Lecture 2

Models With Nominal Rigidities

In this lecture, we will study Keynesian models and their implications. We will assume that prices/wages are sticky in the short run. We will study two types of models time-dependent models and state-dependent models. Before, we do so we need some methodological tools. Below we study how to solve stochastic difference equations such as New Keynesian Phillips curve.

1. Expectations

First, we very briefly review some rules of expectation/forecast. Many economic behavioral relationships require expectations or forecasts of future values of variables (endogenous/exogenous or both). We just saw one example of it in the previous lecture - the New Keynesian Phillips curve: current inflation depends on future inflation and real marginal cost. Similarly, when we buy share we take into account future dividend and capital gains. Decision to make an investment depends on expected future return. Thus, we frequently encounter behavioral relationships in economics which involve forward-looking elements.

Of course, future values of variables are mostly unobservable. Then in order to make forecasts, we need some rules. First assumption that we make is that agents have rational expectation.

What does rational expectation mean? It basically means that agents make optimal forecasts given the information and they know the true structure of economy and true behavioral relationships, i.e., they know how economy works. They make forecasts as statisticians do.

Let us take an example. Suppose that random variable, $y_t$, evolves over time as follows:

$$y_t = \rho y_{t-1} + \xi_t$$

(1.1)
where $\xi_t$ is an i.i.d. shock with mean 0 and variance $\sigma^2$. First of all under rational expectation, agents are assumed to know true $\rho$ as well as the probability distribution of random shock (that is what we mean by agents knowing the structure of economy or they know true values of structural parameters). Now, what is the optimal forecast of $y_t$ at time $t$. It is simply

$$E_t(y_t) = E_t(\rho y_{t-1} + \xi_t) = \rho y_{t-1}$$  \hspace{1cm} (1.2)

At time $t$, an agent is taking into account all the relevant information available to him, i.e., his knowledge of $y_{t-1}, \rho, \xi_t$, and the mean and variance of the random shock.

How much is the forecast error?

$$y_t - E_t(y_t) = \rho y_{t-1} + \xi_t - \rho y_{t-1} = \xi_t.$$  \hspace{1cm} (1.3)

As we can see that forecast error has zero mean. Also forecast error is uncorrelated to any other information, i.e., $E(\xi_t \ast y_{t-1}) = 0$. Non-zero mean of forecast error and correlated error would imply that agents are ignoring some systematic information which can be used to improve the forecast of $E_t(y_t)$. In other words, (1.1) does not correctly characterize the true path of $y_t$ and thus the forecast would not be optimal.

How do agents revise their forecast over time?

Suppose that at time $t$, agents make forecast of $y_{t+1}$, $E_t(y_{t+1})$. At time $t + 1$, more information becomes available and in the light of new information, they again make forecast of $y_{t+1}$, $E_{t+1}(y_{t+1})$. What is the difference between $E_t(y_{t+1})$ and $E_{t+1}(y_{t+1})$.

$$E_{t+1}(y_{t+1}) - E_t(y_{t+1}) = \rho y_t - \rho y_{t+1} \xi_{t+1}.$$  \hspace{1cm} (1.4)

This implies that

$$E_{t+1}(y_{t+1}) - E_t(y_{t+1}) = \rho y_t - \rho E_t(y_t).$$

Further

$$E_{t+1}(y_{t+1}) - E_t(y_{t+1}) = \rho y_t - \rho E_t(y_{t-1} + \xi_t)$$
which implies

\[ E_{t+1}(y_{t+1}) - E_t(y_{t+1}) = \rho y_t - \rho^2 y_{t-1}. \]

From the above equation, we get

\[ E_{t+1}(y_{t+1}) - E_t(y_{t+1}) = \rho \xi_t. \] (1.4)

Finally, how do we make forecast of forecasts? Here we use law of iterated expectation.

\[ E_t(E_{t+1}(y_{t+1})) = E_t(y_{t+1}). \] (1.5)

These rules are not special to first order autoregressive processes but applicable to more general processes such as

\[ y_t = \rho_0 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \beta_1 z_t + \beta_2 z_{t-1} + \nu_t. \] (1.6)

We will use these rules in this lecture and other lectures quite extensively.

