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Performance Pay, Productivity and Morale*

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This paper incorporates the notion of worker morale into an economic model of pay and performance, and examines its implications for the efficacy and design of performance-based pay schemes. A worker's morale is determined by his relative pay status. A contract that rewards only individual performance can therefore undermine the morale of the least skilled workers in a firm and thereby adversely affect their productivity. On the other hand, competition for relative pay status tends to boost the productivity of highly skilled workers in the firm. The net effect on productivity depends on the composition of the firm's workforce. If the workforce is sufficiently heterogeneous then the inclusion of a profit-sharing component in the pay contract, which reduces the pay differential across workers, can sufficiently boost the morale of the least skilled workers as to improve overall productivity and profitability.

I Introduction

Psychologists and sociologists have long recognized the importance of relative pay effects in the determination of worker productivity. A host of studies from the psychology and sociology literatures indicate that paying similar workers differentially can be detrimental to morale and can undermine productivity.[1] Those who design real world pay schemes also seem to be well aware of this fact: personnel management texts routinely stress the importance of 'equitable' pay rates in fostering harmonious and productive work environments.[2] Moreover, there is considerable empirical evidence to suggest that actual pay differentials do not fully reflect productivity differences; workers are paid more equitably than their relative productivities would seem to dictate.[3] One such egalitarian practice is profit-sharing, in which an individual's pay is partly tied to the overall performance of the firm. It is often argued that in contrast to practices which tie pay solely to individual performance, profit-sharing can boost morale by engendering a 'team spirit' among workers. Despite the free-rider problem associated with profit-sharing, it is in widespread use.[4]

There are of course other explanations for the observed discrepancy between pay and individual productivity. Profit-sharing is sometimes used in instances of team production (where individual productivity cannot be identified), for risk-sharing purposes, and to obtain tax advantages. Discrepancies between temporal pay and individual performance can also be sometimes attributed to information asymmetries and implicit contract arrangements. However, it is abundantly clear

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[1]See the studies cited in Vroom (1964), Lawler (1971), Pencavel (1977), Frank (1985) and Akerlof and Yellen (1990).

[2]See Akerlof and Yellen (1990) for some examples.

[3]See Frank (1984a) and the studies cited by Baker, Jensen and Murphy (1988).

[4]See Baker, Jensen and Murphy (1988, p. 605).

from the personnel management literature that considerations for worker morale do play an important role in motivating egalitarian pay practices.

The idea that relative pay status may have important implications for labour market behaviour is already well established in the economics literature.[5] Daniel Hamermesh (1975) constructs a model of equilibrium wage determination when workers have preferences defined over their own income and their wage relative to the average wage. George Akerlof and Janet Yellen (1990) argue that worker productivity is likely to depend on relative wages and that the associated market equilibrium will involve unemployment. The link between relative pay and productivity is not modelled explicitly but the underlying mechanism relates a worker's productivity to the perceived 'fairness' of his wage relative to that of his co-workers.[6] Robert Frank (1984b) constructs a model in which the firm is viewed as a free association of workers who bargain over their relative pay within the firm. Workers have a preference for high relative income status and choose whether or not to work at a firm according to what their relative income status will be in that firm. Highly productive workers want to associate with less productive workers and so earn a higher relative income, but the less productive workers are willing to accept that association only if they are compensated for their lower status with a higher absolute income. The resulting trading equilibrium exhibits wage compression in the sense that the distribution of wages within the firm is much less dispersed than the corresponding distribution of marginal products.

My paper differs most notably from this previous work by examining *optimal pay contracts* in the presence of such effects. In this respect my paper relates quite closely to work by Lazear (1989). He examines a firm's optimal response to adverse relative pay effects but in an explicit tournament setting where rewards are based on relative performance. The relative pay effects stem directly from the nature of the tournament rather than from worker preference for high relative income status.[7] In principle it should be possible to distinguish empirically between these two different sources of relative pay effects. Relativity effects that do not arise from an underlying preference for relative income status should arise only in the presence of tournament-like pay schemes. The relative pay effects that I examine in this paper should arise even when pay is based on individual performance alone.

