Stochastic Confinement of Rossby Waves by Fluctuating Eastward Flows

Adam Hugh Monahan, Lionel Pandolfo, and Peter Imkeller

Abstract. The effect of stochastic fluctuations in the background zonal velocity field on the energy dispersion of stationary wave responses to meridionally localised forcing is considered, using the non-divergent, barotropic vorticity equation. It is found that for small noise levels or large lengthscales in the noise autocovariance function, the oscillatory structure of the solutions is not altered. However, for noise levels (or autocovariance lengthscales) comparable to or larger (smaller) than those observed in the circulation at 300mb, the marginal density functions of the solution process displays a pronounced attenuation away from the stationary wave source. This indicates that fluctuations in the velocity field inhibit the dispersion of wave energy. The symmetry of the marginal PDFs about the source rather than about the equator indicates that the localisation is primarily an integrated effect of backscattering by potential vorticity gradients in regions of real refractive index, and not due to attenuation by regions of imaginary refractive index or by critical lines in the flow.

1. Introduction

As was first noted by Rossby et al. [22], the barotropic (i.e., depth-integrated), linearised equations of motion of the atmosphere admit wave solutions of potentially global extent. The restoring force for these waves arises from the meridional (north-south) variation of the local vertical component of the earth's rotation axis - the so-called beta effect [20]. These planetary, or Rossby, waves have played an important role in shaping our conception of atmospheric dynamics. In particular, in a linear atmosphere they are the fundamental dynamical elements mediating the global-scale response to spatially-localised vorticity sources such as topography or diabatic heating.

Hoskins and Karoly [6] considered the dynamics of planetary waves in a spherical atmosphere, using the nondivergent, barotropic vorticity equation linearised around a constant angular momentum (superrotation) background flow. Employing WKB (Liouville-Green) theory, they found that the meridional dispersion of a stationary (zero-frequency) wave's energy was a function of its zonal (east-west) wavenumber: for their background flow speed of 15ms^{-1}, they found that wavenumbers 1 and 2 were able to propagate from a subtropical or midlatitude source to the near vicinity of the poles, while for higher wavenumbers there are "turning latitudes" beyond which the wave is evanescent. Subsequent modelling studies have considered the dynamics of stationary Rossby waves on zonally-inhomogeneous flows such as the climatological boreal winter 300 mb flow ([1], [5],[8]), and of planetary waves of nonzero frequency on both zonally-homogeneous and inhomogeneous flows ([15],[16]). Observational studies by Kiladis et al. ([10], [11], [12], [13]) demonstrate the potential importance of Rossby waves in mediating the interaction between the tropics and extratropics in both the 6-30 day and 30-70 day frequency bands.

All of the above modelling studies have considered the dynamics of planetary waves on a spatially smooth background flow. Such smooth background flows may represent the mean circulation, but this is not a state whose neighbourhood in phase space is often visited by the atmosphere [18]. In fact, at any time, the atmospheric circulation contains substantial small-scale structure. Pandolfo and Sutera [19] demonstrated using observed zonal-mean 300mb circulation data that the mean of solutions to the nondivergent barotropic vorticity equation linearised around individual realisations of the flow was not identical to the solution of the equation linearised around the mean background flow. Denoting the dynamical operator linearised around the flow U as $\mathcal{L}(U)$, this difference is a reflection of the inequality

$$\hat{\psi} \neq E\{\psi\} \tag{1}$$

where $\mathcal{L}(U)\psi = S$ and $\mathcal{L}(E\{U\})\hat{\psi} = S$ (S is a vorticity source). In particular, Pandolfo and Sutera demonstrated $E\{\psi\}$ displayed less meridional dispersion of wave energy than $\hat{\psi}$.

This result can be understood by applying the quantum theory of Anderson localisation [25] to the problem of classical wave propagation. If the properties of the medium through which the waves propagate fluctuate in a characteristic manner, the waves will be attenuated as they disperse. The restoring force for planetary waves is the background potential vorticity (PV) gradient, which may be affected both by variations in topography and background flow field [20]. A number of studies have demonstrated the asymptotic exponential decay of planetary waves propagating over rapidly fluctuating topography ([23],[24],[26],[27],[28]), but little attention has so far been paid to the effects of random fluctuations in the background flow. Keller and Veronis [9] studied this problem but assumed that the fluctuations in the background potential vorticity gradient are much smaller than the planetary PV gradient. In the real atmosphere, however, the fluctuations can be $O(1)$.

This paper describes a Monte Carlo investigation of the effect of fluctuations in the PV gradient on the energy dispersion of stationary planetary waves in spherical geometry. This work extends that of Pandolfo and Sutera [19], who considered observational zonal-mean flows for which the strength of the fluctuations and their characteristic autocovariance lengthscale are fixed. By generating random background flows and studying the probability distribution of the resulting solution the random linear vorticity equation, we are able to investigate the dependence of this distribution on the strength and characteristic lengthscales of the fluctuations in the background PV gradient. In Section 2, we describe the model used in this study. We present the results in Section 3 and their interpretation in terms of a classical version of the quantum theory of Anderson Localisation in Section 4. Section 5 concludes the paper with a discussion of the physical implications of our results. The companion article by Imkeller, Monahan, and Pandolfo develops an analytic theory of classical wave localisation applicable to the problem of atmospheric planetary waves propagating in fluctuating winds.

