#### On the Birth of the Intentional Orientation to Knowledge

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Constructivist theories of cognition presuppose learners' intentional orientation to the knowledge (object) that they are to construct. But how can learners orient to the construction of an (alien) object that they do not already know? Using illustrative examples from the area of perception, I develop—grounding myself in the works of the phenomenological philosophers Maine de Biran, Merleau-Ponty, Henry, and Marion—a phenomenological description of learning as donation. I provide an extended analysis of a classroom episode from a second-grade mathematics lesson in which students have been asked to build clay models of objects that they can touch but not see. This analysis shows that telling students what they are to learn does not advance the learning process. In the episode, the student eventually learns what is expected when she recognizes the object as it emerges from her own actions: she literally comes to see (perceive, know) as her gaze (seeing the model) and her touch (feeling the object) come to be coordinated.

One cannot explicitly intend [vouloir] that which one does not know in some way. (Maine de Biran, 1859a, p. 226)

[I]ntentionality . . . is not enough. By itself, it does not leave sufficient room for the alien as the alien. (Waldenfels, 2007, p. 22)

The intentional orientation to the alien and therefore to the initially unknown learning object is presupposed in much of the scholarship on knowing and learning (Roth, 2012d). Intention means that there is an object of intention toward which the acting individual orients (Husserl, 1973). Thus, "intentional analysis is the unveiling of the actualities and potentialities, in which objects constitute themselves as sense unities" (p. 19). Moreover, "all analysis of sense accomplishes itself in the transition from real experiences into the intentional horizons that are prefigured in them" (p. 19). Yet we may ask, how learners, who by definition do not yet cognize the knowledge (object) to be known, can orient to that invisible, unseen, and therefore unforeseen object? How does Merleau-Ponty's (1964b) hand that intentionally explores the other hand exploring the world come to be intentional in the first place? In this study I suggest that what the philosopher says about vision—i.e., that "there is in its center a mystery of passivity (Merleau-Ponty, 1964a, p. 52)—also is the case for tact. The question actually belongs to a problem generally referred to as the *learning paradox* in psychology; in philosophy, it pertains to the original, first constitution of the sense (Sinnbildung) (Husserl, 1976). As Husserl realizes, before this initial constitution, the sense that is the *result* of some activity could not have been there as intention. The present study is designed as a contribution to the framing of a learning theory that takes into account such first constitution of sense, that is, a learning theory takes into account the perspective of learners who cannot see, and therefore intentionally orient to, the (subject matter *specific*) knowledge they are to acquire in formal learning settings. In the course of the present inquiry, it will turn out that the "inseparable interlacement" of "world—the world, of which man, of which we speak and can speak—and language, on the other hand" (Husserl, 1976, p. 370) is both resource and produced anew. To achieve this aim, I bring together empirical studies (rarely used in phenomenological works) and phenomenological analyses (rarely used in empirical studies in education).

#### The Learning Paradox and Attempts to Explain it

In this section, I use an analogy to specify the learning paradox and then review two general attempts at explaining it. Readers are encouraged to attend to their experience without preconceptions about *what* is happening to them; this experience serves as a counterpart to the third-person data and analyses presented in the subsequent section. In this way, readers engaging with this text reproduce the very methodological approach I have advocated – following Varela (1996) – to play off one another first- and third-person accounts of cognitive phenomena (e.g., Roth, 2012a).

For a Better Appreciation of the Learning Paradox: An Analogy

The learning paradox frequently is formulated with "knowledge" as its object. In this way, many educational practitioners and researchers appear to have difficulties really appreciating this contradiction (e.g., Roth, 2012c). I therefore created the following analogy for actively orienting towards something that one does not yet know. How do we intentionally look for something that we do not know that is there? How do we look for what the questioner sees when we participate in the children's game *Do you see what I see?* To make available to readers such an experience, I produced Figure 1: "Do you see what I see?"

Take moment looking at the figure before reading on; try monitoring how you are going about the task.

When you engage in this task, you probably look and you look. Your eyes are moving about. You may not see anything and eventually give up and move on. You will notice, above all, that you cannot intend seeing what I (author) see because you do not know what it is; and if you do see some figure, you likely take it to be the one I intended you to see. But once you do in fact see a figure (the one that I see or any other one), you will easily see it again.¹ The figure I (your teacher) intend you to find is in fact so easy for me to see that I am close to being embarrassed using a puzzle that is so easy. You, as I, may be astonished why others do not see it. Yet I know from experience with this and similar figures that even if I name the entity that can be seen in the figure, some members in the audience/readership will not see it.

It is evident that in this game, we are looking for the proverbial needle in the haystack without knowing that we are looking for a needle. When you do find something then it is in excess of what you did and could intend. The intentional eye

 $<sup>^{1}</sup>$  Elsewhere I provided a detailed phenomenological analysis and description of the work of the eye required to make a figure appear (Roth, 2012b).

movement, which makes the figure appear, could not emerge from the impression that the first seeing of a figure bequeaths to you. The hidden figure, which the knowing subject immediately sees again, cannot be the result of the passive impression attributed to the outside. Rather, if perception is active, then there has to be another sensation that arises from the eye movement without this movement showing itself. This is what allows us to envision the object, once we have seen it, even if it is not before the eyes, a phenomenon (visualization) that could not exist as the result of a pure impression.

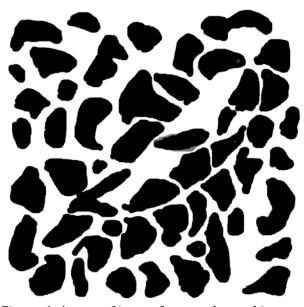


Figure 1. A type of image frequently used in psychology.

When you looking for something in Figure 1, the fact that it appears *here* in the pages of this journal provides readers with sufficient reason to look for something, some *thing*, which might not likely occur if the same patterns of splotches were to exist somewhere in our everyday world. In all likelihood, we would simply bypass it without wondering. That is, our gaze is already motivated to look for something in the visual domain, even though we do not know what it is. Therefore, when some figure does occur in visual perception, it is a good candidate for being an answer to the question, "Do you see what I see?" Moreover, my question itself allows readers to assume that there is some/thing to see; and this itself contributes toward modifying the orientation in the direction of this segment of the world (i.e., Figure 1). In addition, the question "Do you see what I see?" would make little sense if I could not assume that readers already were familiar with the thing that is hidden among the splotches.

I often get the response that a teacher – here me, the author – can tell *what* there is to find and thereby assist learners in their orientation. However, I know from experience that this is not so. In lecture presentations, I have told my audiences to find a . . ., and then produced the sound /ˈɔrsaɪnəs ˈɔrkə/ (which an Anglo-Saxon biologist hears as *Orcinus orca*). But in few cases does naming the entity "assist" the audience members in finding what I see – few people know what to do when they

hear the sound /ˈɔrsaɪnəs ˈɔrkə/. If you do in fact see what I see, then it will be an orca (killer) whale on its back towards the bottom right and with its belly toward the top left. (If you have not seen it, go to the appendix for an outline of the animal.) Once you have seen (the killer whale), try imagining it without actually looking at the figure. In all likelihood, you will not have difficulty imagining the killer whale, its orientation in the image as a whole. But if you do this imagination of the killer whale actively, then it cannot have been on the basis of a *passive* imprint. There would be no relation between the passive imprint left while looking at Figure 1 and the active production of the killer whale in and as imagination.