My paper is quite narrow in its focus. It attempts to shed light on two specific questions: (1) how are the incentives created by a performance-based contract affected by its implications for pay relativities and worker morale; and (2) can a contract comprising a profit-sharing component create better incentives despite its associated free-rider problem? These questions are addressed in the context of a quite specific model and I make no claim of generality in the specific results derived. My objectives are more modest. They are twofold: first, to present an example of the potentially important implications of sociological factors for the design of pay schemes; and second, to illustrate that these sociological factors can be successfully examined in the context of an otherwise standard economic framework.[8]

The rest of the paper is organized as follows. Section II presents a simple model of a relative pay effect. Section III examines the incentives created by a contract based solely on individual performance. Section IV then identifies conditions under which the inclusion of a profit-sharing component can be superior to the individual performance contract. Some summary remarks are made in Section V. An appendix contains all proofs.

II The Model

The model used is the simplest one possible for examining the issues of interest. Specific functional forms are used to facilitate tractability. The starting point is the specification of a utility func-

[5]Arthur Robson (1992) has looked at the importance of relative wealth status in a very different context; he examines its role in explaining observed attitudes towards risk. In particular, he provides a convincing explanation of Friedman-Savage 'concave-convex-concave' utility as a reflection of preference for relative wealth status.

[6]Summers (1988) examines a similar model in which the reference wage is the worker's outside opportunity.

[7]See Holmstrom and Milgrom (1991) for related work where workers have more 'standard' preferences defined only over individual income rather than relative income.

[8]It should be noted that I examine only one particular aspect of worker morale. The determinants of worker morale in reality surely include factors other than relative income status. I have focused on relative income status because of its obvious implications for the design of pay schemes.

tion for workers that captures a preference for relative pay status. The goal is to capture the flavour of psychological and sociological findings regarding relative pay and productivity. As summarized by Baker, Jensen and Murphy (1988, p. 596): 'The notion is that a worker will feel badly if a co-worker gets a bigger bonus, and the net effect is to reduce morale and ultimately productivity'. This is a fairly vague notion and there is no obvious best approach to how it should be represented in a formal model. I have chosen to capture this notion by specifying a model in which the marginal utility of a reward is declining in the rewards paid to co-workers. This means that a particular level of pay will have a relatively weaker motivational effect on a worker if it is the lowest amount paid than if it is the highest amount paid.[9] The following simple utility function has this property:

$$u_i = r_i y_i - c_i e_i^2 \qquad (1)$$

where r_i is relative pay status within the firm for worker i, y_i is his pay, e_i is productive effort and c_i captures the utility cost of effort. Note that utility is increasing in pay for a given relative pay status, but the marginal utility of pay is lower when relative pay is lower. Workers are assumed to judge their relative pay status using the average pay in the firm as a reference. That is, $r_i = y_i/\bar{y}$ where $\bar{y}$ is average pay in the firm. Note that this specification implies that a worker's relative pay status does not change if the pay for all workers increases proportionately.

It must be emphasized that in this formulation relative income status is assumed to affect the marginal utility of income.[10] This special property is important for the results I derive here. In particular, if relative income status has no effect on the marginal utility of income then the 'levelling' effect of the profit-sharing scheme I examine will not have incentive effects at the margin. For example, in Frank's (1984b) model a worker's utility depends only on the percentile ranking of her income. There is a no-marginal utility effect. The absolute pay increments associated with a profit-sharing scheme would have no affect on percentile rankings and so would have no affects on productivity.

The model of the firm is also very simple. There are n workers and total output is a linear function of the total productive effort from those workers. That is: $Q = kE$ where $E = \sum_{i=1}^{n} e_i$ and $k > 0$. The price of output is normalized to one and there are no non-labour costs, so profit is given by $Q - \sum_{i=1}^{n} y_i$. Workers differ only in their utility cost of productive effort c_i. Reference will sometimes be made to a worker's skill or ability, which is simply the inverse of effort cost.

The contract design problem is examined in the context of a principal-agent model with many agents. Each worker's problem is to choose how much effort to supply in response to the contract offered by the firm. The owner's problem is to choose the pay contract to maximize profit, given the Nash equilibrium response of workers. Effort is observable and so pay can feasibly be tied to individual performance. Consideration is confined to the following class of contracts:

$$y_i = \alpha e_i + \beta Q \qquad (2)$$

That is, each worker is paid a wage α per unit of productive effort, plus a share $\beta \in [0, 1/n]$ of total output. The parameters α and β are restricted to be non-negative and independent of c_i. The contract will be called a pure performance contract if $\beta = 0$ and a pure profit-sharing contract if $\alpha = 0$.

