ECONOMIC PROGRESS AND SKILL OBSOLESCENCE WITH NETWORK EFFECTS

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ABSTRACT

We construct an OLG model with network effects to examine skill obsolescence when individuals can choose technological vintages. In the absence of transfer payments, some regions of the parameter space have unique stationary equilibria, others have unique cyclical equilibria, and others have multiple stationary equilibria. All equilibria are Pareto efficient. However, "rat race" equilibria can exist in which all agents currently alive prefer a slower rate of progress than occurs in equilibrium. When contemporaneous transfers are allowed, equilibria are unique everywhere, but a cycle still exists, and a rat race can still arise in equilibrium. Allowing intertemporal transfers (debt) ensures that all equilibria are stationary. In the relevant parameter range, the introduction of debt can eliminate cycles and increase the long-run growth rate. No rat race equilibria exist when debt is allowed.

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1. INTRODUCTION

Skill obsolescence is a major problem that comes with economic progress. Skills that individuals learn when young can become devalued when new technologies arrive that require a different set of skills. This can be a problem particularly for older workers. In traditional societies, where the pace of technological change is relatively slow, elders remain valuable because the skills in which they have experience are still relevant. In contrast, in societies with a fast pace of technological change, seniors can become marginalized. The introduction of new techniques can force older people to either learn the new techniques or face obsolescence, since the productivity of their skills is largely a function of the number of other people who have complementary skills. This externality, inherent in skill choices, can potentially lead to intergenerational conflict and suboptimal outcomes.

Recent analyses of this problem, such as Krusell and Rios-Rull (1996) and Aghion and Howitt (1998, chapter 9) have used the vintage human capital model developed by Chari and Hopenhayn (1991) to argue that this potential conflict can lead to political action by older generations to suppress new technologies. In their framework, uniform technology choices are determined by majority voting. This gives an intergenerational conflict interpretation to Olson's (1982) "vested interests" argument: inefficient periodic occurrences of technological slowdowns may occur due to intergenerational conflicts, through democratic government policy.

In this paper, we examine conditions under which *individual choices* of vintages can lead to similar outcomes, without government policy. We present an overlapping generations model of human capital vintages, similar in spirit to those in the above studies, but with a different approach to modeling skill complementarities which allows for a relatively simple analysis of individual skill choices. We eliminate the distinction between “skilled” and “unskilled” workers and focus on the commonality of technological vintage as the source of skill complementarity. The productivity of each vintage is assumed to be an increasing function of the number of individuals who use that vintage, and hence have compatible skills. This modeling approach draws upon the "network effect" literature, pioneered by Katz and Shapiro (1985), in the context of product complementarities.[1]

Most forms of production involve extensive interaction between agents, and the productivity of that interaction depends to a considerable degree on the compatibility of the skills of the agents involved. This is not to say that productive matches require individuals to have the *same* skills; on the contrary, interaction is often most valuable when it brings together individuals with different expertise. However, if different skill sets are to complement each other, they must be compatible. We believe that these network effects are equally important at a more primitive level. The ability of agents to interact productively relies on a commonality of language, not just in terms of words spoken, but more fundamentally, in terms of the way they think and the methods they use. It is from this perspective that we view vintages of human skills; we think of a technological vintage as akin to a language, in the broadest sense of that word.

In any model with network effects, coordination problems can lead to multiple equilibria and inefficiency. This issue is not our primary concern in this paper. Here, we set aside the usual coordination problems by focusing on coalition-proof Markov perfect equilibria. We examine the properties of such equilibria under different assumptions about the extent to which transfers between agents are possible. One can think of the availability of transfers as an institutional mechanism, the existence of which reflects the degree of social capital in the economy, and in that sense our analysis examines how patterns of technological progress depend on the richness of social capital.

In the absence of transfers, we find that some regions of the parameter space have unique stationary equilibria while others have unique cyclical equilibria, and others have multiple stationary equilibria. All of these equilibria are Pareto efficient. However, "rat race" equilibria can exist in which all agents currently alive prefer a slower rate of progress than occurs in equilibrium. When contemporaneous transfers are allowed, equilibria are unique everywhere in the parameter space, but a cyclical equilibrium still

[1] Katz and Shapiro (1994) also consider the notion of "forming systems": collections of compatible individual goods (such as nuts and bolts) that work together to provide services. Network effects pervade such systems. Languages are another important area where network effects arise: the usefulness of a language increases with the number of people who speak it (Church and King (1993), Lazear (1999)). Shy (2001) provides an excellent survey of the network economics literature.

exists, and a “rat race” can still arise. Allowing intertemporal transfers (through debt) ensures that all equilibria are stationary, and reduces the equilibrium outcome to one of only two types: those where all agents learn a new technology in every period, and those where no-one ever learns a new technology. This affects only the region of the parameter space in which the unique equilibrium would otherwise be cyclical, and changes the nature of the equilibrium in that region to one in which all agents learn the technology in every period. Thus, in the relevant parameter range, the introduction of debt can eliminate cycles and increase the long-run growth rate. Moreover, the introduction of debt eliminates the possibility of a “rat race”.

The closest model to ours in the literature is presented in Shy (2001). The two models differ in the following important ways. First, new innovations occur in Shy’s model regardless of the patterns of adoption whereas, in our model, innovations occur only if newer technologies are adopted. Secondly, we allow both young and old agents to choose technologies. Finally, the precise specification of the network effect is different in the two models. In particular, given the rate of innovation, Shy characterizes the duration of a technology’s usage as a function of the size of the innovation and the number of people in the network. Here, we endogenize the innovation rate, and characterize the equilibrium allocations under different assumptions about the availability of transfers. This allows us to address the interplay of intergenerational conflict and the richness of social capital in the determination of economic progress.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium allocations under different assumptions about the availability of transfer payments. Section 4 presents some concluding remarks. Proofs of all propositions are given in the appendix.

2. THE MODEL

We examine an economy with overlapping generations in which agents live for two periods, and each generation has N (initially identical) agents. When young, agents must choose which technology to learn. When old, they must choose whether to retain the technology they learned when young, or re-tool by learning another technology. The payoff from using a particular technology is subject to a network effect. In particular, the payoff $y_t(\tau)$ to an agent using a particular technology τ in period *t*, is proportional to $x_t(\tau)$ the number of agents using it:

$$y_t(\tau) = \gamma^{\tau} x_t(\tau) \tag{1}$$

where $\gamma > 1$ is a parameter reflecting the productivity of the technology.[2]

Technologies evolve in the following way. If technology τ is learned by the young in period *t*, then a new technology $\tau + 1$ becomes available in period *t*+1. Otherwise, technology τ remains the most productive technology available. The following learning-by-doing rationale underlies this evolution: only when agents adopt a new technology do they discover new ways in which it can be improved. Moreover, technology will only progress if some young agents learn the new technology, since only they have any incentive to develop further technologies.[3]

We assume that in period 0, all old agents enter the period endowed with technology $\tau = 0$. Technology $\tau = 1$ also becomes available in period 0. If any young agents adopt technology $\tau = 1$, then technology $\tau = 2$ becomes available in period 1, in addition to the existing technologies, $\tau = 0$ and $\tau = 1$. Otherwise, only technologies $\tau = 0$ and $\tau = 1$ are available in period 1. This evolution continues with a new, more advanced technology becoming available each time any young agents adopt the previous new technology. Agents can choose any technology from those available.

Learning costs are a key element of our model. We assume that learning is costless for young agents but re-tooling is costly for the old.[4] In particular, any old agent who re-tools must devote a fraction $k \in [0,1]$ of his available productive time to learning.[5] Thus, an old agent in period *t* who re-tools with technology τ receives a payoff equal to $(1-k)y_t(\tau)$ in that period.

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[2] The functional form used in (1) is relatively standard in the network literature. It implies that *total* surplus from a network is proportional to the square of the number of users. See Shy (1995) for a discussion, and Schmalensee (2002) for a recent application to payment systems.

[3] Incentives to *invent* new technologies are not modeled explicitly in this paper. Our specification (that progress is only made if young agents adopt the latest technology) endogenizes technological progress in a very simple way, in the spirit of the learning-by-doing approach used by Jovanovic and Nyarko (1996).

[4] It makes no difference if learning is costly for the young, provided learning costs are independent of which technology they learn.

[5] Our use of the male pronoun throughout was decided by a coin toss.

As in Chari and Hopenhayn (1991), preferences take the following form:

$$u(c_1, c_2) = c_1 + \beta c_2 \tag{2}$$

where c_1 denotes consumption when young, c_2 denotes consumption when old, and $\beta \in [0,1]$ is the discount factor.

3. COALITION-PROOF MARKOV PERFECT EQUILIBRIA

As one would expect, the network effects in this economy give rise to multiple Nash equilibria. Our intention is not to exhaustively characterize all of the equilibria that can occur. Our purpose is to characterize the equilibrium outcomes with respect to the rate of economic progress, under different assumptions about the types of transfer payments that are possible in an economy. We therefore restrict attention to coalition-proof Markov perfect Nash equilibria, under these different assumptions.[6] For brevity, hereafter, we use the term "equilibrium" rather than "coalition-proof Markov perfect Nash equilibrium". We first consider equilibria when no transfer payments are allowed at all. We then allow for transfer payments, but only between agents that are alive at the same time – requiring no external dynamic enforcement mechanism. Finally, we allow for transfer payments among all current and future agents, where future generations can make transfers to current ones through chains of intermediate generations.

We identify six distinct patterns of technological adoption on which our analysis will focus. Four of these are stationary, and two are cyclical.[7]

- *Eternal Stagnation (ES)* outcomes are those in which all agents use the initial technology, and no technological change occurs.

[6] A Markov perfect equilibrium is a profile of Markov strategies that yields a Nash equilibrium in every proper subgame. See Fudenberg and Tirole (1991), chapter 13, for further details.

[7] We have in fact been able to prove that no other technological adoption patterns can arise in equilibrium but the proof is extremely long and could not be included in this paper in a succinct way. Heuristically, the proof proceeds in the following way. A sequence of technology adoption choices can be an equilibrium if and only if every three-period subsequence that forms part of the candidate equilibrium sequence, is itself an *equilibrium* subsequence. There are 64 distinct three-period subsequences to consider. We examined each one, and found that only 32 of these can be equilibrium subsequences. We then examined which of these three-period subsequences could be joined to form a five-period subsequence such that every three-period subsequence in that five-period subsequence could simultaneously be an equilibrium subsequence. By proceeding in this way, we were able to eliminate all non-stationary sequences except the one identified in the paper. A complete proof is available from the authors upon request.