2. Spectral Model

We consider the non-divergent, barotropic vorticity equation in spherical coordinates:

$$\partial_t \nabla^2 \Psi - \frac{\partial_\phi \Psi}{a} \frac{\partial_\lambda \nabla^2 \Psi}{a \cos\phi} + \frac{\partial_\lambda \Psi}{a \cos\phi} \left(\frac{\partial_\phi \nabla^2 \Psi}{a} + \frac{2\Omega}{a} \cos\phi \right) + \mu \nabla^2 \Psi = 0, \qquad (2)$$

where λ and ϕ are the zonal and meridional coordinates, respectively, Ψ is the streamfunction, a is radius of the Earth, Ω is angular frequency of rotation of the Earth, and μ is an Ekman (scale-independent) friction parameter. The zonal and meridional components of the wind field, respectively u and v, are given by

$$u = -\frac{1}{a} \partial_\phi \Psi \qquad (3)$$

$$v = \frac{1}{a \cos\phi} \partial_\lambda \Psi \qquad (4)$$

Equation (2) is a simple model for the depth-averaged dynamics of a shallow fluid on a rotating sphere [20]. We simplify equation (2) by assuming Ψ is composed of a meridionally-varying background zonal flow plus a small perturbation having zero zonal mean:

$$\Psi = -a \int^{\phi} d\phi' \, U(\phi') + \psi'(\lambda, \phi, t). \qquad (5)$$

The derivation of an equation governing the wave perturbation ψ' is done in three steps. Firstly, introduce (5) into (2). Secondly, linearise the resulting equation with respect to the background wind U. Thirdly, subtract from the linearised equation its zonal mean. This yields:

$$\partial_t \nabla^2 \psi' + \frac{1}{a \cos\phi} U \partial_\lambda \nabla^2 \psi' - \frac{1}{a \cos\phi} (\partial_\lambda \psi') \left(\partial_\phi \nabla^2 \int^{\phi} d\phi' \, U(\phi') \right) + \frac{2\Omega}{a^2} \partial_\lambda \psi' + \mu \nabla^2 \psi' = 0. \qquad (6)$$

Assuming a stationary wave form for ψ':

$$\psi'(\lambda, \phi) = \mathrm{R}e(\psi(\phi) \exp il\lambda), \tag{7}$$

and adding a source term $S(\phi)$, the non-divergent barotropic vorticity equation linearised about the background zonal flow U becomes the ODE in ϕ:

$$\begin{aligned} &\left(U - \frac{i\mu a \cos\phi}{l}\right)\left(\cos\phi \frac{d}{d\phi}\left(\cos\phi \frac{d}{d\phi}\psi\right) - l^2\psi\right) \\ &- \left\{\cos^2\phi \frac{d}{d\phi}\left(\frac{1}{\cos\phi}\frac{d}{d\phi}(U\cos\phi)\right)\right\}\psi + (2\Omega a \cos^3\phi)\psi \quad = \quad S(\phi). \end{aligned} \tag{8}$$

This equation is exactly equation (6.12) of Held [4], and for $\mu = 0$ reduces to equation (A10) of Pandolfo and Sutera [19]. Under a transformation to Mercator coordinates, the pole is removed to infinity. It is obvious that there is always a latitude poleward of which the solutions are evanescent, so we obtain the following boundary conditions:

$$\psi(-\pi/2) = \psi(\pi/2) = 0. \tag{9}$$

Equation (8) has been well-studied in the case that $U(\phi)$ is a deterministic function [4]. What has *not* been investigated is the distribution of solution processes $\psi(\phi)$ obtained when $U(\phi)$ is taken to be a stochastic process in ϕ. This paper details the initial effort to address this question.

In equation (8), the stochastic processes $U(\phi)$ and $\psi(\phi)$ are multiplied together. Expressing this equation formally as

$$\mathcal{L}(U)\psi = S, \tag{10}$$

this implies that in general the mean $E\{\psi\}$ of solution processes to the equation $\mathcal{L}(U)\psi = S$ will *not* equal the solution process $\hat{\psi}$ to the equation with the mean operator: $\mathcal{L}(E\{U\})\hat{\psi} = S$. Pandolfo and Sutera [19] pointed out that in consequence the Rossby wave solution to the equation linearised around a climatological mean zonal flow need not bear any resemblance to the average of wave solutions to the equations linearised around individual realisations of the background flow.

Because U is in general not smooth, we cannot use WKB theory to solve (8), and instead turn to numerical methods. Equation (8) is naturally discretised by recasting it in spectral form. The natural Fourier basis on the domain $[-\pi/2, \pi/2]$ is the orthogonal set of functions $\exp 2in\phi$, $n = -\infty, ..., \infty$. The function $\psi(\phi)$ is expanded on this basis as

$$\psi(\phi) = \sum_{m=-\infty}^{\infty} \psi_m e^{2im\phi}, \tag{11}$$

where the expansion coefficients ψ_m are given by

$$\psi_m = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} d\phi \; \psi(\phi) e^{-2im\phi}. \tag{12}$$

Similar expansions hold for U and S. Substituting these expansions into equation (8), we obtain the infinite set of coupled algebraic equations for the ψ_m:

$$\sum_{m=-\infty}^{\infty} M_{nm}\psi_m = S_n, \tag{13}$$

where

$$\begin{aligned} M_{nm} \quad = \quad & (n-1/2)(n-2m-1)U_{n-m-1} + [2(n-m)^2 + 1 - 2m^2 - l^2]U_{n-m} \\ & +(n+1/2)(n-2m+1)U_{n-m+1} + 2\Omega a C_{n-m} \\ & +\frac{i\mu a}{l}\left\{m(m+1/2)D_{n-m-1} + (2m^2+l^2)D_{n-m}\right. \\ & \qquad \left. +m(m-1/2)D_{n-m+1}\right\}. \end{aligned} \tag{14}$$

The coefficients

$$C_m = \frac{12}{\pi}(-1)^m \frac{1}{(1-4m^2)(9-4m^2)} \tag{15}$$

are the Fourier components of $\cos^3\phi$, and

$$D_m = \frac{2}{\pi}(-1)^m \frac{1}{1-4m^2}. \tag{16}$$

are the Fourier components of $\cos\phi$.

For numerical implementation, equation (13) must of course be truncated to a finite number $2N+1$ of Fourier components, yielding the approximate equation

$$\sum_{m=-N}^{N} M_{nm}\psi_m = S_n. \tag{17}$$

Throughout this study, we used a value of $N = 50$. Sensitivity studies indicate that increasing the number of modes N retained does not change the results. It is worthwhile to note that the truncated spectral model we have adopted is superior to, say, a finite difference approximation, because the latter involves errors both due to limited resolution and finite differencing of derivatives. In a spectral model all derivatives are evaluated exactly, and the only error is associated with truncation of the Fourier series.