2. Solving Rational Expectation Models

a. Expectations of Exogenous Variables

Suppose that \( x \) is an exogenous variable and \( y \) is endogenous. \( y \) depends on expected future value of \( x \). More specifically,

\[ y_t = \lambda E_t(x_{t+1}). \] (2.1)

Notice that in order to find out \( y_t \), we need future values of \( x \) which are normally unavailable or unobserved at time \( t \). In order to know the value of \( y_t \), we need to express (2.1) in terms of observable variables at time \( t \). To do this, we need some forecasting rule for \( x_t \). Suppose that \( x \) evolves over time as follows:

\[ x_t = \rho x_{t-1} + \xi_t. \] (2.2)
Using (2.2), we can replace $E_t(x_{t+1})$ by $\rho^2 x_{t-1}$. Thus

\[ y_t = \lambda \rho^2 x_{t-1}. \tag{2.3} \]

(2.3) involves variables which are currently observed. In order to derive (2.3), we need both (2.1) and (2.2). Equations such as (2.3) are known as reduced form equations as endogenous variables are expressed solely in terms of exogenous variables. Equations such as (2.1) and (2.2) are known as structural equations. These equations represent structure of the economy and endogenous variables are functions of both endogenous and exogenous variables.

For solving models which include expectation of exogenous variables, all we have to do is to replace the expectation by the statistical forecast.

**Question 1** How can you identify structural parameters from the estimates of reduced form parameters?

**b. Expectations of Endogenous Variable**

Many economic behavioral relations involve expectation about future endogenous variable. The price of a share today depends on its future price. The current inflation rate may depend on the future inflation rate.

\[ y_t = \beta E_t y_{t+1} + \phi x_t \tag{2.4} \]

with $|\beta| < 1$ and $\phi > 0$. I will briefly discuss two methods to solve it.

**Method I: Undetermined Coefficients**

In this method, we guess a form of the solution and then solve for its coefficients. Suppose the guess is

\[ y_t = k x_t \tag{2.5} \]
where $k$ is some guessed constant. Essentially, we have to find the value of this constant, which will satisfy (2.4). To begin with we will assume that our guess is correct. Then (2.2) and (2.5) imply that

$$E_t y_{t+1} = k E_t x_{t+1} = k \rho x_t.$$  

(2.6)

Putting (2.6) in (2.4), we have

$$y_t = (\beta k \rho + \phi) x_t.$$  

(2.7)

Now compare (2.7) with (2.5). One can immediately see that for our guess to be correct, it must be the case that

$$k = \beta k \rho + \phi.$$  

(2.8)

From (2.8), we have

$$k = \frac{\phi}{1 - \beta \rho}.$$  

(2.9)

Thus the reduced form solution for (2.4) is

$$y_t = \frac{\phi}{1 - \beta \rho} x_t.$$  

(2.10)

The method is quite simple. Only hard part is how to make a guess. Three things need to be sorted out – (i) the functional form, (ii) what variables to include, and (iii) the lag length. Choice of functional form is easy. The structural equation is linear and thus the guessed function should also be linear. Regarding the number of variables to include, we should include all the exogenous variables of the structural equation. Finally, the choice of lag length depends on number of lags included in the laws of motion of exogenous variables. One should make the lag length in the guess one less than in the law of motion of the exogenous variables. In the above example, the law of motion was $x_{t+1} = \rho x_t + \xi_{t+1}$ and thus we needed to include only $x_t$ in our guess.
Method II: Repeated Substitution

This method is equally simple. In order to solve

\[ y_t = \beta E_t y_{t+1} + \phi x_t \]  \hfill (2.11)

repeatedly substitute for \( y_{t+i} \) \( \forall i = 1, \ldots, \infty \). In the first step, we get

\[ y_t = \phi x_t + \beta E_t \phi x_{t+1} + \beta^2 E_t y_{t+2}. \]  \hfill (2.12)

