase in the demand for real money balance, and thus the interest rate rises relatively more. In the case of low L_i, again for a given increase in Y the interest rate rises relatively more in order to restore equilibrium in the financial/money market.

Let us now turn to the relationship between the LM curve, money supply, and price level. Any increase in the money supply for a given price level will require an increase in the money demand for equilibrium to be achieved in the money market. How the money demand will increase? For a given level of output, Y, and price, P, the interest rate, i, has to fall. Thus, a given level of output, Y, is associated with a lower level of interest rate, i, i.e., the LM curve shifts to the right in (i, Y) space. In the case of a decline in money supply, the LM curve shifts to the left.

Question 1. What is the mechanism for the adjustment in the interest rate, i, following an increase in the money supply, M, given output, Y, and price, P?

Given the above reasoning, one can easily work out the relationship between the LM curve and price, for a given money supply and output. A lower price level, P, effectively increases the supply of real money balance, $\frac{M}{P}$. Thus, the LM curve shifts to the right. A higher price level shifts LM curve to the left.

Question 2. What is the mechanism for the adjustment in the interest rate, i, following an increase in price, P, given output, Y, and money supply, M?

2. The IS Curve

The IS curve traces out the combinations of interest rate, i, and income Y, such that the planned expenditure by agents (optimal expenditure given price, interest rate, and income) in the economy is equal to the actual expenditure. Alternatively, the IS curve gives a relationship between the interest rate, i, and income Y, such that the goods market is in equilibrium.

The planned expenditure, E, is assumed to be the function of income, Y, the real rate of interest, $r \equiv i - \pi^e$, where π^e is the expected inflation rate, the government expenditure, G, and tax, T.

$$E = E(Y, r, G, T), \; 0 < E_Y < 1, \; E_r < 0, \; E_G > 0, \; \& \; E_T < 0. \tag{2.1}$$

A higher real rate of interest, r, and tax, T, reduce investment and consumption. A higher income, Y, and the government expenditure, G, lead to a higher planned expenditure.

One important assumption is here is that increase in income, Y, leads to less than proportionate increase in the planned expenditure, E. This is the basis for the famous

Keynesian multiplier. This leads to the conclusion that a given increase in the planned expenditure (autonomous) results in more than proportionate increase in output, in equilibrium.

The equilibrium requires that the planned expenditure, E, is equal to the actual expenditure/output/ income, Y. Thus,

$$Y = E(Y, r, G, T). \tag{2.2}$$

Question 3 What is the intuition behind the Keynesian multiplier?

From (2.2), one can derive the IS curve, which gives a relationship between the interest rate, i, and income, Y. One can easily see that the IS curve is downward sloping, or there is a negative relationship between i and Y. For a given level of expected inflation, π^e, a higher interest rate, i, implies a higher real rate of interest, $r \equiv i - \pi^e$. A higher real rate of interest, r, leads to a lower consumption and investment and thus lower income, Y.

The above equilibrium condition can be written in the form of equality between savings and investment. In a closed economy,

$$E \equiv C + I + G. \tag{2.3}$$

By definition, savings, S, is

$$S = Y - C - G \tag{2.4}$$

where C is consumption. Thus, equilibrium requires that

$$S = I. \tag{2.5}$$

Using (2.5) as well, one can derive a downward sloping IS curve. Intuitively, higher income, Y, leads to more saving, S. More saving results in the lower real rate of interest, r, and thus lower interest rate, i.

By differentiating (2.2) with respect to Y, we can formally derive the slope of the IS curve.

$$1 = E_Y + E_r \frac{di}{dY}. \tag{2.6}$$

Rearranging, we have

$$\frac{di}{dY} = \frac{1 - E_Y}{E_r} < 0 \tag{2.7}$$

since $0 < E_Y < 1$ and $E_r < 0$. (2.7) shows that higher E_Y and E_r imply flatter IS curve, *i.e.*, a given change in output, Y, leads to relatively smaller change in the interest rate, i.

Intuitively, a higher E_r implies that an increase in i reduces E relatively more and thus output Y must fall relatively more in order to achieve equilibrium. A higher E_Y makes the planned expenditure curve steeper in (E, Y) space. Thus, an increase in the interest rate, i, reduces equilibrium output, Y, relatively more.

Question 4 What is the relationship between the Keynesian multiplier and E_Y?