We model the background zonal wind by the equation

$$U(\phi) = \tilde{U}(\phi) + \eta U_{max} R(\phi), \tag{18}$$

where $\tilde{U}(\phi)$ represents the resulting mean background wind profile, η tunes the amplitude of the fluctuations,

$$U_{max} = \max_{\phi} \tilde{U}(\phi) \tag{19}$$

scales the noise strength, and $R(\phi)$ is a stationary, mean-zero, unit-variance stochastic process with an oscillating gaussian autocovariance function:

$$E\{R(\phi)\} = 0, \tag{20}$$

$$E\{R^2(\phi)\} = 1, \tag{21}$$

$$E\{R(\phi)R(\phi+\phi_0)\} = \exp(-\phi_0^2/2\tau^2)\cos(2\alpha\phi_0), \tag{22}$$

The oscillating component of the autocovariance function was introduced to emulate the observed "index cycle" fluctuation in the zonal-mean zonal wind [17]. Realisations of this process were generated directly in Fourier space using an algorithm described in Monahan and Pandolfo [17].

Because this study is primarily interested in the variability of the meridional structure of forced waves, arising from fluctuations in the mean flow, we are not particularly concerned with the details of how the waves are forced. Thus, throughout the study, we employ a simple, narrow Gaussian forcing of unit amplitude:

$$S(\phi) = \frac{1}{\sqrt{2\pi\sigma}}\exp(-(\phi-\phi_F)^2/2\sigma^2) \tag{23}$$

where $\sigma = 1^{\mathrm{O}}$. Because of the narrowness of this source term, the solution processes will essentially correspond to a random Greens function.

3. Superrotation Flow

The mean background zonal wind considered in this study is a flow with constant angular velocity:

$$\tilde{U}(\phi) = U_{max}\cos\phi, \tag{24}$$

also referred to as superrotation. Because it is of such a simple structure, the propagation of Rossby-type waves in such a flow field has been considered several times in the atmospheric dynamics literature (eg. [1], [6], [29]). The associated vorticity field has the same meridional structure as the planetary vorticity, so the superrotation flow affects the propagation of Rossby waves by either amplifying ($U_{max} > 0$) or attenuating ($U_{max} < 0$) the background potential vorticity gradient:

$$\partial_\phi(\zeta + f) = 2\left(\frac{U_{max}}{a} + \Omega\right)\cos\phi \tag{25}$$

(where $\zeta = \nabla^2\left(-a\int^\phi d\phi' U(\phi')\right)$ is the relative vorticity of the background flow and $f = 2\Omega\sin\phi$ is the planetary vorticity). For eastward superrotation flow, the potential vorticity gradient is increased, and wavelike disturbances experience a stronger meridional restoring force [20].

We consider a series of experiments using superrotation flow with $U_{max} = 35\mathrm{ms}^{-1}$; Ekman friction, $\mu = (10\mathrm{days})^{-1}$; and with perturbations having a zonal wavenumber $l = 1$. Figure 1 displays a plot of $|\psi|\cos^{1/2}\phi$ for the $l = 1$ wave, normalised to unity, for a forcing at $18N$ in the limit of zero noise, $\eta = 0$. The product $|\psi|^2\cos\phi$ is a measure of the energy per unit area in the perturbation.

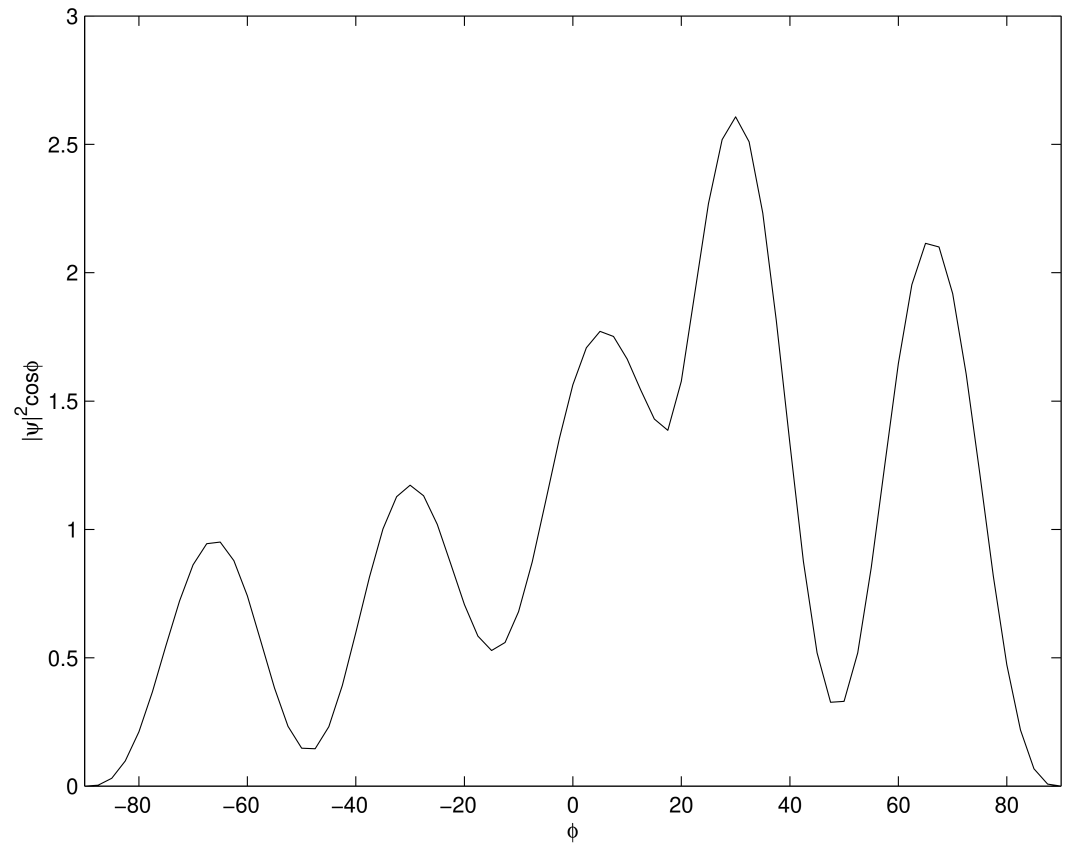

FIGURE 1. Plot of normalised $|\psi| \cos^{1/2} \phi$ for a source located at $18N$.