- *Partial Learning 1 (PL1)* outcomes are those in which each agent learns the new technology when young, but does not re-tool with the subsequent new technology when old.
- *Partial Learning 2 (PL2)* outcomes are those in which only the old learn new technologies.
- *Complete Learning (CL)* outcomes are those in which each agent learns the new technology when young and re-tools with the subsequent new technology when old.
- *Periodic Partial Learning (PPL)* outcomes are those in which old agents never learn new technologies and young agents adopt new technologies every *second* period.
- *Periodic Complete Learning (PCL)* outcomes are those in which *all* agents learn new technologies every *second* period.

3.1 Equilibria Without Transfers

An equilibrium allocation is one in which there does not exist a deviation strategy for any coalition of agents that yields a higher payoff to all members of the coalition than the equilibrium payoff. It is straightforward to show, and intuitively clear, that the network effect in this economy means that any coalition of agents will always do best by including as many agents as possible in the coalition. Thus, any coalition of young agents will include all young agents, and any coalition of old agents will include all old agents. Thus, all agents *within* a generation will make the same technology choice.[8] The key issue of interest is whether or not the old and the young can agree to a joint coalition in which both generations make the same technology choice. It will become clear that this is critical for the determination of equilibria.

It is useful to introduce some additional notation at this point. Let $i_t \in \{0,1\}$ denote the technology choice for the young in period *t*, where $i_t = 1$ indicates choosing

[8] We assume throughout that whenever agents are indifferent between new and old technologies, they choose the new technologies. It makes no difference if we assume the converse, except along the boundaries of the critical parameter regions. We rule out young agents committing to particular learning choices when old.

the new technology, and $i_t = 0$ indicates choosing the old technology. Similarly, let $j_t \in \{0,1\}$ denote the technology choice for the old in period *t*. Thus, in stationary allocations, $\{i_t, j_t\} = \{i_{t+1}, j_{t+1}\} \ \forall t$, and in nonstationary ones, $\{i_t, j_t\} \neq \{i_{t+1}, j_{t+1}\}$ for some *t*.

The following proposition, illustrated in Figure 1, summarizes the stationary equilibria.

Proposition 1

(a) Eternal Stagnation is an equilibrium without transfers if and only if

$$\gamma < [2(1+\beta)/(1+2\beta)] \text{ and } \gamma < [1/(1-k)]$$

(the regions labeled "ES" in Figure 1).

(b) Complete Learning is an equilibrium without transfers if and only if

$$\gamma \geq [1/2(1-k)]$$

(the regions labeled "CL" in Figure 1).

(c) Partial Learning 1 is an equilibrium without transfers if and only if

$$\gamma < [1/2(1-k)] \text{ and } \gamma \geq [(2+\beta)/(1+\beta)]$$

(the region labeled "PL1" in Figure 1).

(d) Partial Learning 2 is not an equilibrium anywhere in the parameter space.

(e) There does not exist a stationary equilibrium without transfers if

$$\gamma < [1/2(1-k)] \text{ and } [2(1+\beta)/(1+2\beta)] \leq \gamma < [(2+\beta)/(1+\beta)]$$

(the region labeled "PPL" in Figure 1).

In Figure 1, the horizontal axis measures $k \in [0,1]$, the fraction of time that each old agent must spend learning if he is to adopt the new technology. The vertical axis measures $\gamma > 1$, the productivity increment associated with each new technology. Given β, these two parameters represent the costs and benefits of economic progress.

It is clear from Figure 1 that stationary equilibria exist everywhere in the parameter space except for the region labeled "PPL". Moreover, these stationary equilibria are unique everywhere except in the region labeled "CL or ES". As expected, CL occurs as the unique stationary equilibrium when γ is sufficiently high relative to *k*, and ES occurs when the converse is true. PL2 is not an equilibrium anywhere in the parameter space: given that the young learn costlessly, and given the network effects, a coalition of all agents that deviates by having everyone learn will always be able to break the candidate equilibrium. However, PL1, where the young learn but the old do not, is an equilibrium outcome when the ratio of γ to k is in the intermediate range. In this range, the new technology is sufficiently productive that it is worthwhile for the young to adopt it even though they work within a smaller network (because the old retain the old technology), but the cost of learning is too high for the old to warrant re-tooling even though they too thereby work within a smaller network.

No stationary equilibrium exists in the region of Figure 1 labeled "PPL". In this region, γ is too high to support ES as an equilibrium because it is worthwhile for the *current* young to deviate from that candidate equilibrium and learn the new technology, if the *next* young are expected to learn that same technology, even if the current old do not re-tool. That is, $i_t = 1$ is the best response to $i_{t+1} = 0$ and $j_t = 0$. Thus, the current young are willing to sacrifice the benefits of working with a universal technology today *if* they expect the new technology they learn today will be used universally when they are old. However, this expectation cannot be fulfilled in a stationary equilibrium; the next young generation have exactly the same incentive to deviate from ES. Similarly, PL1 is not an equilibrium in this region because if the young in period *t* expect the young in period *t*+1 to learn the new technology in that period, then it is not worthwhile for the young in period *t* to learn the new technology, given that the old in period *t* do not re-tool. That is, in the "PPL" region, the best response to $j_t = 0$ and $i_{t+1} = 1$, is $i_t = 0$; the $i_t = 1$ response can only be supported at a higher value of γ (in which case PL1 is the equilibrium). Thus, in the "PPL" region, neither ES nor PL1 are equilibria.

Proposition 2

Periodic Partial Learning is an equilibrium without transfers if and only if

$$\gamma < [1/2(1-k)] \quad \text{and} \quad [2(1+\beta)/(1+2\beta)] \leq \gamma < [(2+\beta)/(1+\beta)]$$

(the region labeled "PPL" in Figure 1).[9]

The existence of an equilibrium with PPL relates directly to the non-existence of a stationary equilibrium in that region. Recall that in the "PPL" region, the optimal choice for the young in period *t* is to learn the new technology $\tau+1$ when the old retain technology τ, only if the young in period *t*+1 also learn technology $\tau+1$. The cost of working in a smaller network when young is only justified by the benefits of using a more advanced technology if that technology is used within a universal network when old. So in order for the adoption of a new technology to be worthwhile, there must be a period of stagnation after its adoption in which that technology is used universally; otherwise those who adopt the new technology cannot reap a return high enough to warrant the initial opportunity cost.

In the intermediate region labeled "CL or ES" in Figure 1, two stationary equilibrium outcomes exist: CL or ES. In this region, coalition-proofness cannot eliminate the multiplicity of equilibria. However, this does *not* mean that young and old agents are necessarily in disagreement over which outcome is better. To understand this point it is useful to characterize the conditions under which there is intergenerational conflict in this economy. The following proposition, illustrated in Figure 2, sets out those conditions.

Proposition 3

(a) Old agents in any period weakly prefer ES to any other technology adoption pattern if $\gamma < 1/(1-k)$ and strictly prefer CL to any other adoption pattern if $\gamma \geq 1/(1-k)$.

(b) Young agents in any period weakly prefer ES to any other technology adoption pattern if

$$k > \frac{\beta\gamma^2 + \gamma - (1+\beta)}{\beta\gamma^2}$$

and strictly prefer CL to any other adoption pattern if

[9] Although PPL is cyclical, it is a Markov perfect equilibrium. In this framework, the state variable is the vintage of the technology that the old agents carry into the current period (i.e., either one period old or two periods old). Given this state, the equilibrium strategies of the young and old agents are uniquely defined.

$$k \le \frac{\beta\gamma^2 + \gamma - (1+\beta)}{\beta\gamma^2}$$

Figure 2 partitions the parameter space into three regions based on the threshold conditions in Proposition 3. For high values of γ relative to *k*, young and old agents are in agreement that CL is the best outcome. That is, if living agents could *impose* a pattern of technology adoption that would be binding on future agents, they would unanimously choose CL. Conversely, for relatively low values of γ, young and old agents agree that ES is the best outcome. For intermediate values of γ there is intergenerational conflict: young agents prefer CL while old agents prefer ES. [10]

Now consider the interplay of intergenerational conflict and coalition-proof equilibria. Figure 3 overlays Figure 2 on Figure 1 to illustrate two key points. First, in the lower shaded region of Figure 3, young and old agents agree that ES is preferred to CL yet both outcomes can arise as equilibria. This multiplicity reflects a coordination problem, but not of the usual form; the lack of coordination is between agents currently alive and the not-yet-living next generation. In particular, if young agents in period *t* expect young agents in period *t*+1 to adopt the new technology in period *t*+1, then their anticipated best response when they old in period *t*+1 is to re-tool with the new technology. The payoff from doing so will be higher if they have adopted the latest technology when they are young in period *t*, because technologies progress only if the young adopt the latest technology, and for the parameter values corresponding to the region of multiple equilibria, this is a dominant strategy for the young. In turn, the best response from the old agents in period *t* is to learn the new technology. Thus, CL is supported as an equilibrium. Conversely, if young agents in period *t* expect young agents in period *t*+1 to learn the old technology, then ES is the equilibrium outcome. Since coalitions cannot be formed between the living and the not-yet-living (and because young agents cannot commit when young to a particular action when old), coalition-proofness

[10] Note that young agents would always prefer {1,1} to {0,0} in the period in which they are young, but Proposition 3 and Figure 2 relate to a preference over technology adoption *patterns*, for which both periods of life are relevant unless $\beta = 0$, in which case young and old agents never agree (unless $k = 0$).

on its own cannot eliminate this multiplicity of equilibria, despite the fact that there is no disagreement among living agents as to which outcome is preferred.

The second key point to note from Figure 3 is that for potentially large regions of the parameter space, the equilibrium outcome involves progress even though all *living* agents would prefer no progress. That is, there can arise "rat race" equilibria in this economy. In the upper shaded region of Figure 3, young agents learn the new technology either in every period (PL1) or in every second period (PPL), and old agents are left behind. Yet both young and old agents in any given period would prefer to retain the old technology. Similarly, in the lower shaded region of Figure 3, progress occurs in every period if CL arises as the equilibrium, even though young and old agents in any given period would prefer no progress.[11] Note that the converse never arises; young and old agents in any given period never agree that there is too little progress, in any region of the parameter space.