3. The FE Line

The FE line or full employment line gives a relationship between the interest rate, i, and output, Y, such that the labor market is in equilibrium *i.e.*, the demand for labor is equal to the supply of labor. The demand for labor being equal to supply does not mean that there is no unemployment. Equilibrium in the labor market allows for the possibility of **frictional** and **structural** unemployment.

Frictional unemployment arises as workers search for suitable jobs and firms search for suitable workers. Since workers and jobs are heterogeneous, and there is imperfect information about workers and jobs, there is always some frictional unemployment as matching workers to appropriate jobs in a dynamic economy is a costly and time-consuming process. Structural unemployment arises when the skill set of a section of workers does not match with skill requirements of jobs. This leads to long term unemployment. In any economy, there is always some frictional and structural unemployment. Frictional and structural unemployment rate together constitute, what is known as the **natural rate of unemployment**. When we say labor market is in equilibrium, we mean that the unemployment rate is at its natural rate.

Let us now derive FE line. We will assume that the labor market is competitive. Let the production function be

$$Y = F(L),\ F_L > 0,\ F_{LL} < 0 \tag{3.1}$$

where L is labor input. A profit maximizing firm employs workers till the point where the marginal product of labor equals real wage.

$$F_L(L) = \frac{W}{P} \tag{3.2}$$

where W is the nominal wage. From (3.2), we can derive a downward sloping labor demand function, $L^D(\frac{W}{P})$. The supply of labor is derived from the utility maximization problem of consumer/workers. Let the utility function be

$$u(C, L),\ u_C > 0,\ u_{CC} < 0,\ u_L < 0,\ u_{CL} \leq 0 \tag{3.3}$$

where C is consumption. The budget constraint faced by a consumer/worker is

$$PC = WL. \tag{3.4}$$

Putting (3.4) in (3.3), and taking the derivative with respect to labor supply, L, we have

$$-\frac{u_L}{u_C} = \frac{W}{P}. \tag{3.5}$$

(3.5) equates the marginal rate of substitution between labor and consumption with real wage. From (3.5), one can derive upward sloping labor supply function, $L^S(\frac{W}{P})$.

The labor market equilibrium is achieved, when

$$L^D\left(\frac{W}{P}\right) = L^S\left(\frac{W}{P}\right). \tag{3.6}$$

Let $\overline{L}$ be the equilibrium employment. Then the associated output, $\overline{Y}$, is given by

$$\overline{Y} = F(\overline{L}). \tag{3.7}$$

$\overline{Y}$ is known as the **potential output** or **full employment output**. This is the profit-maximizing output, which is consistent with the natural rate of unemployment.

Since neither the labor demand nor the labor supply depends on the interest rate, i, equilibrium employment also does not depend on it. Thus, the FE line is vertical in (Y, i) space at the potential output, $\overline{Y}$. Anything that shifts labor demand or supply function shifts the FE line as well. Potential candidates are productivity shocks, taxes, change in population, unemployment insurance etc.

The potential output and the natural rate of unemployment also depend on the market structure of goods and labor markets. The potential output and the natural rate of unemployment in the above example are derived under the assumption of competitive labor market. If we assume that the labor market is imperfect, then we will get another level of the potential output and the natural level of unemployment. The classical economists assume competitive goods and labor markets, while the Keynesian economists assume imperfect goods and/or labor markets. Consequently, the classical potential output (natural rate of unemployment) is higher (lower) than the Keynesian potential output (natural rate of unemployment).

Question 5 Draw the FE line in (Y, i) space and analyze how supply shocks, taxes, change in population etc. affect the FE line.

4. General Equilibrium

The general equilibrium is achieved at a point in (Y, i) space at which all the three curves – IS, LM, and FE intersect each other. At this particular combination of interest rate, i, and output, Y, all the three markets – money/financial, goods, and labor are in equilibrium. The general equilibrium point is also the point of the long run equilibrium.

Any shift in any of the curves takes the economy out of general equilibrium. The possibility that an economy at a point in time can be out of equilibrium raises three questions:

1. When the economy is out of general equilibrium, what determines output, interest rate, employment, price and other variables, or what determines the short-run equilibrium?

2. When the economy is out of general equilibrium or the long-run and the short-run equilibrium points do not coincide, what adjustment processes take the economy back to the general equilibrium point?

3. How fast is the adjustment process or how much time does it take the economy to go back to the general equilibrium point?