The preceding analysis shows that intentional orientation towards the orca, the thing I wanted you to learn and see, is the result of a process where the object initially is given to the learner in perception. After having received the image for a first time, readers (learners) can orient towards it and therefore see the orca again; and you can create a visual image for it in your mind's eye.

It is also possible that some readers have seen some other form while gazing at Figure 1. Formal experiments show that there may be all sorts of entities that people come to see when gazing at such figures. As research on perception in lecture demonstrations has shown, learners may see very different, even contradictory phenomena – such as the presence versus the absence of movement (Roth, McRobbie, Lucas, & Boutonné, 1997). Yet teachers tend to continue the lessons without ascertaining that everyone has actually seen and oriented to the same. Whatever students saw in this experiment, they took as *the* phenomenon that they were supposed to see. Thus, students who saw movement worked from the assumption that they were to see movement; and those students who saw no movement worked from the assumption that they were to have seen no movement. What they saw could not have been the result of an interpretation, for there was nothing initially that could be interpreted. They were confronted with a novel situation, such as readers were confronted with a novel situation in Figure 1. Prior to seeing the orca in Figure 1, the reader has no sign – a segment of the material continuum (Eco, 1984) – that *could* be interpreted. There are other accounts of the emergence of new knowledge that provide much more convincing explanations for such phenomena.

# The Ideology of Cognitive Construction

In educational learning theories, the verb "to construct" has come in vogue especially with the rise of the constructivist epistemology during the late 1980s. Thus, students of all ages now are said to be "constructing knowledge." With the subsequence upsurge of social constructivism came the idea that this knowledge construction first occurs in the social realm before learners appropriate the constructed object to make it their own. The verb "to construct" is transitive and therefore has an object (i.e.,  $subject \xrightarrow{action} object$ ); it does not exist as intransitive verb so that a sentence such as "The student constructs." is both grammatically incorrect and does not make sense. There will always be the question, "The student constructs what?" where the interrogative "what" is in pronominal form, standing in

for the missing object. We may say of the carpenter that she *constructs* a house, because the end result already exists for her as possibility in (individual, collective) consciousness. She can actually compare what she currently has to what she intends to achieve and, thereby, engage in a process that psychologists call metacognition. But we hardly call *constructing* the operations of someone how does not know what he is doing.

There exists in the constructivist literature one attempt to explain the learning paradox. This attempt centers on the Piagetian construct of the *scheme* (von Glasersfeld, 2001). However, in the tripart structure of the scheme—perceived situation, activity, expected result—intentionality is already built in, as the activity is oriented toward an anticipated result. This attempt, therefore, uses the explanandum as the explanans by presupposing "the very movement that makes doing things with objects possible" (Sheets-Johnstone, 2011, p. 227). But from the perspective of the learner, the (unknown) learning object cannot be the expected result, and the schema remains something to be explained once the question about the origin of intentionality has been settled. Piaget's entire work is built on the idea that human beings construct their minds, from the very beginning, based on schemas. He does not question where the schemas come from, making all schemabased theories (including embodiment theories) fall short of an evolutionary sound approach to the question of knowledge (Sheets-Johnstone, 2009). Piaget also suggests that the "elementary forms of the habit grow out of an assimilation of new elements to previous schemata which are in essence reflex schemata" (Piaget, 1950, p. 100) and maintains that the sensori-motor intelligence "organizes the real by constructing, in its very functioning, the great categories of action that are the schemes of the permanent object, space, time and causality, the substructures of the future corresponding notions" (Piaget & Inhelder, 1966, p. 14–15). How, we may ask, does the child, having only experienced Heraclitean flux, construct object permanence if it does not know or can conceive of permanence? Even though Piaget was at the verge of recognizing the primacy of movement (Sheets-Johnstone, 2011), he actually failed to do so because his constructivist account does not take into account the productivity that comes with pregnancy [prégnance] (Merleau-Ponty, 1964b). In salience, "the form has arrived at itself, that is itself, that poses itself by its own means, is the equivalent of the cause in itself" (p. 258). What Piaget fails to see. according to Merleau-Ponty, is the fact that the "body obeys pregnancy, responds to it, it is what renders itself to it [salience]" (p. 259). The philosopher suggests that it is precisely this aspect of a motricity-implying salience that lies completely outside of the alternatives available to the psychologist Piaget.

### Toward a Phenomenological Account

I am not aware of a phenomenological account of the open-endedness of learning events and experiences generally and the learning paradox specifically (other than my own work). However, an interesting account of the emergence of a new form of knowledge and intention was provided in pragmatic philosophy (Rorty, 1989). Here, (scientific, artistic) revolutions amount to the emergence of new forms of expressive (linguistic, artistic) means. These new means are not constructed

because these come with new purposes, making possible the explanation of the latter. This is, as Rorty says, where the breakdown of the constructivist metaphor occurs, because "the craftsman typically knows what job he needs to do before picking or inventing tools with which to do it" (p. 12). The one inventing a new means of expression—Galileo, Yeats, and Hegel are used as examples, individuals engaged in *poiesis* in general—"is typically unable to make clear exactly what it is that he wants to do before developing the language in which he succeeds in doing it" (p. 13). Also concerned with Galileo's accomplishment, Husserl (1976) notes: "for him, the creator of the conception that made physics possible in the first place, that which became self-evident through his deed could not already have been selfevident" (p. 35, emphasis added). This fits with a phenomenological account of the painter and his/her painting: "The unseen that the painter is looking for remains, up to the point of its ultimate appearance, unforeseen – unseen thus unforeseen. The unseen, or the unforeseen par excellence" (Marion, 1996, p. 54, emphasis added). That is, what the painter is looking for is unseen, therefore unforeseen, and, as a result. could not have motivated his/her gaze: "the new is graspable only after the fact" (Waldenfels, 2006, p. 65). Thus, "one is not able in any way to foresee the newly visible in terms of its unseen, by definition invisible" (Marion, 1996, p. 54). Somehow "the visible must itself provoke the aim [visée] that renders it accessible... The visible *precedes* the aim [visée]: it is what must be rendered visable by us, since we did not expect it" (p. 62, emphasis added).<sup>2</sup> What emerges in the phenomenalization of the newly visible is not foreseeable on the basis of the already visible. It is the newly created—the poem, painting, language—that brings its intention (aim) to life, and, with it, the modality of perception related to it. In phenomenological philosophy, the notions of "passive (pre-) constitution [passive (Vor-) Konstitution]" (Husserl, 1939b), "givenness [donation]" (Marion, 1997), and "originary passivity [passivité originaire]" (Henry, 2000) provide categories for answering the open questions that arise from a critical analysis of the constructivist approach to learning something new. In learning, the individual transcends intentionality and rule-based "construction," and this transcendence occurs in responding to a summons (Roth, 2013b). But, as I show in the analysis below. following a thick description of one classroom event, learning is more than passive impression.