Numerical experiments (not shown) show that in the limit of no friction ($\mu \to 0$), the amplitude of the oscillatory function $|\psi| \cos^{1/2} \phi$ is approximately constant on either side of the forcing. This is precisely the result predicted by Hoskins and Karoly [6] using WKB theory. Their approximate result for superrotation flows yielded oscillatory solutions for which

$$\psi \sim \cos^{-1/2} \phi. \tag{26}$$

For the case of a smooth superrotation background flow, then, the $l = 1$ wave is global in meridional extent, and in the limit of vanishing friction, the envelope of the energy per unit area does not diminish poleward. The slight poleward attenuation of energy displayed in Figure 1 results from the use of moderately strong friction $\mu = (10\text{days})^{-1}$.

We now consider the effect of fluctuations in the background wind, with a forcing located at $\phi_F = 18N$. For $\eta \neq 0$, the distribution of ψ at any latitude is no longer a delta function. Figure 2 displays plots of the marginal distribution $P(|\psi| \cos^{1/2} \phi)$ for values of η in increments of 0.05 from 0.05 to 0.5 with autocovariance lengthscale parameters $(\tau, \alpha) = (25^{\mathrm{O}}, 3)$. These marginal distributions were estimated from 1000 realisations of the solution process at each noise level. For each realisation, $|\psi|^2 \cos \phi$ has been normalised to unity, because we are more concerned with the meridional structure of the response than its overall amplitude. For η less than about 0.2, the distribution of $|\psi(\phi)| \cos^{1/2} \phi$ about the noise-free solution broadens as the noise level increases, but retains an essentially oscillatory character. However, at about $\eta = 0.2$, the oscillatory character of the distributions has started to disappear while the poleward decay is accentuated. As the noise level increases further, this poleward decay of the amplitude entirely subsumes the oscillatory structure, characteristic of the $\eta = 0$ solution, until eventually the oscillatory character of the PDF has vanished outright. We refer to the poleward attenuation of stationary wave energy due to the presence of fluctuations in U as *localisation*.

Figure 3 displays the marginal PDF of $|\psi| \cos^{1/2} \phi$ for the solution process associated with a source at $36N$, over the same range of noise levels η as in Figure 2. Again, the same loss of oscillatory structure and increase in poleward attenuation of the marginal distribution of $|\psi| \cos^{1/2} \phi$ with increasing η is seen with the source at this latitude.

Figures 4 and 5 display the marginal PDF of $|\psi| \cos^{1/2} \phi$ for sources at $18N$ and $36N$, respectively, over a range of values of the autocovariance scale parameters $(\tau, \alpha) = (j \times 5^{\mathrm{O}}, 15/j)$ for $j = 3, ..., 10$, with $\eta = 0.2$ fixed. These values were selected so that the decay and oscillation lengthscales of the fluctuations in U were maintained at a constant ratio. The autocovariance function of the observed 300mb zonal-mean zonal winds is reasonably well approximated when $(\tau, \alpha) = (25^{\mathrm{O}}, 3)$ [17]. We observe that reducing the autocovariance lengthscale of the fluctuations has the same effect on the distribution of $|\psi| \cos^{1/2} \phi$ as increasing the noise level, namely, increased confinement around the source. This will be discussed further in the following section.

Thus, the effect of fluctuations in the background zonal wind on the evolution of a stationary wave is an attenuation in the poleward dispersion of its energy. This effect is strengthened by either increasing the amplitude of the fluctuations or decreasing the autocorrelation length scales. The mean effect on the amplitude of $|\psi| \cos^{1/2} \phi$ of fluctuations in U resembles that which would follow from an increase in the dissipation parameter μ. However, the cause of the confinement is a dynamical mechanism that is physically distinct from friction.

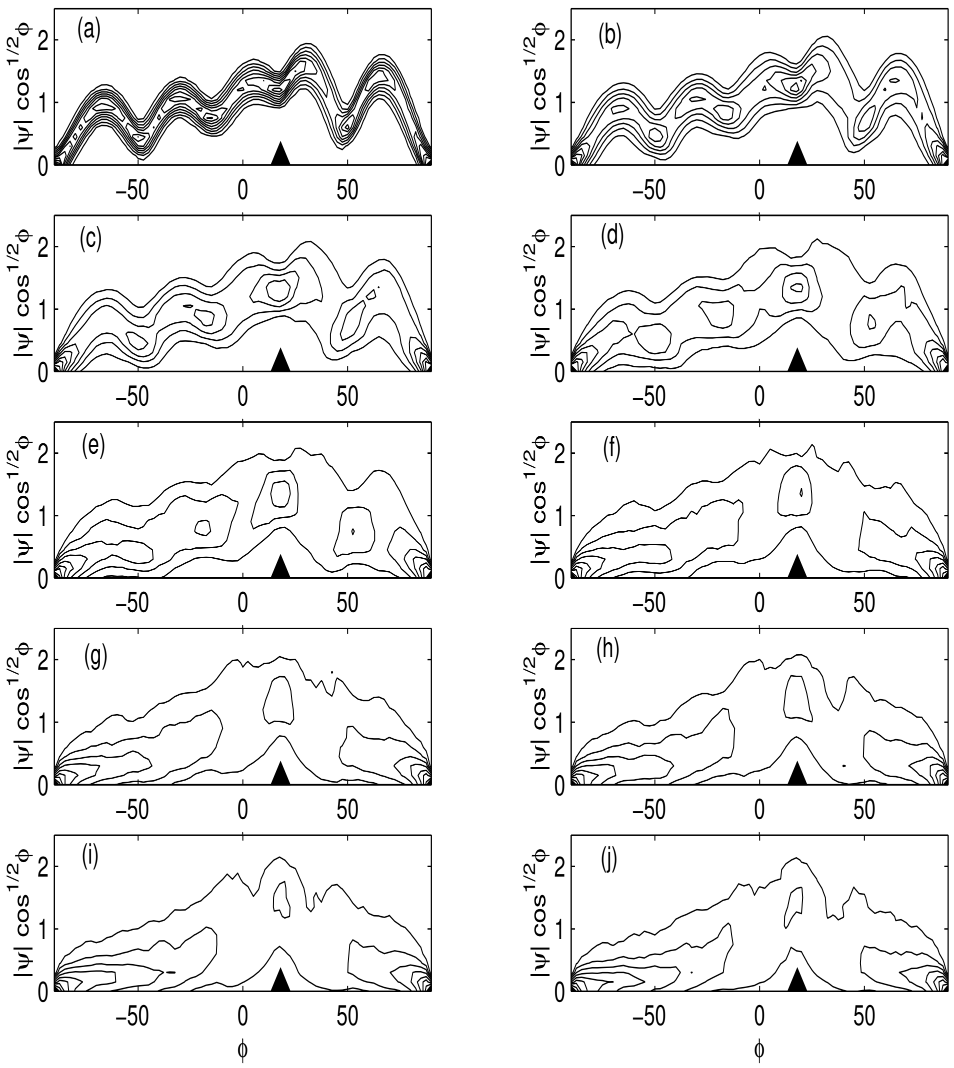

FIGURE 2. Estimated PDF of $|\psi|\cos^{1/2}\phi$ as a function of latitude for $\alpha = 3$, $\tau = 25^{\mathrm{O}}$, and η in increments of 0.05 from (a) $\eta = 0.05$ to (j) $\eta = 0.5$. The source, marked by the triangle, is at $18N$. Contour interval: 0.33.

4. Interpretation

Two mechanisms for the localisation of planetary waves demonstrated above suggest themselves: backscattering of waves by fluctuations in the background PV

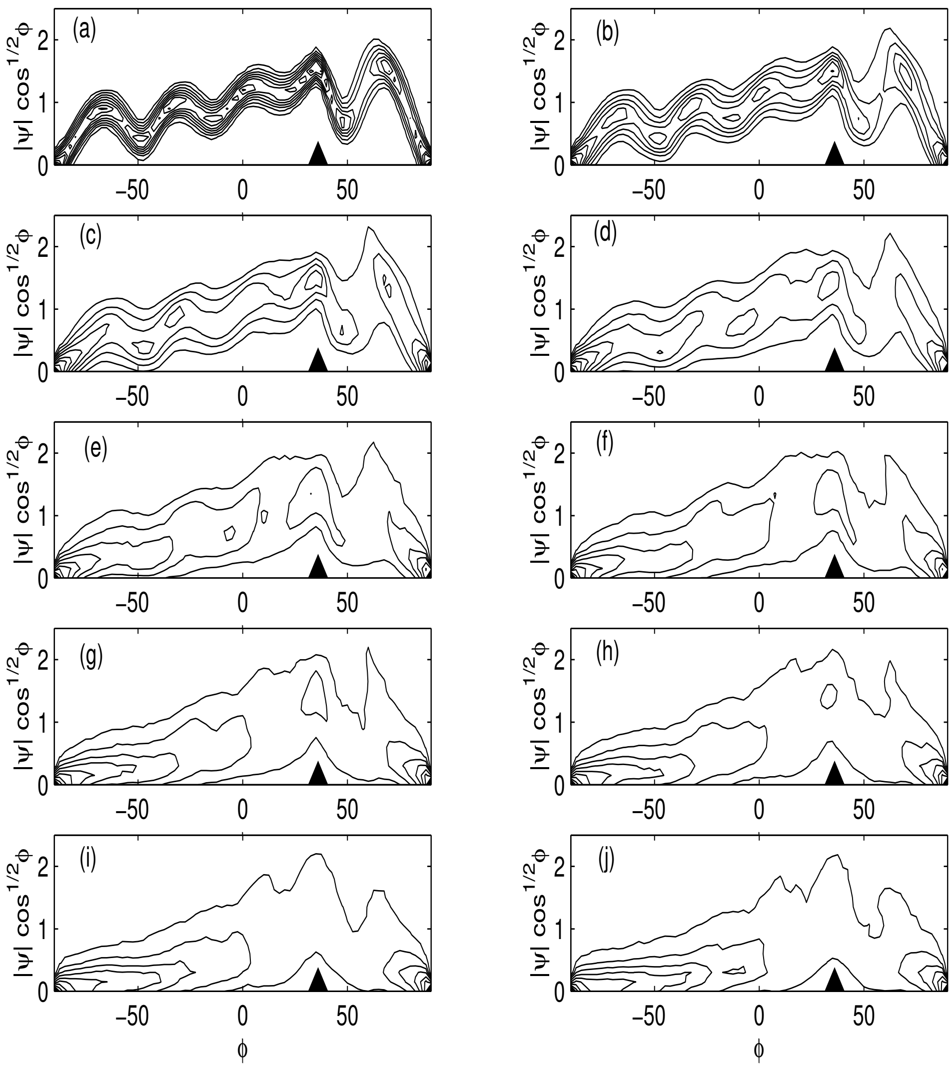

FIGURE 3. Estimated PDF of $|\psi| \cos^{1/2} \phi$ as a function of latitude for $\alpha = 3$, $\tau = 25^{\mathrm{O}}$, and η in increments of 0.05 from (a) $\eta = 0.05$ to (j) $\eta = 0.5$. The source, marked by the triangle, is at $36N$. Contour interval: 0.33.

gradient, and the appearance in the flow of critical lines. Considering the first cause, wave attenuation could occur in regions where wind fluctuations create either an imaginary index of refraction or a highly-fluctuating real index of refraction. It was noted by Pandolfo and Sutera [19] that fluctuations in the background

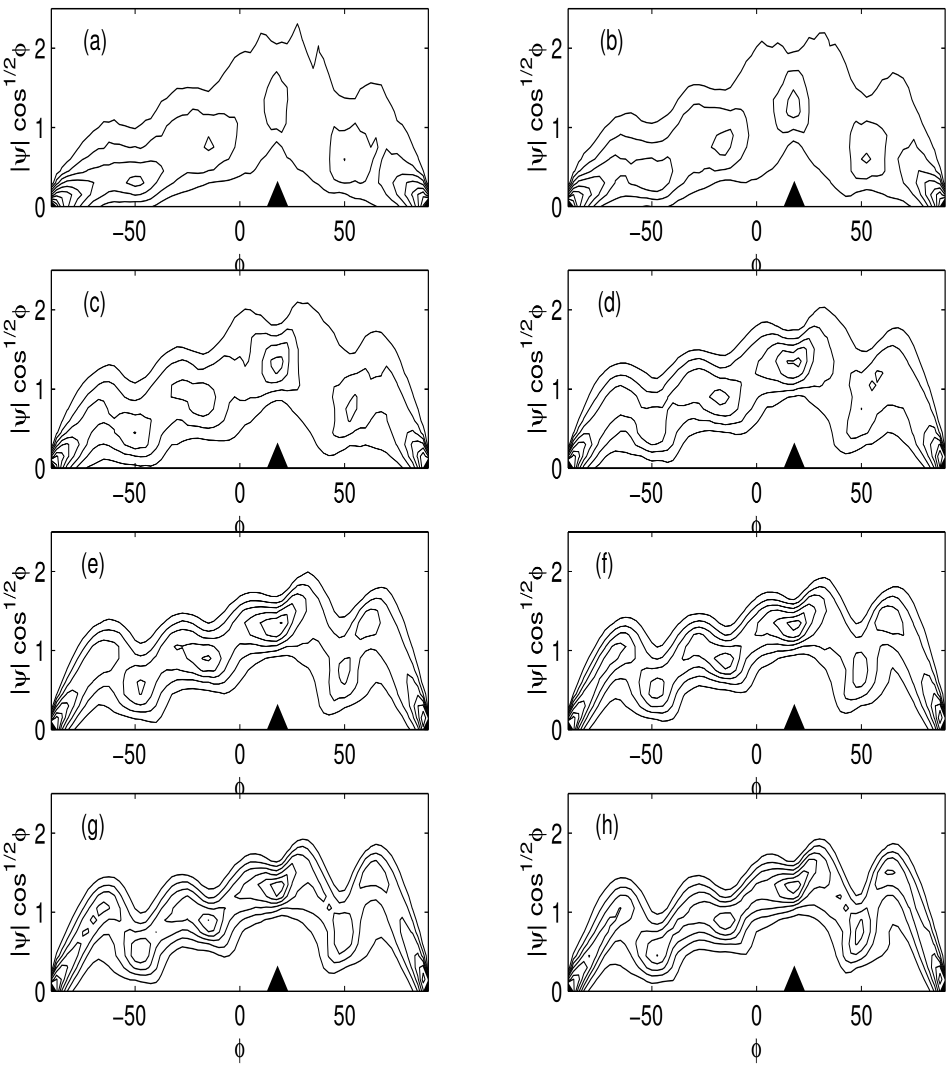

FIGURE 4. Estimated PDF of $|\psi|\cos^{1/2}\phi$ as a function of latitude for $\eta = 0.2$ and $(\tau, \alpha) = (j \times 5^{\rm O}, 15/j)$ from (a) $j = 3$ to (h) $j = 10$. The source, marked by the triangle, is at $18N$. Contour interval: 0.33.

velocity can lead to regions in the flow where the refractive index associated with the wave equation (8) is imaginary. In these regions solutions are not oscillatory but decaying. Passing through one of these, the amplitude of a wave is attenuated. In steady state, these regions are therefore reflective. As the noise level increases

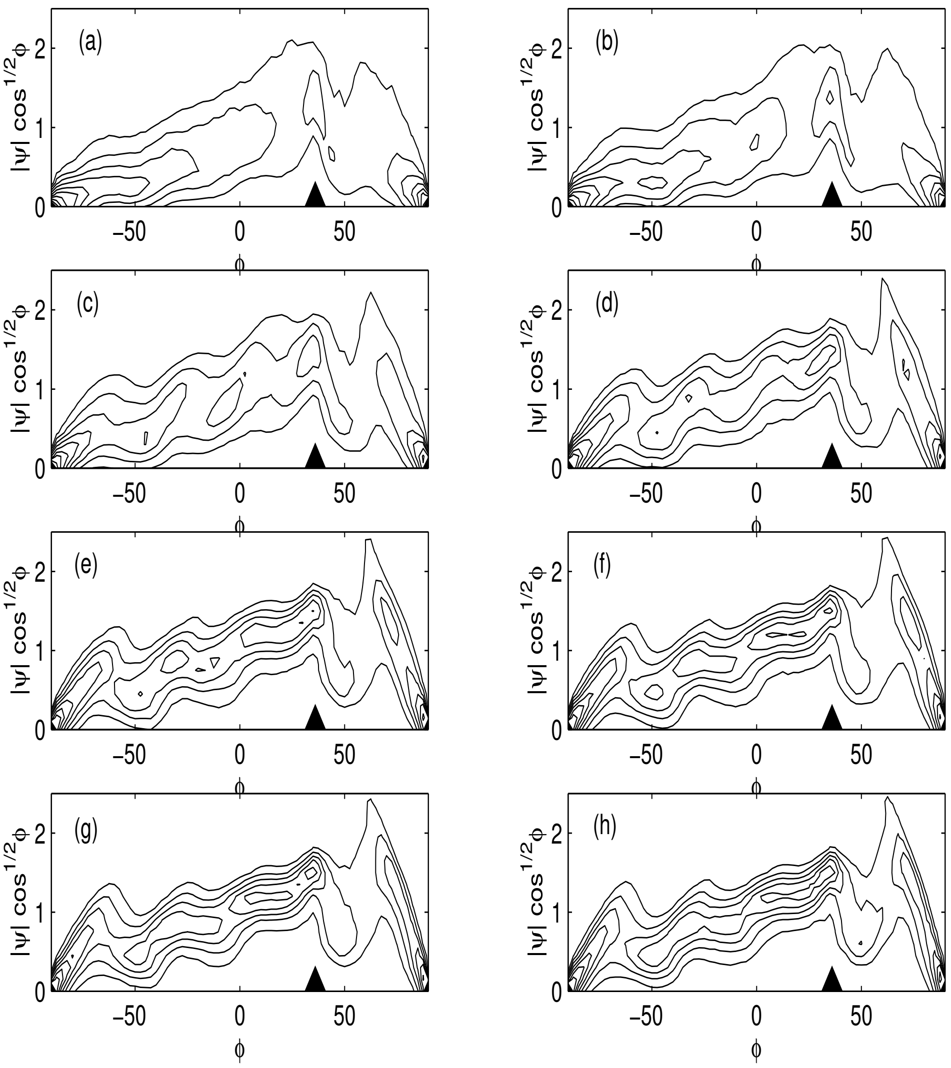

FIGURE 5. Estimated PDF of $|\psi| \cos^{1/2} \phi$ as a function of latitude for $\eta = 0.2$ and $(\tau, \alpha) = (j \times 5^{\mathrm{O}}, 15/j)$ from (a) $j = 3$ to (h) $j = 10$. The source, marked by the triangle, is at $36N$. Contour interval: 0.33.

or the autocovariance lengthscale of the fluctuations decreases, these areas occupy a larger fraction of latitudes, and consequently the planetary wave energy is increasingly trapped in the vicinity of the source. Pandolfo and Sutera also showed that in a real index of refraction exhibiting substantial spatial fluctuations, wave energy becomes localised near its source. The other mechanism which could be important for localisation is the appearance of critical lines. In the problem at hand these will appear at latitudes where the zonal mean zonal velocity vanishes. The extent to which critical lines are absorbing, reflecting, or overreflecting remains controversial (e.g. [1],[2],[3],[14]), but without question they inhibit the dispersion of Rossby waves. The average number of critical lines in a given region increases with increased noise level η and decreased autocovariance length scales; and because of the form given to $\tilde{U}$, the frequency of occurrence of critical lines in the background wind increases toward the poles. Then, as η increases or as the autocovariance lengthscales decrease, it is possible that the concentration of the PDF of $|\psi| \cos^{1/2} \phi$ near zero, starting near the poles and moving equatorward, merely reflects the equatorward movement of the latitudes at which the wave is likely to first encounter a critical line.

Each of the three causes of wave trapping discussed above participates in producing the PDFs of Figures 2 to 5. However, the fundamental mechanism of localisation is neither attenuation by regions of imaginary index of refraction nor attenuation by critical lines. This can be determined by an examination of Figure 6, which contours as a function of latitude and noise level the fraction of realisations for which the background wind $U(\phi)$ is non-positive. Firstly, Figure 6 shows that, even for high noise levels, critical lines are commonplace only in polar and sub-polar regions. The corresponding figure for the index of refraction (not shown) reveals a similar distribution of imaginary realisations. This is because the wave restoring force is dominated by the advection of planetary vorticity by the perturbation field, so the index of refraction is imaginary only where the background wind is westward [19]. Secondly, the symmetry of wind fluctuations with respect to the equator present in Figure 6 would be mirrored by the PDFs of Figures 2 through 5 if critical lines (or imaginary indices of refraction) were important factors in determining the shape of PDFs. This symmetry would exist independently of the position of the forcing as long as the source is situated in subtropical regions. Conversely, if backscattering by fluctuations in the PV gradient (in regions where the index of refraction is real) is the dominant mechanism, then the localisation should appear symmetric about the source, because then the attenuation will be an integrated effect of the distance from the source. Indeed, comparing Figure 2 with Figure 3 and Figure 4 with Figure 5, it is clear that the PDFs are symmetric with respect to the position of the source. Hence, it is unlikely that critical lines or imaginary indices of refraction are responsible for the confinement observed in the PDFs displayed in Figures 2 through 5. This indicates that it is the fluctuations of (mostly) real indices of refraction that are responsible for the wave backscattering that localises wave energy around the source. The intensification of the localisation effect with increasing noise level and with decreasing autocovariance lengthscale can then be understood to result simply from an increase in the density of scattering centres. The symmetry of the PDF with respect to the source and the monotonic decrease of wave amplitude away from the source in

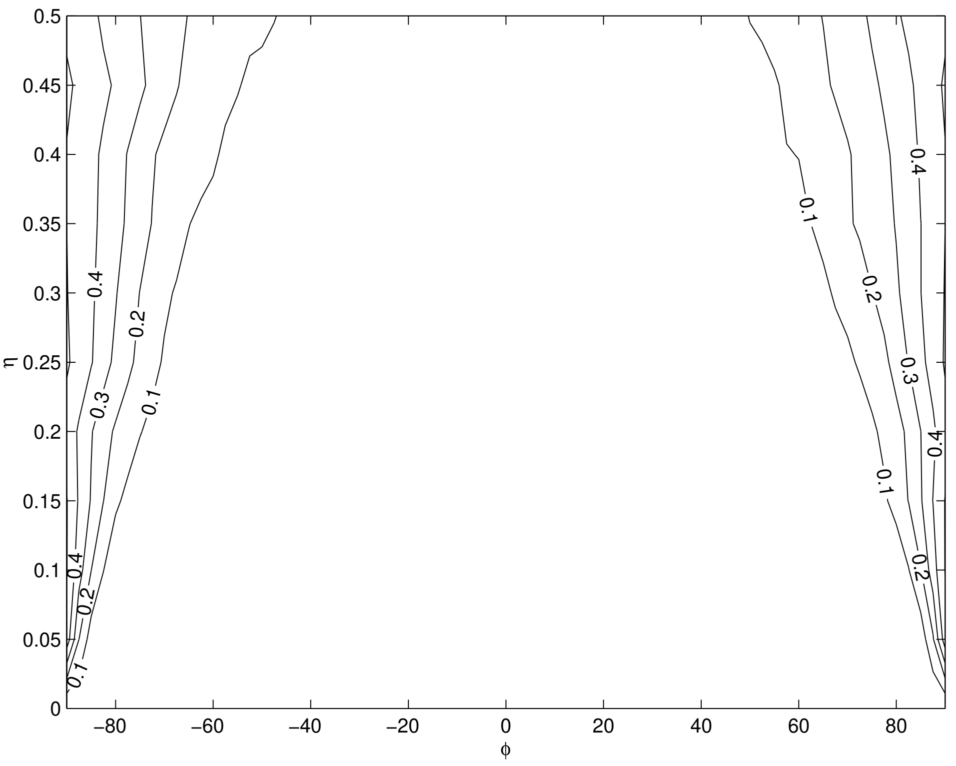

FIGURE 6. Fraction of realisations for which $U(\phi) \leq 0$.

a real index of refraction are the two characteristics of the occurrence of Anderson localisation for the dispersion of (classical) waves in a random medium [25]. A companion paper in this volume presents the mathematical framework for describing the physical mechanisms leading to the localisation effect observed in our numerical experiments [7].

From a different point of view, Penland and Sardeshmukh [21] also consider the effects of fluctuations in the background wind field on the dispersion of planetary wave energy. However, their approach is to use wind fluctuations that are spatially homogeneous but fluctuating in time with a white spectrum. They find that the average effect of these fluctuations is an enhanced dissipation, inhibiting the dispersion of wave energy away from the source. Their study is at the opposite limit to the present work (in which we consider fluctuations fixed in time but spatially varying) and yet arrives at a qualitatively similar conclusion. Thus, it appears that both temporal and spatial fluctuations in the background wind result in confinement of wave energy.

5. Conclusions

Using a Monte Carlo technique to sample realisations of a stochastic zonally-averaged zonal velocity field, we have estimated the joint PDFs of the solution process to the nondivergent, barotropic vorticity equation linearised around these realisations. As was demonstrated in Pandolfo and Sutera [19], fluctuations in the background flow $U(\phi)$ around a smooth mean $E\{U(\phi)\}$ can yield wave solutions with dispersion characteristics that are very different from those of waves propagating on $E\{U(\phi)\}$ itself. In particular, fluctuations can cause wave energy to be trapped around the wave source. The extent of this trapping does not appear to be a simple linear function of the magnitude of the fluctuations. If the standard deviation of $U(\phi)$ is less than about 20% of the maximum of the mean background flow, the waves do not seem to be particularly localised by fluctuations in $U(\phi)$. However, as the noise level increases beyond this point, waves become strongly attenuated in the subpolar region, and in the midlatitudes as the noise increases even further. This result is complementary to that of Penland and Sardeshmukh [21], who find that the mean effect of fluctuations in the background wind field, that are homogeneous in space but rapidly varying in time, is to increase the dissipation in the system. Reducing the characteristic lengthscale of the noise autocovariance function also increases localisation.

The boreal winter data considered by Monahan and Pandolfo [17] indicate that the observed noise level for zonal mean zonal velocity at 300mb is $\eta \sim 0.2$. Hence, linearisation around a smooth climatological flow can only give a rough qualitative description of wave dispersion. As shown in Figure 2d or 3d, the marginal PDF is hardly a delta function at each latitude for this noise level. In this regard, it will be interesting to investigate the propagation characteristics of a two-dimensional (latitude-longitude) atmosphere for which the constraint of zonal averaging is removed.

Branstator [1] addressed the question of what velocity field would be an appropriate background flow around which to linearise the equations of motion. The results presented here indicate that linearisation around a fluctuating background flow may produce results that are different than those obtained by linearising around a smooth flow. Physical intuition based on the latter may not be appropriate when dealing with realistic atmospheric flows.

Finally, the results of this study can be understood in two distinct ways. Firstly, this analysis can be thought of as an investigation of the dispersion characteristics of stationary wavelike disturbances on a non-smooth background flow without wave-mean flow interactions. This interpretation implies a temporal separation of scales such that the adjustment time of the circulation to a stationary forcing source is much less than the time scale of changes in the background wind. If the fluctuations in the background flow are associated with "weather" (in contrast to the "climate" of the smooth background flows), it is not clear that this separation of timescales holds for terrestrial flows. A more complete analysis would involve adding temporal variability to the problem.

Secondly, this study may be regarded as an analysis of the *sensitivity* of the solutions to the structure of the background flow around which the equations of motion are linearised. Generally, smooth background flows are chosen for their simplicity, and *not* for their relevance to the *actual* circulation of the atmosphere. In fact, the long-term climatological average is a circulation whose neighbourhood in phase space is *rarely* visited by the real atmosphere [18]. In general, these background flows are forced to be solutions to the linearised equations of motion through the introduction of appropriate forcings, calculated *a posteriori* after the selection of the zonal mean background state. This begs the question of the sensitivity of the character of the solutions to the basic state chosen, a question which this study addresses. The realisations of the background wind with fluctuations are then seen not so much as representing actual circulations in the atmosphere, but background states around which it is *equally plausible* to linearise as around the smooth background flows. We find that the structure of the background wind can *qualitatively* affect the dispersion of energy away from a localised forcing source for background flows that differ sufficiently from smooth ones.

These two interpretations of the results are distinct. The first is more physical, but involves an approximation concerning atmospheric relaxation and fluctuation timescales. The second is more mathematical, but addresses a question that is of relevance to linearised theories of atmospheric dynamics.

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