We can follow through these steps and derive

\[ y_t = \phi \sum_{i=0}^{n} \beta^i E_t x_{t+i} + \beta^n E_t y_{t+n}. \]  \hfill (2.13)

Now we make a crucial assumption that \( \lim_{n \to \infty} \beta^n E_t y_{t+n} = 0 \). This assumption is known as transversality condition and rules out explosive paths or bubbles. The condition basically says that increase in the expectation of \( y_t \) should not be too fast. If the expectation is rising too fast, then the path of \( y_t \) will not be convergent. We will see below that if the transversality condition is not satisfied then (2.11) has infinite number of solutions.

Once we impose transversality condition then we get

\[ y_t = \phi \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}. \]  \hfill (2.14)

Using the law of motion for \( x_t \) to replace expectation terms in (2.14), we have

\[ y_t = \frac{\phi}{1 - \beta \rho} x_t. \]  \hfill (2.15)

Here you can see the role played by our assumption \( |\beta| < 1 \). It is necessary condition for \( \lim_{n \to \infty} \beta^n E_t y_{t+n+1} = 0 \), for any \( E_t y_{t+n+1} > 0 \). Returning to (2.11), ignoring uncertainties and expectation, the condition \( |\beta| < 1 \) can be stated as the condition that the difference equation that gives \( y_{t+1} \) as a function of \( y_t \) \((y_{t+1} = \frac{1}{\beta} y_t - \frac{\phi}{\beta} x_t)\) should be unstable or have root \( \left| \frac{1}{\beta} \right| > 1 \). In this case, for a given sequence of \( x \), there is a unique value of \( y^* \) for which \( y \) does not explode.
c. Bubbles

In the previous example, we guessed a particular form of solution \( y_t = k x_t \) and found the reduced form. Then we saw that we get the same solution if the transversality condition is satisfied. However, \( y_t = k x_t \) is just one of the solutions of (2.11). Suppose, we guess that

\[
y_t = k x_t + a \left( \frac{1}{\beta} \right)^t.
\]

(2.16)

The guess has two parts – fundamental solution and a term involving inverse of the coefficient on \( y_{t+1} \). Then repeating the steps of method I, you can easily show that

\[
y_t = (\beta k \rho + \phi) x_t + a \left( \frac{1}{\beta} \right)^t.
\]

(2.17)

Then comparing (2.16) with (2.17), we have

\[
k = \frac{\phi}{1 - \beta \rho}
\]

(2.18)

exactly as before.

Additional terms like \( a \left( \frac{1}{\beta} \right)^t \) are known as bubbles. Since \( |\beta| < 1 \), this term rises over time without bound. By varying \( a \), we can get generate any number of bubble paths.

To clarify these things, let us suppose that \( y_t \) is the price of an asset and \( x_t \) is the dividend stream. Then (2.17) suggests that the price of the asset depends on not only its dividend stream (called fundamentals), but also on something extraneous \( a \left( \frac{1}{\beta} \right)^t \). Along the bubble path, you hold an asset because you expect to sell it to someone at higher price, who expect to sell it somebody else and so on. Such bubble path is consistent with the rational expectation. Transversality condition rules out such bubble path. By imposing this condition, we are picking up only fundamental solution.

d. Second Order Stochastic Difference Equation

So far we have learned to solve first order stochastic difference equation with rational expectation. Now we will solve for second order stochastic difference equation.
Let the equation be

\[ y_t = \lambda_1 y_{t-1} + \lambda_2 E_t y_{t+1} + \lambda_3 x_t. \]  

(2.19)

We will use the method of undetermined coefficient. Let the guessed form of solution be

\[ y_t = \alpha_1 y_{t-1} + \alpha_2 x_t. \]  

(2.20)

Then

\[ E_t y_{t+1} = \alpha_1 y_t + \alpha_2 \rho x_t. \]  

(2.21)