Husserl (1973) frequently is credited for articulating the complex relationship of activity and passivity in perception—though Maine de Biran (1859a, 1859b) had anticipated him by more than a century (e.g., Derrida, 2000). At a first level, objects, such as the killer whale in Figure 1, are "passively given [passiv vorgegeben]" to and in experience. But with the object in hand (sound in ear), we are not oriented toward its presence but rather toward the object as unity that presents itself in the flux of experience. This active orientation toward and grip of the continuity of the object in a continuously changing now occurs against a passive coincidence of the one (object) with itself. Thus, the activity of perception "has a complex structure, which is due to the law-like regularity of a constitution of living duration with its

<sup>&</sup>lt;sup>2</sup> "Visable" is a neologism that translators of Marion's work have formed by introducing the corresponding French neologism formed on the verb *viser*, to aim.

own passivity prior to all activity" (Husserl, 1939, p. 118). This passivity is a law of activity itself. There is therefore a double passivity: one that precedes active perception in the givenness with the (unchanging or changing, resting or moving) object [Gegenstand], and another, objectifying one layered above the first that (co-) thematizes objects [Gegenstände]. It is only because this "active-passive keeping-ingrip" (p. 119) that the object can be grasped in/by perception as an enduring one. With this emergence of the object arises the emergence of the intentional orientation towards it. Husserl's insights have been taken up and developed in recent years by phenomenological philosophers—including M. Henry and J.-L. Marion—on whose work I draw in the analyses and discussion of the empirical work in mathematics lessons reported on in the next section.

# How a Rectangular Solid Comes to Emerge from the Movements of the Hands

In the thinking of the body, the body forces thinking always farther, always *too* far . . . This is why it makes no sense to talk about body and thinking apart from each other. (Nancy, 2006, p. 34)

In educational theorizing, the mind and the representations it is said to use tend to be privileged. But, following the quotation from Nancy, there is no sense to talking about body and thinking apart from each other. I therefore present in this section an account of a classroom episode from a second-grade mathematics curriculum specially designed to allow students physical explorations of objects as a context for learning certain aspects of three-dimensional geometry. The curriculum was in advance of what major theories of child development as age appropriate including Piaget's theory on the development of geometrical knowledge (e.g., Roth, 2011; Roth & Thom, 2009). Three researchers with two cameras were present during each daily 70-minute lesson of the 15-day unit. These cameras followed individual groups of students when there when students completed tasks in small groups, and they recorded whole-class sessions from different angles to triangulate actions and sound. The curriculum, designed together by the regular teacher and one of the researchers, was premised on the idea that bodily experiences are constitutive of geometrical knowledge. Among the many tasks, there was one in which children were given mystery objects located inside cardboard boxes, which they could only feel with one hand by entering through a hole but that they could not see. Working in small groups, the children were asked to use plasticine for building models of the mystery object. That is, they were to shape the plasticine so that it resembled the mystery object without actually having seen it. The task is structurally equivalent to the one featuring the killer whale, because readers explore the figure in the same way that the children use their hands and fingures to explore the mystery object. In the following, we witness the events within one of the groups, including Jane, Melissa, and Sylvia. The description pays particular attention to Melissa.

Melissa, after feeling out the mystery object for a while (turn 1), begins to shape what will eventually look like a cube. She announces that the mystery object is "a cube" (turn 2). But Jane and Sylvie object (turns 4, 6, 7). Yet Melissa insists, and provides a reason: She checked the sides in a way that she demonstrates by means of gestures: Grasping her cubical model with the left hand, she uses an unchanging caliper-like configuration of the right thumb and index finger and holds it against three different, orthogonal edges of the cube (Figure 2).

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Fragment 1
001 S: i feel it ((2:46))
002 M: feel it eh? i have felt its a cu:be () <<ff>hU::::::ge.>
003     (1) ((J grimaces, questioningly?))
004 S: no its not a cube ((shakes head while rH in box))
005     ()
006 J: i didnt feel a cube.
007 S: me either.
008     (3)
009 M: i did. (1) i checked the sides like that. ((caliper grip on each of 3 sides, Figure 2))
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Figure 2. Melissa explains why the object is a cube. Using her model, she describes that all the edges of the mystery object have the same length.

She does not merely describe what she feels but also provides a description of what she has done. In so doing, she in fact offers a set of instructions for what one has to do to feel the mystery object in the way she does. This is apparent when she moves her hand and arm through specific configurations (Figure 3). But deploying such movements in order to symbolize movements that have occurred before requires the capacity for and memory of these movements. These movements are the same as when she was feeling out the mystery object, though now they have symbolizing function whereas the original movements had epistemic and ergotic (i.e., work-oriented) function (Roth, 2003). In both instances represented in Figures 2 and 3, therefore, the illustration of what she has done is produced with the same hand: the representing movement is in fact indistinguishable from a presenting movement. The cube exists not primarily in the form of some abstract (geometric) idea but in the form of successive movements that rotate the cube and associated perceptions. In and with these movements, the sameness of the different sides is

instructably made visible in the here-and-now of this situation. They are necessary and sufficient "to warrant the judgment 'here is a cube'" (Merleau-Ponty, 1945, p. 235); and this judgment requires the "displacements to be located in an objective space" (p. 235), which they are in and as of the demonstration and instruction.



Figure 3. Movements of hand and arm that symbolize how the hand moves around the mystery object that leads to the feel of it being a cube.

Sylvia and Jane disagree. While Melissa is again feeling out the mystery object (Figure 4, right), Sylvia suggests to "feel the sides like this," while rubbing the palms of her hands (Figure 4, left). Melissa apparently moves around the box and about the object, looking at Sylvia who is giving her instructions on how to feel the sides (of the mystery object).



Figure 4. Sylvia (left) shows Melissa (right), who is feeling the object, what its shape is. (Artistic rendering)

In the course of the next 16 minutes, Melissa repeatedly explains that the three (orthogonal) faces of the mystery object are the same. While doing so, she uses a gesture whereby an entire edge of her cubical model is grasped between thumb and index finger (Figure 2), producing what we might denote as "caliper configuration." In an exchange with the graduate student filming the group at work, Melissa said

that it "has the same edges, and that feels square, it has the same [faces]. She insisted, "I still think it's [mystery object] still the same as mine [model]."

Sylvia and Jane do not just attempt to convince Melissa by restating what the mystery object feels like or that it resembles their own models rather than what Melissa had formed. They instead provide instructions for what to attend to while feeling the mystery object. For example, Sylvia makes a two-handed gesture to illustrate how the mystery object would fit between the two hands (Figure 5, left) and then acts out for Melissa how she has to use her fingers to feel that there is little height to the mystery object (Figure 5, right) relative to the flatness of the other dimension that she has just illustrated.



Figure 5. Sylvia exhibits the shape of the object by showing how it would fit between the palms of both hands (left). She then shows Melissa how she has to use her fingers to feel that the mystery object is flat.