III A Pure Performance Contract

Before deriving the optimal contract from the class in (2) it is useful to examine the morale-based effects that arise under a pure performance contract. Faced with this contract, the problem for worker i is:

$$\max_{e_i} \; [ne_i/(e_i + E_{-i})] \, \alpha e_i - ce_i^2 \qquad (3)$$

where $E_{-i} = E - e_i$. Rearranging (3) reveals that worker i will derive positive utility from supplying effort if and only if $e_i < (n\alpha/c_i) - E_{-i}$. This means that worker i will be willing to participate in this firm if and only if $c_i < n\alpha/E_{-i}$ in equilibrium. To interpret this condition, consider a Nash

[9] This can explain why employees are sometimes required to keep their incomes secret from their co-workers, particularly when income is based on individual performance. However, such secrecy is rarely perfectly maintained. In my model workers can observe each other's incomes.

[10] Note too that the reduced form specification implies increasing marginal utility in income for given average income co-workers. This implies that the importance of relative income status more than offsets the declining marginal utility of income ordinarily assumed.

equilibrium in which $c_i < n\alpha/E_{-i}$ for all workers. The first-order conditions for the Nash equilibrium are:

$$2e_i([n\alpha/(e_i + E_{-i})] - c_i) - e_i^2\,[n\alpha/(e_i + E_{-i})^2] = 0\ \forall_i \quad (4)$$

If $c_i < n\alpha/E_{-i}$ for worker i then (4) has a unique solution given by:

$$e_i = (2n - 1)n\alpha[2C - (2n - 1)c_i]/2C^2 \quad (5)$$

where $C = \Sigma_{i=1}^{n} c_1$. (See the appendix). This expression can be used to interpret the condition on c_i required for a worker to supply positive effort in equilibrium.

Proposition 1 Let $\bar{c}_{-i} \equiv (C - c_i)/(n - 1)$ denote the average cost of effort for workers other than worker i. All workers supply positive effort under a pure performance contract if and only if $c_i < 2(n - 1)\bar{c}_{-i}/(2n - 3)\ \forall_i$.

This result suggests that a worker will not want to join a firm offering a pure performance contract if the average skill level in that firm is too much higher than his own. In a firm with highly skilled workers, a relatively unskilled worker is disadvantaged in the competition for relative pay status and as a result his morale suffers. This in turn undermines his productivity. If a worker is too disadvantaged then he will not be willing to supply any effort and he will choose not to join the firm. Frank (1985) identifies a similar effect when he argues that no one likes to be 'a small frog in a big pond'.

A natural corollary of proposition 1 is that the firm's workforce cannot be too heterogeneous with respect to ability.[11] To see this, write the condition in proposition 1 as:

$$c_j < 2[(n - 1)\bar{c}_{-i} + c_i]/(2n - 1)\ \forall_j \quad (6)$$

where worker i and worker j are any pair of workers in the firm.[12] Summing over $j \neq i$ and rearranging then yields $c_i > \bar{c}_{-i}/2\ \forall_i$ as a necessary condition for all workers to supply positive effort. This means that the introduction of some highly skilled individuals into a firm can sufficiently undermine the morale of some workers as to cause them to withdraw from the firm. These workers become discouraged by the new higher standards and will choose to seek less competitive standards (a smaller pond) elsewhere. Proposition 1 and its corollary therefore together place a restriction on the extent to which any worker can differ from his co-workers without at least one worker becoming sufficiently discouraged as to withdraw from the firm:

$$\bar{c}_{-i}/2 < c_i < 2(n - 1)\bar{c}_{-i}/(2n - 3)\ \forall_i \quad (7)$$

Attention is hereafter restricted to firms whose workers satisfy this condition.

Proposition 2 If all workers satisfy condition (7), then each worker's effort is increasing in the average cost of effort of his co-workers. That is: $\partial e_i/\partial\bar{c}_{-i} > 0$.