Putting (2.21) in (2.19) and simplifying we have

\[ y_t = \frac{\lambda_1}{1 - \alpha_1 \lambda_2} y_{t-1} + \frac{\rho \lambda_2 \alpha_2 + \lambda_3}{1 - \alpha_1 \lambda_2} x_t. \]  

(2.22)

Comparing (2.22) and (2.20), we see that

\[ \alpha_1 = \frac{\lambda_1}{1 - \alpha_1 \lambda_2} \]  

(2.23)

which is a quadratic equation in \( \alpha_1 \) and

\[ \alpha_2 = \frac{\rho \lambda_2 \alpha_2 + \lambda_3}{1 - \alpha_1 \lambda_2} \]  

(2.24)

(2.24) simplifies to

\[ \alpha_2 = \frac{\lambda_3}{1 - (\rho + \alpha_1) \lambda_2}. \]  

(2.25)

(2.23) can be written as

\[ \lambda_2 \alpha_1^2 - \alpha_1 + \lambda_1 = 0. \]  

(2.26)

It has two solutions or roots given by

\[ \frac{1 \pm \sqrt{1 - 4\lambda_1 \lambda_2}}{2 \lambda_2}. \]  

(2.27)
Now the question is which solution we should pick up. Normally parameters of the model is specified in such a way that one root is less than one and another root is strictly greater than one in absolute value. In this case, the model is said to possess the saddle point property. The path of $y_t$ associated with the root smaller than one in absolute value will be convergent path, while the path associated with the other root will be exploding. We are interested in the convergent path and so we pick up the root with the smaller absolute value.

Now we are in position to analyze models with nominal rigidities. Before, we do so, we will study one of the major building blocks of modern Keynesian models.

3. Monopolistic Competition and the Aggregate Demand

As pointed out in lecture 1, Keynesian models differ from classical models not only with regard to their assumption about the speed for price adjustment, but also with regard to the structure of markets. Classicals believe in competitive markets, while Keynesians believe that monopolistic competition is a better characterization of real world markets. In this section, we will analyze the implications of monopolistic competition with regard to the effects of aggregate demand using simplified version of the model developed by Blanchard and Kiyotaki (1987). It is one of the major building blocks of modern Keynesian models.

Economy

Economy here is modified version of the economy we studied in section 7, lecture 1. Basic difference is that now the product market is characterized by monopolistic competition. Suppose that economy is populated by a large number of households. Each household consists of two agents – a producer and a worker. Each household produces a distinct good and it is the only producer of that particular good in the economy. It enjoys monopoly power in that good and sets it prices.

Suppose that the household’s production function is

$$Y_i = L_i$$

(3.1)
where $L_i$ is the amount of labor input. As opposed to the goods market, the labor market is assumed to be competitive (though this assumption is not essential for results).

Let us suppose that demand for any good $i$ depends on: real income and the good’s relative price. Assume that demand is log-linear. Specifically, the demand for good, $i$, is

$$y_i = y - \eta(p_i - p), \; \eta > 0$$

(3.2)

where $y$ is log of real income and $\eta$ is the elasticity of demand for each good. $y_i$ is the demand per producer of good $i$. $y_i$ is assumed to be equal to the average demand across all goods, and $p$ is the average of $p_i$’s:

$$y = \bar{y}_i$$

(3.3)

and

$$p = \bar{p}_i.$$

(3.4)

Suppose that the aggregate demand side of the model is

$$y = m - p.$$  

(3.5)

(3.5) says that aggregate demand is equal to aggregate real money balance. As discussed earlier, it captures the idea that aggregate demand is negatively related to price and positively related to money supply.