In one instance, a teacher comes to the table where the three girls work to talk to them about their progress. The teacher begins with the question whether they had come to an agreement about the mystery object; and she then invites the three girls to state their ideas. Melissa says it is a cube; Jane categorizes hers as a "rectangular prism"; and Sylvia says that she has forgotten the name (there is *something*, i.e., some *thing*, though it does not yet have a name). After a pause, the teacher says that it "will be a rectangular prism *as well*," and thereby categorizes Jane's model and Sylvia's model as belonging to the same class of geometrical objects. The teacher places the three plasticine models in front of Melissa and herself, grouping the two rectangular prisms together and a bit separate from Melissa's cube (Figure 6). She says, "this is a cube and these are rectangular prisms." But the teacher does not accept the different solutions to the problem; instead she insists that the three girls have to arrive at a consensus about what that *one* mystery object is.



Figure 6. With Sylvia's, Jane's, and Melissa's models on the table, the teacher (hand to the left) talks to the latter to get her to reconsider the model she has built.

Despite this exchange, and pursuing the effort to convince each other, the three continue to talk about the nature of the mystery object. Melissa persists, maintaining that there is a cube. At one point, while Jane reaches into the box, Melissa offers her cube to Jane and asks her to feel it simultaneously with feeling the mystery object. Jane turns the cube while apparently doing the same within the box; she turns the cube and again places her hand on top; and she does so for a third time (Figure 7). Jane turns to Melissa and asserts, "it does not feel the same."



Figure 7. Jane feels the top of the cube while her right hand is in the box apparently doing the same. She turns the cube and again touches the top; and then she does the same with the third side.

The teacher eventually asks the children to bring the task to a close so that the whole class can come together for a discussion. Sylvia announces that they are not done yet and need some more time. She proposes to Melissa to feel the mystery object in one hand while also touching a model with the other hand (turn 225). Melissa asks Jane about using the latter's model to feel it out "to see if it's the same" (turn 226). While Melissa's right hand moves about in the box (Figure 8, left), we observe the left hand touch Jane's rectangular prism. Jane places her palm on the rectangular prism in Melissa's hand (Figure 8, right), instructing her which part to feel while doing the same with the (mystery object) in the box (turn 230). Melissa then feels the prism on one side, rotates and feels it on the other side; she rotates and feels it out three more times (Figure 9). While doing so, she continuously gazes intently toward her left hand and the rectangular prism in it (Figure 8, left).



Figure 8. Melissa feels and turns Jane's model in her left hand while doing the same with the hidden mystery object in the box with her right hand (left). Jane places her palm on the large rectangle part of the model and instructs Melissa how to feel (right).

#### Fragment 2

- 225 S: okay. look what you can do. you can take your object and () and feel like with that one ((hits the box with her left hand)) and take it as you feel like its really the same. ((Shakes each hand separately))
- 226 M: ((M reaches for J's model)) can i feel with this to see its the same? ((Reaches into box with her right hand, holds J's object in her left hand, Figure 8))
- 227 (2)
- 228 S: no i (want?) you to take here in your hand ((holds her model)) ((turns over J's model))
- 229 (3)
- 230 J: feel this part ((touches square face of the model)) and go in here ((touches the box)).
- 231 M: ((Places model on narrow edge, turns it over 90 degrees at a time, touches with index finger; J watches))
- 232 (11)
- 233 M: oh oh ((lifts gaze))

After having rotated and felt out Jane's rectangular prism four times, apparently doing so, as instructed, with the mystery object inside the box, Melissa produces twice the interjection "Oh" and then puckers her lips; her face turns into an expression as if she has been caught doing something inappropriate. "I am making it else," she announces and then begins to reshape her plasticine into what will become a rectangular prism. Now, after having reached into the box and after having felt out the mystery object eight times, for a total of 3 minutes and 10 seconds, the object that the teacher had intended for her to model has emerged from

the movements of her hands, the objects' movement in her hands, and the sensations of self-movement and resistance that the (inside, outside) objects offered to the movement. Although she initially had felt the mystery object in a way consistent with her cubical plasticine model, there was a sufficient level of pregnancy to her tact that allowed something very different to emerge.



Figure 9. Melissa feels out the rectangular prism on one side, then rotates it and feels out another side, and again, and again.

Pregnancy: An Analysis of the Excess of Intuition over Intention

At the beginning of this episode, Melissa, as the other children, was literally in the dark about what she would find out and learning. Her right hand was groping around, without visual access, to find out about the nature of its contents. She could not intend what she will have learned once everything is said and done. The mystery object will have given itself to her in the way the killer whale gave itself (or not) to the reader while gazing at Figure 1. To know, Melissa had to feel out the material, that is, move her fingers and palm over the object to see what the movement yields and what the material gives of itself (Fr. *pour voir ce que cela donne*). What gave itself initially was a «cube».<sup>3</sup> She could know what she had in her hand only after the exploration, opening up a gap between exploring (*Sein, l'Être, Being*) and knowing what will have been explored (*Seiendes, étants, beings*). Eventually, even before trying to build her model, she already gives it a name: "It's a cube." That is, she already knew her way around the world and its language games to associate the feel in/of her hand with something of a similar cubical form as other entities that populate her everyday lifeworld.

Educators and philosophers might be tempted to say that this case is consistent with the Sapir-Whorf hypothesis, whereby existing linguistic categories shape human perception (e.g., Lakoff, 1987). Others might argue that it is an example of existing theories driving the nature and content of observation (Hanson, 1965). But this would be a hasty conclusion, for right up to the instant that the mystery object gives itself in and to her hand, she has nothing to go by for using a linguistic or perceptual filter that makes it a cube (unless cubes are the only thing she knew, in which case everything would have been a cube). Right up to the moment that the mystery object was giving itself as what it was—so that Melissa *could* in fact give it a name—there was not even *something* (some thing). Constructivists tend to suggest that Melissa interpreted *what* she had in hand. Thus, to interpret as a transitive verb

<sup>&</sup>lt;sup>3</sup> I use chevrons to distinguish a thing, e.g., «cube» from its name, "cube," and, thereby, highlight the apophantic relation: something *as* something (Heidegger, 1927/1977).

requires an object. But, here, this object as such, with geometrical properties that could be talked about, did not yet exist until it actually had emerged, here literally from the dark and in her hand. What she "sees" and expresses in the cube is "a conditioned thought, it is born 'at the occasion' of that which arrives in the body, it is 'excited' to think through it" (Merleau-Ponty, 1964a, p. 51). Melissa was in the same situation as the reader staring at Figure 1, who could not construct or interpret a thing until after the object had given itself.