This result states that a worker of given ability will supply less effort when teamed with highly skilled co-workers than when teamed with less able co-workers. The result can be interpreted in terms of the worker's morale. A worker with less ability than his co-workers has little prospect of achieving a high relative pay status within the firm and will tend to be discouraged from trying. The converse is true for a worker of high relative ability who is better positioned to achieve a high relative status.

This phenomenon is analogous to the perverse incentive effects created by tournament promotion schemes when candidates differ in ability.[13] Under a tournament scheme, lower ability workers may be discouraged from competing for the promotion 'prize' because their probability of success is low and effort per se is not rewarded. The implication of proposition 2 is somewhat stronger. It suggests that workers may become discouraged even when they are rewarded for each unit of productive effort regardless of relative performance. It is the worker's perception of the reward in a relative context that introduces a tournament-like aspect to the performance contract.

IV Profit-Sharing

The analysis in the previous section suggests that a pure performance contract may not neces-

[11]An analogous argument is sometimes cited in support of 'talent streaming' in children's sports and in the classroom. Talent streaming places similarly talented children together and can serve to ensure that the least talented children are not discouraged from participating. (Of course this is not the only reason for talent streaming. Streaming in the classroom also allows the teacher to set a pace that most closely matches the skills of the students).

[12]The condition in proposition 1 can be written as $c_i < 2C/(2n - 1)$. This must hold for all workers, including $i = j$. Thus, $c_j < 2C/(2n - 1)$. Replacing C with $(n - 1)\,\bar{c}_{-i} + c_i$ then yields expression (6).

[13]See Lazear and Rosen (1981).

sarily be profit-maximizing when there are relative pay effects. In particular, the inclusion of a profit-sharing component in the contract may serve to 'level the playing field' on which workers compete for status and thereby improve the morale of the least skilled workers. But this improved morale comes at a price. The inclusion of a profit-sharing component dilutes the competitive environment that motivates highly skilled workers, and at the same time introduces an incentive for all workers to free-ride. These problems may more than offset the gains from the improved morale of the least skilled workers. The objective of this section is to identify conditions under which the gains to some profit-sharing outweigh the costs.

Consider a contract of the form in (2) with $\beta > 0$. The problem for worker i is now:

$$\max_{e_i} n[\alpha e_i + \beta k(e_i + E_{-i})]^2 / [(e_i + E_{-i})(\alpha + n\beta k)] - c_i e_i^2 \quad (8)$$

The first-order conditions for a Nash equilibrium are:

$$n\alpha^2 e_i^2 - 2(e_i + E_{-i}) [n\alpha^2 - (e_i + E_{-i})(\alpha + n\beta k)c_i]e_i - n(e_i + E_{-i})^2 \beta k[2\alpha + \beta k] = 0 \ \forall_i \quad (9)$$

The closed form solution for equilibrium e_i is of no particular interest here; to characterize the optimal contract it is necessary only to determine equilibrium E. This is given implicitly by (see the appendix):

$$\sum_{i=1}^{n} [(n\alpha^2 - E[\alpha + n\beta k]c_i)^2 + n^2\alpha^2\beta k[2\alpha + \beta k]^{1/2} - E[\alpha + n\beta k]C + n(n-1)\alpha^2 = 0 \quad (10)$$

It is now possible to characterize the optimal contract. The owner's problem is to choose α and β to maximize profit subject to participation constraints:

$$\max_{\alpha,\beta} E[k - \alpha - n\beta k]$$
$$\text{s.t. } \alpha \geqslant 0 \text{ and } \beta \geqslant 0 \quad (11)$$

The key properties of the solution to this problem are summarized in propositions 3 and 4 below.

Proposition 3 The optimal contract always includes a positive wage component.

This result indicates that a pure profit-sharing arrangement is never optimal. The productivity gains associated with the enhanced morale of the least skilled workers under pure profit-sharing are more than offset by the costs of free-riding and the foregone competition for relative pay status amongst the higher ability workers.

Proposition 4 The optimal contract includes a positive profit-sharing component if and only if:

$$\sum_{i=1}^{n} [2C - (2n-1)c_i]^{-1} > (2n-1)n/C \quad (12)$$

It is instructive to interpret this condition in terms of the heterogeneity of the firm's workforce. For a given aggregate cost of effort C, the right side of (12) is independent of the composition of the workforce. Conversely, for a given C, the left side of (12) is increasing in the variance of the c_i's. Hence, for a sufficiently heterogeneous work force, the optimal contract will include a profit-sharing component.[14] This result is quite intuitive. The adverse morale-based productivity effect associated with a pure performance contract gets worse as the heterogeneity of the workforce increases because the degree of differential treatment across workers necessarily increases. Consequently, the benefits of a profit-sharing component in the contract increase with worker heterogeneity. So if the firm's workforce is sufficiently heterogeneous, the incentive costs of including a profit-sharing component will be more than offset by the productivity gains in the least skilled workers.

Two further remarks are warranted here. First, in response to the adverse productivity effects associated with pay dispersion across heterogeneous workers under a pure performance contract, why would the firm not choose to split its workers into more homogeneous groups rather than introduce a profit-sharing scheme? The obvious answer is that there must be synergies associated with a larger production team that more than offset the costs of heterogeneity within the team. Recall from Section II that output is assumed to be a linear function of aggregate effort: $Q = kE$. The productivity coefficient k is likely to be a quadratic function of n. Such a relationship would reflect initial gains to specialization but eventual increasing coordination costs. A balance of these

[14]It is straightforward to show that conditions (12) and (7) can be satisfied simultaneously. That is, there are feasible workforce compositions for which some profit-sharing is optimal.

two factors, together with the variance of ability across workers, will determine the optimal size of the firm.

The second remark relates to the assumed characterisitcs of the labour market underlying the analysis here. I have focused on a monopoly equilibrium where the firm chooses its contract to maximize profit subject to participation constraints for workers. One could alternatively consider a competitive equilibrium characterized by zero profit for the firm. Such an equilibrium would look something like the equilibrium proposed by Frank (1984b). All firms would prefer to hire workers of high ability with as little heterogeneity as possible, and pay them purely for performance. But the least able workers in such a firm could be bid away by another firm that matches them with less able workers and so provides them with higher relative income status. In equilibrium all firms will have some heterogeneity across workers. If the dispersion of the population of workers is high enough, and the optimal size of firms is large enough, then each firm will have sufficient heterogeneity to warrant a contract with some profit-sharing.

V Conclusion

This paper has examined the implications of a relative pay effect for the efficacy and design of performance-based pay schemes. A contract that rewards only individual performance can undermine the morale of the least skilled workers in a firm and thereby adversely affect their productivity. On the other hand, competition for relative pay status tends to boost the productivity of highly skilled workers in the firm. The net effect on productivity depends on the composition of the firm's workforce. If the workforce is sufficiently heterogeneous then the inclusion of a profit-sharing component in the pay contract, which reduces the pay differential across workers, can sufficiently boost the morale of the least skilled workers as to improve overall productivity and profitability.

These results suggest that observed egalitarian pay practices, and profit-sharing schemes in particular, can be at least partly explained as profit-maximizing behaviour in response to morale-related considerations. In principle it should be possible to examine empirically the importance of these factors. The hypotheses I have proposed would find support from evidence of significant profit-sharing schemes in moderately large firms in which the dispersion of productivity is markedly higher than the dispersion of pay.

The analysis in this paper also lends support to the notion that sociological phenomena can be successfully incorporated into a fairly standard economic framework. Economic models have traditionally ignored many aspects of social interaction that psychologists and sociologists consider so important and obvious. Explicit recognition of some of these social factors may lead to economic models that are better able to explain real world phenomena.

Appendix

Solution of (4) The cubic in (4) has three distinct real roots. They are $e_i = 0$ and:

$$e_i = [\phi \pm (\phi^2 + \varphi)^{1/2}]/4c_i \qquad \text{(A1)}$$

where $\phi = (n\alpha - 4c_iE_{-i})$ and $\varphi = 16c_i\, E_{-i}\,(n\alpha - c_iE_{-i})$. Note that $(\phi^2 + \varphi) = n^2\alpha^2 + 8n\alpha c_iE_{-i}$ which is necessarily positive. Thus, both non-zero roots are real. If $c_i < n\alpha/E_{-i}$ for worker i then (4) has a unique positive root given by the positive branch of (A1). Rearranging yields:

$$e_i = 2E(1 - Ec_i/n\alpha) \qquad \text{(A2)}$$

This yields a global maximum over the non-negative domain of e_i. To see this note that from (4) that:

$$u_i''(e_i) = 2([n\alpha/(e_i + E_{-i})] - c_i) - e_i\,[4(e_i + E_{-i}) - 2e_i]n\alpha/(e_i + E_{-i})^3 \qquad \text{(A3)}$$

Thus, $u_i''(0) > 0$ when $[(n\alpha/E_{-i}) - c_i] > 0$. Hence, $e_i = 0$ is a local minimum. Since there is one and only one positive root to the first-order condition, it follows that it must be a local maximum and a global maximum on the non-negative domain.[15] Summing over i in (A2) yields:

$$E = (2n - 1)n\alpha/2C \qquad \text{(A4)}$$

Substituting (A4) back into (A2) then yields equation (5) in the text.

Proof of proposition 1 Recall that worker i will supply positive effort iff $c_i < n\alpha/E_{-i}$. Noting that $E_{-i} = E - e_i$, and substituting for e_i from (5) and E from (A4) yields the result.)

Proof of proposition 2 Rewrite (5) as:

$$e_i = (2n - 1)\alpha n[2(n - 1)\bar{c}_{-i} - (2n - 3)c_i]/2[(n - 1)\bar{c}_{-i} + c_i]^2$$

[15]Note that this root must be a local maximum and not an inflexion point since $u_i'(e_i) < 0$ for $e_i > 2E(1 - Ec_i/n\alpha)$.

and differentiate with respect to $\bar{c}_{-i}$ to obtain:

$$\partial e_i/\partial \bar{c}_{-i} = (2n-1)(n-1)^2 \alpha n[2c_i - \bar{c}_{-i}]/[(n-1)\bar{c}_{-i} + c_i]^3$$

which is positive since $[2c_i - \bar{c}_{-i}] > 0$ for all workers satisfying (7).[16]

Derivation of (10). Substituting E for $(e_i + E_{-i})$ in (A2) and solving for e_i yields two solutions, only one of which is feasible when $\beta > 0$:

$$e_i = (E/n\alpha^2)(n\alpha^2 - E[\alpha + n\beta k]c_i) + (E/n\alpha^2)[(n\alpha^2 - E[\alpha + n\beta k]c_i)^2 + n^2\alpha^2 \beta k(2\alpha + \beta k)]^{1/2} \quad (A5)$$

(The other root gives rise to an e_i which is always negative). The solution in (A5) yields a maximum. To see this note that the solution in (A5) is unambiguously positive and is the unique positive root. It must therefore represent the global maximum on the feasible domain since $u'(0) > 0$ and $u(0) > 0$ when $\beta > 0$. An implicit solution for total equilibrium effort can be found by summing over i in (A5). Rearranging yields (10).

Proof of proposition 3 The Kuhn-Tucker conditions for (11) are:

$$[\partial E/\partial\alpha][k - \alpha - n\beta k] - E + \lambda_A = 0,\ \lambda_A \geq 0,\ \alpha\lambda_A = 0 \quad (A6)$$

$$[\partial E/\partial\beta][k - \alpha - n\beta k] - nkE + \lambda_B = 0,\ \lambda_B \geq 0,\ \beta\lambda_B = 0 \quad (A7)$$

where $\lambda_A \geq 0$ and $\lambda_B \geq 0$ are the Lagrange multipliers associated with the non-negativity constraints. If $\alpha = 0$ and $\beta > 0$, then $\lambda_B \geq 0$ and $\lambda_B = 0$. Then from (A7):

$$[\partial E/\partial\beta][k - n\beta k] - nkE = 0 \quad (A8)$$

Setting $\alpha = 0$ in (9) and solving for e_i:[17]

$$e_i|_{\alpha=0} = \beta k/2c_i \quad (A9)$$

Then summing across i and differentiating:

$$[\partial E/\partial\beta]|_{\alpha=0} = \Omega k/2 \quad (A10)$$

where $\Omega = \Sigma_{i=1}^{n} c_i^{-1}$. Substitution into (A8) then yields:

$$\beta|_{\alpha=0} = 1/2n \quad (A11)$$

Substituting (A10) and (A11) into (19) yields:

$$\lambda_A|_{\alpha=0} = [k/2]\left[\Omega/2n - [\partial E/\partial\alpha]|_{\alpha=0,\beta=1/2n}\right] \quad (A12)$$

Differentiating (9) to find $\partial e_i/\partial\alpha$, evaluating at $\alpha = 0$ and $\beta = 1/2n$, and then summing across i:

$$[\partial E/\partial\alpha]|_{\alpha=0,\beta=1/2n} = \sum_{i=1}^{n}\left[E(k - 2e_ic_i)/k(Ec_i + 2e_ic_i - k/2n)\right] \quad (A13)$$

Substituting for e_i and E from (A9):

$$[\partial E/\partial\alpha]|_{\alpha=0\,\beta=1/2n} = \Omega(2n-1)/2n \quad (A14)$$

Finally, substituting (A14) into (A12):

$$\lambda_A|_{\alpha=0} = -\Omega k(n-1)/2n \quad (A15)$$

which is negative for $n > 1$ and cannot satisfy the first-order conditions for a maximum.

Proof of proposition 4 If $\alpha > 0$ and $\beta = 0$, then $\lambda_A = 0$ and $\lambda_B \geq 0$. Then from (19):

$$[\partial E/\partial\alpha][k - \alpha] = E \quad (A16)$$

Recall from (10) that $E = (2n-1)n\alpha/2C$ when $\beta = 0$. Solving (A16) for α yields $\alpha = k/2$. Making the relevant substitutions into (A7) then yields:

$$\lambda_B|_{\beta=0} = [k/2]\left[(2n-1)n^2 k/2C - [\partial E/\partial\beta]|_{\alpha=k/2,\beta=0}\right] \quad (A17)$$

Totally differentiate (10) to obtain:

$$\partial E/\partial\beta = \frac{\left[\sum_{i=1}^{n} F(c_i)\left[n^2\alpha^2k[\beta k + \alpha] - (n\alpha^2 - E[\alpha + n\beta k]c_i)Enkc_i\right] - nkEC\right]}{[\alpha + n\beta k]\left[C + \sum_{i=1}^{n} F(c_i)(n\alpha^2 - E[\alpha + n\beta k]c_i)c_i\right]} \quad (A18)$$

where $F(c_i) = \left[(n\alpha^2 - E[\alpha + n\beta k]c_i)^2 + n^2\alpha^2\beta k [2\alpha + \beta k]\right]^{-1/2}$. Setting $\alpha = k/2$ and $\beta = 0$ yields:

$$[\partial E/\partial\beta]_{\alpha=k/2\,\beta=0} = nk\left[\sum_{i=1}^{n}[2C - (2n-1)c_i]^{-1} - (2n-1)n/2C\right] \quad (A19)$$

Substituting (A19) into (A17) then yields:

[16] Note that it should not be inferred that $\partial e_i/\partial \bar{c}_{-i} < 0$ for $[2c_i - \bar{c}_{-i}] < 0$. Equation (11) is derived on the supposition that all workers provide positive effort. If $[2c_i - \bar{c}_{-i}] < 0$ for any worker then some workers provide zero effort and (11) does not reflect true equilibrium effort. In that case, the number of workers who supply positive effort in equilibrium will be some $m < n$, depending on the distribution of c_i in the firm. For certain distributions (for example, if $c_i = \delta i$, where $\delta > 0$ and i = to n), e_i is declining in $\bar{c}_{-i}$ when $c_i < \bar{c}_{-i}/2$. This suggests that very highly skilled workers may 'slack-off' when teamed with much lesser skilled co-workers. While this is intuitively appealing, the result is quite sensitive to the specification of the distribution of c_i in this model.

[17] Expression (10) cannot be used here because its derivation involved division by α.

$$\lambda_B|_{\beta=0} = [nk^2/2] \left[(2n-1)n/C - \sum_{i=1}^{n} [2C - (2n-1)c_i]^{-1}\right] \quad (A20)$$

which is non-negative iff $\sum_{i=1}^{n}[2C - (2n-1)c_i]^{-1} < (2n-1)n/C$.

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