The utility function for the $ith$ household is

$$U_i = C_i - \frac{1}{\gamma}L_i^\gamma, \; \gamma > 1.$$  

(3.6)

The budget constraint faced by the consumer is simply

$$C_i = \frac{(P_i - W)Y_i + WL_i}{P}$$

(3.7)

where $P$ is the average price level. The right hand side of (3.7) consists of two terms - profit and wage receipts. An individual household chooses the price of its good, $P_i$ and its labor supply, $L_i$, in order to maximize (3.6).
Household Behavior

Putting (3.1), (3.2), and (3.7) in (3.6), we have

$$\max_{P_i, L_i} (P_i - W)Y \left( \frac{P_i}{P} \right)^{-\eta} + WL_i - \frac{1}{\gamma} L_i^\gamma.$$ \hfill (3.8)

The first order conditions yield

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{W}{P}$$ \hfill (3.9)

and

$$L_i = \left( \frac{W}{P} \right)^{\frac{1}{\gamma - 1}}.$$ \hfill (3.10)

(3.9) shows that producers set prices as a markup over their marginal cost, which is equal to real wage. The size of markup $\frac{\eta}{\eta - 1}$ depends on the elasticity of demand $\eta$. More elastic the demand (higher $\eta$) less will be the markup. In the limiting case where $\eta \to \infty$ (perfectly competitive market), the markup disappears ($\frac{\eta}{\eta - 1} \to 1$).

(3.10) shows that labor supply is an increasing function of real wage with elasticity $\frac{1}{\gamma - 1}$.

Equilibrium

Given symmetry in the model (i.e. all households are in similar situation), in equilibrium, every household sets the same price, supplies same amount of labor, and thus produces the same quantity. Thus, $Y_i = Y$ for all $i$. Given $L_i = Y$, from (3.1), we have

$$\frac{W}{P} = Y^{\gamma - 1}.$$ \hfill (3.11)

Putting (3.11) in (3.9), we have

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} Y^{\gamma - 1}.$$ \hfill (3.12)

(3.12) shows that the relative price is increasing in output and the elasticity is $\gamma - 1$ which is inverse of the elasticity of labor supply $\frac{1}{\gamma - 1}$. It implies that lower the elasticity...
of labor supply, higher is the increase in relative price for a given change in aggregate demand/output. The reason is that lower elasticity of labor supply implies that real wage has to increase relatively more in order to produce a given increase in output, which increases the marginal cost of production relatively more.

Again given symmetry, since every household sets the same price, the relative price is equal to one i.e.

$$P_i = P, \forall i.$$  \hfill (3.13)

Putting (3.13) in (3.12), we get expression for equilibrium output, \(Y\),

$$Y = \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma - 1}}. \hfill (3.14)$$

(3.14) shows that equilibrium output is fixed and is completely independent of changes in money supply, \(m\). Thus, mere presence of monopolistic competition does not make money non-neutral. However, equilibrium output in this case is less than what it would be under perfect competition. In order to derive output under perfect competition just let \(\eta \to \infty\), then from (3.14) one can immediately see that output will be unity.

Why output under monopolistic competition is less than that in perfect competition? The reason is that real wage of workers in this economy is less than their marginal product. Given the linear production function, the marginal product of labor is unity, but the real wage here is (see 3.9)

$$\frac{W}{P} = \frac{\eta - 1}{\eta} < 1. \hfill (3.15)$$

The result is that labor supply and thus output in this economy is less than that of perfect competition.

Finally, using the aggregate demand equation, \(Y = \frac{M}{P}\), and (3.12), we get expression for equilibrium price level, \(P\),

$$P = \frac{M}{\left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma - 1}}}. \hfill (3.15)$$
(3.15) shows that money is neutral in this model. Any change in money supply is reflected just in prices.

Can money be made non-neutral in this model? Blanchard and Kiyotaki (1987) show that if we introduce small menu-cost (sellers incur some fixed cost in changing prices), then money becomes non-neutral. In case of small changes in money supply, sellers do not change their prices. The reason is that profit forgone by not changing the price is more than offset by the cost of changing price because of the presence of menu cost.