At this point, Melissa was convinced that what she had felt was consistent with a kind of cultural object known under the name of cube. She did not just believe but had evidence in the form of descriptive properties of a thing that can be felt and that her telling instructed others to feel when reaching into the box. In her account, there were three kinds of sides, with edges of the same length. She exhibited those invisible but sensible properties by using her model, turning it about, and holding the thumb and index finger in a caliper configuration that suggested a measurement. Melissa was convinced of the mystery object's cubical form right up to the instant marked by surprise—"Oh, oh" (turn 233) together with a facial expression that culturally competent individuals see as surprise—that a new and different kind of object had emerged into her hand. Again, intentionally oriented towards a cube, she did not intend this new and different form of the object. But the mystery object gave itself in a new way; and Melissa expressed this new way in a reconfigured plasticine material, as a rectangular prism with a pair of large faces separated by two pairs of narrow sides. There was therefore a qualitative shift in the experience, where an intentional orientation towards a cube was overturned and overturned itself to become the intentional orientation toward a rectangular prism. This new experience arose from her tact, which was, without knowing itself as such, already pregnant with this new form. In fact, there is Husserlian continuity in the materiality of the mystery object, in its active-passive constitution, and discontinuity of form. It is in and through her grasping feeling that she discovered in her hands the very object others had wanted to tell her about. It was in and through her feel right before and at the moment of surprise that she came to have in hand the rectangular prism that allowed her to discover the relevance of the instructions to see (e.g., Figure 3, left: Figure 4, left) and move/feel (Figure 4, right; Figure 6). That is, these descriptions that Jane and Sylvia had provided made sense *after* the mystery object had given itself as a «rectangular prism».

Before the first («cube») and second gift («rectangular prism»), there was a lot of talk intended to convince Melissa that she was wrong in asserting the mystery object to have cubical shape. However, all the previous verbal descriptions and perceptions of the models others had built did not convinced Melissa of the incorrectness of her own sensations on the basis of which she had built a cube as a model of the mystery object. Now, as she turned Jane's model over and over to feel it out with her left hand while doing the same with the mystery object in her right hand, a new shape suddenly emerged. The relevance of others' descriptions and models could only emerge from her actions and sensations in a manner that was not anticipated. She could know what the others were talking about when she had this what in hand. But because she did not know it, she could not intentionally orient toward the «rectangular prism» until it was in her hand as a new sensation, which

emerged in excess of what she had perceived and, at the time, could perceive. Inside and outside the box, Melissa moved the mystery and model objects, respectively; and their (non-) correspondence emerged from the associate feel and feelings. The particular shapes («cube», «rectangular prism») emerged from and above a texture of movements, which constituted their invisible ground. These first phenomena of perception were passive, "because they are absolutely foreign to the activity of the will or the me" (Maine de Biran, 1859b, p. 28). Even the specific form of the activity was foreign, as readers could notice in the qualitative shift in the experience of Figure 1 from a mass of splotches to a killer whale against an indifferent ground. To be able to be repeated (e.g., to visualize it), any movement had to have arisen from a self-affection, which in turn made it possible to recognize the repetition in the absence of intention. No *me* would be possible and recognize itself if it were not for the immediate, internal sensation or apperception of the flesh (*Leib*, *chair*).

Between the first («cube») and second emergence of form («rectangular prism»), Melissa was presented with talk inviting her to see and interpret differently what she was feeling. She was encouraged to reconsider the nature of the object in the light of the experiences of others, expressed in the material forms of their models. She was invited to reconsider what she had felt in the explicit confrontation of her model with those that Sylvia and Jane had shaped (Figure 5). Although there were perceptible differences, and although Sylvia considered her model to be (slightly) different from the one Jane had formed, the teacher explicitly named (and therefore classified) these as the same. The teacher not only said to Sylvia that hers "will be a rectangular prism as well [as Jane's]," but she also visually grouped the models by collocating Sylvia's and Jane's separately from Melissa's. The teacher insisted that they come to an agreement as to whether the mystery object felt more like Sylvia's or Jane's (while holding up each of the two models) or more like Melissa's. The teacher emphasized that there was only *one* mystery object so that there could not be different models. She said, "as a group you have to say, 'We think it is . . . '."

Despite the insistent calls on Melissa to change her position on the nature of the mystery object—necessitated by the task condition to come to an agreement about what the *one* thing in the box is—the one she perceived resisted being felt as something else: as what Jane and the teacher named a "rectangular prism," as what Jane and Sylvia perceptually articulated in their plasticine models, as what Sylvia exhibited to see in her iconic hand gestures and in her specific instructions of how to go about feeling the mystery object.

It was not as if Melissa had not been trying. Substantially more frequent and longer than Sylvia and Jane, she had reached into the box (as in Figures 4, right, and 8, left) to move the object in her hand (as shown in Figure 3) and had felt it out. She had made apparent her efforts to follow the instructions Jane and Sylvia provided to guide her explorations. But right up until the end of the eighth time she had reached into the box, the mystery object first became and then remained what she expressly articulated as cube by shaping her plasticine mass into a cubiform shape. In this shaping, her right hand, the one that has moved about the box and felt out the mystery object, also came to move about, feel out, and give shape to the mass of plasticine. Although she apparently changed her mind in the end, initially and for the longest amount of time, what would turn out to be a «rectangular prism» (in the

right hand) initially did feel like the plasticine cube in the left hand. But it was not her mind that had changed the thing. The mystery object had shown itself in a different way; and when it showed itself differently than what it initially appeared to be. Melissa announced this novel phenomenalization with and as a surprise.

#### Discussion

This study was designed as an investigation into the learning paradox and the emergence of intentional orientation towards knowledge. Methodologically, the study draws on a phenomenological approach to address a problem in the cognitive domain. Both the case of the figure containing a killer whale and the empirical study of learning in a second-grade mathematics class exemplify that there is something fundamentally wrong with the constructivist account of learning: it presupposes the intentional orientation towards a learning object that is only the outcome of the learning process. The origins of those very schemas that are said to explain the learning paradox in constructivist accounts require explanation, because the intentional orientation already is present in them.

Emergence of Tangible Forms: Constructivist and Phenomenological Accounts

The permanence of colors and objects therefore is *not constructed* by intelligence . . . (Merleau-Ponty, 1964a, p. 65, emphasis added)

In what would be her first attempt, Melissa named, gesturally described, and modeled the mystery object (as) a cube. In a constructivist account, she might be said to incorrectly "interpret" the object, or to have a misconception about threedimensional objects. Thus, (related to two-dimensional geometrical objects) children are said to go through a stage where they recognize the things they touch but are "at first unable to abstract shape for want of sufficient exploration" (Piaget & Inhelder, 1967, p. 22). This has been attributed to a "want of sufficient" and "inadequate exploration." As a result of "the lack of exploration" there is "a general deficiency in perceptual activity itself" (p. 24), which is taken to mean "that the child's perceptions are still passive or static instead of being integrated in a system of sensori-motor co-ordinations tending to bind them together" (p. 24). The same kind of language is used by the authors to describe the emergence of threedimensional geometrical concepts. What is required, according to Piaget, is a passage from one centration to another, a process of decentration. But as we can see in the movements of Melissa, this process is already present. We can see how the hand and arm movement is from centration to centration, where the mystery object is felt in a specific way depending on its orientation. She is right at the «cube» itself: and it manifests itself in what she feels and sees. The mystery object initially is a «cube», denoted by the word ("cube") and the model. It is so not because of a *lack* of exploration—of which there were a considerable number over an extended period of time. But the very process of centration requires an object, and it, as material and form (shape), is given rather than intended. Moreover, as Merleau-Ponty (1996) notes, Piaget's descriptions are quasi-empiricist such that the "passage to a higher

type of perception and conduct could be simply explained by a more complete and more exact registering of experience" (p. 104). Piaget does so when development in fact constitutes a "reorganization of the perceptual field and the *advent* of clearly articulated forms" (p. 104), which, here, are the «cube» and the «rectangular prism».

Piaget also notes that the child's image of the three-dimensional object "is a product of imitation. It is in fact, an internalized imitation, one that can be made without resort to external gestures" (Piaget & Inhelder, 1967, p. 40). It is quite apparent that the object that Melissa reproduced on the outside of the box—we do not see or know anything that might occur in her mind—was not the product of an imitation, just as the killer whale emerged for the reader not as a product of mimesis, at least not the product of an intentional one. This is so because readers could imitate what they did not see but eventually came to be a killer whale. Thus, what we see when looking at a painting, the image, is not the result of an imitation. Rather, it is an "entering and exiting, that is what makes the image: appearing and disappearing. Not first representing, but first being or making 'a time, une fois' a first and last time, the time [temps] of making or taking an image, the time of time itself, which opens the eyes" (Nancy, 2005, p. 98). The imitation would be a second presentation, a representation of something that precedes it. In the phenomenological approach, the visible image or tactile figure is born into presence of the ectype (*Abbild*) without and in the absence of an archetype (*Urbild*). There is also an auto-affection of and associated with the movement, which is inseparable from the experience of the passive reception. It is this auto-affection (of life) from which the passive constitution of objects arises (Henry, 2000). But what is received could not be the source of the originary memory. This is related to the auto-affection of the sensing hand itself, which thereby is enabled to remember, without any mediation, itself in its own replication. Melissa could intentionally show others what she had done inside the box (Figures 2, 3) because her body knew how to move in the way we see.4

Piaget's position, though based on careful observational studies of children, therefore, does not help us understand learning and development or change in perception. He only articulates a lack rather than a developmental movement. Development is not the result of a lack being reduced, in an apparent teleological process whereby a void comes to be filled (i.e., the child developing *towards* adult rationality [e.g., Meyer-Drawe, 1986]). Rather, what we need to understand is the fact that from the perspective of Melissa (and children like her) there is not a lack but a plenitude of experience that exceeds intention. Our actions—e.g., the reader's eye movements in gazing at Figure 1—are pregnant with new forms that exceed their intentions. What we need to understand is how out of any plenitude a new plenitude arises in/as an apparent excess of intuition over intention. Even and especially in the second phenomenalization of the mystery object as a «rectangular prism» there was an excess of intuition over the intention that was initially oriented intentionally to perceiving a cubiform object. She finds *that* object again in the same

<sup>&</sup>lt;sup>4</sup> Some readers might want to suggest that we can want (intend) something without actually being able to do it (e.g., jump 2.5 m). But then we would not actually observe the actions that realize the intention, its praxis, which is never symbolic.

way that we find and see the killer whale again without trouble once we have seen in it for a first time. During the second qualitative shift of perception, a new object phenomenalized itself *even though* and *despite* the initial intentional orientation towards a cube. As soon as Melissa had felt the mystery object as «rectangular prism», a path was laid towards remembering it as such (see next subsection). How something felt to be a «cube» can all of a sudden be felt as a «rectangular prism» is itself a problematic to be addressed by research.

In the embodiment literature, similar to the constructivist approach, hand-arm movements related to specific mathematical objects (e.g., a line, curve) are theorized in terms of schema theory (e.g., Núñez, 2009). The empirical data in this study show, however, consistent with a phenomenological perspective, that there is a primacy of movement over all other, sign-mediated forms of knowing. It is not, however, because of *schemas* enacted by the body. Rather, the very possibility of intentional movement described by a schema, and the transcendental forms of knowing built on these, has its roots in originary self-movement that does not presuppose intentionality (Henry, 2000; Maine de Biran, 1859a, b; Sheets-Johnstone, 2011).

In a remarkable analysis that anticipates similar inquiries in phenomenological philosophy by two centuries, Maine de Biran (1859b) critiques the constructivist (schema-theoretic) account of the emergence of form in/from touch. When the hand explores a surface of an object, there are two aspects: one related to the voluntary movement of the hand, the other related to the resistance offered to the hand on the part of the object. The two aspects are simultaneous. The double aspect related to the hand movement requires the consent of two parts of the same body, one of which has to be voluntarily movable. If we were to discover forms in this manner whether the resistance is experienced as proper or foreign of the body—the intelligent being would appear to be the basis of geometry constructed because of capacities within itself. Maine de Biran suggests that when the feeling hand is stopped in its movement, the individual knows or feels immediately an obstacle on the outside of me, a cause outside of its will. The resistance to the free movement of hand or eye, arising within and from the organ itself, is at the origin of the ability to remember, imagine, and, therefore, to intend. The organ follows the object. *entrained* in its movement by the object, and in this movement experiences resistance to itself in an auto-affection of the flesh. This movement and the associated experienced resistance (which diminishes with habitude), the kinesthesia, are at the origin of the re/cognition of the object (Sheets-Johnstone, 2009).

It is this self-affection that allows us to understand the episode involving Melissa. If cognizing the «cube» had been a simple issue of passive reception, then she should have been in a situation to recognize the resemblance of mystery object and those that her peers showed her. She should have been able to follow their instructions and perceive what they had perceived. If this was not the case then it was that in and as result of her actions, the «rectangular prism» did not emerge from the auto-affection related to the exploring hand-arm movements. But once the hand and arm constituted the appropriate movement, then that new object emerged in and from it.

How do the tangible forms that the children turned their attention to emerge? Maine de Biran (1859b) describes what he calls the memory of tangible forms in and through movement. Taking the mystery object (the children's, or Figure 1), the hand (eyes) explores it. What the hand remembers, however, are not the external forms it feels but its own movements in the form of "I can." Moreover, there is no memory of movement (e.g., in the form of a schema, representation, or sign) that then brings about the movement. Rather, it is in and as of these movements themselves that memory exists—kinetic dynamics and kinesthetic memory are two aspects of the same phenomenon (e.g., Sheets-Johnstone, 2009). This is why the verbal instructions and descriptions that Melissa hears cannot bring about the actions and perceptions in her inquiry. Thus, in the absence of the object, we can relive in an internal sense the traces that the movement has left. "All movements executed by the hand, all positions that it has taken by traveling over the solid, can be repeated voluntarily in the absence of the object" (Maine de Biran, 1959b, p. 147). More importantly, these movements are the signs pointing backward to the elementary perceptions and the primary qualities felt in the hand. These signs, "therefore, could serve to recall the ideas, and this recall, effectuated by means of the available signs, constitute the memory properly speaking" (p. 147). In his development of a phenomenology of the flesh, Henry (2000) refers directly to Maine de Biran's description and provides his own example, the memory associated with a statuette that he was given. It is by recalling the act of taking the statuette in, traveling over it with, his hands that he not only remembers its shape but also the sense of beauty associated with it. He concludes: "It is the memory of a body that remembers each time the manner of taking the statuette, to move towards to take hold of it" (p. 206, original emphasis).

In the present situation, we see that the situation is not simple, for Melissa repeatedly took the hidden object into her hand, felt it, and turned it about to be able to build a model of it. This model, too, she held in her hands. In fact, when asked to explain why she thinks the mystery object to be a cube, she took her cubiform model into the hands and showed, by means of a constant caliper configuration of her right hand, that the different edges of the object were of the same length (Figure 3). She maintained her position on numerous occasions during this lesson, despite repeated attempts on the part of Sylvia and Jane to convince her otherwise, and despite the attempts on the part of the graduate student and teacher interacting with her and requesting explications. That is, an abyss opened between kinesthetic memory of the past and present kinesthesis. This is particularly interesting in light of the fact that Melissa's right hand and arm moved in the way she also told to have done within the box—and where she had felt a cube. She not only said that it was a cube but she built one, so that both the movement and the feel existed in the presence outside the box. The body itself was doing again what it had been doing inside (Figure 3). Such a "doing again" recalls an earlier experience, but it is the idea in the present tense rather than as one represented in a different modality. This, too, points us to an intentionality, for to be able to produce the hand-arm movements as part of the communicative act, the body itself has to recall the movement as an "I

can." It is only when the body already knows to move in a particular way that there can be an intention to do so in the movement we observe. It is precisely in this way that ergotic, work-related movements become symbolic movements (Roth & Lawless, 2002a, b, c). The first movements over the mystery object, which came forth without intention, were the very material from which intention emerged (e.g., the intentions observable in her gestures). The case study shows, however, that there is an obstacle or resistance to this sort of memory. Here, there was a delay between the first and second instance—i.e., the movement and sensation concerning the mystery object *inside* the box followed by movement and sensation of the model object *outside* the box—so that what turns out to be different is actually feeling the same.

#### Phenomenalization of the New

Learning means coming to know something that one has not known before, and which therefore is of the realm of the alien. Because this subsequently known does not exist before, the learner cannot intentionally orient towards it. The learning object, the outcome of the learning process, cannot be used (teleologically) to explain what the learner does. It is only after the fact—when they actually know what they were intended to come to know—that learners can articulate what they were oriented to (the killer whale, the rectangular prism). In the course of learning, therefore, the knowledge and the learning intention are unseen and unforeseen: these exist in the future perfect tense and arrive (as surprise). Whereas phenomenological philosophers have been attuned to the role of passivity in the first constitution of knowledge, educators generally do not attend to the essence of learning (acquiring the new): how the absolutely new phenomenalizes itself on the ground of experiences that may be antithetical to it. It is the same kind of question that Husserl (1976) pursued with his interest in understanding the origin of geometry, which has emerged from the everyday experiences of the Greek that were not geometrical at all. In fact, in their experiences, the  $\kappa \dot{\nu} \beta o \varsigma$  (kubos, cube),  $\pi \nu \rho \alpha \mu i \varsigma$  (pyramis, pyramid), or κύλινδρος (kulindros, cylinder) have been everyday objects that only in their unobtainable limit are the objects of geometry as a science.

In the preceding analysis of the empirical data, I note that Melissa does not come to understand the mystery object because of the descriptions and instructions that others provide. Instead, the relevance of these descriptions and instructions arrives when the mystery object gives itself to Melissa's exploring hand as a «rectangular prism». That is, the traditional account of how learners learn is turned on its head. It is because Melissa already acts and perceives consistent with the «rectangular prism», even though unintended, that instructions and descriptions come to make sense. As Nancy (2006) suggests, the sense of the body becomes the body of sense.

Piaget and modern-day constructivists characterize children's knowing negatively: as lack, deficit (with respect to adult), or deviance (e.g., "misconception," "alternative framework," or "naïve conception") (cf. Merleau-Ponty, 1964b; Meyer-

<sup>&</sup>lt;sup>5</sup> This requires us to think the person not only as the subject of an (learning) event but also as the *advenant*, the one to whom something advenes (arrives) (Roth, 2013a).

Drawe, 1986). However, when we observe children in action—such as when Jane. Melissa, and Sylvia explore their mystery object and build models thereof—there is nothing that should lead us to the sense of a lack. Learning and development is not the filling of a conceptual or cognitive void, but an engagement with the world as given to the children in their experience. The «cube» in Melissa's hand is no less real than the «rectangular prism» in Jane's. Moreover, Melissa's hand, too, comes to experience a «rectangular prism». But it does so not because of a lack. Rather, the empirical materials show that there is an excess of the new (plenitude) over the old plenitude, which, up to that point, was sufficient in and to itself. Learning and development then is the result of the realization of a possibility (perceiving the mystery object as a «cube» from not perceiving it at all; perceiving the mystery object as a «rectangular prism» when having perceived it as a «cube» so far). The possibility of this new is already contained in what was before and emerges from it, thereby also sublating it. Sublating, in the dialectical tradition, means both doing away with and keeping something. In the constructivist literature, we can frequently read that misconceptions (here the «cube») have to be eradicated. In a phenomenological appreciation of the learning process, on the other hand, the sense-giving transition that corresponds to the phenomenalization of the new is sedimented in what is emerging; but as sedimentation, it also constitutes a forgetting (Husserl, 1976). We know from a subsequent conversation with the teacher that she wanted Melissa to "construct" on her own a rectangular prism (ideally, as embodied mental structure and materially, in plasticine). Constructivist researchers, much in the way Piaget had done, focus on children's (learners') errors and faults, the *misconceptions* that they have about some aspect of the world presented to them in lessons or clinical interviews. However, the «cube» arises for Melissa not as the result of a misinterpretation or a default. It arises in her originary perception, in and as sensual experience, intimately joined to her exploratory handarm movements. Those very capacities that lead to the initial emergence of the «cube», the realization of the cube from its possibility, are the same that also lead to the emergence of the «rectangular prism», and, with it, to the destruction / disappearance of the «cube». That new experience is the possible but unseen, the encounter of the familiar with the alien (Waldenfels, 2006). The possibility of the new is contained in the present even though it also means an overturning.

In the preceding empirical study, forms of matter emerged («cube», «rectangular prism») and subsequently were associated with (actual, forgotten) names ("cube," "rectangular prism"). The «cube» and «rectangular prism» are ectypes (Nachbild) of the mystery object to which there are no archetypes. As Nancy (2005) suggests, what emerges is both a first and a last, an original (Urbild, archetype) and a copy (Nachbild, ectype). Moreover, "between the strictly unseen and the purely visible ectype . . . remains the bottom/background [fond] itself" (Marion, 1996, p. 70). This bottomless background constitutes the possibility of the transition from the strictly unseen to the perceived, a transition that for Melissa gives rise to the «cube» and then the «rectangular prism». What makes the first and second crossing possible?