It is important to recognize that the potential for a "rat race" in this economy does not necessarily indicate inefficiency. Any "rat race" that does arise is viewed as such by all *living* agents but an assessment of efficiency must also take account of future agents. The following proposition describes the efficiency properties of the equilibria.

Proposition 4

All the equilibria are Pareto efficient.

This is perhaps a surprising result, in view of the presence of multiple equilibria, and the potential for a "rat race". However, as the proof makes clear, any deviation from equilibrium anywhere in the parameter space causes either the current old to be made worse off (if the deviation involves more progress) or future agents to be made worse off (if the deviation involves less progress). Thus, all equilibria are Pareto efficient. However, this does not mean that there are no *potential* Pareto improvements in this

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[11] The range of the parameter space over which this type of "rat race" can occur depends on the size of β. The shaded regions in Figure 3 shrink as β falls, and while the lower region persists to some degree for any positive β, the upper region vanishes for any $\beta < 1/2$.

economy.[12] Introducing the possibility of transfers in this economy means that potential Pareto improvements can be exploited by coalitions to the mutual benefit of the agents involved, giving rise to very different equilibrium outcomes. In the next section we examine equilibria when contemporaneous transfers are possible.

3.2 Equilibria with Contemporaneous Transfers

The following proposition, illustrated in Figure 4, characterizes the equilibria that can be supported with contemporaneous transfers.

Proposition 5

(a) Eternal Stagnation is an equilibrium with contemporaneous transfers if and only if

$$\gamma < [(2+\beta)/(2+\beta-k)]$$

(the region labeled "ES" in Figure 4).

(b) Complete Learning is an equilibrium with contemporaneous transfers if and only if

$$\beta\gamma^2(1-k)+\gamma[1+(1-k)(1-\beta)] \geq 2$$

(the region labeled "CL" in Figure 4).

(c) Periodic Complete Learning is an equilibrium[13] with contemporaneous transfers if and only if

$$\gamma \geq [(2+\beta)/(2+\beta-k)] \quad \text{and} \quad \beta\gamma^2(1-k)+\gamma[1+(1-k)(1-\beta)] < 2$$

(the region labeled "PCL" in Figure 4).

(d) No other equilibria exist when contemporaneous transfers are possible.

These equilibria have a number of noteworthy properties. First, there is a unique equilibrium outcome in every region of the parameter space; the multiplicity of equilibria is eliminated when contemporaneous transfers are possible. Transfers allow either the young to compensate the old for agreeing to re-tool (for relatively high values of γ), or

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[12] A potential Pareto improvement can be made whenever an action can be taken which makes no agent worse off but makes at least one agent better off once appropriate redistributions have been made. See Boadway and Bruce (1984), chapter 4, for further details.

[13] Although PCL is cyclical, it is a Markov perfect equilibrium, for the same reason that PPL is (see footnote 9 above).

the old to compensate the young for agreeing to learn the old technology (for relatively low values of γ). The co-existence of CL and ES is thereby eliminated.

Second, there exists a nonstationary equilibrium, with an endogenous cycle. This cycle involves PCL: all agents learn the new technology in one period, and no agents learn the new technology in the next. In the region of Figure 4 labeled "PCL", re-tooling by the old is to their advantage only if they are partly compensated by the young for the costs of that re-tooling. It is in the interests of the young to make the compensating transfer to the old only if they do not have to incur re-tooling costs when they are old themselves. That is, investing in the new technology, and paying off the old to do the same, is worthwhile for the young only if a period of stagnation follows in which the rewards of that investment can be reaped when they are old.

This cycle is importantly distinct from the cycle that can occur in an economy without transfers. In *that* case the old never re-tool; the learning of the new technology alternates only between one young generation and the next. This means that in every second period the young and the old use different technologies. This splitting of the economy into two competing networks never occurs in equilibria with contemporaneous transfers: all agents alive in any given period always use the same technology. The transfer opportunity between young and old ensures that the productivity benefits of a universal network are fully exploited.

Third, in contrast to the case without transfers, CL is never an equilibrium in the region where the current young and the current old both prefer ES to CL.[14] However, there still exists the potential for a "rat race" in this economy: PCL can exist as an equilibrium even when all living agents prefer ES to any other outcome.[15] In contrast, neither ES nor PCL are equilibria in the region where the current young and the current old both prefer CL to ES; that is, there is never too little progress from the perspective of all living agents.

[14] It is straightforward to show that the lower threshold in Figure 2 lies everywhere below the upper threshold in Figure 4 except when $\beta = 1$, in which case the two thresholds coincide. Thus, CL is never an equilibrium with contemporaneous transfers if the current young and current old both agree that ES is a better outcome.

It is straightforward to show, and intuitively clear, that the equilibria with contemporaneous transfers are Pareto efficient. (Recall from Proposition 4 that the equilibria without transfers are efficient; the introduction of transfers to exploit potential Pareto improvements cannot *induce* inefficiency). Nonetheless, in some regions of the parameter space, equilibrium outcomes that involve progress are strictly inferior to eternal stagnation from the perspective of living agents. This reflects the fact that the equilibrium choice of the current young in any period depends on their expectation of the choice of the next generation, with whom they are unable to make agreements or exchange transfers. In the next section we introduce the possibility of such transfers across time.

3.3 Equilibria with Intertemporal Transfers

We now allow for intertemporal transfers, where future generations can make transfers to current ones through chains of intermediate generations. Since the young in any period have contact with both the current old and the next young generation, they are able to make transfers to the current old on behalf of the next generation, and may be willing to do so if they are assured that next generation will honor the promise of a transfer to them when they are old. Since all future generations prefer more learning now, these transfer payments would always be made in return for the current old choosing to adopt the new technology. Of course, the potential size of these transfer payments is constrained by the aggregate product of the young in all periods of any such chain, and this limits the degree to which such transfers can foster progress.

The primary obstacle to intertemporal transfers is that young agents in any given period may find it in their interests to *not* honor the promise made on their behalf by the current old in the prior period. We assume that a commitment mechanism exists to ensure that the future young always pay when these promises are made. In particular, these transfer payments resemble debt commitments for future generations, and their enforceability can be viewed as a form of social capital in this economy which is absent from the economy examined in the previous section, where intertemporal transfers are

[15] This “rat race” will occur if and only if $\beta^2 + \beta > 1$.

not possible. The following proposition, along with Figure 5, summarizes the equilibria in this economy.

Proposition 6

(a) Eternal Stagnation is an equilibrium with intertemporal transfers if and only if

$$\gamma < [(2+\beta)/(2+\beta-k)]$$

(the region labeled "ES" in Figure 5).

(b) Complete Learning is an equilibrium with intertemporal transfers if and only if

$$\gamma \geq [(2+\beta)/(2+\beta-k)]$$

(the region labeled "CL" in Figure 5).

(c) No other equilibria exist when intertemporal transfers are possible.

It is clear from a comparison of Figures 4 and 5 that the equilibrium outcomes in this economy are identical to those in an economy in which only contemporaneous transfers are possible, except in the region denoted "PCL" in Figure 4. When only contemporaneous transfers are allowed, PCL is an equilibrium outcome for intermediate combinations of γ and k, but this outcome never arises in equilibrium when intertemporal transfers are allowed. In that region of the parameter space, CL is the only equilibrium outcome in this case. This leaves only two possible equilibrium outcomes: CL and ES.

The elimination of the cyclical equilibrium requires that in each period, transfers be made from future generations to the current old generation (see the proof of Proposition 6). As mentioned above, this type of transfer necessarily involves debt obligations being passed on to future generations. Thus, in the region of the parameter space labeled "PCL", the introduction of such a debt mechanism allows the economy to move from a cyclical equilibrium to one with constant growth and double the long run growth rate.

The introduction of intertemporal transfers also eliminates the potential for a "rat race" in this economy. It is straightforward to show that there exists a feasible transfer from the young to the old in every period that ensures indifference between ES and CL on the part of the old in every period, and a strict preference for CL over ES on the part of the young in every period, if and only if CL is the equilibrium outcome.[16] Conversely, there exists a feasible transfer from the old to the young in every period that ensures indifference between ES and CL on the part of the young in every period, and a strict preference for ES over CL on the part of the old in every period, if and only if ES is the equilibrium outcome. That is, there exist feasible transfers that ensure that all living agents, in every period, can agree that the equilibrium outcome is the preferred outcome.

4. CONCLUSION

Our analysis has examined the interplay of intergenerational conflict and the richness of social capital in the determination of economic progress in an economy with network effects in worker skills. We find that equilibrium patterns of progress depend critically on the extent to which transfers are available. In the absence of transfers, the economy can exhibit endogenous cyclical equilibria, multiple stationary equilibria, and “rat race” equilibria in which all living agents prefer a slower rate of progress than occurs in equilibrium. If contemporaneous transfers are available then all equilibria are unique, but a cyclical equilibrium still exists, and a “rate race” can still arise. In contrast, if intertemporal transfers are available, equilibria are unique and stationary, and no “rat race” can arise.

Our analysis highlights that technological change has benefits and costs, that tend to be distributed asymmetrically between the young and the old, giving rise to intergenerational conflict over the value of progress. Moreover, the fear of being left behind technologically when agents expect progress to occur, can lead to a technological “rat race” from the perspective of those currently alive. However, future generations are often the main beneficiary of progress today, as technological innovations tend to build on previous ones. Allowing some of those benefits to accrue to current generations

[16] The transfer required in period t is $R(t)=\gamma^{t}2N-\gamma^{t+1}2N(1-k)$. This is feasible if and only if $\gamma\geq 1/(2-k)$, which holds everywhere in the parameter space. Young agents in period t are better off in the CL outcome, after making a transfer of $R(t)$ in period t and receiving a transfer of $R(t+1)$ in period

through intertemporal transfers can eliminate the technological “rat race” in which living agents may feel trapped, and at the same time foster faster economic growth.

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t+1, than in the ES outcome, if and only if $\gamma \geq (2+\beta)/(2-k+\beta)$, in which case CL is the equilibrium outcome.

APPENDIX

Proof of Proposition 1

We begin by presenting the payoffs for young and old agents alive in period *t*, for different technology choices $\{i_t, j_t\}$ and $\{i_{t+1}, j_{t+1}\}$ in table form. Table 1 presents the payoffs for the old agents, which are independent of $\{i_{t+1}, j_{t+1}\}$ but do depend on $\{i_t, j_t\}$. The first row of the table shows the possible combinations of $\{i_t, j_t\}$, and the second row shows the corresponding payoffs to the old, divided by *N*.

{0,0}	{0,1}	{1,0}	{1,1}
$2\gamma^{\tau}$	$\gamma^{\tau+1}(1-k)$	γ^{τ}	$2\gamma^{\tau+1}(1-k)$

Table 1: Payoffs to the Old Agents

Table 2 shows the payoffs to the young agents (also divided by *N*). These are functions of both $\{i_t, j_t\}$ and $\{i_{t+1}, j_{t+1}\}$. As in Table 1, the first row of the table shows the different possible combinations of $\{i_t, j_t\}$; the first column shows the possible combinations of $\{i_{t+1}, j_{t+1}\}$. The payoffs are then shown for different combinations of $\{i_t, j_t\}$ and $\{i_{t+1}, j_{t+1}\}$.

	{0,0}	{0,1}	{1,0}	{1,1}
{0,0}	$2\gamma^{\tau}(1+\beta)$	$\gamma^{\tau}(1+2\beta)$	$\gamma^{\tau+1}(1+2\beta)$	$2\gamma^{\tau+1}(1+\beta)$
{0,1}	$\gamma^{\tau}(2+\beta\gamma(1-k))$	$\gamma^{\tau}(1+\beta\gamma(1-k))$	$\gamma^{\tau+1}(1+\beta\gamma(1-k))$	$\gamma^{\tau+1}(2+\beta\gamma(1-k))$
{1,0}	$\gamma^{\tau}(2+\beta)$	$\gamma^{\tau}(1+\beta)$	$\gamma^{\tau+1}(1+\beta)$	$\gamma^{\tau+1}(2+\beta)$
{1,1}	$2\gamma^{\tau}(1+\beta\gamma(1-k))$	$\gamma^{\tau}(1+2\beta\gamma(1-k))$	$\gamma^{\tau+1}(1+2\beta\gamma(1-k))$	$2\gamma^{\tau+1}(1+\beta\gamma(1-k))$

Table 2: Payoffs for the Young Agents

We now prove the five parts of Proposition 1.

(a) Recall that ES is characterized by $\{i_t, j_t\} = \{0,0\}\ \forall t$. We will first show that ES is an equilibrium in the specified region. By Table 1, $\{i_t, j_t\} = \{0,0\}$ is strictly preferred to $\{i_t, j_t\} = \{1,0\}$ by all old agents in period *t*. Thus, no coalition of old agents will defect unilaterally from the proposed equilibrium. If $\gamma < [2(1+\beta)/(1+2\beta)]$ then by Table 2, $\{i_t, j_t\} = \{0,0\}$ is strictly preferred to $\{i_t, j_t\} = \{1,0\}$ by all young agents in period *t*. Thus, no coalition of young agents will defect unilaterally from the proposed equilibrium. Finally, if $\gamma < 1/(1-k)$ then by Table 1, old agents will not be willing to join a coalition with young agents and switch from $\{i_t, j_t\} = \{0,0\}$ to $\{i_t, j_t\} = \{1,1\}$. Thus, ES is an equilibrium in the specified region. We will next show that ES is *not* an equilibrium in any other region. First, if $\gamma \geq [2(1+\beta)/(1+2\beta)]$ then by Table 2, $\{i_t, j_t\} = \{1,0\}$ is preferred to $\{i_t, j_t\} = \{0,0\}$) by young agents in period *t* when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$; thus, ES can be broken by a coalition of all young agents, given the candidate equilibrium choice by the old. Second, if $\gamma \geq 1/(1-k)$ then by Table 1 and Table 2, ES can be broken by a joint coalition of all young and old agents alive in period *t*. This proves part (a) of the Proposition.

(b) Recall that CL is characterized by $\{i_t, j_t\} = \{1,1\}\ \forall t$. We will first show that CL is an equilibrium in the specified region. If $\gamma \geq 1/[2(1-k)]$ then by Table 1, $\{i_t, j_t\} = \{1,1\}$ is weakly preferred to $\{i_t, j_t\} = \{1,0\}$ by all old agents in period *t*. Thus, no coalition of old agents will defect unilaterally from the proposed equilibrium. By Table 2, $\{i_t, j_t\} = \{1,1\}$ is strictly preferred to $\{i_t, j_t\} = \{0,1\}$ by all young agents in period *t*. Thus, no coalition of young agents will defect unilaterally from the proposed equilibrium. Finally, by Table 2, young agents will not be willing to join a coalition with old agents and switch from $\{i_t, j_t\} = \{1,1\}$ to $\{i_t, j_t\} = \{0,0\}$. Thus, CL is an equilibrium in the specified region. We will next show that CL is *not* an equilibrium in any other region. If $\gamma < 1/[2(1-k)]$ then by Table 1, $\{i_t, j_t\} = \{1,0\}$ is preferred to $\{i_t, j_t\} = \{1,1\}$ by old agents in period *t*; thus, CL can be broken by a coalition of all old agents, given the candidate equilibrium choice by the young. This proves part (b) of the Proposition.

(c) Recall that PL1 is characterized by $\{i_t, j_t\} = \{1,0\}\ \forall t$. We will first show that PL1 is an equilibrium in the specified region. If $\gamma < 1/[2(1-k)]$ then by Table 1, $\{i_t, j_t\} = \{1,0\}$ is weakly preferred to $\{i_t, j_t\} = \{1,1\}$ by all old agents in period *t*. Thus, no coalition of old agents will defect unilaterally from the proposed equilibrium, or be willing to join a coalition with young agents and switch from $\{i_t, j_t\} = \{1,0\}$ to $\{i_t, j_t\} = \{1,1\}$. If $\gamma \geq [(2+\beta)/(1+\beta)]$ then by Table 2, $\{i_t, j_t\} = \{1,0\}$ is weakly preferred to $\{i_t, j_t\} = \{0,0\}$ by all young agents in period *t*. Thus, no coalition of young agents will defect unilaterally from the proposed equilibrium or be willing to join a coalition with old agents and switch from $\{i_t, j_t\} = \{1,0\}$ to $\{i_t, j_t\} = \{0,0\}$. Thus, PL1 is an equilibrium in the specified region. We will next show that PL1 is *not* an equilibrium in any other region. If $\gamma \geq 1/[2(1-k)]$ then by Table 1, $\{i_t, j_t\} = \{1,1\}$ is preferred to $\{i_t, j_t\} = \{1,0\}$ by all old agents in period *t*; thus, PL1 can be broken by a coalition of all old agents, given the candidate equilibrium choice by the young. If $\gamma < [(2+\beta)/(1+\beta)]$ then by Table 2, $\{i_t, j_t\} = \{0,0\}$ is preferred to $\{i_t, j_t\} = \{1,0\}$ by young agents in period *t*; thus, PL1 can be broken by a coalition of all young agents, given the candidate equilibrium choice by the old. This proves part (c) of the Proposition.

(d) Recall that PL2 is characterized by $\{i_t, j_t\} = \{0,1\}\ \forall t$. By Table 2, $\{i_t, j_t\} = \{1,1\}$ is strictly preferred to $\{i_t, j_t\} = \{0,1\}$ by all young agents in period *t* when $\{i_{t+1}, j_{t+1}\} = \{0,1\}$. Thus, PL2 can always be broken by a coalition of all young agents, given the candidate equilibrium choice by the old. This proves part (d) of the Proposition.

(e) This follows directly from parts (a) – (d) of the Proposition. ■

Proof of Proposition 2

The candidate equilibrium comprises the sequence $\{1,0\} \rightarrow \{0,0\} \rightarrow \{1,0\} \rightarrow \{0,0\}$ etc. To fix period notation, let $\{i_t, j_t\} = \{1,0\}$. We will first show that PPL is an equilibrium in the specified region. First consider the choices by old agents. If $\gamma < 1/[2(1-k)]$ then by Table 1, $\{i_t, j_t\} = \{1,0\}$ is weakly preferred to $\{i_t, j_t\} = \{1,1\}$ by all old agents in period *t*. Thus, no coalition of old agents will defect unilaterally from the proposed equilibrium in period *t*. Moreover, by Table 1, $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ is strictly preferred to $\{i_{t+1}, j_{t+1}\} = \{0,1\}$ by all old agents in period *t*+1, since $\gamma < 1/[(1-k)]$ when $\gamma < 1/[2(1-k)]$. Thus, no coalition of old agents will defect unilaterally from the proposed equilibrium in period *t*+1. Finally, by Table 1, $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ is strictly preferred to $\{i_{t+1}, j_{t+1}\} = \{1,1\}$ by all old agents in period *t*+1, since $\gamma < 1/[(1-k)]$ when $\gamma < 1/[2(1-k)]$. Thus, old agents in period *t*+1 will not be willing to join a coalition with the young in period *t*+1 and switch from $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ to $\{i_{t+1}, j_{t+1}\} = \{1,1\}$.

Next consider the choices by young agents. If $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ and $\gamma \geq [2(1+\beta)/(1+2\beta)]$ then by Table 2, $\{i_t, j_t\} = \{1,0\}$ is weakly preferred to $\{i_t, j_t\} = \{0,0\}$ by all young agents in period *t*. Thus, no coalition of young agents will defect unilaterally from the proposed equilibrium in period *t*. Moreover, if $\{i_{t+2}, j_{t+2}\} = \{1,0\}$ and $\gamma < [(2+\beta)/(1+\beta)]$ then by Table 2, $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ is strictly preferred to $\{i_{t+1}, j_{t+1}\} = \{1,0\}$ by all young agents in period *t*+1. Thus, no coalition of young agents will defect unilaterally from the proposed equilibrium in period *t*+1. Thus, since no coalition of the young nor coalition of the old nor joint coalition of the young and old will defect from the candidate equilibrium in either period *t* or period *t*+1, PPL is an equilibrium in the specified region.

Next we will show that PPL is *not* an equilibrium in any other region. If $\gamma \geq [1/2(1-k)]$ then by Table 1, $\{i_t, j_t\} = \{1,1\}$ is weakly preferred to $\{i_t, j_t\} = \{1,0\}$ by old agents in period *t*, and thus they will defect unilaterally from the candidate equilibrium. Similarly, if $\gamma < [2(1+\beta)/(1+2\beta)]$ then by Table 2, $\{i_t, j_t\} = \{0,0\}$ is strictly preferred to $\{i_t, j_t\} = \{1,0\}$ by young agents in period *t* when they expect $\{i_{t+1}, j_{t+1}\} = \{0,0\}$, and thus they will defect unilaterally from the candidate equilibrium. Finally, if $\gamma \geq [(2+\beta)/(1+\beta)]$ then by Table 2, $\{i_{t+1}, j_{t+1}\} = \{1,0\}$ is strictly preferred to $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ by young agents in period *t*+1 when they expect $\{i_{t+2}, j_{t+2}\} = \{1,0\}$. Thus, they will defect unilaterally from the candidate equilibrium. Thus, PPL cannot be an equilibrium outside the specified region. ■

Proof of Proposition 3

Part (a) follows directly from Table 1. Consider part (b). The lifetime payoff to a young agent under CL is $\gamma^{\tau+1}2N + \beta\gamma^{\tau+2}2N(1-k)$, under PL1 is $\gamma^{\tau+1}N + \beta\gamma^{\tau+1}N$, under ES is $\gamma^{\tau}N + \beta\gamma^{\tau}N$, and under PPL is either $\gamma^{\tau+1}N + \beta\gamma^{\tau+1}N$ or $\gamma^{\tau}N + \beta\gamma^{\tau}N$. The payoff to CL is greater than the payoff to PL1 if $\gamma \geq (\beta-1)/[2\beta(1-k)]$, which holds everywhere in the parameter space. Thus, CL is always strictly preferred to PL1. The payoff to CL is greater than the payoff to ES if $k \leq [\beta\gamma^2 + \gamma - (1+\beta)]/\beta\gamma^2$, which is the condition in the proposition. If this condition holds then CL is strictly preferred to ES, and to PPL, since the payoff to PPL is never greater than the payoff to either PL1 or ES. ■

Proof of Proposition 4

(a) Consider the ES equilibrium, in which $\{i_t, j_t\} = \{0,0\}$. There are three alternative outcomes in period *t*: $\{i_t, j_t\} = \{1,0\}$, $\{i_t, j_t\} = \{0,1\}$ and $\{i_t, j_t\} = \{1,1\}$. If $\{i_t, j_t\} = \{1,0\}$, then by Table 1, old agents in period *t* are worse off than in the equilibrium outcome $\forall \gamma > 1$ and $k \leq 1$, regardless of the outcome in any future period. Thus, no alternative outcome involving $\{i_t, j_t\} = \{1,0\}$ for any *t* can Pareto dominate the equilibrium. If $\{i_t, j_t\} = \{0,1\}$, then by Table 1, old agents in period *t* are worse off than in the equilibrium outcome, regardless of the outcome in any future period, unless $\gamma \geq 2/(1-k)$, but by Proposition 1(a), ES is not an equilibrium in that region of the parameter space. Thus, no alternative outcome involving $\{i_t, j_t\} = \{0,1\}$ for any *t* can

Pareto dominate the equilibrium. Finally, if $\{i_t, j_t\} = \{1,1\}$, then by Lemma 1(d), old agents in period *t* are worse off than in the equilibrium outcome, regardless of the outcome in any future period, unless $\gamma \geq 1/(1-k)$, but by Proposition 1(a), ES is not an equilibrium in that region of the parameter space. Thus, no alternative outcome involving $\{i_t, j_t\} = \{1,1\}$ for any *t* can Pareto dominate the equilibrium.

(b) Consider the CL equilibrium, in which $\{i_t, j_t\} = \{1,1\}$. There are three alternative outcomes in period *t*: $\{i_t, j_t\} = \{1,0\}$, $\{i_t, j_t\} = \{0,1\}$ and $\{i_t, j_t\} = \{0,0\}$. If $\{i_t, j_t\} = \{1,0\}$, then by Table 1, old agents in period *t* are worse off than in the equilibrium outcome, regardless of the outcome in any future period, unless $\gamma < 2/(1-k)$, but by Proposition 1(b), CL is not an equilibrium in that region of the parameter space. Thus, no alternative outcome involving $\{i_t, j_t\} = \{1,0\}$ for any *t* can Pareto dominate the equilibrium. If $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{0,0\}$, then no new technology arrives in period *t*+1, because the young agents in period *t* do not learn the new technology in that period, so young agents in period *t*+1 are worse off than in the equilibrium outcome. This cost to young agents in period *t*+1 cannot be offset by any deviation from the equilibrium outcome in period *t*+1 since that would require either that the old agents in period *t*+1 are made worse off (if the deviation involves $\{i_{t+1}, j_{t+1}\} = \{1,0\}$) or that young agents in period *t*+2 are made worse off (if the deviation involves $\{i_{t+1}, j_{t+1}\} = \{0,1\}$ or $\{i_{t+1}, j_{t+1}\} = \{0,0\}$). Thus, no alternative outcome involving $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{0,0\}$ for any *t* can Pareto dominate the equilibrium.

(c) Consider the PL1 equilibrium, in which $\{i_t, j_t\} = \{1,0\}$. There are three alternative outcomes in period *t*: $\{i_t, j_t\} = \{1,1\}$, $\{i_t, j_t\} = \{0,1\}$ and $\{i_t, j_t\} = \{0,0\}$. If $\{i_t, j_t\} = \{1,1\}$, then by Table 1, old agents in period *t* are worse off than in the equilibrium outcome, regardless of the outcome in any future period, unless $\gamma \geq 2/(1-k)$, but by Proposition 1(c), PL1 is not an equilibrium in that region of the parameter space. Thus, no alternative outcome involving $\{i_t, j_t\} = \{1,1\}$ for any *t* can Pareto dominate the equilibrium. Using the same reasoning as for part (b) above, no alternative outcome involving $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{0,0\}$ for any t can Pareto dominate the equilibrium.

(d) Consider the PPL equilibrium, in which $\{1,0\} \to \{0,0\} \to \{1,0\} \to \{0,0\}$ etc. To fix period notation, let $\{i_t, j_t\} = \{1,0\}$. There are three alternative outcomes in period *t*: $\{i_t, j_t\} = \{1,1\}$, $\{i_t, j_t\} = \{0,1\}$ and $\{i_t, j_t\} = \{0,0\}$. If $\{i_t, j_t\} = \{1,1\}$ then by Table 1, old agents in period *t* are worse off than in the equilibrium outcome, regardless of the outcome in any future period, unless $\gamma \geq 2/(1-k)$, but by Proposition 2, PPL is not an equilibrium in that region of the parameter space. Thus, no alternative outcome involving $\{i_t, j_t\} = \{1,1\}$ in place of $\{i_t, j_t\} = \{1,0\}$ for any *t* can Pareto dominate the equilibrium. Using the same reasoning as for part (b) above, no alternative outcome involving $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{0,0\}$ in place of $\{i_t, j_t\} = \{1,0\}$ for any *t* can Pareto dominate the equilibrium. There are also three alternative outcomes in period *t*+1: $\{i_{t+1}, j_{t+1}\} = \{1,0\}$, $\{i_{t+1}, j_{t+1}\} = \{0,1\}$ and $\{i_{t+1}, j_{t+1}\} = \{1,1\}$. Using the same reasoning as for part (a) above, no alternative outcome involving any of these three alternatives in place of $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ for any *t* can Pareto dominate the equilibrium. ■

Proof of Proposition 5

We begin by deriving the best technology choices for agents alive in period *t* in response to each of the possible expected choices of agents alive in period *t*+1. These are described in the following four lemmas.

Lemma 1. If agents alive in period *t* expect $\{i_{t+1}, j_{t+1}\} = \{0,0\}$, then the best choice for agents in period *t* is

(i) $\{i_t, j_t\} = \{0,0\}$ if and only if $\gamma < [(2+\beta)/(2+\beta-k)]$

(ii) $\{i_t, j_t\} = \{1,1\}$ if and only if $\gamma \geq [(2+\beta)/(2+\beta-k)]$

Proof. We begin by showing that under $\{i_t, j_t\} = \{1,1\}$, old agents in period *t* can be at least as well-off, and young agents in period *t* can be strictly better-off, than under $\{i_t, j_t\} = \{1,0\}$ or $\{i_t, j_t\} = \{0,1\}$, $\forall \gamma > 1$ and $k \leq 1$.

First, consider the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{1,0\}$. The payoff to an old agent in period *t* under $\{i_t, j_t\} = \{1,0\}$ is $\gamma^{\tau} N$. To receive this same payoff under $\{i_t, j_t\} = \{1,1\}$, old agents would have to receive a transfer *T* from the young:

$$T = \gamma^{\tau} N - \gamma^{\tau+1} 2N(1-k)$$

(Note that *T* could be negative). The lifetime payoff to a young agent in period *t* under $\{i_t, j_t\} = \{1,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$, is

$$v_t(\{1,0\} \rightarrow \{0,0\}) = \gamma^{\tau+1} N + \beta\gamma^{\tau+1} 2N$$

The lifetime payoff to a young agent in period *t* under $\{i_t, j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \rightarrow \{0,0\}) = \gamma^{\tau+1} 2N + \beta\gamma^{\tau+1} 2N$$

It is *feasible* for the young to transfer *T* to the old, since $\gamma^{\tau+1} 2N \geq T \quad \forall \gamma > 1$ and $k \leq 1$. So old agents can be at least as well-off, and young agents strictly better-off under $\{i_t, j_t\} = \{1,1\}$ relative to $\{i_t, j_t\} = \{1,0\}$ if and only if

$$v_t(\{1,1\} \rightarrow \{0,0\}) - T \geq v_t(\{1,0\} \rightarrow \{0,0\})$$

This condition is satisfied $\forall \gamma > 1$ and $k \leq 1$.

Next consider the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,1\}$. The payoff to an old agent in period *t* under $\{i_t, j_t\} = \{0,1\}$ is $\gamma^{\tau+1} N(1-k)$. The payoff to an old agent under $\{i_t, j_t\} = \{1,1\}$ is $\gamma^{\tau+1} 2N(1-k)$. Therefore, old agents are better-off under $\{i_t, j_t\} = \{1,1\}$ $\forall \gamma > 1$ and $k \leq 1$. The lifetime payoff to a young agent in period *t* under $\{i_t, j_t\} = \{0,1\}$ when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$, is

$$v_t(\{0,1\} \rightarrow \{0,0\}) = \gamma^{\tau} N + \beta\gamma^{\tau} 2N$$

The lifetime payoff to a young agent in period *t* under $\{i_t, j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \rightarrow \{0,0\}) = \gamma^{\tau+1} 2N + \beta\gamma^{\tau+1} 2N$$

Therefore, young agents are also better-off under $\{i_t, j_t\} = \{1,1\}$ $\forall \gamma > 1$ and $k \leq 1$.

Since $\{i_t, j_t\} = \{0,1\}$ and $\{i_t, j_t\} = \{1,0\}$ are both dominated $\{i_t, j_t\} = \{1,1\}$, we can focus on the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$. The payoff to an old agent in period *t* under $\{i_t, j_t\} = \{0,0\}$ is $\gamma^{\tau} 2N$. To receive this same payoff under $\{i_t, j_t\} = \{1,1\}$, old agents would have to receive a transfer T' from the young:

$$T' = \gamma^{\tau} 2N - \gamma^{\tau+1} 2N(1-k)$$

(Note that T' could be negative). The lifetime payoff to a young agent under $\{i_t, j_t\} = \{0,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ is

$$v_t(\{0,0\} \to \{0,0\}) = \gamma^{\tau} 2N + \beta\gamma^{\tau} 2N$$

The lifetime payoff to a young agent under $\{i_t, j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \to \{0,0\}) = \gamma^{\tau+1} 2N + \beta\gamma^{\tau+1} 2N$$

It is *feasible* for young agents to transfer T' to the old, since $\gamma^{\tau+1} 2N \geq T' \ \ \forall \gamma > 1$ and $k \leq 1$. So old agents can be at least as well-off, and young agents better-off, under $\{i_t, j_t\} = \{1,1\}$ if and only if

$$v_t(\{1,1\} \to \{0,0\}) - T' \geq v_t(\{0,0\} \to \{0,0\})$$

This is the condition in Lemma 1. ■

Lemma 2. If agents alive in period *t* expect $\{i_{t+1}, j_{t+1}\} = \{1,1\}$, then the best choice for agents in period *t* is

(i) $\{i_t, j_t\} = \{0,0\}$ if and only if $\beta\gamma^2(1-k) + \gamma(2 + \beta k - k - \beta) < 2$

(ii) $\{i_t, j_t\} = \{1,1\}$ if and only if $\beta\gamma^2(1-k) + \gamma(2 + \beta k - k - \beta) \geq 2$

Proof. Using the same approach as for the first part of lemma 1, it is straightforward to show that under $\{i_t, j_t\} = \{1,1\}$, old agents in period *t* can be at least as well-off, and young agents strictly better-off, than under $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{1,0\}$ $\forall \gamma > 1$ and $k \leq 1$. We therefore focus on the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$.

Note that the expected choices of agents in period *t*+1 have no direct effect on the old in period *t*. Their payoffs under $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{1,1\}$ are the same as when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$. So from the proof of lemma 1, we know that old agents are indifferent between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$ if under $\{i_t, j_t\} = \{1,1\}$ they receive a transfer T' from the young:

$$T' = \gamma^{\tau} 2N - \gamma^{\tau+1} 2N(1-k)$$

(Again note that T' could be negative). The lifetime payoff to a young agent under $\{i_t, j_t\} = \{0,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{1,1\}$ is

$$v_t(\{0,0\} \to \{1,1\}) = \gamma^{\tau} 2N + \beta\gamma^{\tau+1} 2N(1-k)$$

The lifetime payoff to a young agent under $\{i_t, j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \to \{1,1\}) = \gamma^{\tau+1} 2N + \beta\gamma^{\tau+2} 2N(1-k)$$

Therefore, old agents can be at least as well-off, and young agents strictly better-off, under $\{i_t, j_t\} = \{1,1\}$ if and only if

$$v_t(\{1,1\} \to \{1,1\}) - T' \geq v_t(\{0,0\} \to \{1,1\})$$

This is the condition in lemma 2. ■

Lemma 3. If agents alive in period *t* expect $\{i_{t+1}, j_{t+1}\} = \{1,0\}$, then the best choice for agents in period *t* is either $\{i_t, j_t\} = \{1,1\}$ or $\{i_t, j_t\} = \{0,0\}$.

Proof. Using the same approach as for the first part of lemma 1, it is straightforward to show that under $\{i_t, j_t\} = \{1,1\}$, old agents in period *t* can be at least as well-off, and young agents strictly better-off, as under $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{1,0\}$ $\forall \gamma > 1$ and $k \leq 1$. So the choice for agents in period *t* reduces to that between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$. ■

Lemma 4. If agents alive in period *t* expect $\{i_{t+1}, j_{t+1}\} = \{0,1\}$, then the best choice for agents in period *t* is either $\{i_t, j_t\} = \{1,1\}$ or $\{i_t, j_t\} = \{0,0\}$.

Proof. As for Lemma 3.

We can now prove the four parts of Proposition 5.

(a) This follows directly from part (i) of Lemma 1.

(b) This follows directly from part (ii) of Lemma 2.

(c) This follows directly from part (ii) of Lemma 1 and part (i) of Lemma 2.

(d) From Lemmas 1 to 4 together, we know that neither $\{i_t, j_t\} = \{0,1\}$ nor $\{i_t, j_t\} = \{1,0\}$ are best choices in period *t* for *any* given expected choices in period *t*+1. Therefore, neither choice will be observed in equilibrium in any period. The only possible equilibrium choices are $\{i_t, j_t\} = \{1,1\}$ or $\{i_t, j_t\} = \{0,0\}$, and these are characterized fully in parts (a) to (c). ■

Proof of Proposition 6

Consider the payoffs in the CL outcome when an old agent in period *t* receives from a young agent in period *t*, a debt transfer equal to a fraction $\alpha_t \in [0,1]$ of the production by that young agent in that period. Under this scheme, the payoff to an old agent in period 1 is

$$\gamma 2N(1-k) + \alpha_1 \gamma 2N$$

and the lifetime payoff to a representative young agent in period *t* is

$$\gamma^t 2N(1-\alpha_t) + \beta[\gamma^{t+1} 2N(1-k) + \alpha_{t+1}\gamma^{t+1} 2N]$$

In comparison, the payoff to an old agent in period 1 in the PCL equilibrium is

$$\gamma 2N(1-k)$$

and the lifetime payoff to a representative young agent in period *t* in that equilibrium is

$$\gamma^{\frac{t+1}{2}} 2N + \beta\gamma^{\frac{t+1}{2}} 2N$$

for $t \geq 1$ odd, and

$$\gamma^{\frac{t}{2}} 2N + \beta\gamma^{\frac{t+2}{2}} 2N(1-k)$$

for $t \geq 2$ even. Thus, the CL outcome, with debt transfers, Pareto dominates the PCL equilibrium if and only if the following conditions hold:

(A14) $$\gamma 2N(1-k) + \alpha_1 \gamma 2N \geq \gamma 2N(1-k)$$

and

(A15) $$\gamma^t 2N(1-\alpha_t) + \beta[\gamma^{t+1} 2N(1-k) + \alpha_{t+1}\gamma^{t+1} 2N] \geq \gamma^{\frac{t+1}{2}} 2N + \beta\gamma^{\frac{t+1}{2}} 2N$$

for $t \geq 1$ odd; and

(A16) $$\gamma^t 2N(1-\alpha_t) + \beta[\gamma^{t+1} 2N(1-k) + \alpha_{t+1}\gamma^{t+1} 2N] \geq \gamma^{\frac{t}{2}} 2N + \beta\gamma^{\frac{t+2}{2}} 2N(1-k)$$

for $t \geq 2$ even. First consider condition (A14). This is clearly satisfied for any $\alpha_1 \geq 0$. So without loss of generality, set $\alpha_1 = 0$. Then evaluating (A15) at $t = 1$, we have

(A17) $$\alpha_2 \geq \overline{\alpha} \equiv \frac{1}{\gamma} + k - 1$$

This is feasible ($\overline{\alpha} \leq 1$) if and only if $\gamma \geq [1/(2-k)]$, which holds everywhere in the parameter space. So without loss of generality, set $\alpha_2 = \overline{\alpha}$. Next consider (A15) for $t \geq 3$. Evaluating (A15) at $\alpha_t = \alpha_{t+1} = \overline{\alpha}$, we have

(A18) $$\gamma^{\frac{t-1}{2}}[2+\beta-k-\frac{1}{\gamma}] \geq 1+\beta \quad \text{for } t \geq 3 \text{ odd}$$

The LHS of (A18) is increasing in *t*, so a necessary and sufficient condition for (A18) is that it hold at $t = 3$. Setting $t = 3$, and rearranging yields

(A19) $$\gamma \geq [(2+\beta)/(2+\beta-k)]$$

which is the lower boundary of the PCL equilibrium. (See proposition 5). Finally, consider (A16). Evaluating (A16) at $\alpha_t = \alpha_{t+1} = \overline{\alpha}$, we have

(A20) $$\gamma^{\frac{t}{2}}[2+\beta-k-\frac{1}{\gamma}] \geq 1+\beta\gamma(1-k) \quad \text{for } t \geq 2 \text{ even}$$

Since the LHS of (A20) is increasing in *t*, a necessary and sufficient condition for (A20) is that it hold at $t = 2$. Setting $t = 2$, and rearranging, yields $\gamma \geq (2/[2-k(1-\beta)])$. This must hold if (A19) holds. Thus, there exists a feasible set of transfers to implement CL from the PCL equilibrium.

Next consider the ES equilibrium. The payoff to an old agent in period 1 in this equilibrium is $2N$, and the lifetime payoff to a representative young agent is $2N(1+\beta)$. Thus, the CL outcome with debt transfers Pareto dominates the ES equilibrium if and only if the following hold:

(A21) $$\gamma 2N(1-k) + \alpha_1 \gamma 2N \geq 2N$$

and

(A22) $$\gamma^t 2N(1-\alpha_t) + \beta[\gamma^{t+1} 2N(1-k) + \alpha_{t+1}\gamma^{t+1} 2N] \geq 2N(1+\beta) \quad \forall t \geq 1$$

Rearranging (A21) yields

$$\alpha_1 \geq \overline{\alpha} \equiv \frac{1}{\gamma} + k - 1$$

Evaluating (A22) at $\alpha_t = \alpha_{t+1} = \overline{\alpha}$, we have

(A23) $$\gamma^t [2 + \beta - k - \frac{1}{\gamma}] \geq 1 + \beta \quad \forall t \geq 1$$

A necessary and sufficient condition for (A23) is that it hold at $t = 1$. Setting $t = 1$, and rearranging yields

$$\gamma \geq [(2 + \beta) / (2 + \beta - k)]$$

ES is not an equilibrium with transfers for this range of γ . (See Proposition 5). ■

REFERENCES

Aghion, Philippe and Howitt, Peter, (1998) *Endogenous Growth Theory*, MIT Press, Cambridge.

Boadway, Robin, and Bruce, Neil, (1984) *Welfare Economics*, Basil Blackwell, Oxford.

Chari, V. V., and Hopenhayn, Hugo, (1991) "Vintage Human Capital, Growth, and the Diffusion of New Technology." *Journal of Political Economy*, 99(6), pp. 1142-65.

Church, Jeffrey and King, Ian (1993) "Bilingualism and Network Externalities." *Canadian Journal of Economics*, 26(2), pp. 337-45.

Fudenberg, Drew, and Tirole, Jean, (1991) *Game Theory*, MIT Press, Cambridge.

Jovanovic, Boyan and Nyarko, Yaw, (1996) "Learning-by-Doing and the Choice of Technology." *Econometrica*, 64(6), pp. 1299-1310.

Katz, Michael and Shapiro, Carl, (1985) "Network Externalities, Competition, and Compatibility." *American Economic Review*, 75, pp. 424-40.

Katz, Michael and Shapiro, Carl, (1994) "Systems Competition and Network Effects." *Journal of Economic Perspectives*, 8(2), pp. 93-115.

Krusell, Per and Rios-Rull, Jose-Victor, (1996) "Vested Interests in a Positive Theory of Stagnation and Growth." *Review of Economic Studies*, 63(2), pp. 301-329.

Lazear, Edward., (1999), "Culture and Language", *Journal of Political Economy*, 107, S95-S127.

Olson, Mancur, (1982) *The Rise and Decline of Nations*, Yale University Press, New Haven.

Schmalensee, Richard, (2002) "Payment Systems and Interchange Fees", *Journal of Industrial Economics*, 50 (2), pp. 103-122.

Shy, Oz., (1995) *Industrial Organization: Theory and Applications*, MIT Press, Cambridge.

Shy, Oz., (2001) *The Economics of Network Industries*, Cambridge University Press, Cambridge.

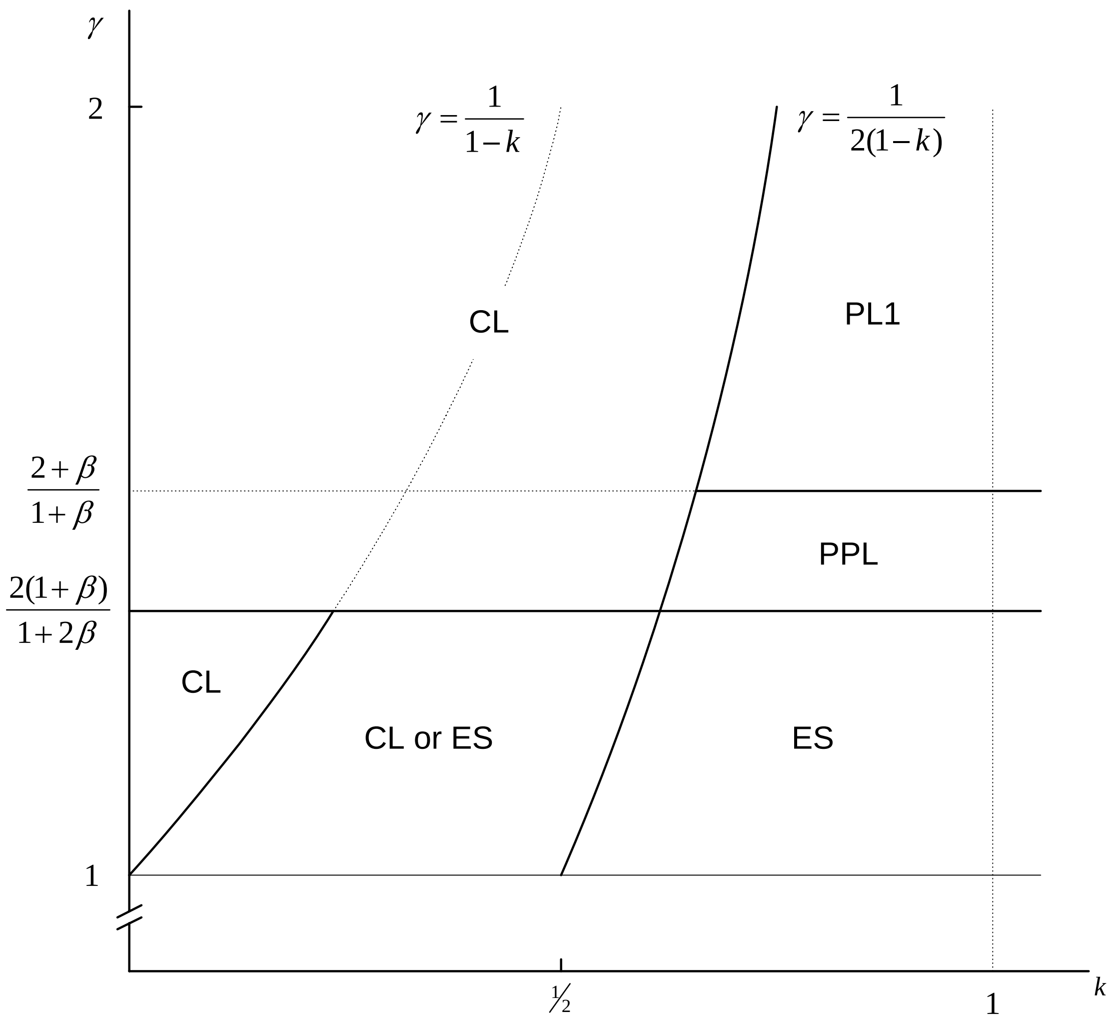

Figure 1

Equilibria without Transfers

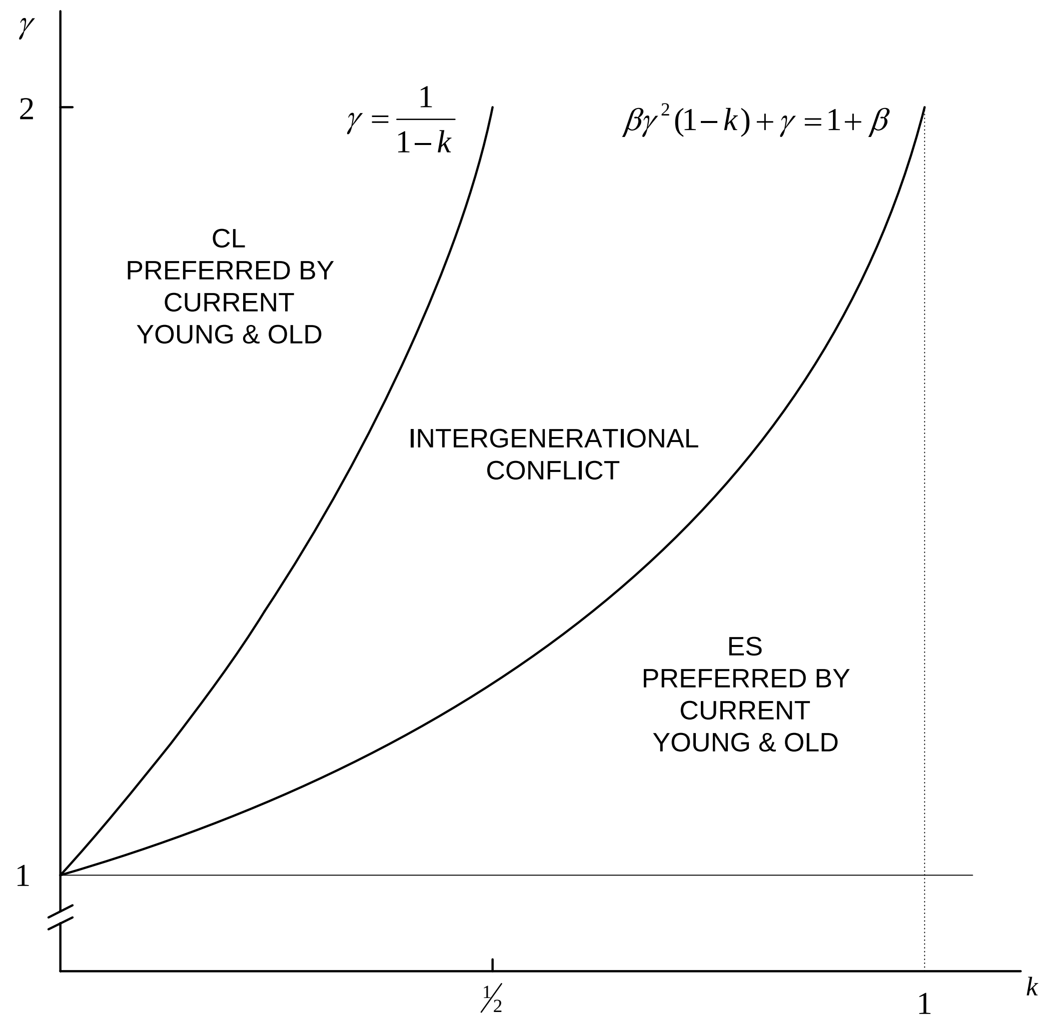

Figure 2

Intergenerational Conflict without Transfers

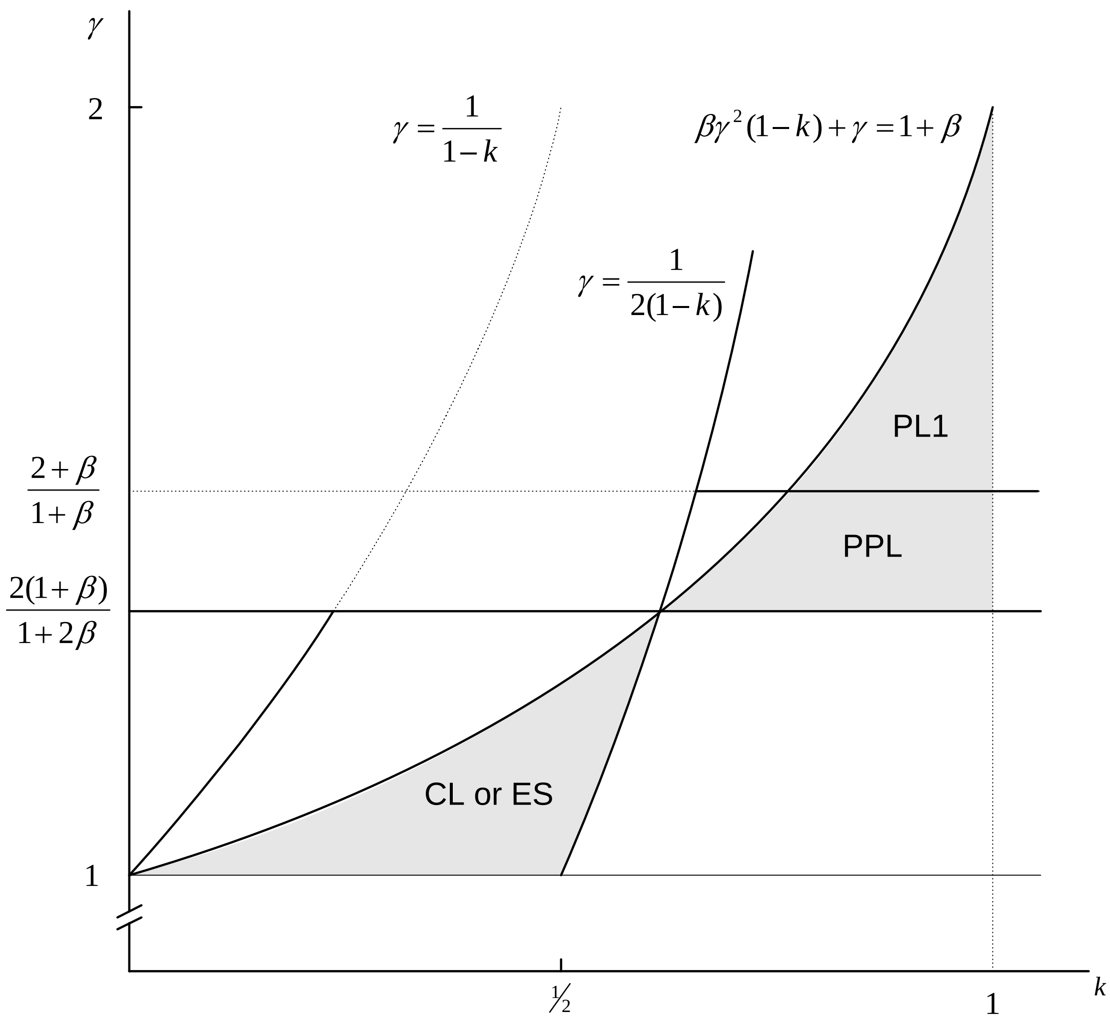

Figure 3

"Rat Race" Equilibria without Transfers

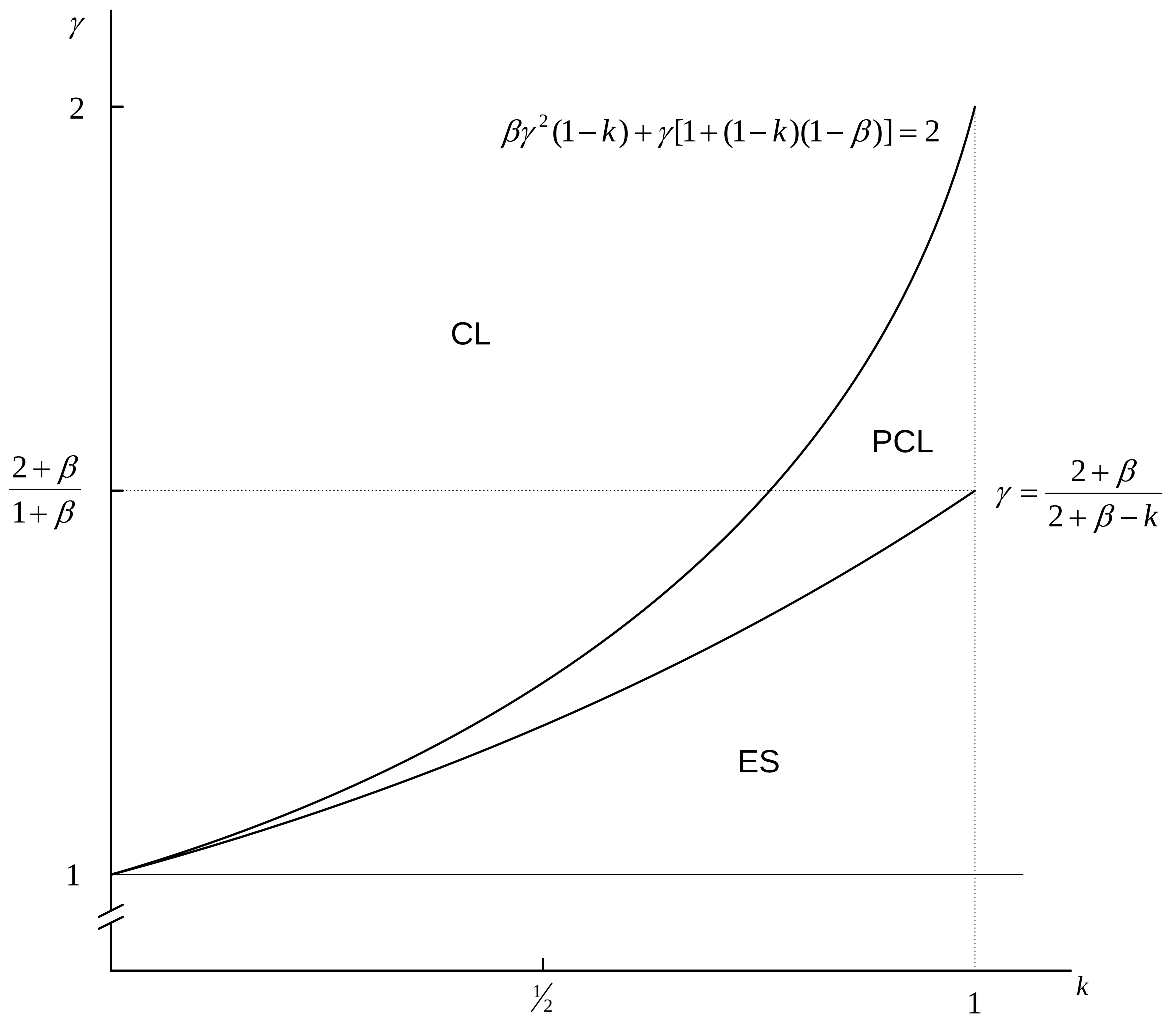

Figure 4

Equilibria with Contemporaneous Transfers

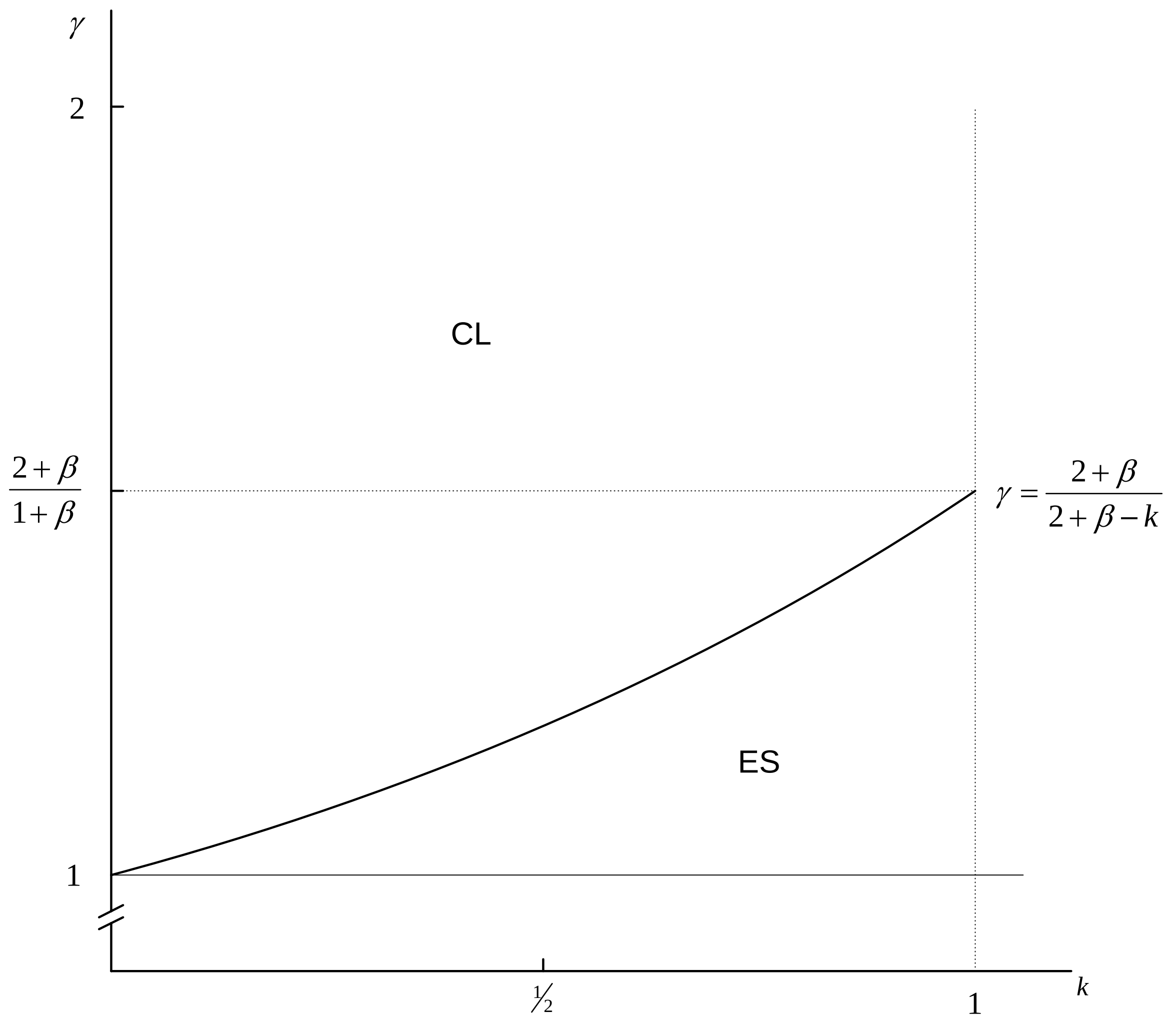

Figure 5

Equilibria with Intertemporal Transfers