The model also tells us what is the optimal price setting rule under monopolistic competition. Denote the optimal price by $P^*_i$, then (3.12) implies that

$$\frac{P^*_i}{P} = \frac{\eta}{\eta - 1} Y^{\gamma - 1}.$$  (3.16)

Taking log and using the fact that $y = m - p$, we have

$$p^*_i = k + (1 - \phi)p + \phi m$$  (3.17)

where $k = \ln \frac{\eta}{\eta - 1}$ and $\phi = \gamma - 1$. Recall that the elasticity of labor supply with respect to real wage is $\frac{1}{\gamma - 1}$ (see 3.10). Thus, $\phi$ is the inverse of the elasticity of labor supply. (3.17) shows that when the elasticity of labor supply is low, relative price is more sensitive to the money supply. We will use the optimal price setting rule given in (3.17) extensively in following sections. For convenience, we will normalize $k = 0$.

### 4 Models With Nominal Rigidities

Now, we turn to analyze the implications of nominal rigidities. In particular, we will work out the dynamics of real output and price level. We will see that results crucially depend on the forms of nominal rigidities assumed. Two distinctions are important in this case.

The first is between **time and state-dependent price rules**. Under time-dependent rules, the price is changed as a function of time. Under state-dependent rules, it is changed as a function of state or underlying conditions of the economy. Examples of both types of pricing rule are easily observed in real life. Contract wages are normally changed at fixed
intervals of time, when contracts expire (e.g. negotiation between CAW and automobile companies). The length of time between changes for most list prices appears to be largely random and presumably a function of state.

The second distinction, within the class of time-dependent rules, is between those rules that predetermine the path of prices and those which fix the price for a given length of time. Under pre-determined pricing rule, sellers specify at time $t$ prices for $t$ to $t+i$ for some $i > 1$. Under fixed price rule, prices are not only predetermined but also constant over a given length of time.

We will first study models with time-dependent pricing rules and then models with state-dependent pricing rules. Among these two types of models, time-dependent models are more widely used mainly because of their tractability.

A. Time-Dependent Models

Fisher Model

Fisher model is an example of pricing rule in which prices are predetermined. In the Blanchard and Kiyotaki model, price setters can change their prices any time they like. Suppose that it is not the case. More specifically suppose that producers set their prices for next two periods. Also change in the price is not synchronized. Suppose that half the producers change their prices in odd periods and half in even periods. Denote period $t$ price set by a producer at time $t-1$ by $p_{1t}^1$. Similarly, denote period $t$ price set by a producer at time $t-2$ as $p_{1t}^2$. The average price at time $t$ then is given by

$$p_t = \frac{p_{1t}^1 + p_{1t}^2}{2}. \quad (4.1)$$

Suppose that producers can choose different prices for two periods. The desired price of a producer $i$ in period $t$ is

$$p_{it}^* = (1-\phi)p_t + \phi m_t. \quad (4.2)$$
At time $t - 1$, producers do not know the actual money supply at time $t$ and $t + 1$. Thus, they set their prices equal to the expected desired prices. This implies

$$p_{t1} = E_{t-1}p_{t1}^*$$  \hspace{1cm} (4.3)

and

$$p_{t2} = E_{t-2}p_{t2}^*.$$  \hspace{1cm} (4.4)

Putting (4.2) in (4.3), we have

$$p_{t1} = \frac{2\phi}{1 + \phi} E_{t-1}m_t + \frac{1 - \phi}{1 + \phi} p_{t2}.$$  \hspace{1cm} (4.5)

Taking expectation of both sides, we have

$$E_{t-2}p_{t1} = \frac{2\phi}{1 + \phi} E_{t-2}m_t + \frac{1 - \phi}{1 + \phi} p_{t2}.$$  \hspace{1cm} (4.6)

Putting (4.2) and (4.6) in (4.4), we have

$$p_{t2} = E_{t-2}m_t.$$  \hspace{1cm} (4.7)

(4.5) and (4.7) imply that

$$p_{t1} = E_{t-2}m_t + \frac{2\phi}{1 + \phi} (E_{t-1}m_t - E_{t-2}m_t).$$  \hspace{1cm} (4.8)