With respect to a painting, which may serve us as a perceptual analogy to the mystery object, we can say that *it* "brings the gaze to life, not the other way around" (Marion, 1996, p. 76). It is "the painting that accords to the gaze this crossing, and

also the ascendance of the unseen towards the visible, the liberation of the ectypes, and the exhausted relapse of the fund [fonds]" (p. 76). In our situation, it is the mystery object itself that accords to Melissa—as to her peers—the crossing from the invisible to the ectypes, leading first to the emergence of the «cube» and then to that of the «rectangular prism». These ectypes, emerging from the unseen, are not and cannot be the results of intentional construction, as constructivist educators (and philosophers) want to have it. Rather, it is the object (painting, mystery object) itself that gives rise to ectypes. But how does the object do this? In the case of the painting, "it transmits to our gaze its own movement as the imprescriptible condition to be able, precisely, to perceptually follow in it the ascent of the unseen into the visible" (p. 79). By analogy, it is the mystery object in the present case that transmits to Melissa's hand its own movement. This movement is the imprescriptible condition for the unforeseeable «cube» and «rectangular prism» to emerge from the unfelt into the feelable; and, via the shaping of the plasticine model, from the invisible and unseen into the visible. "To feel [the «cube», «rectangular prism»]" is "to receive [the «cube», «rectangular prism»]" in the same way as "to see [voir] is to receive [recevoir], since to appear is to give (itself) to be seen  $[\dot{a}\ voir]$ " (p. 80).

In the end, however, it is the result of the simultaneous feeling of the mystery object in her right hand and her own model in the left hand from which emerges a new perception: the mystery object feels, and therefore is, like the model that Sylvia had built and given to her for verification. The difference between the hidden mystery object and her cubiform model did not emerge from the movements separated in time. The movements felt the same when she first held the one (original) and then the other object (model) in her hand(s). The movements and sensation felt differently when she had the opportunity to touch, move, and feel the two objects simultaneously.

#### *Children and the Learning of Geometry*

Tact [is] the geometrical sense. It alone, in effect, can give a base to these originary synthetic observations of the geometer, which recompose an intelligible solid with the point, line, surface. (Maine de Biran, 1859b, p. 145–146)

The goal of education is tradition, the handing-down of ways of knowing typical of the culture. But the handing-down of cultural knowing does not occur in the same way as we trade objects in the marketplace, where these pass unaffected from one hand to another. Rather, it is in and through their actions with material forms that new generations come to relive, in the present tense, the sense-constituting [sinnstiftende] acts that lie at the origin of specific forms of knowledge such as geometry (Husserl, 1976); it is in and through their actions that new generations bring to life geometry, and, simultaneously, keep geometry alive. The lessons from which the empirical materials derive were explicitly designed to provide children with experiences of living through such sense-constituting acts. Because geometry is grounded fundamentally in our bodies (cf., introductory quotation to this section),

in experiences such as those we observe in the episodes, the children are doing rather than preparing for geometry. The task of modeling a mystery object exhibited an interesting phenomenon all-too-often glossed over in education and educational research: The relation between the different perceptual modalities (gaze, tact).

The episode featured here foregrounds the relation between the visible and the tangible. The mystery object inside the box is invisible to the three girls, and they are required to build something in its image without actually having seen it. On the basis of tact, they initially have different experiences, which materialize in the three models (Figure 6). These models ground the different experiences in a way that is visually accessible to all (rather than somehow inaccessibly hidden in the minds and as interpretations). There is therefore an interlacing of the visible and the tangible (Merleau-Ponty, 1964b). Using as his analytic phenomenon the hand that touches the other hand in the course of exploring the world, the philosopher notes that one's own body is a thing of the same kind as the material world. The movements of hand, as the movements of the eye, are tied to the sensible and visible that they produce. Merleau-Ponty concludes: "And because, inversely, all experience of the visible was given to me in the context of the movements of the gaze, the visible spectacle belongs to the touch no more and no less than the 'tactile qualities'" (p. 174–175). We therefore have to get used to the idea that the visible is carved into the tangible. However, whereas both the tactile and visible maps are complete, they are not confounded. "The two parts are complete parts and yet they are not superposable" (p. 175). This episode shows the complex relations between the visible and tangible maps, and event the relation between the tangible maps of two objects.

Whereas the teachers of the class wanted Melissa to shape her plasticine into a rectangular prism (i.e., they wanted Melissa to learn geometry rather than disadvantage her by leaving her with the "incorrect" model), the issue is more complicated from the perspective of mathematics. Thus, for example, whether Melissa's model and the hidden mystery object—or the three objects Melissa, Sylvia, and Jane produced—are part of the same class is a matter of perspective. This is so because different contexts can be specified whereby the cube and the rectangular prisms are "the same" and others can be specified where even Jane's and Sylvia's models are different. For example, the cube is in fact a special case of the "rectangular prism," so that Melissa had indeed constructed a model thereof. Sylvia's and Jane's models are different when the proportions are considered; and both would have been inappropriate when the task had been to build "scale models" of the mystery object. All objects considered here further would have been members of the same class if topological relations had been the organizing feature—most mystery objects under consideration in the lesson, including cubes, (triangular, rectangular, hexagonal) prisms, spheres, pyramids, and (solid) cylinders, would have been in one class (homotopically speaking, these have 0 holes), whereas cups. mugs, rings, tubes, and doughnuts would have been part of another (homotopically speaking, these have 1 hole).

#### Coda

In psychology and education, as in philosophy, the visible and seen are often privileged. Learning and development, as shown in the work of Piaget, are thought from the perspective of the seen and known relative to which the learner or child is in a situation of a deficit. For the learner, this known does not exist and is invisible and ungraspable. It cannot be shown or told in the same way that the teacher, Sylvia, or Jane could not show the «rectangular prism» to Melissa. Even the teacher's verbal articulation of the difference, "cube" versus "rectangular prism," was insufficient for Melissa to "see" (understand, feel) the intended object. The «rectangular prism» eventually emerged for Melissa from the invisible and unseen, requiring the girl's readiness and willingness to receive that which gives itself from itself. But to study learning generally and the paradoxes the phenomenon involved requires more than simply observing children at work. This is so because, taking the example of painting, "only the painter can sense the transition, more than he can see it. and he can say nothing, since to show it would be to mask it" (Marion, 1996, p. 54). This is why I paired here—as I frequently do—a detailed empirical study of learning with a situation where the reader experiences the phenomenalization of the new (figure, knowledge), such as the process of coming to see in the case of the killer whale (Figure 1). The phenomenological account leads us to a different appreciation of the learning process, which generates not only the knowledge object but also the intentional orientation towards it. Once educators fully take into account the learning paradox, they have to rethink their approaches to teaching and to designing curriculum. They will understand and empathize with the child or student who asks, "Teacher, is this what you want me to do (see, know)?" or "Teacher, am I on the right track?" For when we do not know what we are to learn, we have no means of evaluating whether we have arrived (a question of cognition) or whether we are on the correct way (a question of metacognition). Not only issues of cognition have to be rethought but also the entire agenda on metacognition and self-directed learning.

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# Appendix

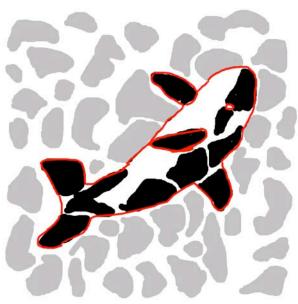


Figure A1. The hidden killer whale stands out when some of the splotches are drawn in grey.