There is no controversy regarding the answers to first two questions. When the economy is out of general equilibrium, output, interest rate, employment, price and other variables are given by the **intersection** of the **IS and LM curves**. In other words, the short-run equilibrium is given by the intersection of the IS and the LM curves. Thus, employment can be higher or lower than the full employment level temporarily. The long run equilibrium occurs at the point of the intersection between the IS curve and the FE line.

When the economy is out of general equilibrium, prices in all three markets adjust, which brings the economy back to the general equilibrium, such that short-run and long-run equilibrium coincide. If for instance, the intersection of the IS and the LM curve occurs at output level $Y > \overline{Y}$, then price, P, rises which shifts the LM curve up. Price rises because firms are producing more than their long term profit-maximizing level of output. This process goes on till all the three curves intersect each other, and the economy goes back to the general equilibrium point.

Question 6 Draw the IS, the LM, and the FE curves and show a general equilibrium point. What happens if there are shocks to these three curves?

Question 7 How do changes in the money supply and the expected inflation rate affect equilibrium Y and i?

The third question is highly controversial in macroeconomics. It is with respect to this question the **Classical** and the **Keynesian** economists greatly differ in their viewpoints. The classical economists, generally, believe that the process of adjustment is very fast. The economy through its price mechanism adjusts to any shock very rapidly, and thus for all practical purpose one can assume that economy is always in the general equilibrium.

The Keynesian economists believe that the adjustment process is slow due to various types of nominal and real rigidities in the markets, and the economy can be out of general equilibrium for a long periods of time. The other major difference between the Keynesian and the Classical economists is with respect to their assumption about the market structure. The Keynesians believe that markets are characterized by the monopolistic competition. Such market structure along with the nominal and real rigidities slow down the process of adjustment. The Classicals believe that perfect competition is better characterization of the real world markets.

These two contrasting viewpoints have strong policy implications. According to the Classicals, since economy always remains at the general equilibrium point, the government intervention is unnecessary from the point of view of enhancing social welfare and improving the efficiency of allocations. Many believe that the government interventions actually make economy worse off. The Keynesians, on the other hand, believe that the government interventions may improve welfare and the efficiency of allocations.

The speed of adjustment has implications regarding the effects of changes in the money supply on real variables. Money is said to be **neutral** if changes in the money supply has no real effects. Both the Keynesians and the Classicals believe that money is neutral in the long run. But the Keynesians also believe that money is not neutral in the short run, *i.e.*, changes in money supply have real effects in the short run. The Classicals on the other hand believe that money is neutral even in the short run, since the process of adjustment is very fast.

Question 8 Show the effects of changes in the money supply in the IS-LM-FE framework?

B. The AD-AS Model

Having analyzed the IS-LM model, let us now turn to the AD-AS model. We will begin with the derivation of the AD curve.

5. The AD (Aggregate Demand) Curve

AD curve gives relationship between the price level, P, and the desired real expenditure, Y, such that both the goods and the money markets are in equilibrium. In other words, it is given by the intersection of the IS and the LM curves. In order to derive the AD curve all we have to do is to change the level of price P and using the IS and the LM curves find out the equilibrium Y.

Suppose, price level is increased. From (2.2), it is clear that the IS curve is not affected, as it does not affect the planned expenditure. However, it shifts the LM curve to the left (1.4) as the supply of real money balance falls, $\frac{M}{P}$. The result is that output, Y, falls and the interest rate, i, rises. Thus there is a negative relationship between price and output.

The intuition is that a higher price reduces supply of real money balance, which increases the interest rate. A higher interest rate reduces consumption and investment and thus output falls.

To formally derive the slope of the AD curve, take the derivative of (1.4) and (2.2) with respect to price, P. The derivative of (1.4) gives

$$-\frac{M}{P^2} = L_i \frac{di}{dP} + L_Y \frac{dY}{dP}. \tag{5.1}$$

The derivative of (2.2) gives

$$\frac{dY}{dP} = E_Y \frac{dY}{dP} + E_r \frac{di}{dP}. \tag{5.2}$$

Combining (5.1) and (5.2), we have

$$\frac{dY}{dP} = -\frac{M/P^2}{L_Y + \frac{(1-E_Y)L_i}{E_r}} < 0. \tag{5.3}$$

The AD curve is derived under the assumption that the money supply, M, the government expenditure, G, and tax, T, are constant. Any change in these variables shifts the AD curve. For example, an increase in the money supply, M, reduces the interest rate, i, which in turn increases consumption and investment for any given level of price, P. Thus the AD curve shifts up to the right, implying that a given price level is associated with a higher output. In addition, the AD curve becomes steeper (see equation 5.3). Any increase in the expected inflation also shifts the AD curve up and to the right.

Question 9 How do changes in the government expenditure and tax affect the AD curve?

6. The AS (Aggregate Supply) Curve

As discussed earlier, the AD curve gives relationship between price, P, and output, Y. In order to pin down the equilibrium level of price and output, we need an additional equation in price and output. This additional equation is provided by the AS curve. The AS curve gives the relation between price level, P, and the aggregate amount of output, Y, which firms wish to supply.

There are two types of aggregate supply curve – **Long Run Aggregate Supply Curve (LRAS)** and **Short Run Aggregate Supply Curve (SRAS)**. As names suggest LRAS is long run relationship between price, P, and the aggregate supply, Y, while SRAS is the short run relationship between the two.

The LRAS is vertical in (Y, P) space at the level of potential output, $\overline{Y}$, *i.e.*, changes in price has no effect on the aggregate supply in the long run. This is because the potential output is the profit maximizing output in the long run. Unless the profit maximizing employment changes or the FE line shifts there is not going to be any change in the potential output.

The general equilibrium is achieved at the point where the AD curve intersects the LRAS curve. At this combination of (Y, P), all the three markets are in equilibrium and the aggregate planned expenditure is equal to the LRAS. Given that the LRAS curve is vertical at the potential output, one can immediately see that changes in the aggregate demand has no effect on output, employment, and any other real variables in the long run. Any change in the aggregate demand curve is reflected simply in prices or nominal variables.

The shape of the SRAS curve and reasons behind it are subject of controversy between the Keynesians and the Classical economists. This controversy can be traced to their views regarding the speed of adjustment process and the market structure. The shape of the SRAS curve is quite important from both the positive and the normative point of view. For example, if the SRAS curve is vertical just as the LRAS curve, it follows immediately that any shift in the AD curve will not have any effect on real variables. The variations in the aggregate demand will just affect nominal variables. One immediate implication is that money is **neutral** in the short-run. However, if the SRAS curve is non-vertical, then any change in the aggregate demand has real effects in the short-run.

Below, we discuss different Keynesian models which generate different types of SRAS curve.

7. The Keynesian View and the SRAS Curve

We will derive the Keynesian SRAS curve under two different assumptions. First, we will assume that nominal wage is fixed. This model generates an upward sloping SRAS curve. Then we will assume that price is fixed. This model generates a horizontal SRAS curve. In both models, expected and unexpected monetary shocks affect real output.

i. Nominally Rigid Wages, Flexible Prices, And a Imperfect Labor Market

Suppose that nominal wage is fixed at a ceratin level

$$W = \overline{W}. \tag{7.1}$$

Also suppose that $\overline{W}$ is higher than the nominal wage that will prevail in the case of no nominal rigidity *i.e.*, there is involuntary unemployment. Suppose that production function is

$$Y = F(l),\ F_l > 0 \ \&\ F_{ll} < 0. \tag{7.2}$$

Profit maximizing firm will employ workers such that

$$F_l(l) = \frac{\overline{W}}{P}. \tag{7.3}$$

(7.3) equates the marginal product of labor to the real wage. In this case, any increase in the aggregate demand increases price and lowers the real wage. This induces firms to hire more workers and output goes up. This implies upward sloping short-run aggregate supply curve. Thus, employment and output are pro-cyclical. However, the model predicts anti-cyclical real wage, which is not supported by data. Data suggests that real wage is either acyclical or mildly pro-cyclical.

ii. Rigid Prices, Flexible Wages, and a Competitive Labor Market

Suppose instead that price is rigid over certain range of supply but wage is flexible.

$$P = \overline{P},\ \forall\ Y \leq Y_{max}. \tag{7.4}$$

(7.4) basically assumes that firms are willing to supply as much output as demanded as long as the aggregate demand is less than Y_{max}. In other words, the short-run aggregate supply curve is horizontal over range of output $Y \in [0, Y_{max}]$.

Y_{max} is the level of output at which price, $\overline{P}$, equals the marginal cost of production, $mc(W)$. The marginal cost of production depends on wage, W. So for any $Y < Y_{max}$, price, $\overline{P}$, exceeds the marginal cost. Thus it is profitable for the firm to meet all the demand as long as $Y < Y_{max}$. Notice that the level of Y_{max} generally depends on price, P, and wage, W. A lower price and a higher wage will reduce the level of Y_{max} as the first reduces the marginal revenue and the second increases the marginal cost of production.

These assumptions have implication for the shape of the labor demand curve. It is no longer just given by (7.3). As long as $Y < Y_{max}$, the demand for labor is given by

$$l = F^{-1}(Y), \tag{7.5}$$

since the firm meets all the demand and employs just enough workers to produce, Y. For example, if $Y = l^\alpha$ then $l = Y^{\frac{1}{\alpha}}$. If demand exceeds $Y > Y_{max}$, then the demand for labor is given by (7.3). Such labor demand is called the **effective labor demand**. Labor demand depends on the quantity of goods firm is able to sell. The equilibrium real wage is given by the intersection of the labor supply curve and the effective labor demand.

In this model, any decline in the aggregate demand leads to lower output, and thus lower employment, and lower real wage. This model implies a pro-cyclical real wage and an anti-cyclical mark-up (the ratio of price to the marginal cost). A fall in demand reduces the marginal cost both because real wage falls and the marginal product of labor rises.

Having discussed the aggregate supply curves, we now turn to the discussion of the Phillips curve. The shape of Phillips curve is closely related to the shape of aggregate supply curve.

8. The Phillips Curve

The Phillips curve refers to the relationship between output/unemployment rate and the inflation rate. The exact shape of the Phillips curve is a subject of controversy. Analogous to the two types of aggregate supply curves, two types of Phillips curves are distinguished – long run and short run. The general consensus is that the long run Phillips curve is vertical in (Y, π) space. In other words, there is no long run trade-off between output/unemployment rate and the inflation rate. It simply reflects the fact that the LRAS curve is vertical at the potential output and changes in the money supply do not affect it.

The short run Phillips curve is either vertical or upward sloping depending on assumptions regarding the market structure (perfect or imperfect competition), the presence or absence of rigidities, perfect or imperfect information regarding prices. The shape of short-run Phillips curve depends on the shape of the SRAS curve. If the SRAS curve is vertical, then the short run Phillips curve is vertical too. If the SRAS curve is upward sloping or horizontal, then the short run Phillips curve is upward sloping in the (Y, π) space or downward sloping in the unemployment-inflation space.

i. The Traditional Phillips Curve

We will work with the model of nominally rigid wage. Suppose that

$$Y = F(L) = \frac{1}{\alpha} L^{\alpha},\ 0 < \alpha < 1. \tag{8.1}$$

Wage is rigid at $W = \overline{W}$. Labor employed is given by

$$l = \left(\frac{\overline{W}}{P}\right)^{\frac{1}{\alpha - 1}}. \tag{8.2}$$

Putting it in (8.1), we have

$$Y = \frac{1}{\alpha}\left(\frac{\overline{W}}{P}\right)^{\frac{\alpha}{(\alpha - 1)}}. \tag{8.3}$$

Let the lower case denote the log of a variable (e.g. $y = \ln Y$). Then, taking log we have

$$y = Const + \frac{\alpha}{(\alpha - 1)}(\overline{w} - p). \tag{8.4}$$

Now suppose that $\overline{W}$ is equal to last period price, P_{t-1}. Then (8.4) can be written as

$$y_t = Const + \frac{\alpha}{(1 - \alpha)}(p_t - p_{t-1}). \tag{8.5}$$

which gives an expression for the traditional Phillips curve

$$y_t = Const + \frac{\alpha}{(1 - \alpha)}\pi_t \tag{8.6}$$

where π_t is the inflation rate. (8.6) suggests that there is a permanent trade-off between output and inflation. By increasing inflation, policy makers can permanently raise output level.

This conclusion crucially depends on the assumption regarding how firms/workers adjust the nominal wage. Here by assumption, current period wage is indexed to the last period price level. Workers/firms do not take into account the fact that the government is following a permanent expansionary policy, which leads to lower real wage. However, sucha behavior is not rational. For instance, in this example if wage is indexed to the expected current price, P_t, then from (8.6), one can immediately see that $y_t = 0$ and it is independent of the inflation rate (or output is at its natural rate).

ii. The Expectation Augmented Phillips Curve

The idea that firms/workers take into account any permanent or expected inflation rate while fixing wages and prices, leads to the concept of the expectation-augmented Phillips curve. Let π_t^* be the core or underlying inflation (or inflation which is consistent with the natural rate of unemployment). A typical modern Phillips curve is

$$\pi_t = \pi_t^* + \lambda(y_t - \overline{y}_t) + \xi_t^S \tag{8.7}$$

where $\overline{y}_t$ is the potential output and ξ_t^S is the supply shock. (8.7) suggests that there is a trade-off only when the actual inflation deviates from the core inflation.

What should be the core inflation, π_t^*? This is subject of controversy and has strong policy implications. In the case of the classical economists, the core inflation is simply equal to the expected inflation π_t^e. In this case, (8.7) becomes

$$\pi_t - \pi_t^e = \lambda(y_t - \overline{y}_t) + \xi_t^S. \tag{8.8}$$

Only a higher unexpected inflation leads to a higher output. In this case, there is a trade-off between the unexpected inflation and output, but that is something which the policy makers cannot exploit. Of course, when $\pi_t = \pi_t^e$, then the deviation of the actual output from the potential output occurs only due to the supply shocks. The neo-classical theorists believe that for all practical purpose this is the case.

Modern Keynesian economists believe that the core inflation depends on the expected inflation as well as the past inflation, *i.e.*,

$$\pi_t^* = \phi\pi_t^e + (1 - \phi)\pi_{t-1}. \tag{8.9}$$

Idea is that there is some inertia in wage and price inflation. There is some link between the past and the future inflation beyond the effects operating through expectation. Putting (8.9) in (8.7), we have

$$\pi_t = \phi\pi_t^e + (1-\phi)\pi_{t-1} + \lambda(y_t - \overline{y}_t) + \xi_t^S. \tag{8.10}$$

(8.10) can be written as

$$\pi_t - \pi_{t-1} = \phi(\pi_t^e - \pi_{t-1}) + \lambda(y_t - \overline{y}_t) + \xi_t^S. \tag{8.11}$$

From (8.11), one can immediately see that for a given past inflation rate, any increase in the current inflation rate raises current output. Thus, there is a exploitable trade-off between inflation and output.

A special case arises when $\phi = 0$. Then we have the case of adaptive expectation. Individuals form expectation about the future inflation by extrapolating the past trend. In this case,

$$\pi_t - \pi_{t-1} = \lambda(y_t - \overline{y}_t) + \xi_t^S. \tag{8.12}$$

Now the government can raise output above the natural rate, if it is willing to tolerate ever increasing inflation rate. This is the famous **Friedman-Phelps Phillips curve**. In this case, there is a trade-off between change in inflation and output.

iii. The New Keynesian Phillips Curve

Till early nineties, the standard Keynesian models used for most monetary analysis combined the assumption of nominal rigidity with a simple structure linking the quantity of money to the aggregate spending. The quantity of money is linked to the aggregate spending either through the quantity theory of money or the traditional text-book IS-LM model, which we discussed in the previous sections.

Now the standard approach is to build a dynamic, stochastic, general equilibrium model based on the optimizing behavior of agents, combined with some kind of nominal wage and/or price rigidity. Also it is assumed that there is monopolistic competition in the product market and/or monopsonistic competition in the labor market and expectations are rational. These assumptions typically yield relationship between the current inflation rate, the expected future inflation rate, and the real marginal cost, which is known as the **New Keynesian Phillips Curve**.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \tag{8.13}$$

where mc_t is the deviation of real marginal cost from its steady state value and κ is a constant. (8.13) is an example of the **stochastic difference equation**.

The new Keynesian Phillips curve implies that the real marginal cost is the driving variable for the inflation process and also it is forward-looking, with current inflation a function of the expected future inflation. In empirical work, mc_t is generally proxied by the deviation of output from its potential level.

$$mc_t = \gamma(y_t - \overline{y}_t). \tag{8.14}$$

With this modification, the New Keynesian Phillips curve becomes

$$\pi_t = \beta E_t \pi_{t+1} + A(y_t - \overline{y}_t) \tag{8.15}$$

where A is some constant.

For estimational purpose, (8.15) can be written as follows

$$\pi_{t+1} = \frac{1}{\beta}\pi_t + \frac{A}{\beta}(y_t - \overline{y}_t) + \xi_{t+1} \tag{8.16}$$

where ξ_{t+1} is the error term. By assuming that the potential output $\overline{y}_t$ is constant, one can estimate the following regression:

$$\pi_{t+1} = Const \ + \ a\pi_t + by_t + \xi_{t+1}. \tag{8.17}$$