Now we have got expressions for $p_{t1}^1$ and $p_{t2}^2$, then by using the definition of average price, $p_t$, and the aggregate demand condition $y_t = m_t - p_t$, we can get expressions for $p_t$ and $y_t$ solely in terms of exogenous variables. They are

$$p_t = E_{t-2}m_t + \frac{\phi}{1 + \phi} (E_{t-1}m_t - E_{t-2}m_t)$$  \hspace{1cm} (4.9)

and

$$y_t = (m_t - E_{t-1}m_t) + \frac{1}{1 + \phi} (E_{t-1}m_t - E_{t-2}m_t).$$  \hspace{1cm} (4.10)
The first term on the right hand side of (4.10) is the unexpected change in money supply. Thus in this model as well any unexpected change in money supply has real effect just as in the case of Lucas imperfect-information model. However, there is an additional term in the RHS, which shows that any expected or known change in money supply between periods \( t - 1 \) and \( t - 2 \) also affect real output, which is unlike the Lucas model. This happens because half the producers in this model set their price in time \( t - 2 \). Only the other half of the producers who set their price in time \( t - 1 \) can react to any information becoming available between time \( t - 1 \) and \( t - 2 \). In other words for producers setting prices in time \( t - 2 \), \((E_{t-1}m_t - E_{t-2}m_t)\), is unexpected and thus it has real effect.

The fact that information available between period \( t - 1 \) and \( t - 2 \) has real effect on output at time \( t \) opens the possibility of policy intervention. The central bank by following appropriate monetary policy can reduce fluctuations (or variance) in output. To illustrate this idea suppose that the actual money supply \( m_t \) is given by

\[
m_t = m_t^* + v_t
\]

where \( m_t^* \) is the money supply chosen by the central bank and \( v_t \) is some non-policy velocity shock. Suppose that \( v_t \) follows random walk

\[
v_t = v_{t-1} + \xi_t
\]

where \( \xi_t \) is mean zero \( i.i.d \) shock. Essentially we are looking for money supply rule or choice of \( m_t^* \) which reduces the variance of \( y_t \). Now consider following money supply rule

\[
m_t^* = a_1 \xi_{t-1} + a_2 \xi_{t-2}.
\]

Basically we are looking for values of \( a_1 \) and \( a_2 \) which will minimize the variance of \( y_t \). Putting (4.11)-(4.13) in (4.10), one can show that

\[
y_t = \xi_t + \left( \frac{1 + a_1}{1 + \phi} \right) \xi_{t-1}.
\]

From (4.14) one can immediately see that by setting \( a_1 = -1 \) and \( a_2 = 0 \), the central bank can completely nullify the effects of \( \xi_{t-1} \) and \( \xi_{t-2} \) on the current output \( y_t \).
One implication of this model is that any shock which is anticipated more than two period ago has no effect on current output. In the absence of any other source of persistence, the effect of aggregate demand on output lasts only for a period equal to the period for which prices are predetermined. Now we turn to a model in which changes in aggregate demand affects output for a longer period than the period for which prices are predetermined.

Taylor Model

Now we change our assumption about price setting a bit. Now suppose that a producer sets the same price for two periods. In other words, prices are not only predetermined but also fixed. We will see that this change has important implications regarding the dynamics of output and price. To simplify matters, suppose that money supply follows a random walk

\[ m_t = m_{t-1} + u_t \]  (4.15)

where \( u_t \) is mean zero i.i.d shock. Suppose that a producer at time \( t \) sets price for time \( t \) and \( t + 1 \). Let \( x_t \) denote the price chosen by producers who set their prices at time \( t \). Then

\[ x_t = \frac{1}{2} (p_{it}^* + E_t p_{it+1}^*) \]  (4.16)

where \( p_{it}^* \) is the optimal price at time \( t \). Using (4.2) one can write down (4.16) as

\[ x_t = \phi m_t + \frac{1}{4} (1 - \phi) (x_{t-1} + 2x_t + E_t x_{t+1}). \]  (4.17)

(4.17) implies that

\[ x_t = A (x_{t-1} + E_t x_{t+1}) + (1 - 2A) m_t \]  (4.18)

where \( A = \frac{1}{2} \frac{1 - \phi}{1 + \phi} \).

One can show that the solution of (4.18) has the form

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\[ x_t = \lambda x_{t-1} + (1 - \lambda)m_t \]  \hspace{1cm} (4.19)

where

\[ \lambda = \frac{1 \pm \sqrt{1 - 4A^2}}{2A}. \]  \hspace{1cm} (4.20)

More specifically,

\[ \lambda_1 = \frac{1 - \sqrt{\phi}}{1 + \sqrt{\phi}} \]  \hspace{1cm} (4.21a)

and

\[ \lambda_2 = \frac{1 + \sqrt{\phi}}{1 - \sqrt{\phi}} \]  \hspace{1cm} (4.21b)

We use smaller of the root. Using (4.19), \( y_t = m_t - p_t \), and \( p_t = \frac{x_t + x_{t-1}}{2} \), one can show that

\[ y_t = \lambda y_{t-1} + \frac{1 + \lambda}{2}u_t. \]  \hspace{1cm} (4.22)

The key implication of (4.21) is that shocks to aggregate demand is long lasting \( i.e. \) their effects persist even after all producers have changed their prices as long as \( 0 < \lambda \leq 1 \). The effect is long lasting because of interdependence between price decisions. From (4.22), we can see that if \( \lambda = 1 \), then the effect is permanent. Smaller the value of \( \lambda \) less persistent the effects. In that sense, \( \lambda \) captures the degree of inertia of nominal prices.

The value of \( \lambda \) or the degree of inertia depends on the value of \( \phi \) (see 4.21a). When \( \phi = 0 \), then \( \lambda = 1 \), and the aggregate demand shock will have permanent effect. When \( \phi = 1 \), then \( \lambda = 0 \) and the aggregate demand will have purely transitory effect.

What is the intuition behind it? Remember that \( \phi \) is the inverse of the elasticity of labor supply. Thus, higher \( \phi \) implies lower elasticity of supply of labor. Thus, any increase in aggregate demand leads to increase in real wage relatively more. Thus given the mark-up pricing rule, prices rise relatively more. Opposite is the case, when \( \phi \) is low.
and thus the elasticity of labor supply is high. This can be easily seen by inspecting the optimal pricing decision rule (3.16) which in logarithmic form can be written as

\[ p_i^* - p_t = \ln \frac{\eta}{\eta - 1} + \phi y_t. \] (4.23)

In a nutshell, higher \( \lambda \) is associated with higher elasticity of labor supply and thus aggregate demand shocks have long lasting effect on output. Lower \( \lambda \) is associated with lower elasticity of labor supply and thus aggregate demand shocks have relatively short-lived effects on output.

State-Dependent Models

In Fischer and Taylor models, firms are constrained to change price infrequently. Firms by their actions cannot change the timing of price changes. In state-dependent models, the timing of price change is optimally chosen by firms i.e it is endogenous.

Sheshinski and Weiss (1977) showed that in an environment where inflation rate is steady, aggregate output constant, and firms face fixed cost in changing nominal prices, it is optimal for firms not to change their prices continuously. In fact, firms follow an \( Ss \) pricing strategy. Specifically, whenever a firm adjusts its price, it sets the price in such a way that the difference between the actual price and the optimal price at that time, \( p_i - p_i^* \), equals some target level, \( S \). The firm then keeps the nominal price, \( p_i \), fixed until money growth has raised the optimal price, \( p_i^* \), sufficiently so that \( p_i - p_i^* \) has fallen to some trigger level, \( s \). The firm then resets the price so that \( p_i - p_i^* \) equals \( S \).

Both the upper bound, \( S \), and lower bound, \( s \), on price band are endogenously determined. In that sense, the timing of price change becomes endogenous and depends on parameters of model. Because of such endogeneity, these models are known as state-dependent models.

Dotsey and King (2005) show that the dynamic responses of output and prices to monetary shocks in state-dependent models are quite different compared to time-dependent models. In particular, there are complicated oscillatory dynamics in price and output. The reason is that monetary shocks affect the frequency (or timing) of price adjustments.
References:

